ON THE RECONCILIATION OF SIMULTANEOUS MICROWAVE IMAGING AND HARD X-RAY OBSERVATIONS OF A SOLAR FLARE

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Abstract. We have compared microwave imaging data for a small flare with simultaneous hard X-ray spectral observations. The X-ray data suggest that the power-law index $\delta$ of the energy distribution of the radiating electrons is 5.3 (thick-target) which differs significantly from the estimate ($\delta = 1.4$) from a homogeneous optically-thin gyrosynchrotron model which fits the radio observations well. In order to reconcile these results, we explore a number of options. We investigate a double power-law energy spectrum for the energetic electrons in the flare, as assumed by other authors: the power law is steep at low energies and much flatter at the higher energies which produce the bulk of the microwaves. The deduced break energy is about 230 keV if we tentatively ignore the X-ray emission from the radio-emitting electrons: however, the emission of soft photons by the flat tail strongly contributes to the observed hard X-ray range and would flatten the spectrum there. A thin-target model for the X-ray emission is also inconsistent with radio data. An inhomogeneous gyrosynchrotron model with a number of free parameters and containing an electron distribution given by the thick-target X-ray model could be made to fit the radio data.

1. Introduction

Since the early recognition of the similarity of the hard X-ray and microwave time profiles of solar flares (Kundu, 1961), much effort has gone into combining information from the two domains to derive the physical parameters of the source. Initially this was done under the assumption that the X-ray and radio sources were identical, but with increasing spatial resolution in both domains it was realized that this is not the case, and that more sophisticated treatments are therefore needed. Moreover, it has been found that the sources of hard and soft X-rays tend to be different, as are the sources of optically-thick and optically-thin radio emission. Multi-wavelength observations in each domain are then required to be confident of the interpretation of an event.

A related difficulty has been the reconciliation of the numbers of energetic electrons needed to explain the hard X-ray and radio observations, respectively. It was long held that the number of X-ray emitting electrons was several orders of magnitude larger than the number of radio-emitting electrons, but recently this problem has been ameliorated by the use of a more appropriate model for the X-ray source (thick target rather than thin target; e.g., see Gary, 1985; Kai, 1986; Nitta and Kosugi, 1986; Lu and Petrosian, 1988).

The subject we address in this paper is the relationship between the X-ray and optically-thin radio spectral indices. Non-imaging studies have shown that the radio spectral indices imply flatter electron energy spectra than are deduced from the corresponding X-ray observations with the thick-target model. However, there have generally been ways to avoid this conflict, largely because of insufficient data. In particular, the radio emission is complicated by the fact that it depends not only on the energy spectrum of the electrons but also on the strength and geometry of the magnetic field. In one event Marsh et al. (1981) concluded from Very Large Array (VLA) observations at 15 GHz and a hard X-ray spectrum from the Hard X-Ray Burst Spectrometer (HXRBS) on the Solar Maximum Mission Satellite that the radio and X-rays could not come from a single-power-law electron distribution. They deduced that the 15 GHz radio emission was optically thin from the spectrum of the event. Schmahl, Kundu, and Dennis (1986) reached a similar conclusion for another event. However, Gary (1985) has argued that the use of radio spectral indices can be misleading because of the possible contamination of high-frequency radio emission by optically-thick emission from part of the source, which effectively flattens the spectrum.

In this paper we present observations of another flare in which the radio and X-ray data predict different spectral indices for the electron energy spectrum. However, in this case there is adequate information to reject the hypothesis that the radio spectrum is contaminated by optically-thick emission and therefore inappropriate. The radio observations consist of simultaneous imaging data at two frequencies, which show a co-spatial, highly polarized source. We explore other interpretations of the radio data, and conclude that the radio data can only be reconciled with the X-ray data by using an ad hoc inhomogeneous model for the emitting region containing a number of free parameters.

2. Summary of Observations

A small flare occurred at 15:07 UT on 1980 October 1 and was observed with the VLA in the C array and the HXRBS instrument on board SMM. Details of the observations at the VLA are described in Kundu, Velusamy, and White (1987; hereafter referred to as KVW). In Figure 1 we present the HXRBS time profile in the 29–503 keV range (upper panel). The time of the burst peak is 15:07:16 UT. The total HXRBS counting rate is only around 150 s^{-1}. The hard X-ray spectrum obtained with HXRBS during the peak period (15:07:11–15:07:19 UT) is shown in Figure 2. This is a fairly common hard X-ray spectrum and the fits to it give a power-law index γ of 3.8. Note that the occasional high points above ~200 keV are not highly significant, since the error bars are at the 1σ level.

The lower panel of Figure 1 shows the time variation of the brightness temperature $T_b$ and degree of circular polarization $r_c$ of the radio source at 5 and 15 GHz. This plot is reproduced from Figure 6 of KVW with a shift introduced to convert from International Astronomical Time (IAT) to Universal Time (UT). The burst peak is at ~15:07:20 UT (to within 10 s). We have inspected total flux records from both
Fig. 1. *Upper panel:* HXRBS time profile of the 1980 October 1 flare. The energy range is 29–503 keV. *Lower panel:* Time profile of the VLA peak brightness temperature and degree of polarization at both 5 and 15 GHz during the burst (from KVV).

Sagamore Hill and Bern, which were taken in the form of chart records. They are accordingly incapable of determining the peak time with an accuracy that is much better than the VLA integration time, i.e., 10 s, but are consistent with the VLA profile (E. Cliver and A. Magun, private communications). We therefore conclude that the main peak in microwaves coincides exactly with that in hard X-rays.

The VLA total flux at each of the two wavelengths integrated over the source area is consistent with fluxes obtained with the patrol telescopes at Sagamore Hill and Bern, which, however, have \( \sim 20\% \) uncertainty (E. Cliver and A. Magun, private communi-
cations). The VLA maps show a simple compact source at both 5 and 15 GHz, apparently of identical position at the two wavelengths. The source is sufficiently small and simple that the VLA data should not be affected by any 'missing flux' problems. The polarization is high at both wavelengths, which together with the flat spectrum led KVV to conclude that the radio source was optically thin. This assumption allows a complete determination of parameters assuming a homogeneous source and using the non-thermal gyrosynchrotron formulae of Dulk (1985), which leads to the energy spectral index of 1.4.

On the other hand, a hard X-ray spectrum with a power-law index γ is generated from a population of electrons with δ = γ + 1.5 or δ = γ − 0.5, depending upon whether we assume thick-target or thin-target X-ray production, respectively (Brown, 1971). Note that for the thick-target case γ + 1.0 is the slope of the flux of electrons interacting with the target. In order to get the spectrum we need to multiply the flux by a factor proportional to (velocity)^−1, which is asymptotically proportional to E^−0.5 as E becomes small. For the present flare the deduced value of δ is 5.3 for the thick-target case and 3.3 for the thin-target case. Both of these values differ greatly from the value of δ obtained from the microwave observations. In Section 3 we use accurate non-thermal gyrosynchrotron calculations to check the radio source parameters derived by KVV, and specifically δ. In the following section we will discuss ways to reconcile the spectral index of electrons from the microwave images with that from the hard X-ray data.
3. Homogeneous Radio Source Model

As KVV noted, the resulting magnetic field strength of 390 G implies that the 5 GHz emission is at the 5th harmonic of the gyrofrequency, and Dulk's (1985) formulae are not intended to be accurate at low harmonics. We have, therefore, used accurate numerical calculations (described in Schmahl, Kundu, and Dennis, 1990) of the gyro-synchrotron opacity in order to check the best parameters for a homogeneous source, and to confirm that the electron energy spectrum is rather flat.

We find that in general the parameters derived by KVV are appropriate, except for the angle \( \theta \) between the line of sight and the source: we derive a value closer to 30°, rather than the 60° found by KVV. Otherwise, a field strength in the range 320–350 G and an energy spectral index 1.1–1.4 seems appropriate.

However, we were unable to exactly reproduce the observed parameters in our calculations. The principal difficulty is in simultaneously achieving 60% polarization at 5 GHz and 35% at 15 GHz while the fluxes at the two frequencies are identical. In fact, with the above range of parameters, the best achievable result for the polarization was 58% at 5 GHz and 37% at 15 GHz. But then the 15 GHz flux was invariably 30–50% smaller than the 5 GHz flux. If parameters were chosen to make the fluxes at the two frequencies equal (larger field or larger density), then the polarizations at the two frequencies were too close together. We also note that a very large upper cutoff to the electron energy distribution (i.e., over 10 MeV) was necessary in order to get close to the observed polarizations.

4. Reconciliation of Radio and X-Ray Data

One solution to the discrepancy between the radio and X-ray spectral indices is to assume that the radio-emitting electrons and the X-ray-emitting electrons are completely independent: however, this contradicts many other lines of evidence. Instead, here we explore other possible solutions: re-interpretation of the radio data, and the possibility of a double power-law energy spectrum.

4.1. Line-Dipole Inhomogeneous Models

One can ask whether an inhomogeneous gyrosynchrotron model will provide a better solution than the homogeneous model discussed above. This is unlikely to be the case here, because it is very difficult to achieve higher degrees of polarization in an inhomogeneous model than in a homogeneous one. We have analyzed filled east–west loop models using the code developed by Schmahl, Kundu, and Dennis (1990). In these models the magnetic field is that of a line dipole, so that they represent arcades of loops along a neutral line rather than isolated loops and the magnetic field falls off quadratically with distance, rather than cubically as in a dipole field. The dimensions of the model arcade were chosen to coincide with the source observed by the VLA. A range of footpoint magnetic field strengths, energetic electron number densities and spectral indices, and loop thicknesses were considered. In effect this model contains one more
free parameter than the homogeneous model (namely six: $\delta$, $N$, $B$, $\theta$, $L$, and the choice of the magnetic field model, which may be parametrized by the dependence of $B$ on height).

For this particular type of loop model we were unable to achieve results better than in the homogeneous case. At disk centre, a symmetric loop always gives zero net polarization by symmetry arguments. As the apparent longitude of the loop increases, the degree of polarization increases, but due to the large degree of cancellation which still occurs we found that it is difficult to achieve net polarizations much greater than 10–20%. An asymmetric loop can give rise to greater net polarization, but seems unlikely to give greater polarization than a homogeneous source.

4.2. **Optically thick emission at 5 GHz**

Here we will investigate the possibility that the emission is at least partly optically thick at 5 GHz. (To some extent we have also covered this situation with the discussion on inhomogeneous models.) If so, the deduced energy spectral index will be much steeper than one derives if the 5 GHz emission is optically thin. However, we must reconcile optically thick emission with a degree of polarization of 60%.

A high polarization can be attained if the emission is optically thick for the prevailing extraordinary mode but optically thin for the ordinary mode. A familiar expression relating the effective temperature $T_{\text{eff}}$ and brightness temperature $T_b$,

$$T_b = T_{\text{eff}}(1 - e^{-\tau})$$

holds for both the ordinary and extraordinary modes. The ratio of the optical depths for the ordinary and extraordinary modes is

$$\frac{\tau_o}{\tau_{ex}} = \frac{\kappa_o}{\kappa_{ex}}$$

(2)

where $\kappa_o$ and $\kappa_{ex}$ are absorption coefficients due to self-absorption for the ordinary and extraordinary modes, respectively.

In Figure 3, we plot $T_{\text{eff}}$ and $\kappa_o/\kappa_{ex}$ against harmonic number $\nu/\nu_B$ for different values of the angle $\theta$ between the magnetic field and line of sight. We have used the formulae for gyrosynchrotron emission given by Takakura (1972), inserting the spectrum of electrons which produces the actual hard X-ray spectrum with $\gamma = 3.8$ through the thick-target process. We note that values of $T_{\text{eff}}$ at a given $\nu/\nu_B$ are not much different for the ordinary and extraordinary modes, and that for $\nu/\nu_B < 10$, $\kappa_o/\kappa_{ex}$ lies in the range $10^{-3}$–0.3. The latter argument is little changed if we introduce a much harder electron spectrum that leads to $\gamma = 1.8$ (Figure 3(d)). According to Equation (2) and Figure 3, a situation can easily take place in which $\tau_{ex} > 1$ and $\tau_o < 1$. In that case, since $T_{b,ex} \approx T_{\text{eff},ex}$, $T_{b,o} \approx \tau_o T_{\text{eff},o}$ and $T_{\text{eff},ex} \approx T_{\text{eff},o}$, the degree of polarization becomes

$$r_e = \frac{T_{b,ex} - T_{b,o}}{T_{b,ex} + T_{b,o}} \approx \frac{1 - \tau_o}{1 + \tau_o}.$$
Fig. 3. Plot of effective temperatures due to gyrosynchrotron emission for the ordinary and extraordinary modes against the harmonic number $\nu/\nu_B$ (scale shown on the left axis). Also, the ratio of absorption coefficients for both modes is plotted (scale shown on the right axis). The angle $\theta$ between the magnetic field and line of sight, and the power-law index of X-rays emitted by the electrons through the thick-target process is displayed at the top of each map.
This can be close to unity. Thus, a high degree of polarization does not necessarily mean optically thin emission.

There is, however, one severe difficulty with this interpretation of this event. We must assume that the impulsive peak of the flare decays in time due to a decrease in the number of emitting electrons in the loop (the radio observations show no change in source dimension). If at the flare peak the \( x \)-mode is optically thick and the \( o \)-mode is optically thin, then in the initial stage of decay, the \( x \)-mode emission will not decrease much since it will remain optically thick, but the \( o \)-mode emission is proportional to the number of electrons present and should decrease accordingly. It follows that the degree of polarization at 5 GHz should increase during the decay of the flare, until the \( x \)-mode becomes optically thin. This is not observed (Figure 1): rather, the degree of polarization remains quite constant at both 5 and 15 GHz, as one would expect of optically thin emission.

It is also possible that absorption of the flare radiation by cooler overlying material alters the polarization and the spectrum of the radio emission. Such absorption would need to be only marginally optically thick in order for us to still see the burst, and would induce a polarization in the sense of the \( o \)-mode. The main argument against such absorption is that the X-ray data already do not provide enough electrons to produce the observed radio emission, and any absorption present will exacerbate this problem. Note that a population of electrons with \( \delta = 5.3 \) would lead to the ratio \( T_{b, 5 \text{ GHz}}/T_{b, 15 \text{ GHz}} = 455 \) if both wavelengths are optically thin, i.e., a factor of 50 greater than observed, according to the formula given by Dulk (1985).

### 4.3. Two-power-law spectrum

The X-ray spectrum (Figure 2) suggests a flattening at energies above 200 keV, although as noted the statistics at high energies are poor. Individual cases of spectra where such flattening is clearly present have been presented by Suri et al. (1975), Yoshimori, Watanabe, and Nitta (1985a, b), and Dennis (1988). This leads us to consider the possibility of a flattening of the energy spectrum, such that below some break energy the spectral index is \( -5.3 \), as suggested by the X-ray data, and above the break energy the spectral index is about \( -1.5 \) as suggested by the radio data. It is straightforward to show that, provided the magnetic field is not excessively strong, the electrons in the range 10–200 keV do not contribute significantly to the microwave flux above 5 GHz. Most of the flux will come from the higher energy electrons. This is the conclusion reached by Marsh et al. (1981) in their study of a much smaller flare based on images at a single frequency.

We can estimate the break energy by equating the deduced hard-X-ray spectrum and the deduced microwave spectrum. We first consider the case of thick-target emission, which is generally regarded as the more likely case (e.g., Wu et al., 1989). The peak flux spectrum of the energetic electrons producing the hard X-rays may be deduced from the observed photon spectrum at the flare peak (Brown, 1971; Hudson, Canfield, and Kane, 1978). Tentatively neglecting the flattening at higher energies, we find:

\[
F(E) = 5.48 \times 10^{39} E_{\text{keV}}^{-4.8} \text{ s}^{-1} \text{ keV}^{-1}.
\]
To convert this to a number density, we use the simple formula $F(E) = A v n_e$ where $v$ is the parallel velocity corresponding to the energy $E$; the cross-section area $A$ of the loop may be estimated from the dimension of the radio source ($2 \times 10^8$ cm). With these parameters we find

$$\frac{d^2N(E)}{dE \, dV} = 1.2 \times 10^{14} E_{\text{keV}}^{-5.3} \text{ cm}^{-3} \text{ keV}^{-1}. \quad (5)$$

The energy density spectrum found from the radio data is (KVW)

$$\frac{d^2N(E)}{dE \, dV} = 7 \times 10^4 E_{\text{keV}}^{-1.4} \text{ cm}^{-3} \text{ keV}^{-1}. \quad (6)$$

Equating the X-ray and radio number densities gives a crossover energy of 230 keV. This energy is actually fairly insensitive to the parameters: thus if the radio spectral index is actually 2, then the normalization factor in (6) is larger and this offsets the steeper spectrum when calculating the transition energy. Note that the X-ray emission actually arises low in the loop where the loop cross-section may be small compared with its dimension at the site of the radio source: by calculating the X-ray number density using a dimension for the source area which is derived from radio observations, we have in a loose sense ‘transported’ the number density up the loop to the location of the radio source where the comparison should be carried out. We have, however, ignored the fact that the thick-target electrons may be only a small part of the energy distribution of the radio-emitting electrons higher in the loop (that part with small pitch angles).

With a break energy so low, and with the high-energy spectrum so flat, we have to take into account the contribution of the flat-spectrum tail to the observed hard-X-ray range of energies. Kosugi, Dennis, and Kai (1988) discuss the relative contributions of electrons in different energy ranges to the observed hard X-rays for a thick target model. They find that for a steep energy spectrum most of the X-ray flux at a given photon energy $\varepsilon$ is produced by electrons with energies less than $3\varepsilon$, whereas for a flat energy spectrum the contribution of electrons with energies much greater than the photon energy can be significant. Thus, they find that for an energy index of $\delta = 4.5$, 30% of the photons at 200 keV are contributed by electrons with energies above 600 keV. We have calculated the relevant function for the $\delta = 1.4$ spectrum energy distribution. In this case also an upper limit to the particle energies must be imposed in order to avoid a singularity. We find the bulk of the emission at any given energy comes from electrons with energies much larger than the photon energy (we used the non-relativistic bremsstrahlung formula, which will not strictly be accurate for energetic electrons in the tail of the distribution, but the conclusion is not likely to be affected). Thus the break deduced above, 230 keV, is apparently inconsistent with the observed spectrum below 200 keV.

Due to this discrepancy we also consider the possibility that the X-ray emission is produced by the thin-target process. In this case we deduce an electron energy spectrum
of (Hudson, Canfield, and Kane, 1978)

\[
\frac{dN(E)}{dE} = 2.16 \times 10^{47} E_{\text{keV}}^{-3.3} n_i^{-1} \text{keV}^{-1},
\]

where \(n_i\) is the target ion density in units of \(\text{cm}^{-3}\). Normalizing the density to a typical coronal value of \(10^9 \text{ cm}^{-3}\) and assuming a target volume which is a cylinder with diameter and length both \(2 \times 10^8 \text{ cm}\), we deduce an electron energy density spectrum of

\[
\frac{d^2N(E)}{dE \, dV} = 3.44 \times 10^{13} \frac{E_{\text{keV}}^{-3.3}}{n_i \, 9} \text{ cm}^{-3} \text{keV}^{-1}.
\]

This spectrum contains many more electrons than the thick-target spectrum (5). When we now equate (8) with (6) to determine the break energy in the spectrum, we find a much larger energy of \(38 \text{ MeV}\). This break energy too is unacceptable, because now the radio emission should be dominated by the \(7.5 \times 10^{10}\) electrons in the X-ray spectrum above \(10 \text{ keV}\). They are more than adequate to make gyrosynchrotron emission optically thick at both 5 and 15 GHz, and would produce a greater brightness temperature at 15 GHz (\(4 \times 10^8 \text{ K}\)) than is observed. Thus we must also rule out thin-target emission.

4.4. Multi-parameter solutions

In the previous section we considered a homogeneous model for the radio emission and an arcade model containing one extra free parameter. Finally in this section we consider the possibility of a model with even more free parameters. Specifically, we will assume an electron energy spectrum as deduced from the thick-target model for the X-ray emission, suppose the 5 and 15 GHz emission to come from different layers, and allow the magnetic field and the angle \(\theta\) between the magnetic field and the line of sight to be variables in both the 5 and 15 GHz-emitting layers. We then have four observational constraints (two brightness temperatures and two degrees of polarization), but eight free parameters (the two field strengths, the two angles and two line-of-sight depths through the layers in addition to \(\delta\) and \(N\)). We will, therefore, try to determine \(B\) and \(\theta\) at each frequency for various plausible values of the line-of-sight depth \(L\). We find that at 5 GHz, \(L\) must exceed \(2 \times 10^8 \text{ cm}\) to obtain a solution (for short path lengths the values of \(B\) and \(\theta\) must be large to achieve the required brightness temperature, but then the degree of polarization cannot be achieved). At \(L = 3 \times 10^8 \text{ cm}\) the required value of \(\theta\) is \(52^\circ\) and \(B = 270 \text{ G}\). At 15 GHz, we find that \(\theta\) remains constant at about \(70^\circ\) as we increase \(L\) from \(10^8 \text{ cm}\) to \(3 \times 10^8 \text{ cm}\), and the magnetic field drops from 740 to 630 G. Thus we find that the 15 GHz emission arises from a higher-field layer as expected, but that the field is nearly perpendicular to the line of sight in the higher-frequency layer and the angle \(\theta\) decreases as we go from the higher to the lower frequency (i.e., upwards). The parameters deduced in the 15 GHz layer would make the 5 GHz frequency optically thick there with a brightness temperature up to twice that observed, depending on \(L\): the optically-thick temperature at 5 GHz in the 15 GHz-emitting layer would be.
2 \times 10^8 \text{ K} for L = 10^8 \text{ cm} but 3 \times 10^7 \text{ K} for L = 3 \times 10^8 \text{ cm}. Therefore, a larger value of L is favoured. The brightness temperature contributed at 15 GHz by the 5 GHz-emitting layer is less than 10^5 \text{ K}.

On the basis of these parameters it is, therefore, possible in principle to construct the following model in which electrons with the energy spectrum deduced from the X-ray observations (\(\delta = 5.3\)) can produce the observed radio emission. The effective thickness of the radio source is 3–6 \times 10^8 \text{ cm}, and both B and \(\theta\) increase with line-of-sight depth from about 250 to 700 G and from about 50° to 70°, respectively. A better fit could be achieved if \(N\) and \(\delta\) were also allowed to vary with depth. Note that a value of \(\delta > 3\) above 200 keV would not conflict with the X-ray spectrum. However, detailed parameter-fitting to such a model is beyond the scope of this paper.

5. Conclusion

We have attempted to reconcile the radio and hard X-ray observations of a simple impulsive flare. Two lines of evidence suggest that the radio emission arises from a very hard spectrum of electrons: the flat spectrum seen by microwave patrol telescopes; and the VLA imaging observations showing highly polarized radiation. However, the X-ray spectrum suggests a much steeper energy spectrum for the radiating electrons. We have tried to find a model which encompasses both sets of measurements. Thick target X-ray emission and gyrosynchrotron models with a small number of free parameters can be ruled out, because the flat high-energy part of the electron spectrum would still radiate the bulk of the X-rays and produce a different X-ray spectrum. Optically-thick radio emission is inconsistent with the polarization behaviour seen in the radio observations, and thin-target X-ray emission would require too many electrons to be present. The only model we can find which comes close to fitting the data is an ad hoc gyrosynchrotron model with eight free parameters, which requires that there be two layers in the source which contain different magnetic fields.

Previous comparisons of imaging radio data and X-ray data have often reached the conclusion that the radio and X-ray observations see different populations of non-thermal electrons, e.g., Marsh et al. (1981) and Schmahl, Kundu, and Dennis (1986). They have not received much attention because of doubts about the true value of the radio spectral index in the optically-thin regime (e.g., Garry, 1985). Such spectra, flattening at high energies, have also been noted by X-ray observers from observations of the spectra of some large individual flares. Vestrand (1988) has compared SMM–GRS spectra in the range 25–200 keV with those in the range 300–1000 keV, and found that on average there is a flattening of about 0.5 in spectral index, which can however be explained without a change in the electron energy distribution. However, he also found flattening of up to 2 in many spectra which cannot be so explained. The reason that this explanation does not work in this event is that the deduced radio spectrum is much flatter than in previous studies. Such flat spectra in the impulsive phase are not common, but we hope that further studies of such events may provide a more satisfying explanation than we have found here. Clearly the difficulty we have
in explaining this apparently simple event implies that comparisons of non-imaging X-ray and radio data in the context of homogeneous or simple inhomogeneous models are unlikely to give correct results for the energetics and magnetic fields of flares.

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