THE EFFECT OF KELVIN–HELMHOLTZ INSTABILITY ON RISING FLUX TUBES IN THE CONVECTION ZONE

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Abstract. If the solar dynamo operates at the bottom of the convection zone, then the magnetic flux created there has to rise to the surface. When the convection zone is regarded as passive, the rising flux is deflected by the Coriolis force to emerge at rather high latitudes, poleward of typical sunspot zones (Choudhuri and Gilman, 1987; Choudhuri, 1989). Choudhuri and D’Silva (1990) included the effects of convective turbulence on the rising flux through (a) giant cell drag and (b) momentum exchange by small-scale turbulence. The momentum exchange mechanism could enable flux tubes of radii not more than a few hundred km to emerge radially at low latitudes, but the giant cell drag mechanism required unrealistically small flux tube radii (a few meters for a reasonable giant cell upflow) to counteract the Coriolis force. We now include the additional effect of Kelvin–Helmholtz instability in a symmetrical flux ring caused by the azimuthal flow induced during its rise. The azimuthal flow crosses the threshold for the instability only if there is a giant cell upflow to drag the flux tubes appreciably. In the absence of such a drag, as in the case of a passive convection zone or in the case of momentum exchange by small-scale turbulence, the azimuthal velocity never becomes large enough to cause the instability, leaving the results of the previous calculations unaltered. The giant cell drag, aided by Kelvin–Helmholtz instability, however, becomes now a viable mechanism for curbing the Coriolis force $- 10^4$ G flux tubes with radii of a few hundred km being dragged radially by upflows of 70 m s$^{-1}$.

1. Introduction

It used to be believed that the solar dynamo operates in the convection zone just beneath the photosphere and the magnetic features on the Sun were taken to be direct signatures of this dynamo action. Lately, the hypothesis that is getting increasingly popular is that the dynamo resides in a thin layer at the bottom of the convection zone. The scientific background for this hypothesis with reference to the relevant literature is given in Section I of Choudhuri and Gilman (1987, henceforth Paper I; see also Choudhuri, 1990a). If this hypothesis is true, then the magnetic features that we see on the photosphere can no longer be direct signatures of the dynamo action, because the magnetic fields that are generated at the bottom of the convection zone have to traverse the entire turbulent convection zone before they emerge at the surface. Paper I dealt with the question as to what would be the effect of rotation of the Sun on the emerging magnetic flux tubes. Choudhuri and D’Silva (1990, henceforth Paper II) studied how turbulence in the convection zone would influence the emergence of these flux tubes.

Paper I showed that Coriolis force has a profound effect on the trajectories of flux tubes emerging from the bottom of the convection zone. A flux tube symmetric about the rotation axis in the form of a flux ring was taken. This flux ring was released from some particular latitude, close to the equator, at the bottom of the convection zone and allowed to rise due to magnetic buoyancy. The Coriolis force was found to dominate the magnetic buoyancy force and make the flux rings appear at very high latitudes well
above the latitudes where sunspots are seen, unless the magnetic field had unreasonably high value of the order of $10^5$ G. Choudhuri (1989) added a non-axisymmetry in these flux rings and again found the same difficulty with the tops of the flux loops emerging at latitudes higher than the sunspot latitudes.

Since the calculations of Paper I failed to make the flux tubes starting from the bottom of the convection zone appear at the typical sunspot latitudes, Paper II included the effect of turbulence on these rising flux tubes to see whether they could now be made to emerge at the sunspot latitudes. The effect of turbulence in the convection zone was modeled as (a) a drag due to the updraught of giant cells, and (b) an angular momentum exchange due to small-scale turbulence. It is theoretically expected that giant convection cells extending from the bottom of the convection zone to the top exist (Glatzmaier and Gilman, 1981; van Ballegooijen, 1986). These giant cells could conceivably drag the flux tubes in their updraught and make them emerge at the typical sunspot latitudes. The results of Paper II showed that if these flux rings had to be brought out radially by dominating over the Coriolis force, then either the updraught velocities in these giant cells had to be unreasonably large ($\approx 300$ m s$^{-1}$) or the cross-sectional radius of the flux tubes had to be unreasonably small ($\approx$ a few meters). This ruled out the possibility of giant cells aiding the flux tubes in emerging at lower latitudes. On the other hand, the small-scale turbulence on the size of the flux tubes, which is expected to exchange angular momentum with the flux tubes, was found more effective in suppressing the Coriolis force and led to a radial emergence of the flux rings from the bottom of the convection zone provided they had radii not larger than a few hundred km. The final conclusion of Paper II was that the flux tubes of sizes of the order of a few hundred km can be made to emerge radially at the typical sunspot latitudes if there is enough turbulence in the convection zone at those small length scales.

Both Paper I and Paper II ignored the stability of the flux ring as it emerged. When two fluids in contact with each other are in relative motion, then the interface separating them can become unstable to perturbations. This instability is popularly known as the Kelvin–Helmholtz instability (henceforth KHI). The surface of the rising flux tube forms an interface between the interior fluid and the surrounding convection zone. The relative motions between these two fluids can be resolved into two components. One is the flow around the cross-section of the flux tube which is transverse to the axial magnetic field in the ring, and the other is the azimuthal flow in the ring. The transverse flow grazes the surface of the flux tube and is like the laminar flow around an infinite cylinder moving in the direction perpendicular to its axis. If the flux tube contains only an axial magnetic field, then as explained in Section 2, the flux tube will be totally unstable to all perturbations, which means the flux tube has to split into several flux tubes of smaller cross-sectional area. The presence of a small amount of twist, however, can drastically alter the situation and impart a cohesiveness to the flux tube against such splitting instabilities (Tsinganos, 1980). Since flux tubes come up through the convection zone, it seems likely that those which survive have enough twist to resist splitting and hence we do not consider this type of instabilities in this paper. The second source of KHI is the azimuthal flow inside the flux ring generated due to the conservation of angular
momentum. When the flux ring begins to rise from the bottom of the convection zone, the fluid elements inside the flux ring move away from the rotational axis and hence develop an azimuthal velocity with respect to the surroundings. This azimuthal flow is parallel to the axial field of the flux ring and, as described in Section 2, the interface becomes unstable to KHI if the azimuthal velocity exceeds $\sqrt{2}$ times the Alfvén velocity $v_A$ of the flux tube. When the interface is unstable to KHI, the energy in the azimuthal flow gets into transverse motions at the interface and hence the energy in the azimuthal flow reduces. This in turn reduces the azimuthal velocity between the flux tube and surroundings, suppressing the Coriolis force. The suppression of the Coriolis force can have a major effect on the emergence of the magnetic flux tubes from the bottom of the convection zone.

Section 2 summarizes the main results pertaining to KHI which are of interest to us here. In Section 3, we describe the basic equations used in Paper I and Paper II, and then discuss how KHI can be incorporated in the problem by adding some extra terms in the basic equations. The results of the numerical runs are presented in the three subsequent sections. We first study the effect of incorporating KHI when the convection zone is regarded as passive. In other words, we repeat the calculations of Paper I with the extra term due to KHI put in. The results can be found in Section 5. We find that the Coriolis force makes the flux rings turn parallel to the rotation axis and keeps the azimuthal velocity well below the critical value beyond which KHI is excited. Hence, even though the KHI term is present in the equations, its value always remains zero, and the final trajectories are identical to those in Paper I. Section 4 presents the calculations which incorporate KHI to the case of the flux tubes interacting with small-scale turbulence. We had seen in Paper II that such small-scale turbulence is capable of bringing the flux radially by suppressing the Coriolis force through the exchange of angular momentum. We again find that this exchange of angular momentum process keeps the azimuthal velocity too low to excite KHI, and consequently the trajectories are again the same as in Paper II. Only when we study the drag due to giant cells in Section 6, we find that the tubes are dragged out enough for the azimuthal velocity to cross the critical value for KHI. Once the KHI is excited, it can have drastic effects on the subsequent evolution of the system, and we find that much more reasonable values of giant cell updraught (compared to Paper II) are now sufficient for dragging the flux tubes out radially. The final conclusions are summarized in the last section.

2. Kelvin–Helmholtz Instability

When two fluid media in contact with each other are in relative motion, the interface between the two media can become unstable to perturbations depending on the physical properties of the two media and their interface. This instability is well known as the Kelvin–Helmholtz instability and discussed in detail for incompressible fluids by Chandrasekhar (1961).

We shall consider a magnetic–nonmagnetic interface with no surface tension or gravity, formed by two uniform fluids of densities $\rho_1$ and $\rho_2$ at $z = 0$ as shown in Figure 1.
Medium 1 is in the region $z < 0$ with a uniform magnetic field $B$ in the $x$-direction, and medium 2 is in the region $z > 0$. $U_1$ and $U_2$ are the flow velocities in the $x$-direction of medium 1 and medium 2, respectively. The dispersion relation for this system is given in Equation (202) of Chandrasekhar (1961, §106), with $g = 0$, and reproduced below with a change in notation of the frequency of the perturbations as $\omega$ instead of $n$:
\[ \rho_2(\omega + k_x U_2)^2 + \rho_1(\omega + k_x U_1)^2 = k_x^2 \rho_1 v_A^2, \]

where $k_x$ is the wave number of the perturbations along the $x$-direction. The roots of this equation are
\[ \omega = -\frac{k_x(\rho_1 U_1 + \rho_2 U_2)}{\rho_1 + \rho_2} \pm \frac{k_x \sqrt{\rho_1(\rho_1 + \rho_2)v_A^2 - \rho_1 \rho_2(U_2 - U_1)^2}}{\rho_1 + \rho_2}. \]

If the frequency of the perturbations is complex, then the amplitude of the perturbations grow exponentially and the system is unstable. Hence, the condition for KHI is
\[ \rho_1(\rho_1 + \rho_2)v_A^2 < \rho_1 \rho_2(U_2 - U_1)^2. \]

The system is thus unstable if the relative velocity of the two media exceeds $\sqrt{(\rho_1 + \rho_2)/\rho_2}$ times the Alfvén speed of medium 1.

The discussion above was for KHI for relative flows and perturbation modes parallel to the magnetic field. In general, if the perturbation modes make an angle $\theta$ with $B$ and...
if \( U_1 \) remains always along \( B \) but \( U_2 \) makes an angle \( \phi \) with the mode as shown in Figure 2, we can write down the dispersion relation as

\[
\rho_2(\omega + k U_2 \cos \phi)^2 + \rho_1(\omega + k U_1 \cos \theta)^2 = k^2 \rho_1 v_A^2 \cos^2 \theta,
\]

\[
k = (k_x, k_y, 0).
\]  

The roots of the above equation are

\[
\omega = \frac{-k(\rho_1 U_1 \cos \theta + \rho_2 U_2 \cos \phi)}{\rho_1 + \rho_2} \pm \frac{k \sqrt{\rho_1(\rho_1 + \rho_2)v_A^2 \cos^2 \theta - \rho_1 \rho_2(U_2 \cos \phi - U_1 \cos \theta)^2}}{\rho_1 + \rho_2}.
\]  

Hence, the condition for KHI turns out to be

\[
\rho_1(\rho_1 + \rho_2)v_A^2 \cos^2 \theta \leq \rho_1 \rho_2(U_2 \cos \phi - U_1 \cos \theta)^2.
\]  

In the previous case we had considered the stability of only those modes parallel \( B \) (\( \theta = 0 \), with \( U_1 \) and \( U_2 \) also parallel to \( B \) (\( \phi = 0 \)). Note that Equation (6) reduces to Equation (3) when \( \theta \) and \( \phi = 0 \). On the other hand, we can consider the stability of a mode which makes any angle \( \theta \) with \( B \), with \( U_1 \) and \( U_2 \) parallel to \( B \) (i.e., \( \theta = -\phi \)). Equation (6) shows that the condition for KHI is exactly Equation (3). If \( U_1 \) and \( U_2 \) are...
parallel to $B$, then irrespective of the angle the mode makes with the field the condition for KHI remains as that of Equation (3). However, if $U_2$ makes an angle with $B$ ($U_1$ being parallel to $B$), the condition for KHI is sensitive to the direction the mode makes with $U_2$. To illustrate the point for $U_2$ perpendicular to $B$, we shall take two cases (i) the mode is parallel to $U_2$ (i.e., perpendicular to $B$; $\theta = 90^\circ$ and $\phi = 0$), (ii) the mode is perpendicular to $U_2$ (i.e., parallel to $B$; $\theta = 0$ and $\phi = 90^\circ$). In case (i), the condition for KHI becomes $U_2^2 > 0$, implying that all modes parallel to $U_2$ are unstable. In case (ii), the condition for KHI becomes

$$U_1^2 > \frac{(\rho_1 + \rho_2)}{\rho_2} v_A^2.$$  

We have total stability if $U_1 = 0$.

We saw that in the cases above, the growth rate of the instability which is the imaginary part of the complex frequency increases monotonically with the wave number $k$. This happens because the system in consideration is an infinite system. If we consider a finite system, e.g., a cylindrical magnetic flux tube with a flow inside it parallel to its axis, then the growth rate has a maximum for wave numbers with wavelengths of the size of the tube radius. For a cylindrical jet with an axial magnetic field, Ray (1981) shows that the maximum growth rate for KHI occurs for the sausage modes for wavelengths around $kR \approx 1.0$, where $R$ is the radius of the cross-section of the jet, and for helical modes for wavelengths around $kR \approx 2.0$. Ray (1981) also finds that the growth rate for sausage modes is larger than for helical modes.

3. Introduction of KHI in the Mathematical Equations

The mathematical equations needed to find the trajectory of a flux ring through the convection zone are taken from Equations (4)–(6) of Paper II. Here we reproduce these equations below:

$$2 \left[ \frac{d^2 r}{dt^2} - r \left( \frac{d \theta}{dt} \right)^2 \right] + \left[ -r \left( \frac{d \phi}{dt} \right)^2 \sin^2 \theta - 2r \Omega \left( \frac{d \phi}{dt} \right) \sin \theta \cos \theta \right] = -\frac{\Delta \rho}{\rho_e} \frac{g}{g} - \frac{B^2}{4\pi \rho_e} + \frac{D_r}{\pi \sigma^2 \rho_e},$$  

$$2 \left[ r \frac{d^2 \theta}{dt^2} + 2 \frac{dr}{dt} \frac{d \theta}{dt} \right] + \left[ -r \left( \frac{d \phi}{dt} \right)^2 \sin \theta \cos \theta - 2r \Omega \left( \frac{d \phi}{dt} \right) \sin \theta \cos \theta \right] = -\frac{B^2}{4\pi \rho_e} \cot \theta + \frac{D_\theta}{\pi \sigma^2 \rho_e},$$  

$$r \frac{d^2 \phi}{dt^2} \sin \theta + 2 \frac{dr}{dt} \frac{d \phi}{dt} \sin \theta + 2r \frac{d \theta}{dt} \frac{d \phi}{dt} \cos \theta +$$

$$+ 2\Omega \left( r \frac{d \theta}{dt} \cos \theta + \frac{dr}{dt} \sin \theta \right) = \frac{D_\phi}{\pi \sigma^2 \rho_e},$$  

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where \((r, \theta, \phi)\) is the position of a fluid particle inside the flux ring, with magnetic field \(B\), radius of cross-section \(\sigma\) and density \(\rho\) in a medium of density \(\rho_e\). Paper I gives the details how to specify the quantities like \(\Omega, g\), and \(\rho_e\), and then integrate Equations (8)–(10). \(D_r, D_\theta, \text{and} \ D_\phi\) are components of drag per unit length. We shall see later that KHI can be incorporated through the azimuthal drag \(D_\phi\).

Here we carry out all the calculations assuming that the flux tube is in thermal equilibrium with the surroundings, which means

\[
\frac{\Delta \rho}{\rho} = \frac{B^2}{8 \pi p_e},
\]

where \(p_e\) is the gas pressure of the external medium. The calculations in Paper I included various other thermal conditions and showed that the results were qualitatively similar. Hence, we do not consider more general thermal conditions in this paper.

Sections 2.2 and 2.3 of Paper II describe how the effects of both giant cells and small-scale turbulence can be included through the drag term. To simplify the problem, the effect of giant cells was studied by considering a radial upflow \(u_r\) everywhere, since this is the most favourable type of flow for dragging flux rings radially. The drag due to such a flow as given in Equation (9) of Paper II is

\[
\frac{D}{\pi \sigma^2 \rho_e} = - \frac{C_D}{2 \pi \sigma} \left[ (\dot{r} - u_r)^2 + r^2 \dot{\theta}^2 \right]^{1/2} \left[ (\dot{r} - u_r) \dot{e}_r + r \dot{\theta} \dot{e}_\theta \right],
\]

where \(C_D\) is a dimensionless coefficient which has roughly a constant value of 0.4 for large Reynolds numbers (Goldstein, 1938). When small-scale turbulence alone is present, the angular momentum exchange due to it can be modelled as shown in Equations (10) and (11) of Paper II:

\[
\frac{D_\phi}{\pi \sigma^2 \rho_e} = \frac{r}{\tau_v} \frac{d\phi}{dt} \sin \theta, \quad D_r = D_\theta = 0,
\]

where \(\tau_v\) is the angular momentum exchange time.

We now describe how to introduce the effect of KHI in our equations. We had already mentioned in the Introduction that we are going to consider only KHI due to the azimuthal velocity and not KHI due to the transverse flow, which is probably stabilized by the twist. To estimate the growth time for the appropriate KHI, we note from (2) that for the flow parallel to the magnetic field at a magnetic–nonmagnetic interface the imaginary part of the complex frequency is

\[
\text{Im}(\omega) = \frac{k_x}{\rho_1 + \rho_2} \sqrt{\rho_1 \rho_2 (U_2 - U_1)^2 - \rho_1 (\rho_1 + \rho_2) v_\lambda^2},
\]

\[
\rho_1 \rho_2 (U_2 - U_1)^2 > \rho_1 (\rho_1 + \rho_2) v_\lambda^2.
\]
For the azimuthal motion of the flux ring,

\[ U_1 = v_\phi, \quad U_2 = 0, \]

and

\[ \rho_i \approx \rho_e = \rho. \]

Hence, Equation (14) becomes

\[
\text{Im}(\omega) = \frac{k_x}{2} \sqrt{v_\phi^2 - 2v_A^2}, \quad v_\phi^2 > 2v_A^2, \tag{15}
\]

where \( k_x \) is the wave number along the \( x \)-direction. As discussed in the previous section, maximum growth rates are attained for wavelengths comparable to the radius of the cylinder. We thus take \( k_x \) to be \( \sigma^{-1} \). Hence, \( \text{Im}(\omega) \) becomes

\[
\text{Im}(\omega) = \frac{1}{2\sigma} \sqrt{(v_\phi^2 - 2v_A^2)}, \quad v_\phi^2 > 2v_A^2. \tag{16}
\]

If \( v_\phi \) exceeds \( |\sqrt{2} v_A| \) the perturbations at the tube-surrounding interface grow exponentially, and the energy corresponding to the excess velocity \( (v_\phi - \sqrt{2} v_A) \) gets diffused by being fed into the growing perturbations. One can take the inverse of \( \text{Im}(\omega) \) as a diffusion \( \tau_{KH} \) for this excess energy, i.e.,

\[
\tau_{KH} = \frac{2\sigma}{\sqrt{v_\phi^2 - 2v_A^2}}. \tag{17}
\]

The flux ring, hence, experiences an effective drag in the azimuthal direction as a result of the diffusion of energy from its azimuthal motion. Thus the effect of KHI can be incorporated through the drag term in Equation (10) taken to be of the form

\[
\frac{D_\phi}{\pi \sigma^2 \rho_e} = \begin{cases} 
\frac{r \frac{d\phi}{dt} \sin \theta}{\tau_{KH}} & \frac{r \frac{d\phi}{dt} \sin \theta}{\tau_{KH}} = \frac{\sqrt{v_\phi^2 - 2v_A^2}}{2\sigma}, \quad v_\phi^2 > 2v_A^2, \\
0, & v_\phi^2 \leq 2v_A^2.
\end{cases} \tag{18}
\]

This implies that when the azimuthal velocity of the flux ring exceeds \( |\sqrt{2} v_A| \), the flux ring begins losing the energy contained in its azimuthal motion. The reduction in the azimuthal velocity of the flux ring suppresses the growth of the Coriolis force.

To find the trajectory of a flux ring through the convection zone we integrate Equations (8)-(10) numerically with the initial velocity \( v = (0, 0, 0) \) and initial position \( r = 0.7 R_\odot \) and \( \theta = \) some co-latitude (say 85°, i.e., 5° latitude). For various cases the drag terms chosen are in (12), (13), and (18) or various combinations of these as described in the following sections.
4. Importance of KHI in Flux Tubes Rising through a Passive Convection Zone

Here we demonstrate that KHI does not become important when the magnetic flux ring rises through the convection zone taken to be passive as in Paper I. The equations we use here are (8)–(10) with the drag terms as given by Equations (12) and (18) with the flow velocity \( u_F = 0 \). It is to be noted that if we put \( u_F = 0 \) in Equation (12), then we get the drag experienced in a passive convection zone. The inclusion of Equation (18) allows for the possibility of KHI. As in Paper I, we release flux rings of \( B_0 = 17 \) kG, \( \sigma_0 = 100 \) km placed at latitudes \( 5^\circ, 10^\circ, 20^\circ, 30^\circ, 45^\circ, \) and \( 60^\circ \) at the bottom of the convection zone (taken at \( 0.7 \, R_\odot \)) and allow them to rise to the surface of the Sun. We point out that the equations used in Paper I were slightly incorrect as explained in Choudhuri (1990b), though it had negligible effects on the trajectories. The trajectories we now obtain turn out to be exactly the same as Figure 5(h) of Paper I, implying that KHI has not been excited. To understand the reason for this, the values of \( v_\phi \) and \( \sqrt{2} \, v_A \) for the flux ring released at \( 5^\circ \) are plotted in Figure 3. Note that \( v_\phi \) (dashed line) remains well below the value of \( \sqrt{2} \, v_A \) (solid lines) throughout the trajectory of the flux ring in the convection zone. Hence, KHI is not excited, and the term (18) has remained zero throughout.

![Diagram](image)

Fig. 3. The evolution \( v_\phi \) (dashed line) of a flux ring with \( B_0 = 17 \) kG as it rises from the bottom of the convection zone at \( r/R_\odot = 0.7 \) to the surface at \( r/R_\odot = 1 \), when released at \( 5^\circ \) latitude, in a passive convection zone. The solid lines represent the change of \( \sqrt{2} \, v_A \) with depth.

The reason why KHI does not become important in the above case is not difficult to understand. The flux ring, when released from the bottom of the convection zone, initially moves out radially due to magnetic buoyancy and gains an azimuthal velocity as a virtue of the conservation of angular momentum. Thus the Coriolis force given by
2v_\phi \Omega_s increases as the flux ring rises until it becomes comparable to the magnetic buoyancy force. Thereafter, the flux ring moves parallel to the rotation axis, and the Coriolis force 2v_\phi \Omega_s remains comparable to the magnetic buoyancy force (\Delta \rho/\rho)g. Substituting for (\Delta \rho/\rho) from Equation (11),

\[ 2v_\phi \Omega_s = \frac{B^2}{8\pi p_e} \cdot g. \]

For KHI to become important, the maximum saturation value of v_\phi should exceed \sqrt{2} v_A, i.e.,

\[ v_\phi > \sqrt{2} v_A. \]

Substituting for v_\phi from above and using v_A = B^2/\sqrt{4\pi \rho}, we get

\[ \frac{B^2}{8\pi p_e} \frac{g}{2\Omega} > \frac{B}{\sqrt{2\pi \rho}}, \]

i.e.,

\[ B > 2 \times 10^5 \text{ G}, \]

when we use values appropriate at the bottom of the convection zone.

Thus KHI in principle can get excited for flux tubes with magnetic fields > 2 \times 10^5 \text{ G}. Flux tubes with such high fields would anyway emerge radially even without the aid of KHI, as seen in Paper I. We are, however, not interested in such high fields, as they are orders of magnitude stronger than the equipartition fields and are unlikely to be produced by the dynamo. Flux tubes with reasonable magnetic fields moving through a passive convection zone would not experience KHI.

5. Importance of KHI in Flux Tubes Exchanging Angular Momentum through Small-Scale Turbulence

Here we show that KHI does not become important when magnetic flux tubes are brought out radially by the angular momentum exchange through small-scale turbulence as proposed in Paper II. The equations we use are (8)–(10) with the drag terms D_{\perp} = 0, and D_\phi being the sum of D_\phi in Equation (13) and Equation (18). Equation (13) introduces the effect of the angular momentum exchange due to small-scale turbulence and Equation (18) that of KHI in the equations of motion. As in Paper II we release flux rings of B_0 = 17 \text{ kG} from latitudes 5^\circ, 10^\circ, 20^\circ, 30^\circ, 45^\circ, and 60^\circ at the bottom of the convection zone. The angular momentum diffusion time needed for these flux rings to emerge radially is \tau_v \leq 13 \text{ hr} as seen from Figure 8 of Paper II. We again find that the trajectories of the flux rings for \tau_v = 13 \text{ hr} are the same as given in Figure 6(a) of Paper II. The values of v_\phi and |\sqrt{2} v_A| for the flux ring released at 5^\circ are plotted in Figure 4. Figure 4 clearly shows that v_\phi (dashed line) remains well below |\sqrt{2} v_A| (solid lines) throughout its trajectory so that KHI cannot get excited in this case also.
Fig. 4. The evolution of $v_\phi$ (dashed line) of a flux ring with $B_0 = 17$ kG when released at 5° latitude from the bottom of the convection zone (at $r/R_\odot = 0.7$) as it is brought out radially by suppressing the Coriolis force via the small-scale angular momentum exchange mechanism with $\tau_v = 13$ hr. The solid lines represent the change of $|\sqrt{2} v_\alpha|$ with depth.

The reason why this happens is because the angular momentum exchange mechanism brings the flux tube radially out by suppressing the Coriolis force. The Coriolis force is suppressed by reducing $v_\phi$, which is done by extracting angular momentum from the flux tube. This angular exchange mechanism does not allow $v_\phi$ to exceed $|\sqrt{2} v_\alpha|$ and, hence, KHI is not allowed to get excited.

6. Giant Cell-Drag and KHI

We now present the calculations to demonstrate the effect of KHI on the radial emergence of flux tubes when they are dragged by the giant cell updraught as in Paper II. We will show that KHI plays a very important role in this process and makes the giant cell drag mechanism of Paper II a promising one. The equations of motion used are (8)–(10), with the drag terms as given by (12) and (18). Equation (12) introduces the drag due to the motion of the tube perpendicular to its axis and Equation (18) introduces the effect of KHI in the equations of motion. By putting various values for $u_F$ we can make various giant cell upflows to drag the flux rings to the surface. The flux rings are placed at the bottom of the convection zone at latitudes of 5°, 10°, 20°, 30°, 45°, and 60° and are allowed to rise to the surface. For a given initial field $B_0$ and initial cross-sectional radius $\sigma_0$, the upflow velocity $u_F$ is increased in steps for different runs. As $u_F$ is increased, the flux rings emerge at lower and lower latitudes until they eventually
emerge at the latitudes at which they were released. In this way one can find the minimum value of upflow velocity needed to drag a given flux tube radially.

Figure 5(a) shows the trajectory of flux tubes with $B_0 = 17$ kG and $\sigma_0 = 100$ km when $u_F = 43$ m s$^{-1}$. Note that the flux tubes have emerged radially. Figure 5(b) shows the trajectories for the same flux tubes when $u_F$ is slightly decreased to $u_F = 22$ m s$^{-1}$, the tubes fail to emerge radially. Hence, the upflow velocities $u_F \geq 43$ m s$^{-1}$ can drag these flux tubes radially to the surface. Figure 5(c) shows the evolution of $v_\phi$ and $|\sqrt{2} v_A|$ for the flux ring released from $5^\circ$ latitude at the bottom of the convection zone when $u_F = 43$ m s$^{-1}$. $v_\phi$ crosses $-\sqrt{2} v_A$ very early in the trajectory. This is the point at

Fig. 5a, b. Trajectories of flux rings released at $5^\circ$, $10^\circ$, $20^\circ$, $30^\circ$, $45^\circ$, and $60^\circ$ latitudes. $B_0 = 17$ kG and $\sigma_0 = 100$ km. (a) $u_F = 43$ m s$^{-1}$, (b) $u_F = 22$ m s$^{-1}$. Crosses in (a) and (b) represent time steps of 43.2 hr. (c) The evolution of $v_\phi$ (dashed line) of the flux ring released at $5^\circ$ latitude with the solid lines representing the change of $|\sqrt{2} v_A|$ with depth.
which the KHI term of Equation (18) begins playing an active role in the further evolution of the tube. As is clear from Figure 5(c), once \( v_\phi \) crosses the critical velocity for KHI (viz., \(|\sqrt{2} \, v_\Lambda|\)), the growth of \( v_\phi \) is heavily curbed and it closely hugs the value of \(-\sqrt{2} \, v_\Lambda\) for the rest of its journey to the surface. This suppression of \( v_\phi \) via KHI seems to have a drastic effect on the emergence of these flux rings. The graph in Figure 5 of Paper II shows that for the same flux tubes as above with \( B_0 = 17 \, \text{kG} \) and \( \sigma_0 = 100 \, \text{km} \), \( u_F \) is needed is \( \geq 200 \, \text{m \, s}^{-1} \) to bring them radially out to the surface. The introduction of the effect of KHI reduces this minimum \( u_F \) to a more realistic value of \( 43 \, \text{m \, s}^{-1} \) making the giant cell drag mechanism a more promising one than was thought of in Paper II.

In Paper II, when KHI effects were not included in the giant cell drag mechanism, the minimum upflow velocity needed to drag the flux tubes (of a given \( B_0 \) and \( \sigma_0 \)) radially to the surface was found to be independent of the magnetic field. Here, in contrast we find that, when KHI effects are included, this minimum upflow velocity is a function of the magnetic field and increases rapidly with \( B_0 \). This is mainly because the critical value of the azimuthal velocity at which the KHI term of Equation (18) becomes important is a function of \( B_0 \). Figures 6(a–c) show the trajectories as before for \( B_0 = 5.4 \, \text{kG} \) and \( \sigma_0 = 100 \, \text{km} \). The upflow velocity needed to bring these flux tubes radially out from the bottom of the convection zone in this case is \( 22 \, \text{m \, s}^{-1} \). This means that it is easier to drag flux tubes with weaker magnetic fields compared to the stronger fields. A comparison of Figure 5(c) and Figure 6(c) clearly indicates the reason why this happens. KHI is excited much earlier in flux tubes with weaker magnetic fields, merely because the critical velocity for KHI to occur is smaller and hence the level in the convection zone at which KHI gets excited is lower for these flux tubes with weaker magnetic fields. Hence, for a given flux tube cross-sectional radius the smaller the
Fig. 6. (a-b) Trajectories of flux rings released at 5°, 10°, 20°, 30°, 45°, and 60° latitudes. $B_0 = 5.4$ kG and $\sigma_0 = 100$ km. (a) $U_F = 17$ m s$^{-1}$, (b) $U_F = 26$ m s$^{-1}$. Crosses in (a) and (b) represent time steps of 43.2 hr. (c) The evolution of $v_\phi$ (dashed line) of the flux ring released at 5° latitude with the solid lines representing the change of $\sqrt{2} v_A$ with depth.
magnetic field, the lesser is the minimum upflow velocity needed to drag the flux tubes radially out.

We shall now define a quantity called the critical upflow velocity \( u_c \). A magnetic flux tube of a particular \( B_0 \) and \( \sigma_0 \) when released at the bottom of the convection zone at some particular latitude (say 5°) emerges at very high latitudes (\( \approx 45° \)) when \( u_F = 0 \). When \( u_F \) is increased, the latitude (90° − \( \theta_f \)) at which it emerges decreases, \( \theta_f \) being the co-latitude of emergence. That is to say that as \( u_F \) is increased, \( \theta_f \) keeps increasing until the flux tube emerges at the latitude at which it was released. The second curve from left in Figure 7 shows the graph of \( \theta_f \) as a function of \( u_F \) for a flux ring of \( B_0 = 17 \text{ kG} \), \( \sigma_0 = 100 \text{ km} \) when released from 5° latitude at the bottom of the convection zone. Starting with \( \theta_f = 45° \) when \( u_F = 0 \), it rises rapidly in the vicinity of \( u_F = 32.5 \text{ m s}^{-1} \) after which it flattens out at \( \theta_f = 85° \) (i.e., 5° latitude). There are two asymptotic values of \( \theta_f \), one at \( \theta_f = 45° \) at low \( u_F \) values and the other at 85°, at high values of \( u_F \). Hence, we define the critical upflow velocity \( u_c \) as that value of \( u_F \) corresponding to the value of \( \theta_f \) mid-way between these asymptotic values (viz., 63°). Note that in Figure 7 the curves for a larger \( \sigma_0 \) lie more to the right. \( u_c \) is therefore a monotonically increasing function of \( \sigma_0 \) for a given field strength. The reason is simple, because drag is inversely proportional to \( \sigma_0 \) (as given in Equation (12)), it requires a larger \( u_c \) to drag larger size tubes radially out.

![Graph showing \( \theta_f \) vs \( u_F \)](image)

Fig. 7. A plot of the co-latitude \( \theta_f \) at which the flux ring emerges vs the upflow velocity \( u_F \). The flux ring with \( B_0 = 17 \text{ kG} \) is released at the bottom of the convection zone at a latitude of 5° (co-latitude = 85°).

The numbers at the curves correspond to different values of \( \sigma_0 \) in units of km.

The dependence of \( u_c \) on \( B_0 \) and \( \sigma_0 \) can be estimated by a rough calculation as below. Figures 5(c) and 6(c) show that once KHI is excited, the growth of \( v_A \) is totally curbed and its further evolution follows that of \(- \sqrt{2} v_A \) very closely. Hence, the Coriolis force can be roughly taken to be \( 2(\sqrt{2} v_A) \Omega_z \), and the further evolution of the tube depends
solely on the balance of this Coriolis force and the drag due the giant cell updraught experienced by it. The giant cell drag can be taken as \( \frac{C_D}{4\pi\sigma_0}u_c^2 \). Therefore, balancing the two, we get

\[
2\sqrt{2} v_A \Omega_s \approx \frac{C_D}{4\pi\sigma_0} u_c^2.
\]

Substituting \( v_A = B/\sqrt{4\pi \rho} \) with \( \rho = 0.2 \text{ g cm}^{-3} \) appropriate for the bottom of the convection zone,

\[
u_c^2 \approx 1.57 \times 10^{-4} \sigma_0 B_0.
\]

If \( B = 1.7 \times 10^4 \text{ kG} \), then

\[
u_c^2 \approx 2.7\sigma_0.
\]

This theoretical curve (dashed line) of \( u_c \) vs \( \sigma_0 \) is plotted in Figure 8 for \( 1.7 \times 10^4 \text{ kG} \) along with the computed values for various \( B_0 \) (solid lines). The theoretical curve follows the same trend as the computed curve but lies well above the latter, because the \( v_A \) used in the theoretical estimate is its maximum value at the bottom of the convection zone. However, \( v_A \) decreases rapidly when the flux tube rises giving a lower value for the computed \( u_c \) than the theoretical one. The curves for higher \( B_0 \) lie above those of lower \( B_0 \), the reason for which is clearly explained above.

Since the typical giant cell velocities are expected to be \( \approx 70 \text{ m s}^{-1} \), we have drawn

---

**Fig. 8.** The plot of the critical upflow velocity \( u_c \) vs \( \sigma_0 \) for different field strengths. The numbers next to the curves indicate the values of magnetic fields in kG. The dashed curve is the theoretical curve for \( B_0 = 17 \text{ kG} \). The horizontal line is drawn at the expected giant cell velocity (70 m s\(^{-1}\)).
Fig. 9. The plot of the critical flux tube radius $\sigma_{0,c}$ vs the magnetic field strength $B_0$.

a horizontal line corresponding to that value in Figure 8, and the points at which the
different curves intersect this line give us the maximum flux tube radius $\sigma_{0,c}$ for a given
$B_0$ that can be dragged out radially from the bottom of the convection zone. This critical
tube size $\sigma_{0,c}$ is plotted vs $B_0$ in Figure 9. Flux tubes which lie above this curve fail to
emerge radially and are still dominated over by the Coriolis force, which makes them
emerge at high latitudes well above the latitudes where sunspots appear. Figure 9 shows
that flux tubes of fairly large cross-sectional area can be dragged out radially provided
their magnetic field is made sufficiently small! The regularities of bipolar regions on the
solar surface, however, imply that the magnetic field rising through the convection zone
must have been strong enough to avoid being mixed up by the turbulence, and hence
too small values of $B_0$ are ruled out. If $B_0$ is assumed to have the typical equipartition
value of $10^4$ G then tubes of $\sigma_0 \approx 800$ km can be brought out radially to the surface. This
makes the giant cell drag mechanism with KHI effects put in a viable mechanism in
making flux tubes emerge at the typical sunspot latitudes.

7. Conclusion

We have studied the effects of KHI on flux tubes emerging from the bottom of the
convection zone. We find that only when the flux tubes are dragged out by giant cells,
the azimuthal velocities in the flux tubes may become large enough to excite KHI. If
the flux tubes rise through a passive convection zone as in Paper I or if they are brought
out radially to the surface by the angular momentum exchange mechanism as in
Paper II, then the azimuthal velocities never cross the critical value for exciting KHI.

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We had concluded in Paper II that the giant cell drag was not a viable mechanism for bringing the flux out radially, because in the absence of KHI one needed unreasonably high upflow velocities or unreasonably small flux tube radii to make the drag sufficiently strong. On including KHI, we find that it is now possible to use more reasonable combinations of various parameters to drag flux tubes radially by giant cells. It should, however, be remembered that we had studied the effects of giant cells by using the rather unrealistic flow pattern which, being radial everywhere, was particularly suitable for dragging out the flux tubes radially. With more realistic flow patterns, one may have to use somewhat stronger upflows or smaller radii for the flux tubes to be dragged radially. We are now in the process of extending the non-axisymmetric code of Choudhuri (1989) to include more realistic giant cell drags. It is, however, highly unlikely that more realistic flows would change our qualitative conclusion that giant cell drags aided by KHI can counteract the Coriolis force to bring the flux tubes up radially.

By exploring completely axisymmetric flux ring models, we have been able to come up with two viable mechanisms to curtail the overwhelming dominance of the Coriolis force found in Paper I. The first mechanism proposed in Paper II was the angular momentum exchange with small-scale turbulence. The second mechanism found now is the drag by giant cells aided by the incidence of KHI. These two mechanisms require sufficient power respectively at the small and at the large length scales of convective turbulence. Since we know very little about the conditions of turbulence in the convection zone, we are right now not in a position to conclude whether the operating requirements for either of the mechanisms or for both are met. The flux tubes are probably brought out radially by either of these mechanisms or by a combination of both. We are now studying the effects of both these mechanisms on non-axisymmetric flux tubes. The first job obviously is to stop the tops of the non-axisymmetric loops from moving parallel to the rotation axis as seen in the calculations of Choudhuri (1989). In other words, the Coriolis force has to be suppressed to bring the tops of the flux loops out radially. The initial orientations of the emerging bipolar regions (Garcia de la Rosa, 1986), however, suggest that there must be some leftover Coriolis force to twist the tops of the flux loops before they emerge through the solar surface. Hence, whatever be the mechanism for suppressing the Coriolis force, it should on the one hand suppress the Coriolis force enough to make the tops of the loops come out radially, but it should on the other hand also allow for enough Coriolis force to orient the emerging flux loops by the appropriate amount. Though both the mechanisms described above are viable for bringing axisymmetric flux rings radially, it remains to be seen whether both of them can fill the necessary bill for the non-axisymmetric flux loops, or whether it will be possible to rule one of them out. We hope to be able to present our results in a few months.

Finally we point out that both these viable mechanisms require the flux tube radii to be not more than a few hundred km. Though this is much smaller than the different length scales at the bottom of the convection zone, such values for the tube radii may not be unreasonable. From completely different considerations, Choudhuri (1990a) concluded that the length scale of turbulence in the dynamo region has to be about the same if the dynamo has to have the correct period and wavelengths. Another interesting
fact to note is that a flux tube with a radius of 600 km and a magnetic field of $10^4$ G would carry a flux of $10^{20}$ maxwells. This is about the same order as the flux in a typical fibril flux tube in the solar photosphere (Parker, 1979). It seems likely that even within the convection zone the magnetic flux rises in the form of such thin flux tubes. One obvious implication of this is that all the flux in a large sunspot could not have risen through the convection zone as a single flux tube. This is not surprising in view of the fact that a sunspot has the characteristics of a collection of fibril flux tubes rather than a single monolithic flux tube (Parker, 1979; Choudhuri, 1986). Observations of sunspot formations also indicate that large sunspots do not rise to the surface as whole entities, but rather form on the surface due to the coalescence of smaller pores (Zwaan, 1985).

If the solar dynamo really operates at the bottom of the convection zone and if the flux tube picture is a roughly correct description of the dynamics of magnetic fields in the convection zone, then we conclude that the flux rises in the form of thin tubes, with the giant cell drag aided by KHI and the momentum exchange by small-scale turbulence being the two viable mechanisms for stopping the flux from being diverted by the Coriolis force.

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