CONVECTIVE FLOWS AROUND SUNSPOT-LIKE OBJECTS

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Abstract. Results are given for calculations of convective flows around objects in the outer layers of the Sun that have similar characteristics to small sunspots. These objects are allowed to radiatively (diffusively) exchange heat with their surroundings, but convective motions and exchange are absent. This assumption is based on the simple presumption that a sunspot magnetic field maintains pressure equilibrium with the surrounding medium and prevents convective exchange with that medium.

The flow structure around the object, and the question of the overall balance or redistribution of the emerging heat flux as suggested by earlier empirical models, are studied and discussed.

After a period of adjustment, shortly after the sunspot-like object is placed into the domain, the layer readjusts itself so that most of the heat flux actually reappears at the surface, although some fraction of the flux is carried horizontally far from the object. There is no indication of long term storage of the heat flux that would normally appear in the place where the object resides. Finally, when the object is removed, the surrounding medium responds very quickly and soon returns to the undisturbed state before the object was in place.

The present numerical treatment includes restrictions that may influence aspects of the heat redistribution, convective flows and time scales. In particular, the shape of the object and its size (somewhat smaller than a sunspot) are important, as is the number of spatial dimensions and the treatment of some boundary conditions. Since all of these issues require further investigation, some discussion is presented regarding the applicability of our results to real sunspots.

1. Introduction

Upon analyzing the first few months of data from the ACRIM experiment on SMM, it became clear that the total solar irradiance decreased when a sunspot area crossed the solar limb, reaching a minimum when the area crossed the central meridian (Willson, Hudson, and Chapman, 1981). It was in fact possible to explain the irradiance depression from the size of the projected sunspot area and the contrast between photospheric intensity and spot intensity as a function of location on the solar disk. Oster, Schatten, and Sofia (1982) were able to explain the entire variability of the total irradiance by adding the positive contribution of faculae according to intensity contrasts with the photosphere (as a function of distance from disk center) measured by Frazier (1971) and Chapman (1980). Since facular areas are not routinely available on a daily basis, they assumed that facular areas were approximately equal to the readily available areas and locations of Ca II plages. This scheme was used by Sofia, Oster, and Schatten (1982) to successfully simulate what the ACRIM experiment should have measured for the rest of 1981 when the data analysis was completed. They concluded that within

an error margin of approximately 10%, the average depression of the spot deficit was offset by the average facular excess. Because faculae last longer than sunspots, the spot depression during a typical solar rotation dominates over the facular excess during that rotation, so that an energy storage is suggested. On the other hand, because of the average behavior on the longer time scale, the storage should not exceed the lifetime of the faculae, which is of the order of one month.

Subsequent models of the total irradiance involving longer observation times (Schatten et al., 1985) showed that because of large uncertainties in the observed values of sunspots and facular sizes, the balance between the energy deficit due to sunspots and the energy excess due to faculae within the 10% limit obtained in the first modelling exercise was fortuitous, and that in fact the balance was within approximately 50%. On the other hand, competing empirical models concluded that the facular contribution was no larger than 10% of the sunspot depression, so that much longer term storage was indicated (cf. Hoyt and Eddy, 1982). To investigate this discrepancy it is useful to model the energy and material flow around a sunspot to see what happens to the blocked energy.

The magnetic field in a sunspot impedes convective energy transport and leads to the reduced specific intensity emerging through the sunspot (Biermann, 1941). This causes an entropy increase below the sunspot region which sets up an enhanced energy flux and material motions around the spot in an attempt to re-establish quasi-hydrostatic and thermal equilibrium between the disturbed region and its surroundings.

To treat this problem in a realistic way it is necessary to know the detailed configuration of the spot and faculae magnetic fields below the photosphere since it appears that the interaction between the flow and the magnetic field affects the surface structure of the active region producing a 'depression' (Spruit, 1976; Zwaan, 1978) or a 'hillock' (Schatten et al., 1986). These two descriptions were postulated to account for the directionality of the facular contrast. Since information does not currently exist to accomplish this, it is hoped that an approximation assuming simple spot geometry and no surface geometric effects, e.g., as done by Spruit (1977), will provide useful results.

The earliest models of this type were carried out by Spruit (1977) where time-dependence of the problem as well as systematic flows were neglected, and by Foukal, Fowler, and Livshits (1983) and Chiang and Foukal (1984), where time-dependence was considered, but still no flows were allowed. The results of these investigations indicated that, because of the much stronger thermal contact between the perturbed region and the deep convection zone than between the perturbed region and the surface, the large majority of the sunspot-blocked energy would be diffused into the entire solar convective region to be released on a thermal time scale of this region (i.e., about \(10^5\) years). More recently, Nye, Bruning, and LaBonte (1988) considered this same problem, but included both energy and mass flow in the linear approximation. They found that significant mass flows were generated by the thermal energy blocked by the spot, but that these flows were not accurately in agreement with observations. In this paper we will consider almost the same problem, but in the fully nonlinear compressible regime. In particular, we wish to ascertain the flow around a sunspot-like object at various depths in the solar
convection zone, as well as the energy balance in both the immediate area and far from the object as a function of time. This allows us to determine where heat flux may be stored as a result of the blocking, and thus address the question of the energy balance of active regions. The present paper will concentrate on the flow around the object and the general properties of the redistribution of heat flux and a subsequent paper will deal with the details of the energy balance.

The model and its formulation are given in Section 2. The results of some simulations are presented in Section 3. In Section 4 we discuss the implications of our results and outline what further work is necessary to address the problem of sunspot energy blocking with an increasing degree of realism.

2. Model Formulation

2.1. Our approach

Regions on the solar surface around sunspots, active regions, pores, plages, etc., exhibit very complex magnetic field behavior. The task of modelling, in detail, the interaction of the convective region and the magnetic field region is still in its early stages of development. At present we choose to proceed to model the region of interest without explicitly including the effects of a magnetic field in our calculations. The sunspot-like object is modelled as a region where convective/turbulent flow is prohibited but diffusive heat exchange due to radiation (and conduction) is allowed. This is analogous to a strong magnetic field inhibiting convective flows – a feature which is apparent in laboratory experiments (Nakagawa, 1955, 1957), observationally in stars (e.g., see the discussion in Gray, 1988) and from theoretical linear stability calculations (e.g., Danielson, 1963, 1964; Kato, 1966; Fox, 1985), however the object we consider is much too simple to realistically represent the actual solar surface features.

Sunspots are regions of intense magnetic field (0.1 to 0.2 Tesla, or 1 to 2 kG) with mean radii of about 30 Mm. Pores are much smaller sunspot-like regions, of the order of 1–2 Mm in radius and intermediate strength magnetic fields (0.05 to 0.10 Tesla, or 500 to 1000 G) which have considerably shorter lifetimes (up to several hours) and probably smaller time scales for their influence on the surrounding flow and thermodynamic structure (see Schröter, 1962; Bray and Loughhead, 1964; Simon and Weiss, 1970). Although we wish to model sunspots, our present models more closely resemble pores which, both in domain size and time scale, can be modelled with more affordable computational resources. Unfortunately very little is known about the structure of sunspots or pores beneath the solar surface and any study of heat flow around such objects must operate with this uncertainty, although a range of finite possibilities exists (Simon and Weiss, 1970; Meyer et al., 1974; Piddington, 1976; Parker, 1979). Some details of our present study will be sensitive to the sub-surface field geometry (and, hence, size and shape of our object), nevertheless, besides the smallness of the object, we have chosen a simple shape, namely a rectangle that may be placed at different positions within the convective region.
The particular depth of the object from the solar optical surface has special relevance to the questions of bright rings, thermal shadows and the moat flow. In addition, the depth of the object is important when we make conclusions about convective efficiency around the object.

When the upper boundary of the domain and the object are at the solar optical surface (see Figure 1) the object represents a visible magnetic region (flux blocking) whose characteristics may be compared to observation. The object can also be placed below the surface to address the question of thermal shadows as a result of the diverted heat flux.

![Diagram showing the position of the flux blocking object in relation to the computational domain when both upper boundaries are at the solar optical surface. It also shows the details of the treatment of the object in the staggered mesh formulation of the ADISM method.](image)

In this paper we will concentrate on the general properties of our models of the diversion of heat flux around a small sunspot-like object. We will discuss the implications of our calculations in the context of the solar surface and sunspots in a later section.

In a non-rotating frame of reference, the equations representing the stratified compressible fluid flow are

\[
\frac{\partial \rho}{\partial t} = - \nabla \cdot \mathbf{M},
\]  

\[
\frac{\partial \mathbf{M}}{\partial t} = - \nabla \left( \frac{\mathbf{M}}{\rho} \right) - \nabla p + \nabla \cdot \mathbf{\Sigma} + \rho \mathbf{g},
\]  

\[
\frac{\partial e}{\partial t} = - \nabla \left( e \frac{\mathbf{M}}{\rho} \right) - p \nabla \cdot \left( \frac{\mathbf{M}}{\rho} \right) - \nabla \cdot \mathbf{f} + \Phi;
\]
\( \rho \) is the density, \( \mathbf{M} \) is the mass flux vector \((\rho \mathbf{V})\), \( e \) is the specific internal energy, \( p \) is the pressure, \( \Sigma \) is the viscous stress tensor, \( \mathbf{f} \) represents the diffusive heat flux, \( \mathbf{g} \) is the gravitational acceleration, \( \Phi \) represents the effects of viscous dissipation and \( t \) represents time.

In this study we will use an ideal gas equation of state, with constant mean molecular weight, \( p = \rho T \) (in the appropriate units), such that \( p = (\gamma - 1)e \), \( \gamma \) being the ratio of specific heats. The temperature is then given by \( T = (\gamma - 1)e / \rho \).

The total flux \( \mathbf{F} \) is made up of \( \mathbf{F} = \mathbf{F}_v + \mathbf{F}_k + \mathbf{F}_e + \mathbf{f} \), where the enthalpy, kinetic energy, viscous and diffusive fluxes, respectively, are

\[
\mathbf{F}_e = (e + p) \frac{\mathbf{M}}{\rho},
\]

\[
\mathbf{F}_k = (\frac{1}{2} \rho \mathbf{V}^2) \mathbf{V},
\]

\[
\mathbf{F}_v = -\mathbf{V} \cdot \Sigma,
\]

\[
\mathbf{f} = -K_T \nabla T + K_p \nabla p.
\]

The thermal conductivities \( K_T = K_{SGS} + K_{turb} + K_{rad} \), and \( K_p \) may be written as

\[
K_T = \frac{\mu}{Pr} c_p + K_{turb} + \frac{4acT^3}{3k\rho},
\]

\[
K_p = \frac{\mu}{Pr} c_p \nabla_a \frac{T}{p};
\]

\( c_p \) is the specific heat at constant pressure, \( \nabla_a \) is the adiabatic gradient \( \equiv \partial \ln T / \partial \ln p |_{ad} \), \( Pr \) is the Prandtl number, \( \mu \) is the dynamic viscosity and radiative transfer is treated with the diffusion approximation, \( k \) being the opacity, \( a \) the Boltzmann constant, and \( c \) the speed of light. \( K_{turb} \) is known as the bulk (or turbulent) conductivity which is used to provide a minimum contribution to the conductivity in the absence of a radiation term. This term is usually included at the upper boundary of the domain to enable the star’s heat flux to escape from the domain.

The thermal conductivity of the sunspot-like object in the region of interest is the radiative conductivity (in the diffusion approximation) and is very important in the energy balance as the radiative flux becomes comparable to the convective flux at the solar surface (top of the layer). Most magnetic regions on the solar surface exhibit considerable modulations in specific intensity compared to the surrounding undisturbed regions. In the case of pores or sunspots, the typical reduction is between 10 and 65\% (Bray, 1981). One prediction to be expected from the simulations is this reduction in the output surface flux over the object.

The viscous stress tensor \( \Sigma \) and dynamic viscosity \( \mu \) may be written as

\[
\Sigma = 2\mu \sigma + \lambda (\nabla \cdot \mathbf{V}) \mathbf{I},
\]

\[
\mu = \rho (c_{\mu} \Delta)^2 (2\sigma : \sigma)^{1/2};
\]
$\sigma$ is the usual rate of strain tensor and the discussion of $c_\mu$ and $A$ is contained in Chan and Sofia (1986). For a perfect gas the diffusive flux $f$ may be expressed in terms of the entropy $S$:

$$S = c_\rho (\ln T - \nabla_a \ln p) ,$$

(12)

$$f = - \frac{\mu}{Pr} T \nabla S .$$

(13)

In our current models the sunspot-like object is stationary, implying that it is in pressure equilibrium with its surroundings. This suggests a 'magnetic energy' density within the region that is comparable to the surrounding energy density of the gas and so a mean magnetic field strength for the region can be estimated as approximately 1 kG.

For two-dimensional models, the domain is defined in radius and latitude by $r_1 \leq r \leq r_2$, $\theta_1 \leq \theta \leq \theta_2$, and the boundary conditions are $F_r = F_\theta = (at bottom only)$, $S = S_\tau = constant (at top only)$, $V_r = 0$ and $\partial(V_\theta/r)/\partial r = 0$ at $r = r_1, r_2$, and $F_\theta = \partial S/\partial r = 0$, $V_\theta = 0$ and $\partial V_r/\partial r = 0$ at $\theta = \theta_1, \theta_2$.

When the upper boundary of the domain is placed below the solar surface the object is an obstacle to heat flow, like a submerged flux tube. Naturally the size of the object in relation to the local energy carrying scale could be important in this region since the surrounding convection is very efficient whereas inside the object the relative heat carrying capacity (due to conduction and radiation) is very low. For the results of our calculations to be applied at larger depths the positional stability of the object must again be assumed, requiring that the hypothetical magnetic energy density of the sunspot-like object increase to compensate for the increased pressure in the surrounding medium. This increase, applied to the same object area, implies a proportionally larger magnetic field strength (since the density is increasing as well). A larger field strength is plausible although, as we have mentioned, the structure of subsurface magnetic fields is uncertain. Conversely, if the magnetic energy density does not increase, then our models will grossly overestimate the blocking of heat flux around the object as we do not allow the object to expand. The loss of stability of the object in the domain means that it would move subject to the overall forces applied and its buoyancy. These points are mentioned only for placing the present results in the context of the solar environment.

2.2. Method

The particular type of information we expect to derive from these theoretical calculations requires detailed models of the convective medium surrounding the flux blocking object. As a consequence it is desirable to have sufficient resolution to examine those features in detail. It is important to use a domain size that is large compared to the size of the object (typically 100 to 1 in area for a two-dimensional model) because the domain must thermally readjust every time a disturbance is introduced and this time is proportional to the size, placement and strength of the disturbance. In addition, the assumptions under which these simulations are made, i.e., the completely convectively isolated object, the boundary conditions on the domain and the object, etc., also place a
restriction on what features may be studied and over what time period. In attempting to introduce an increasing degree of realism into the numerical models we must be very careful not to exceed the present limitations of the formulation. Further discussion on this point is presented in Section 4.

The above requirements (and numerical limitations) dictate that we confine our study to two-dimensional models of the flows and domain sizes to be described below. In a two-dimensional calculation the object could be thought of as a magnetic flux tube with its square cross section in the \( r - \theta \) plane.

Specific information on the equations of compressible convection and the solution methods we employ may be found in Chan and Sofia (1986, 1989) along with discussions of the dependence on parameters, initial conditions and more details on the general properties of the convection models (see also Fox and Sofia, 1991). In this paper we will discuss only the alterations made to those models and concentrate on the application to flow around a sunspot-like object.

In order to accommodate the geometry of the sunspot-like object in the ADISM method some modifications were necessary to the flow variables defined on the staggered mesh (see Chan and Wolff, 1982) contained within the object. Figure 1 shows how the object relates to the staggered mesh of the ADISM method. The boundary of the object is on the velocity grid which makes it very easy and accurate to calculate thermodynamic fluxes, that is, the density and internal energy are not constrained by the object and heat is allowed to flow into or out of the object by radiation or conduction. No modifications are made to the equations inside the object except that flow is not allowed (hence there is no SGS viscosity or diffusivity). For other calculations, such as the stress tensor, the object boundaries are identical to the domain boundaries.

Initially the basic structure of the layer is prescribed by a polytropic model with temperature stratification parameter \( Z \) taken equal to 3.0, ratio of specific heats \( \gamma = \frac{5}{4} \) (thus \( c_p = 5.0 \) and \( V_{ad} = 0.2 \)) and polytropic index \( n = 3.80 \) (see Chan and Sofia, 1986, for details). The grid ratio \( Z_c \) is discussed in Chan and Sofia (1986) and is usually chosen to allocate equal numbers of grid points per pressure scale height (in this case 3.0). The Prandtl number is taken as \( \frac{1}{4} \). The width of the domain is 24.4 Mm and the depth is 2.0 Mm. The number of pressure scale heights (PSH) in the layer is 6.654. The three particular cases discussed in this paper, denoted I, II, and III are distinguished by the positioning and size of the sunspot-like object. Case I has no object, i.e., the undisturbed case. Case II has an object placed in the top center of the domain (as in Figure 1) of size 1.2 Mm wide and 0.132 Mm deep. Case III has an object submerged at the center of the domain of size 1.2 Mm wide and 0.159 Mm deep. Each case had 180 grid points in the horizontal and 40 or 80 points vertically. The object in cases II and III was 10 by 5 grid points.

Only a small number of possible combinations are presented in this paper (although many more were explored) due to the degree of qualitative similarity between many cases and also due to computer time limitations. Each of the cases to be studied required approximately ten to fifteen hours of integration time on a Cyber 205 after the initial relaxation.
Cases I–III represent a typical surface convection region near the solar equator which exhibits granulation and other larger scale features which we will not discuss here (see Fox and Sofia, 1991). The percentage of the physical domain (computational domain based on grid point distribution) occupied by the object is 0.35% (0.70%) for case II and 0.43% (0.70%) for case III. Since the aspect ratio for the domain in these cases is as much as 12:1 many of the figures we present will be subsets of the entire domain, usually close to the object.

In each of the cases that we investigated we began by computing the structure and dynamics of the convective region in the absence of the sunspot-like object (some 24.6 hours of integration in solar time). This undisturbed system must be fully relaxed in order to remove the effects of initial conditions on the subsequent behavior of the system, see Chan and Sofia (1986, 1989) and Fox and Sofia (1991) for more details. The sunspot-like object is then placed into the domain and the resulting flow and thermodynamic disturbances are studied as a function of time, making sure that the time step of the numerical scheme is reduced to maintain temporal accuracy.

Once the object is in place, a short time of relaxation occurs as the domain readjusts to the presence of the object. Before the object is present, the lateral boundary condition on the whole domain permits no heat flux across it. Once the relaxed system is obtained to give the mean structure, the temperature and density are fixed at the lateral boundaries allowing heat to flow into or out of the domain (note that this is typically a distance of about 10 Mm from the object). The aim of this procedure is to ascertain how much of the heat blocked by the object is diverted laterally out of the domain, as compared to re-emerging at the surface. We should also be able to determine what the time scale for the redistribution into different parts of the domain is.

We also monitor, in time, the total surface flux both as a function of latitude (horizontally), and latitudinally averaged to determine what, if any, diverted heat flux appears at the surface and what time scales and delay factors are involved.

On a slightly longer time scale, we monitor the regions close to the lateral boundaries of the domain far from the object for any net heat flux (note that the outward kinetic energy flux is zero on all boundaries) as a function of depth and time.

Since magnetic features on the solar surface have a finite lifetime, we also chose case II for an extended integration so that we could remove the object and follow the subsequent readjustment. This is especially relevant since the positive contributions to the irradiance by faculae persist after the sunspots producing the deficit disappear and continue throughout the entire duration of the faculae (some 70 to 120% of the sunspot contribution; Lawrence, Chapman, and Herzog, 1988).

In addition to the more visible quantities mentioned above, we are also interested in particular features of the convective flows around the object. The components of the total heat flux, defined by Equations (4) to (7), in a region directly around the object both before and after its placement will give an indication of how the convective flow reacts to a flux blocking object.
3. Results

3.1. Flow features

Figure 2 is an example of the instantaneous (pseudo-)streamlines of the fluctuating velocity field for case I in the absence of an object and Figure 3 shows the analogous streamlines for case II at 12.75 hours after placement of the object. The vertical axis shows the depth in units of \(\log(\bar{p})\) and the horizontal axis is distance in Mm. The region has only been partially shown to highlight the flows around the object. Figure 4 shows the instantaneous (pseudo-)streamlines for case III at 4.91 hours after placement of the object. In case II, when the object is at the top of the domain, the flows adjacent to the object mostly seem to be downward directed even to the extent that small vortices are generated close to the object to ensure this downflow. The abrupt boundary of the object, coupled with the fact that the temperature inside the object is lower than outside, causes the downflow. This gradient in temperature is usually accompanied by changes/gradients in pressure and/or density as well. This is illustrated in Figures 5 and 6, where the instantaneous velocity vectors are overlayed on contours of the temperature fluctuation about the horizontally averaged temperature and pressure fluctuation, respectively. Near the boundary of the object, the pressure is lower, and the flows are

Fig. 2. Instantaneous (pseudo-)streamlines of the fluctuating velocity field for case I (no object). The vertical axis shows the depth in fractional radius (it is important to note that the vertical distribution is approximately equally spaced in units of \(\log(\bar{p})\)) and the horizontal axis is horizontal distance across the region (in Mm).
Fig. 3. Instantaneous (pseudo-)streamlines of the fluctuating velocity field for case II at 12.75 hours after placement of the object. Only a portion of the domain has been shown to highlight the flows around the object.

Fig. 4. Instantaneous (pseudo-)streamlines as in Figure 3 for case III at 4.1 hours after placement of the object. See text for details.

clearly downward directed; this is representative of the general flow close to the object in case II. In case III, where the object is submerged (see Figure 8), the surrounding flows are also mostly downward directed, even though there is a flow above the object. Since the distance to the surface (5 grid points) is less than a pressure scale height and
Fig. 5. Instantaneous velocity vectors overlayed on contours of the temperature fluctuation about the horizontally averaged temperature for case II at 9.42 hours after the object is positioned. The vertical axis shows the depth in fractional radius (vertical distribution is again in units of log(\(\rho\))) and the horizontal axis is horizontal distance across the region (in Mm). The velocity scale is in non-dimensional units where, for reference, the sound speed at the surface is 1.118. The temperature scale factor is 5800 K.

Fig. 6. Instantaneous velocity vectors overlayed on contours of the pressure fluctuation about the horizontally averaged pressure for case II at 12.75 hours after the object is positioned. The pressure scale factor is 1.48 \times 10^5 \text{ dynes cm}^{-2}.

Heat transport is inefficient it is unlikely that circulation patterns could exist for any length of time. The region above the object acts as a thermal buffer between the top of the object and the surface with fixed temperature.

The temperature fluctuations near the object correlate quite well with the velocity field as shown in Figure 5, with hotter than average regions (solid lines) having upflow and colder than average (dash lines) having downflow. The density fluctuations for the same time and case, shown in Figure 7, also confirm this with lighter elements rising and heavier falling.

There are, however, certain times when there is upflow next to the object (typically only on one side). This occurs when a small element of hotter than average material is diverted very close to one edge of the object and causes a short term upflow as it rises.
to the surface. These events are not predominant though (see Figure 5). The appearance of hotter than average elements around the object is accompanied by excess heat diffusing through the object – recall that the object has some finite conductivity. Since this is a diffusive process, the time scale for releasing this heat to the surface and thus modulating the surface heat flux is on a different time scale to the eddy turnover time which governs nearby flows.

A good example of this is shown in Figure 8 where the instantaneous velocity vectors are overlaid on the temperature fluctuation at 5.89 hours after the object is positioned for case III. Note that just underneath the object there is upflow with excess temperature, a short time later the heat enters the object and diffuses to the surface. This type of behavior occurs when an upflow region persists for a few turnover times below the
object. Upflow regions are usually advected horizontally as they approach the object, and do not get the chance to release their heat into the object.

One important check on the treatment of the object is that the surface region above the object should have a reduction in intensity compatible with those observed for magnetic surface features. The details of the heat flux will be presented in a later section, but for cases II and III the mean temperature deficit of the object from its surroundings was approximately 850 K, giving a reduction in output flux of about 20%.

In all cases there are numerous smaller scale flows (close to the vertical size of the object), near the object, that are of lower speed than the general circulation. This is most apparent just beneath the object.

The temperature distribution for any of the cases considered (see Figure 5 or 8) in and around the object does not seem to indicate long term (longer than the characteristic turnover time) storage of the heat flux below the object or in the lower part of the domain as earlier studies had suggested. The general temperature distribution for a particular time (5.89 hours after placement) in case III is shown in Figure 9. The general density and pressure (not shown here) distributions also indicate that the mean thermodynamic structure does not undergo a noticeable change.

![Diagram](https://example.com/diagram.png)

Fig. 9. Isotherms for case III in a region around the object at 5.89 hours after the object is positioned. See text for details.

What is apparent, and will be discussed in the following sections, is that there is a dynamic readjustment of the layer surrounding the object to accommodate the altered heat carrying capacity of the object. The fluctuation in the entropy distribution shown in Figure 10 overlaid with the kinetic energy flux vectors also indicates significant lateral flows, especially in comparison to the undisturbed case I shown in Figure 11 (note the change in magnitude of the kinetic energy flux).

In earlier work on this subject (e.g., Nye, Bruning, and LaBonte, 1988) the predominant heat flux component (the only one except diffusion) was due to the internal energy of the medium – the enthalpy flux and consequently was the quantity most affected by flux blocking, in addition, this term was treated linearly. The calculations
Fig. 10. Instantaneous kinetic energy flux vectors overlayed on contours of the fluctuating entropy for case II at 9.42 hours after the object is positioned. See text for details. The kinetic energy flux is in non-dimensional units where the surface solar flux is 0.677.

Fig. 11. Instantaneous kinetic energy flux vectors overlayed on contours of the fluctuating entropy for case I (without the object) at a similar time to that of Figure 10. Note the difference in fluctuation of entropy and the much lower kinetic energy flux vectors.

presented in this paper indicate that indeed the enthalpy flux is modified close to the object but not to completely block the emerging flux, rather it is converted into kinetic energy near the object and redistributed in the entire domain and much of it is allowed to reach the surface, both as diverted flow around the object and as radiation through the object. These two aspects are discussed in the following section.

Chan and Sofia (1989) discuss the properties of deep and efficient convection, especially the proportion of the total flux carried in different ways. Their Figure 13 indicates that the greater heat carrying capacity is in the lower part of the domain: that is: as the scale height decreases convection becomes less efficient. With this in mind it is important to note that the redistribution of heat in the current calculations occurs close to the object, within a few pressure scale heights and not multiple scale heights.
below it. Once the heat is diverted though, there is a proportion of flux that continues its rise to the surface and another that seems to be advected laterally and slightly downward where the heat capacity is higher. This is shown (for a region close to the object) by the kinetic energy flux vectors in Figure 10, particularly the left edge of the figure.

3.2. Heat Flux Distributions

Figures 12 and 13 show the time-dependence of the horizontally-averaged emerging flux at the upper boundary of the domain; normalized into units of the solar flux $F_\odot$ for cases I and II and I and III, respectively. The thick solid line in each figure is case I (undisturbed) and the thin solid line is case II or III. The horizontal axis is time in hours relative to the placement of the object into the domain. After an initial period of time when the layer readjusts to the presence of the object it is clear that the average output flux almost returns to its undisturbed level. The fact that it does not return completely is important, as is the increased amplitude of the variation (or pulsing) in the average flux. The time-integrated average flux is 99.022% of the solar flux, that is, 0.978% lower than the total. For case III, the average is 98.79% of the solar flux.

![Graph](image_url)

Fig. 12. Time-dependence of the horizontally-averaged emerging flux at the upper boundary of the domain normalized into units of the solar flux $F_\odot$ for cases I and II. The thick solid line is case I (undisturbed) and the thin solid line is case II. The horizontal axis is time in hours.

* All time integrations are performed over at least 5 hours which includes 30 min before the object is in place.
Fig. 13. Time-dependence of the horizontally-averaged emerging flux at the upper boundary of the domain normalized into units of the solar flux $F_\odot$ for cases I and III. The thick solid line is case I (undisturbed) and the thin solid line is case III. The horizontal axis is time in hours.

For this particular case the maximum decrease in flux we would expect due to complete blocking based on the surface extent of the object is 2.33\% (2.33\% for case III), which is represented by a short dash line in Figure 12 and 13 for each case. This suggests that about half of the flux being blocked in cases II and III re-appears at the surface in the average sense and that the remainder is either stored within the layer somewhere or leaves the domain at the lateral boundaries, a point we will return to soon. It is important to remember that the absolute percentage reductions reflect the effect of the blocking based on a small object in a large area, whereas the relative percentage differences reflect the local changes.

The increase in amplitude of the average flux about a mean value is related to the diversion of the blocked emerging heat flux. The blocked flux requires a certain amount of time to reappear at the surface; a time delay which is related to the size and extent of the object (essentially the time it takes to go ‘around’ it). In addition, the object is allowed to radiatively exchange heat with its surroundings and can absorb or release heat, but on a different time scale, since there is no convection operating. Since the general temperature inside is lower than the surrounding regions, heat is drawn into the object and then released at its upper or side boundaries on that altered time scale, the radiative diffusion time. In fact, it can temporarily ‘store’ heat. The oscillation in the surface output flux is almost solely from the surface of the object. The amount of heat flux that it actually releases at the surface is not, however, the dominant component in raising the surface flux value above that of total blocking. In all the cases, the contri-
bution from the surrounding regions is close to, or exceeds 100%. One important point to note is that the amplitude of the oscillation in the surface flux about its mean is around ± 2–3%, which would be barely observable, especially against the general intensity fluctuations of the surrounding granulation (some 7–22%), the fact that its period is 16 to 20 min (and this is for pore size objects) and also since there are also likely to be other fluctuations at this percentage level (associated with mesogranulation, perhaps, see Deubner, 1989).

For a surface object the details of the heat exchange with its surroundings would depend on an adequate treatment of the effects of radiative transfer. Since these effects are treated crudely here, we will not present a detailed discussion in this paper.

It is interesting to note the 'period' of the variation in the surface fluxes, which ranges between 15 and 17 min for case II. This time scale or period is approximately the radiative diffusion time for the object based on vertical propagation (shortest distance). On the other hand, the undisturbed case does not have this type of regular variation and is more stochastic in nature.

When the object is submerged as in case III, a slightly different time dependence in the flux occurs. Although this case does allow flow above the object, the depth is such that convection is not very efficient there (as previously mentioned) and no real circulation occurs. Figure 13 indicates that the average level of flux falls below the undisturbed value although not by the amount expected if the flux were totally blocked. This case was not studied for an extended time integration, and so we cannot comment on whether the time integrated value would approach that of case II.

Another way to examine the surface output flux is by calculating its temporal autocorrelation, to determine on what time scale the flux variations occur and whether they are regular. Although we do not include the details, the phase shift in the output flux is very irregular for the undisturbed case indicating the stochastic nature of the convective flow, whereas for cases II and III where the object is present the phase shift is regular and maintains a good correlation. The typical phase shifts are 16.5 min for case II and 17 min for case III. These times, as previously mentioned, are associated with radiative diffusion within the object.

Since all solar magnetic surface features have a finite lifetime, then so should the flux blocking objects that we are modeling. Thus there is another important aspect of the time evolution of the surface flux to be considered, i.e., what happens when the object is withdrawn from the layer? Specifically, we no longer disallow flow inside the region where the object previously was positioned. This removal is after 13.1 hours with the object in place and we then follow the flow for another 3.1 hours. The resulting time history of the complete sequence of the horizontally averaged surface flux is shown in Figure 14, again for cases I and II (note that the integration of case I has not been extended).

The re-adjustment after the removal of the object is very rapid; in fact the amplitude of the variations in the surface flux decreases within one hour and the time scale for its variation returns close to what the undisturbed value was. In addition, even after this short time, the time-integrated surface flux is 99.68% meaning that a certain amount
Fig. 14. Time-dependence of the horizontally-averaged emerging flux at the upper boundary of the domain normalized into units of the solar flux $F_{\odot}$ for cases I and II. The thick solid line is case I (undisturbed) and the thin solid line is case II. The horizontal axis is time in hours.

of the heat being stored is actually released shortly after the object is removed. Naturally, the details of what proportion of the blocked flux reappears and the time scale involved will depend on the size and position of the object.

As the object is removed, the layer will respond by releasing heat below the object, but for a short time the amount which gets out is still below the long term average we would expect (i.e., 100%). That means that, (1) there is a natural heat storage in the layer that needs a ‘relaxation time’ to revert to what it was before the object was present, (2) heat flux has left the domain at the lateral boundaries (sufficient to reduce the total flux) and so it will also require ‘relaxation’ to restore the balance again, or (3) a combination of (1) and (2). It seems likely that (3) is true – however, it is hard to distinguish between the two contributions. The time scales may be different, but it seems unlikely we can separate them. Perhaps the important point is that for some time after the removal there is a general excess at the surface for a period of time that depends on the size and position of the previously disturbed region (approximately 30–40 min for case II).

Since it appears that not all of the heat flux being diverted around a flux blocking object reappears immediately at the surface the next logical question is: where does it go? To answer this question a detailed study of the energy flux components is required over many turnover times. The details of the interactions are quite complex and will not be presented in this paper; however we will summarize the gross behavior.

The two major components to the heat flux in the interior of the domain are the
enthalpy \( (F_e) \) and kinetic energy \( (F_k) \) fluxes as defined in (4) and (5), respectively. Since Figure 10 suggests an increase in \( F_k \) close to the object, volume integrated quantities can be used to evaluate the relative changes in the components of the heat flux (actually area integrated since this is two-dimensional). We have examined the kinetic energy \( (1/2 \rho V^2) \), the enthalpy \( (c_p p = e + p) \) and the \( r \)- and \( \theta \)-components of \( F_e \) and \( F_k \). The figures we present compare some of these quantities for cases I and II.

If we integrate these quantities over the entire domain and compare cases I and II, we notice distinct changes in the components as a function of time. Recall that since the area of the object is small, there is a re-adjustment time, over which the entire layer responds to the object and changes in the heat flow. For our purposes, a better illustration is obtained by arbitrarily separating the domain into two parts, one 'close' to the object and the other 'far'; the remainder of the domain. The 'close' area almost corresponds to the subsets of the domain we show in many of the figures. The ratio of 'close' to 'far' is about 1:10 (1:6 in terms of grid points).

Figures 15 to 22 show the comparison between \( F_e | \theta, F_e | r, F_k | \theta, \) and \( F_{ek} | r \) for the 'close' and 'far' parts (weighted by area) as a function of time in hours after the object is introduced. The solid lines in these figures are the undisturbed models (case I) and the dashed lines are with the object (case II).

There are two distinctive effects, the first is the time delay over which the heat flux is modified in response to the object. This is evident by noting that the two cases always

![Graph](image)

**Fig. 15.** Time-dependence (in hours) of the area-integrated horizontal component of the enthalpy flux (in scaled units) close to the object for cases I (solid line) and II (dashed line). (Same notation in Figures 16–22.)
Fig. 16. Time-dependence of the integrated horizontal component of the enthalpy flux far from the object for cases I and II.

Fig. 17. Time-dependence of the integrated radial component of the enthalpy flux close to the object for cases I and II.
Fig. 18. Time-dependence of the integrated radial component of the enthalpy flux far from the object for cases I and II.

Fig. 19. Time-dependence of the integrated horizontal component of the kinetic energy flux close to the object for cases I and II.
Fig. 20. Time-dependence of the integrated horizontal component of the kinetic energy flux far from the object for cases I and II.

Fig. 21. Time-dependence of the integrated radial component of the kinetic energy flux close to the object for cases I and II.
Fig. 22. Time-dependence of the integrated radial component of the kinetic energy flux far from the object for cases I and II.

diverge for the ‘close’ integration before they do for the ‘far’ integration. The time-delay is slightly different for the enthalpy and the kinetic energy fluxes and also for the vertical and horizontal components.

The next effect is the relative changes of the enthalpy flux and kinetic energy flux components and how these can be interpreted as a global indication of modified heat transport. It is important to recall that the sign of the flux gives the general direction of the flux, positive meaning up or to the right (increasing latitude) and negative meaning down or to the left.

The magnitude of the differences in the components close to the object are typically larger than those far from the object. In each case except Figure 20, the horizontal component for the disturbed region is smaller than for the undisturbed region. This is a little misleading, since an inspection of the contours indicates an enhanced kinetic energy flux in the disturbed region. However, this energy is distributed in both directions, indicating that there is a mix of small scale motions either to the left or right. The integration cancels them. In Figure 20, which shows the horizontal kinetic energy flux in the ‘far’ region, we could interpret larger variations about the undisturbed profile as evidence for transfer of the energy into the enthalpy term or into another component. This could be thought of as the response of a layer to an external influence, in this case diverted energy from the region around the object.

The radial components are especially interesting since they contribute to the emergent
heat flux distribution. Again, each radial flux is reduced in comparison to the undisturbed case but some clarification is required. For case I, Figures 17 and 21 show positive fluxes and Figures 18 and 22 show negative fluxes. This is primarily because the 'close' region is weighted toward the surface regions and the 'far' region to the deeper layers (the downward kinetic energy flux is discussed in Chan and Sofia, 1986). One very important factor is displayed in Figure 21, which shows that the kinetic energy flux close to the object changes sign compared to case I. This is due to the fact that there is a net downward, or near zero, flux close to the object, which is associated with downflows beside and away from the object, as distinct from the broad upflows that occur in the absence of the object. Further from the object both radial and kinetic energy fluxes are different for the two cases but essentially they are responding to different forcings as mentioned previously.

The interpretation of the characteristics of the heat flow is difficult and this global approach only shows (and hides) certain features. One problem is that the arbitrary division of 'close' and 'far' regions will tend to smooth the localized changes in the flux components. This is one area of the analysis that must be developed further.

The time-integrated surface flux (but with horizontal dependence) is shown in Figure 23 for cases I, II, and III which gives an idea of where the diverted flux appears in relation to the object. The presence of the object is clearly seen as a decrease in the flux. The typical reduction factor is 15–20% over the object compared to the undisturbed region, while there is a corresponding increase in flux spread over the surrounding

![Graph](image-url)  

**Fig. 23.** Time-integrated surface flux (but with horizontal dependence) for cases I, II, and III clearly shows the decrease in heat flux over the object and the general increase in the surrounding regions without radically altering the features.
region due to the diverted heat flux. Note that since this object is not of sunspot size we would not expect the much larger decrease in intensity. Instead the present value is consistent with observations of pores (Schröter, 1962; Bray and Loughhead, 1964). A number of transient surface features are also present in all of the cases and are part of the general convective flow and do not seem to influence the redistribution of the heat flow near the object (these features are discussed in Fox and Sofia, 1991).

4. Discussion

In the preceding sections we examined a limited number of calculations of compressible convective flow around a sunspot-like object placed in the upper solar convection zone. The aim of these calculations was to discover where and how the emerging heat flux would be diverted away from the object.

In all the cases studied there was a significant increase in the kinetic energy flux and thus the lateral transport of the energy flux surrounding the object. This mainly occurred below the object to allow the heat to appear at the surface, with some time delay. It also allowed it to be transported horizontally far from the object, and either be stored in the internal fluid circulations or escape from the lateral boundaries. The extent to which these three possibilities may occur depends somewhat on the size and position of the object, but for all the cases we computed the tendency was for the heat to try and re-emerge at the surface.

Most of the time the flows adjacent to the object (in all cases) were directed downwards except when a hotter than average region was diverted from just below the object to the bottom edge. In that case there was a short term upflow to bring the heat to the surface. In addition, most of the time the correlations between flow and thermodynamic quantities that usually characterize convective flow (such as the vertical velocity-temperature fluctuation relation, see Fox, 1989; Chan and Sofia, 1989) were not altered to any noticeable degree indicating that the readjustment to the presence of the object was on a convective rather than diffusive time scale.

Given the importance of the kinetic energy in the redistribution of the heat flux, it is not surprising that previous studies found markedly different behavior. Most of the previous work relied only on a diffusion treatment of the heat flow, or used linearized equations, and thus excluded the process that seems to be important in these calculations.

One encouraging aspect of the present calculations is that the percentage reduction in the specific intensity agrees well with the observed deficits for pores or small sunspot-type regions. Another important change in the nature of the convective flow is the time variation of the horizontally averaged surface flux. This quantity increases its amplitude about a slightly reduced mean value, and the time scale for its variation is quite different compared to an undisturbed region. Some of the variation in the surface flux occurs when heat is conducted into the object and diffused to the surface. Unfortunately there are limitations in our present treatment of radiative effects, and so a detailed discussion
is beyond the scope of this paper. In addition, another limitation is the geometry of the object which we have taken to be a simple rectangle and somewhat shallow.

An important result is that the diverted heat flow was not totally blocked by the object. The main effect was in the time-delay for redistribution. Many of the details of how and where the local flux is diverted depend on the particular depth at which the object is, and how its size compares with the local overturning (or convective) scale, which is usually some multiple of a pressure scale height.

By integrating the average surface flux in time we were able to determine that most of the blocked heat actually re-appears at the surface (after a short period of relaxation when the object is first positioned). In addition, when the object in case II is removed, the layer reacts very quickly and after about one hour the surface flux almost returns to its solar value. Another few hours is required though for the layer to fully relax indicating that the object caused some storage of heat within the layer and some heat flux to leave the domain laterally. Naturally, these time scales depend on the size and position of the blocking.

At this stage it is important to recall that faculae are not included (either explicitly or implicitly) in the current models. They are important components in the energy balance from an observational standpoint. This particular study examines the general emergence of heat flux around an object that resembles a small spot. Faculae may well provide ‘preferred points’ to re-radiate the blocked heat but we elected to concentrate on the subsurface heat diversion, rather than the surface distribution. In the present study, the lack of a facular component may not be as important for small-scale objects as it is for sunspots.

There are some other limitations to these first calculations, apart from the ones we have already mentioned (sub-surface geometry and depth extent of the object, the size and lifetime of the objects so that we do not perturb the whole domain too much and some details of stability of such objects). For a surface object (the ones we directly observe) the depth extent may be an important factor. As the depth of the lower boundary of the object increases, so does the local convective efficiency, indicating that convection should deal even more efficiently in redistributing the heat flux and that the conduction of heat into the object would decrease and its time scale for diffusion would increase. The important factor now in the redistribution is the width to depth ratio, i.e., how long it takes the flow to go around. Also, because the pressure scale height is also increasing, the object may be ‘felt’ at a larger distance, which would alter the way heat is transported. These considerations could be very important for sunspots.

As the size of the object is increased (i.e., for real sunspots), and thus the domain size, the importance of larger scale convective flows may also become important (mesogranulation and supergranulation). Unfortunately detailed numerical models are still being developed for these scales and it should be some time before we understand the details of their contribution. It is likely however, that the granulation scale still provides an adequate description of energy transport, especially in the surface layers where the diverted heat flux seems to be appearing. Some obvious improvements to the present formulation may include a better treatment of the surface boundary conditions.
and better radiative transfer within the object. It is important to remember, however, that the size limitation of the object compared to the domain size is a major factor in modelling real solar features with any degree of realism.

Because the sunspot-like object is really magnetic in nature and the convective flow closely surrounds it, it is likely that there will be some conversion between internal, kinetic and magnetic energy (Fox, Theobald, and Sofia, 1991). This process has important consequences for atmospheric heating and related phenomena and it may also alter the specific intensity of the object (smooth it out perhaps). In particular in localized regions where the magnetic field and current are large, the Poynting flux can be an important component of the total flux. The current assumption of the passive existence of the object is only somewhat valid based on surface observations; however the lack of any knowledge on subsurface fields can be seen as an obvious drawback.

Work is in progress, which includes the effects of magnetic fields explicitly (Nordlund and Stein, 1989; Fox, Theobald, and Sofia, 1991). Unfortunately, our understanding of the statistical properties of highly time dependent magnetoconvection is far below that of the non-magnetic case. Thus, we also intend to pursue the approach we have adopted in this paper, as we expect to learn a great deal about the heat flow distribution from refined analysis techniques and in much larger convective regions, perhaps in three dimensions and at different depths in the convection zone and will report the findings of any significant differences to the present work.

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