CONVRECTIVE PENETRATION IN THE SUN

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ABSTRACT Convective motions penetrate beyond the level where the
Schwarzschild criterion predicts the base of a convective envelope. Such
nearly adiabatic penetration is expected to be of the order of the scale
height of the radiative conductivity (Zahn 1991). When comparing models,
with and without penetration, that are calibrated with the present Sun, we
find that they have differences in their sound speed, and therefore in their
acoustic frequencies, which are characteristic of penetration, and which
can be distinguished from those due to other causes. We use this property
to estimate the extent of penetration by matching the eigenfrequencies of
models including convective penetration with those derived by Libbrecht
et al. (1990) from their solar observations.

Keywords: Solar convection zone; convective penetration; helioseismology.

INTRODUCTION

Stellar models rarely include the effect of convective penetration, although there
is overwhelming evidence for it to occur: in the atmosphere, the oceans, in the
laboratory, and now also in numerical simulations. The problem is thus not
whether there is penetration in stellar interiors, but how important it is.

A theoretical prediction for the extent of penetration has been made
recently by one of us (Zahn 1991): it should be of the order of a scale height
of the radiative conductivity \( \chi \) (= \( 16\sigma T^3/3\rho k \) with the usual notations). We
thus write \( L_p = \zeta/(\partial \log \chi/\partial z)_{ad} \), where \( \zeta \) is a parameter which depends on some
properties of the convective motions. It cannot be determined \textit{ab initio}, but it
is presumably of order unity. Our goal is to calibrate it with the Sun, so that it
may be implemented in other stellar models possessing a convective envelope.

THE EFFECT OF PENETRATION

A series of solar models have been computed with the code CESAM built by
one of us (see Morel et al. 1990), which is particularly well suited to deal with
discontinuities. These models include an increasing amount penetration at the
base of the convection zone, the parameter \( \zeta \) varying from 0 to 1. They were
evolved from a ZAMS homogeneous model to 4.75 Gyrs, and both the mixing
length parameter and the helium abundance were iterated until the radius and the luminosity fitted those of the Sun within better than $10^{-4}$.

One consequence of that calibration is that these models have also nearly the same sound velocity in the deep convection zone, which is a well known property of polytropic stars. But they differ of course at the base of the convective envelope, as illustrated in fig. 1, where we compare models for $\zeta = 0.5$ and 1.0 with one that has no penetration. Note that the differences in sound speed $\delta c/c$ are localized in a rather small region, and that they taper off quickly with depth, to vanish at the center of the Sun. This property too is related to a well known tendency of an atmosphere in radiative equilibrium to settle in a polytropic stratification. Consequently, we find virtually the same helium abundance in all models; therefore, had we included penetration without allowing a change in chemical composition, the adjustments in radius and luminosity would have been of order $10^{-4}$, which is the accuracy of our calibration.

![Diagram](image)

**Fig. 1:** The differences in sound velocity between models with penetration ($\zeta = 0.5$ and 1) and a standard model without penetration. These differences occur only at the base of the convection zone.

The modest variations in the solar model that we have seen when implementing convective penetration are in sharp contrast with the impact of an opacity change on such a model, which has been analyzed by Christensen-Dalsgaard (1988). An opacity increase below the convection zone induces there a steeper temperature gradient, and hence it would lead to a temperature rise in the central region. That rise would produce a luminosity increase, which is prohibited by the calibration, and can only be prevented by an adjustment in the chemical composition. But such change in the helium content is felt in the whole star, and therefore the resulting model is quite different everywhere. In particular, the sound velocity of the more opaque model is higher below the
convection zone, as one expects, but it becomes smaller in the central region, due to the increase in molecular weight.

In order to reproduce the differences displayed in fig. 1, one would have to increase the opacity in the convection zone (including the penetration region) by a suitable amount, while keeping it unchanged in the radiation zone just below. Such a modification is unlikely to occur. Thus we conclude that convective penetration has a distinct signature, and that one should be able to separate it from an opacity increase when comparing the sound speed profiles of two models.

ESTIMATING THE EXTENT OF PENETRATION

The acoustic eigenfrequencies of a stellar model reflect the variation of the sound speed with depth. Therefore the comparison of the eigenfrequencies of two models reveals the differences in their sound speed. This powerful diagnostic tool has been developed by Christensen-Dalsgaard et al. (1988).

Likewise, one may compare the eigenfrequencies of a given model with those observed on the Sun, and again interpret the differences in terms of their respective sound velocity. We have done this with our models including convective penetration, and using the acoustic frequencies measured by Libbrecht et al. (1990). The details of the method will be explained elsewhere. Let us just state here that when we compare the solar frequencies with those of a model that has no convective penetration, we find the unmistakable signature of penetration, implying that such penetration indeed occurs in the Sun. But not all differences can be interpreted by the lack of penetration in that model; a small fraction seems still to be due to a deficiency in the opacity, although we have taken the latest data of Iglesias and Rogers (1991). We have also examined the effect of an imperfect equation of state.

At this writing, our best estimate for the penetration parameter is $\zeta \approx 0.70$, and we are working to refine it. That number would mean that the extent of penetration amounts to about 20,000 km. The depth of the convection zone (including penetration) would vary accordingly.

ACKNOWLEDGEMENTS

This work was supported by the Centre National de la Recherche Scientifique (GDR 131), by the U.S. Air Force through grant AFORS 89.0012 to Columbia University and by NASA through grant NSG-7511 to the University of Colorado.

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