Hydrodynamics of the Solar Photosphere: Model Calculations and Spectroscopic Observations

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1. Introduction

A close-up view of the solar photosphere under good conditions of atmospheric seeing shows a characteristic cellular pattern covering the entire surface (except for sunspots), the so called *solar granulation*. Bright isolated elements, the granules, are surrounded by a network of interconnected dark lanes. In contrast to W. Herschel, who first reported the existence of some small-scale inhomogeneities on the solar surface in 1801, we now have a basic physical understanding of the origin of the solar granulation. The essential foundation was laid by Unsöld [1], who pointed out that because of the sharp increase with depth of opacity and ionization of hydrogen, the Schwarzschild criterion for convective instability should be satisfied in the deep photospheric layers. According to current mixing length models of the solar atmosphere (Kurucz [2]), the dividing level between stable (radiative) and unstable (convective) layers is located somewhere between $\tau_{5000} = 0.8$ and $\tau_{5000} = 1$. Consequently, granulation is identified with convection cells in the uppermost layers of the hydrogen convection zone, forming the observed continually changing cellular pattern at the interface between optically thick and optically thin conditions. *Granules* represent hot rising gas elements while the *intergranular lanes* consist of cooler sinking gas. This picture has been verified spectroscopically, confirming the convective origin of the solar granulation.

Although granulation is basically understood as a convective phenomenon, our quantitative knowledge about its dynamical and thermal structure is still incomplete. From current observational material it seems impossible to derive a detailed quantitative picture of granular convection by a purely empirical analysis. To make progress empirically, extensive spectroscopic observations with very high spatial resolution ($\approx 0.1''$) would be required. Lacking such observations, we have to rely on theoretical models. While local mixing length theory may be adequate to model the mean stratification of
deep convective layers, it certainly cannot provide a reasonable description of the details of photospheric convection. Apart from the problem that the mixing length is a free parameter which has to be calibrated somehow, such models give only an average stratification, while we are interested in the effect of horizontal inhomogeneities. Moreover, they assume an abrupt transition between stable and unstable conditions, while actually convective motions must be expected to extend into the stable atmospheric layers to some degree. Although several non-local mixing length theories have been developed (e.g. Shaviv and Salpeter [3], Roxburgh [4]), there is no commonly accepted formalism to model this so called convective overshoot in a simple way.

The only possibility to take into account the complex physics of stellar photospheric convection seems to be a direct numerical simulation of the photospheric gas flows. This approach requires the use of powerful computers and generates large amounts of data.

In the following we shall describe such detailed hydrodynamical calculations for the solar photosphere. Based on the resulting physically self-consistent, parameter-free models we calculate synthetic spectra in order to be able to make a direct comparison with various spectroscopic observations. We conclude that our time-dependent radiation-hydrodynamics models seem to be a reasonable representation of the real solar photosphere, which, on the other hand, is obviously quite different from the picture suggested by classical mixing length models.

2. Modelling the Solar Photosphere

2.1 Physics of Photospheric Convection

Before setting up numerical simulations of convective flows in stellar photospheres, it is important to realize the complexity of the physics involved. In the following we emphasize some important aspects.

The basic processes of photospheric convection are governed by time-dependent, non-linear hydrodynamics in 3 dimensions. Gravity causes the gas to be stratified, implying large variations of gas pressure and density with depth. Apart from the stratification, compressibility of the medium will affect the characteristics of the convective flow as well as lead to additional features like pressure waves.

The stellar gas is a plasma, consisting of different elements (essentially hydrogen and helium) which begin to ionize at different temperatures. Ionization has important consequences for the energy balance in the plasma and increases the degree of convective instability.
While in the deep, optically thick layers radiation can be described as a diffusive process, this is no longer possible in the optically thin photospheric regions. Depending on the magnitude of the frequency-dependent opacity, the radiation field eventually is controlled by non-local transfer beyond different levels in the photosphere. Radiative cooling rates can be substantial, causing a strong coupling between hydrodynamics and radiative transfer.

The very low viscosity of the stellar gas gives rise to another complexity. In view of the correspondingly high Reynolds number \((Re \approx 10^9)\) we must expect the flow to be fully turbulent. Among other things this implies the existence of an enormous range of spatial scales, between some 1000 km for the largest granular dimensions and a few cm, where small-scale turbulent energy is eventually dissipated.

Finally, in convective stellar atmospheres magnetic fields may play an important role. At least where locally concentrated, they may even dominate the gas dynamics.

From this brief overview it is clear that it is impossible to model the complete physics of the problem in a single numerical simulation. Rather, it is necessary to introduce simplifications and approximations. The hope is, however, that the simplified problem can still account for the essential features of photospheric convection.

### 2.2 Numerical Simulations

For our numerical simulations of photospheric convection we have adopted the following major simplifications of the problem. First, we ignore magnetic fields altogether. Second, we restrict the flow to 2 dimensions, assuming axial symmetry in cylindrical coordinates (\(r\): parallel to the stellar surface, \(z\): upwards, antiparallel to the direction of gravity). Finally, every numerical simulation of turbulent flows has to treat the effect of turbulence on subgrid scales in some schematic way. For this purpose we use the concept of *turbulent viscosity* according to Smagorinsky [5] (see also Deardorff [6], [7]). Application of this scheme results in a so called "large eddy simulation approach", where one resolves only the largest (granular) scales, without trying to model the various smaller scales in detail. The idea is that the large-scale behaviour of a turbulent system becomes insensitive to the details of the physics at the smaller scales if \(Re\) is sufficiently large (see Chan and Sofia [8] for a discussion of this concept in the context of numerical simulations of turbulent stellar convection).

Within the framework of these simplifications we try to include as much realistic background physics as possible. Details about the input physics used for our numerical simulations have been described elsewhere (Steffen et al. [9] and Steffen [10]). In the following we briefly reiterate the main points.
To account for the basic physics of compressible convection in a stratified, turbulent atmosphere, we solve the Navier Stokes equations, prescribing the conservation of mass and momentum, together with some energy equation appropriate for the conditions in stellar atmospheres. Turbulence, represented by subgrid scale eddy viscosity, has several effects. *Turbulent exchange of momentum* gives rise to an additional force in the momentum equation and leads to *turbulent dissipation* of large-scale kinetic energy into small-scale (turbulent) kinetic energy and eventually into heat, a process which is taken into account in our energy equation. A second, more important effect on the energy equation is due to the *turbulent exchange of heat*. The corresponding heat flux is modelled as being proportional to the local entropy gradient.

The most important term in the energy balance of the photosphere is the *exchange of radiation*. Hence, a realistic description of radiative transfer effects is essential. Avoiding approximations like the diffusion or Eddington approximation, we use a system of several thousand rays covering the entire model volume under different angles of inclination. Tracing each ray from the top to the bottom of the model, assuming the lateral boundary of the cylindrical computational domain to be reflective, we solve the equation of radiative transfer using a modified Feautrier technique. Assuming LTE, there is no coupling between the rays and the Feautrier scheme can be vectorized efficiently. Although we have recently modified the code to take into account the frequency-dependence of the radiation field in a simplified way [10], the results reported here refer to models calculated in the grey approximation, using a realistic Rosseland mean opacity $\kappa_{Ross}(P,T)$.

The gas is taken to be a mixture of hydrogen and helium with variable degree of ionization, and a representative neutral metal, with relative abundances according to solar composition. Excitation and ionization of H, He, He$^+$ and the formation of H$_2$ molecules are calculated in LTE (Boltzmann and Saha formulae). With this realistic equation of state there are no longer simple relations for e.g. density and entropy as a function of pressure and temperature, and the mean molecular weight and specific heats cannot be regarded as constant ($C_p$ and $C_v$ vary by a factor of 20 or more in the solar photosphere!).

Our models are designed to simulate *photospheric convection* and cannot comprise the entire solar convection zone. Including only the uppermost superadiabatic layers in the computational domain makes the problem of specifying a reasonable lower boundary condition difficult since simple assumptions like a rigid wall are not realistic in this situation. To restrict the flow as little as possible, we have adopted an open lower boundary, allowing a free flow of gas out of and into the model. The gas pressure at the bottom is continuously adjusted in such a way as to conserve the total mass within the model volume. The temperature of the inflowing gas is regulated such that the time averaged radiative surface flux conforms to the specified ef-
fective temperature. The flow velocity at the bottom can then be computed from the hydrodynamical relations, giving the flow the freedom to chose the regions of up and downflows wherever it likes.

The upper boundary is treated as a closed top by requiring the vertical velocity to vanish, while the horizontal velocity has a zero $z$-derivative at the top. At the axis of symmetry as well as at the side walls the horizontal velocity and the $r$-derivatives of all other variables are taken to be zero.

3. Results of the Numerical Simulations

3.1 Steady State Models

Depending on the adopted size of the computational domain, the numerical simulations lead to steady state and instationary solutions, respectively. The flow develops towards a steady state whenever the model diameter is smaller than a critical upper limit, roughly 1500 km. An example of a steady state solution in a cell measuring 1050 km in diameter is displayed in Fig. 1.

The properties of the cylindrical steady state flows have been described in detail by Steffen et al. [9]. Here we just summarize the main characteristics.

Fast narrow converging downflows occur at the axis of symmetry with downflow velocities near the speed of sound ($\approx 8$ km/s), and at the lateral boundary with somewhat lower velocities.

The upflows are slower, not faster than 2 to 3 km/s. In the ascending regions, which are distinctly broader than the downflows, the flow diverges on its way up.

Convection generates large horizontal temperature differences between up and downflows in order to carry the solar flux. A maximum temperature difference of 4800 K (at constant geometrical height) is found about 70 km below the visible surface where the average temperature is approximately 9200 K. Another conspicuous feature is the occurrence of very steep temperature gradients at the boundary between convectively unstable and stable regions. The main reason is efficient radiative cooling once the ascending hot gas becomes optically thin.

The convective "overshoot layers" extend about 300 km above $r = 1$, but the velocity and temperature fluctuations induced in these layers are not very large. Note that the temperature fluctuations change sign in the overshoot layers, i.e. it is relatively hotter above the downflows (intergranular lanes) and cooler above the upflows (granules), a consequence of adiabatic cooling of expanding gas rising in convectively stable layers.
Fig. 1. Steady state flow in a granular convection cell measuring 1050 km in diameter. This simulation uses 40 vertical grid points (spacing $\Delta z = 15$ km in the lower and $\Delta z = 35$ km in the upper part) and 36 equidistant horizontal grid points ($\Delta r = 15$ km). $z = 0$ corresponds to $\tau_{\text{abs}} \approx 1$. Temperature contours are shown in steps of 200 K, while arrows indicate the velocity field. Maximum temperature is 11960 K, maximum flow velocity is 8.6 km/s.

In the highest layers of the model, the stratification is essentially static, plane-parallel and in radiative equilibrium. Here temperature is almost constant with height and the boundary temperature is that of a grey radiative equilibrium atmosphere, $T_{\text{bound}} \approx 4700$ K.

3.2 Instationary Models

If the model diameter is increased beyond a critical limit ($D \gtrsim 1500$ km) the resulting solutions become highly instationary. While many of the basic characteristics of granular convection found in the steady state models are retained, the important new feature is that now the topology of the convective flow pattern changes with time. These changes may be interpreted as a continuous splitting and merging of “granules” on time scales of the order of 10 to 30 minutes, comparable to observed granular lifetimes. Although the
Fig. 2. Snapshot from a numerical simulation of instationary photospheric convection in a cell measuring 2625 km in diameter. This simulation uses 40 vertical grid points (spacing $\Delta z = 15$ km in the lower and $\Delta z = 35$ km in the upper part) and 36 equidistant horizontal grid points ($\Delta r = 37.5$ km). $z = 0$ corresponds to $\tau_{ross} \approx 1$. Temperature contours are shown in steps of 200 K, while arrows indicate the velocity field. Maximum temperature is 11780 K, maximum flow velocity is 8.3 km/s.

Flow goes through similar cycles again and again, these are not really periodic in time but occur in irregular intervals, such that the overall behaviour of the convective part of the atmosphere may rather be characterized as chaotic. But superimposed on the convective flow we find quite regular oscillations, probably related to the observed solar 5-minute oscillations. Most prominent periods in the simulations range between 150 and 250 sec.

The important point to be learned from the time-dependent simulations is that the continually changing flow topology affects the upper atmosphere much more strongly than a steady state flow does. It seems that in stationary flows the convectively stable layers can adjust to a steady state convective flow in the deeper layers in a way that results in only minor spatial fluctuations of velocity and temperature. This is no longer possible in the time-dependent situation. If, for example, a granule collapses and a downflow develops at a point where a short time before there was an upflow (a situation actually found in the instationary models) or if a granule grows as it rises from below, the overlying layers, which are convectively stable in the classical models, must inevitably undergo substantial temporal variations and exhibit corresponding spatial fluctuations. For this reason we
find an “extended overshoot” in the time-dependent case. In the snapshot displayed in Fig. 2 we see much more pronounced velocity fields and temperature inhomogeneities in the photosphere than in the steady state model (Fig. 1). Indeed, the pressure waves excited by the instationary convection carry a substantial amount of mechanical energy into the higher layers. We have estimated the corresponding acoustic flux to be of the order of some $10^7$ erg/cm$^2$/s (Steffen et al. [11]).

Sometimes even shocks develop, preferentially associated with strong downdrafts, as in the example situation displayed in Fig. 3, showing two different shock fronts at the same time. One is almost vertical, travelling away from the intermediate downdraft in horizontal direction for some time until it finally dissipates, the other front extends horizontally and moves upwards. Remarkably, similar shocks have also been found in more idealized convection models by Cattaneo et al. [12], [13] who used a completely different hydrodynamics code and assumed a different geometry.

![Diagram showing solar granular convection](image)

Fig. 3. Snapshot from another instationary simulation of solar granular convection, using 40 vertical ($\Delta z$ between 40 km in the lower and 20 km in the upper part) and 50 equidistant horizontal grid points ($\Delta r = 37.5$ km). This model is somewhat deeper than the one shown in Fig. 2 and has a larger diameter ($D = 3675$ km). The almost vertical shock front at $r \approx 1160$ km in the upper atmosphere is travelling to the left, while the front at the axis of symmetry moves upwards. Maximum temperature is $13720$ K, maximum flow velocity is $11.4$ km/s.
4. Observed and Synthetic Spectra

In order to confront the hydrodynamical granulation models described in the preceding sections with reality, we have compared spectroscopic observations of the solar granulation with synthetic spectra calculated from the simulated convective atmospheres. To obtain spectra corresponding to observations at solar disk-center, we integrate the transfer equation along a set of vertical rays, taking into account the depth dependence of temperature, pressure and velocity in the calculation of the source function and the monochromatic continuum and line opacities. These spectrum synthesis calculations, which are independent of the more simplified treatment of radiative transfer of the hydrodynamical simulations, yield the emergent intensity in the continuum at any prescribed wavelength as well as the profiles of arbitrary spectrum lines as a function of horizontal position within the model.

In the following sections we discuss the calculated continuum intensity contrast as well as spatially resolved and unresolved synthetic line profiles with respect to corresponding observations.

4.1 Continuum Intensity Contrast

In the continuum we see horizontal intensity variations corresponding to temperature fluctuations near \( \tau = 1 \). The rms contrast of a given intensity pattern is defined as \( \delta I_{rms} = \sqrt{\sum_i f_i \left( \frac{I_i}{I_0} - 1 \right)^2} \), where \( f_i \) is the fractional area having intensity \( I_i \), and \( I_0 = \sum_i f_i I_i \) is the average intensity.

For the steady state models, \( \delta I_{rms} \) is found to increase with cell size, saturating towards the largest diameters (Steffen et al. [9]). An upper limit of \( \delta I_{rms} \approx 14\% \) at \( \lambda \ 5000 \ \text{Å} \) is approached for the largest steady state models (\( D \approx 1500 \ \text{km} \)).

Evaluation of the rms intensity contrast becomes more complicated for the time-dependent models. There are now different possibilities to define \( \delta I_{rms} \). For each instant of time we can calculate the rms contrast of the intensity pattern, \( \delta I_{rms}(t_n) \), as in the steady state case. The average rms contrast of the intensity pattern is then calculated as \( \overline{\delta I_{rms}} = \sqrt{\frac{1}{N} \sum_n ^N (\delta I_{rms}(t_n))^2} \). This quantity does not include possible time variations of the horizontally averaged intensity \( I_0(t_n) \). To take this additional contribution into account we define the total rms intensity contrast, \( \langle \delta I_{rms} \rangle \), including spatial as well as temporal intensity variations.

Depending on the size of the model, the treatment of radiative transfer (grey or frequency dependent), and on other numerical details, the instationary models yield \( \overline{\delta I_{rms}} \) values between 16% and 20%, corresponding to \( \langle \delta I_{rms} \rangle \) values between 18% and 22%.
These results may be compared with those from Nordlund's compressible 3-D models of solar granulation, as reported by Lites et al. [14]. Depending on details of their models, the rms intensity contrast at \( \lambda \) 6303 Å lies between 14.4% and 17.5%, which can be translated to corresponding values at \( \lambda \) 5000 Å between 18.2% and 22.1%. We note that the 3-D simulations give essentially the same answer as our instationary 2-D models.

Published data for observed \( \delta I_{rms} \) values, corrected for degradations by atmospheric seeing and instrumental effects, range between 8% and 19% at \( \lambda \) 5000 Å (see e.g. Bray et al. [15]). Comparing observed and calculated intensity contrasts, one should keep in mind that observations of the solar granulation contrast are severely affected by the earth's atmosphere and by instrumental effects, making large corrections necessary to derive the true contrast from the observed one. In view of the substantial observational uncertainties, reflected by the large range of \( \delta I_{rms} \) values derived for the solar granulation contrast by different observers, this quantity can hardly be used as a stringent constraint for theoretical granulation models.

### 4.2 Spatially averaged Line Profiles

The shape of the spectral line profiles is more suitable for a quantitative comparison. In usual solar spectroscopy the light of many granules enters the entrance aperture of the spectrograph and the resulting line profiles refer to a spatial average. Corresponding line profiles are obtained from granulation models by horizontally averaging the emergent intensity. Although the results presented in this section refer to one particular spectral line, they may be regarded as representative in the sense that lines of different ions or of different strengths behave qualitatively similar.

**4.2.1 Steady State Models.** As an arbitrary example we compare in Fig. 4a synthetic and observed line profile of the medium-strong Fe II line at \( \lambda \) 5197 Å. The synthetic profile has been computed from the steady state model displayed in Fig. 1 without introducing any additional broadening like classical micro- and macroturbulence. The non-thermal broadening is provided exclusively by the simulated velocity field. Clearly, the calculated line profile is too narrow and too deep. The obvious explanation is that the simulated steady state velocity field is not sufficiently vehement to account for the observed line broadening.

On the other hand, looking at the line asymmetry we can notify an excellent agreement between the shape of calculated and observed bisectors (Fig. 4b), despite the differences in the profiles themselves. Finally it is worth noting that the line core is essentially unshifted in our example, in contrast to observational evidence calling for a convective blueshift of several hundred m/s (cf. Dravins et al. [16]).

**4.2.2 Instationary Models.** In the case of the time-dependent models the effort needed to compute average spectra is much greater than in the steady
Fig. 4. (a) Comparison of calculated (thick line) and observed (thin line) profile of Fe II, λ 5197.6 Å at solar disk-center. The synthetic line profile has been calculated from the steady state model shown in Fig. 1. (b) Comparison of the corresponding line bisectors. The observed bisector (right) is arbitrarily displaced relative to the calculated bisector (left), which is shown on an absolute velocity scale in the solar reference frame. Differences near the continuum are due to a weak blend in the observed spectrum. (FTS observation courtesy of W. Livingston, National Solar Observatory, Tucson, U.S.A.)

state case, because the spatially resolved spectrum synthesis has to be performed for a time series of many models. The synthetic profile for Fe II, λ 5197 Å shown in Fig. 5a has been obtained as a spatial and temporal average of 2450 individual profiles, calculated from a non-stationary simulation covering almost 2 hours of real time (requiring 7 hours of CPU time on a CRAY X-MP 216). For each of 70 snapshots taken at intervals of 100 sec, 35 spatially resolved line profiles contribute to the total average.

Fig. 5. Same kind of comparison as in Fig. 4, but for a synthetic line profile based on a non-stationary simulation in a cell measuring 2100 km in diameter. The calculated profile was obtained as the combined spatial and temporal average of 2450 individual profiles. Again the observed bisector (right) is arbitrarily displaced relative to the calculated bisector (left).

Agreement with observation is much better than for the steady state profile, implying that the velocity field due to the simulated convective and oscillatory motions is sufficient to provide the observed line broadening. A
remarkable result! It means that we can explain why classical spectroscopic analyses have to introduce the ad hoc parameters micro- and macroturbulence: these quantities obviously represent the combined effects of the granular velocity field and the associated oscillations. A similar suggestion has been made by Holweger et al. [17]. This explanation was substantiated numerically by Nordlund [18] from 3-D anelastic models and by Lites et al. [14] from compressible 3-D simulations.

As in the case of the steady state model, the shape of observed and synthetic line bisector looks very similar (Fig. 5b). Moreover, now a net blueshift is predicted for the line core, resulting in an improved agreement between calculated and observed line shift.

4.3 Spatially resolved Line Profiles

4.3.1 Observed Spectra. Observations of spatially resolved spectra of the solar granulation were carried out by one of the authors (M.S.) in November 1989 at the German Vacuum-tower-telescope (VTT) at Izaña, Tenerife. The 70 cm telescope gives a scale of 4.6″/mm at the focal plane where the entrance slit of the Echelle spectrograph is located. Choosing a slit width of 100 μ allows a maximum spatial resolution of 0.5″ and corresponds to a spectral resolving power of ≈ 500000. In the focal plane of the spectrograph the scale is 4.7″/mm and the dispersion at λ 4912 Å is about 83 mÅ/mm. The detector used was a RCA CCD camera with 512 by 320 pixels, with a pixel size of 30 μ by 30 μ. This chip covers a total of about 72″, or roughly 50 granules, along its larger dimension (0.14″/pixel) and approximately 800 mÅ in the direction of dispersion (2.5 mÅ/pixel).

Spectrograms at wavelengths between 4909 Å and 4913 Å were taken in quiet regions near the center of the solar disk. Exposure times were kept as short as possible to minimize the degrading influence of atmospheric seeing. For most of the frames we used 0.5 sec, the minimum exposure time still giving a reasonable signal to noise ratio. Atmospheric conditions were such that spectrograms taken with longer exposure times suffered a distinct deterioration of contrast and spatial resolution.

The basic reductions were carried out on a SUN workstation at the Kiepenheuer-Institut in Freiburg, using the IDL graphics package to run the standard reduction procedures, developed and made available to visiting observers by the Kiepenheuer-Institut. Final reductions and analysis were performed at Kiel with software written by Freytag [19].

Fig. 6a shows one of the best spectra of Fe I, λ 4909.4 Å, after having gone through the whole series of reduction steps. The granular structure is clearly visible. We estimate the spatial resolution of this spectrogram to be somewhere between 0.5″ and 0.8″. The measured intensity contrast in the continuum is 6.5% and the Doppler shift of the line core indicates peak to peak velocity differences of almost 2 km/s. The corresponding rms velocity fluctuation is about 400 m/s in this example. A strong correlation
Fig. 6. Left-hand panel (a): spatially resolved spectrogram of the solar granulation, centered on Fe I, $\lambda$ 4900.4 Å as taken with the German vacuum-tower-telescope at Izaña on November 7, 1989, using the RCA CCD detector covering 72$''$ in the vertical (spatial) direction and 800 mÅ in the horizontal direction (wavelength increasing to the right). Right-hand panel (b): spatially resolved spectrum as calculated from numerical simulations of the solar granulation for the same spectral line, reproduced to be directly comparable to the observed spectrum (a). Degrading by atmospheric seeing and instrumental effects was modelled in a schematic way. For details see text.

between continuum intensity and line shift is evident, clearly indicating hot rising and cool sinking material. Similar results have been reported by other observers (e.g. Wiehr and Kneer [20]).

Fig. 7 displays the variations of the spatially resolved line profiles and their corresponding bisectors for a small but typical section of the spectrogram shown in Fig. 6a covering 2.8$''$. Evidently, granulation is associated with considerable spatial variations of the line profiles. In particular, the Doppler velocity of the line core changes by more than 1.5 km/s over granular scales, directly demonstrating that substantial velocity fields must exist even in the upper photosphere where the line core is formed.

4.3.2 Simulated Spectra. The first step for the simulation of spatially resolved spectra is to generate a representative granulation pattern from a time series of instationary hydrodynamical models, assuming it is permitted to replace temporal for spatial variations. To circumvent the problem
Fig. 7. Left-hand panel (a): Spatially resolved line profiles of Fe I, \( \lambda 4909.4 \) Å, obtained by scanning the observed spectrogram shown in Fig. 6a over a small subsection extending 2.8″. Right-hand panel (b): Bisectors of the line profiles shown in (a), plotted on an extended scale. Heavy line: bisector of the line profile averaged over the complete spectrogram (72″).

Fig. 8. Artificial granulation pattern, generated from an instationary simulation in a cell measuring 2625 km in diameter, as seen at disk-center in the continuum at \( \lambda 4909 \) Å, taking into account instrumental and atmospheric smearing. The total area covers 52000 km by 32500 km (72″ by 45″).

of uniformly covering a surface with models having a cylindrical cross section, we transform the cross sections into squares of equal area. In this way the fractional areas of dark and bright surface elements are retained and we can easily cover a rectangular surface section without having a problem with interstices in between the actual models. We arrange a sequence of 48 snapshots from an instationary simulation in an essentially random way to cover an area of 52000 km by 32500 km (72″ by 45″) as illustrated in Fig. 8.
showing the artificial granulation pattern in continuum light at \( \lambda \) 4909 Å. To simulate seeing and instrumental effects, the original 2-dimensional intensity pattern has been convolved with an axially symmetric point spread function, given by 
\[
PSF(r) = (1 - p)L(r, a) + pL(r, b),
\]
where
\[
L(r, a) = \frac{a}{2\pi(a^2 + r^2)^{3/2}}
\]
is the inverse Abel transform of the Lorentz-Function. Following Nordlund [21] we used \( p = 0.4, a = 180 \) km and \( b = 1800 \) km.

Obviously the resulting artificial granulation pattern is topologically different from the real granulation and looks more like an assembly of "exploding granules". Nevertheless we use it as the basis for the calculation of spatially resolved synthetic spectra, presuming that it is primarily the vertical structure of individual granulation elements that determines the characteristics of the resolved spectra.

To generate a simulated spectrum, a numerical slit is put onto the artificial granulation pattern, oriented in such a way as to intersect the "granules" in a largely random, non-repetitive way. Scanning the slit in steps of 0.14 " (to match the pixel structure of the observations), we calculate the line profile corresponding to the respective position within the artificial granulation pattern in steps of 2.5 mÅ (= 1 pixel in the observed spectrogram), applying a smearing due to seeing and instrumental effects according to \( PSF(r) \) for every wavelength point of the profile and in addition accounting for the finite slit width of 100μ. The resulting simulated spectrogram is shown in Fig. 6b.

The first impression is that observed and simulated spectra look very similar, at least qualitatively. There are some quantitative differences, though. The simulated intensity contrast in the continuum is 8.2%, while the value measured from the observed spectrogram is 6.5%. Similarly, the simulated rms velocity fluctuation is 575 m/s, somewhat higher than the observed value of 400 m/s. These systematic differences suggest that the adopted smearing probably underestimates the effects of seeing adequate for the actual observing conditions. It is unlikely that the reason lies in the hydrodynamical granulation model, because the simulated velocity field has been found to reproduce the observed broadening of spatially averaged line profiles correctly (see 4.2.2).

Looking for a section in the simulated spectrogram that is similar to the observed detail shown in Fig. 7, we came across the example displayed in Fig. 9. Variation of continuum intensity and Doppler shift as well as the arrangement of the line profiles is strikingly similar in observation and simulation. Although there are differences in the shape of the resolved line bisectors, we note a good agreement between the average bisectors.

If the simulated spectrogram shown in Fig. 6b had not been degraded to account for seeing and instrumental effects, the resulting rms intensity
Fig. 9. Left-hand panel (a): Same as Fig. 7a, but obtained by scanning a similar subsection of the simulated spectrogram shown in Fig. 6b extending over 3.5″. Right-hand panel (b): Corresponding line bisectors. Heavy line: spatially averaged bisector.

Fig. 10. Same as Fig. 9, except that the simulated spectrogram was not artificially degraded by atmospheric seeing and instrumental effects. Per definitionem, the spatially averaged bisector (heavy line) is unaffected. Note that some bisectors fall completely outside the plotted range.

contrast in the continuum would be as high as 21.7%, and the rms velocity fluctuation in the line core would amount to 1.1 km/s. Fig. 10 illustrates how the scans displayed in Fig. 9 would appear in this case. Our models predict that such enormous fluctuations should actually be seen if all observational limitations could be overcome. From the striking differences between Fig. 9 and Fig 10 we conclude that present-day spectroscopic observations are far from really resolving the solar granulation!

5. Conclusions

Numerical simulations of 2-dimensional, axially symmetric compressible convection under conditions appropriate for the solar photosphere, including a detailed treatment of non-local radiative transfer, result in two different kinds of solutions, depending on the adopted size of the convection cell. For small model diameters, the convective flow develops towards a steady state.
solution, whereas for the larger models we obtain time-dependent convection that may be characterized as chaotic.

Comparison with various spectroscopic observations strongly suggests that the non-stationary granulation models are a more realistic approximation to the real solar photosphere than the steady state models. Accepting the time-dependent solutions as representative for the hydrodynamical conditions prevailing in the solar atmosphere, we conclude that the solar photosphere up to the level of the temperature minimum is neither static nor plane-parallel nor in radiative equilibrium. Disturbances generated by the continually changing topology of the flow pattern in the underlying convective layers carry a substantial amount of acoustic flux and have considerable effects on the higher layers. This is at variance with the classical models which consider the solar photosphere as static and in radiative equilibrium.

Observational evidence in fact indicates the presence of substantial velocity fields in the higher photospheric layers. It is well known that non-thermal velocities of the order of 2 km/s are needed to explain the observed broadening of spatially averaged line profiles, the reason for introducing classical micro- and macroturbulence (without understanding their physical meanings). In spatially resolved spectra, even the cores of stronger lines show Doppler shift variations of at least 1 km/s on granular scales, directly illustrating the dynamical state of the solar photosphere. We have shown that our simulations are able to reproduce such spectroscopic observations, replacing both micro- and macroturbulence by the effect of convective granular flows overshooting into the higher photosphere.

From the favourable comparison between observed and synthetic spectra we may conclude that many basic properties of photospheric (stellar) convection can be modelled even quantitatively by 2-dimensional numerical simulations. Moreover, we found basic agreement between the results obtained from 2-dimensional and 3-dimensional granulation models, respectively, suggesting that for many purposes 2-dimensional simulations can be a reasonable alternative, requiring considerably less computational effort than fully 3-dimensional calculations. In view of the extensive work that has yet to be done in the field of stellar convection, this is an important aspect.

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References

1. A. Unsöld: Z. Astrophysik 1 138 (1930)