A NONRADIAL PULSATION MODEL FOR THE RAPIDLY ROTATING δ SCUTI STAR κ² BOOTIS

E. J. KENNELLY AND G. A. H. WALKER

Department of Geophysics and Astronomy, University of British Columbia, Vancouver, BC, V6T 1Z4, Canada

AND

I. HUBENY

Universities Space Research Association, NASA/GSFC, Code 681, Greenbelt, Maryland 20771

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ABSTRACT

A 2.5-hour time series of high-resolution CFHT spectra of the rapidly rotating δ Scuti star κ² Boo exhibits a progression of subfeatures moving from blue to red through the absorption lines. We can reproduce these and other variable features with a geometrical model which imposes sectorial nonradial pulsations (NRPs) on the surface of the star. Synthetic spectra generated with the appropriate Teff and log g are used as input. The entire wavelength region covered by the observations can be reproduced and the effects of line blending are included explicitly. Identification of low-degree modes is guided by radial-velocity variations and the known photometric variations. We find that the data can be reproduced by the combination of a high-degree ℓ = 12 mode with P_{osc} = 0.071 day, and a low-degree mode, ℓ = 0–2 with P_{osc} = 0.071–0.079 day. The projected rotational velocity (υ sin i = 115 ± 5 km s⁻¹) was determined by fitting synthetic line profiles to the observed spectra. By including in our treatment the limit to possible resolution of the stellar surface set by the intrinsic linewidth and υ sin i, we estimate the velocity amplitude of the high-degree oscillations to be ~ 3.5 km s⁻¹. We find that the ratio of the horizontal and radial pulsation amplitudes is small (k ~ 0.02) and consistent with p-mode oscillations. Comparisons are made with models invoking starspots, and we find that it is impossible to fit the observations of κ² Boo by a starspot model without assuming unrealistic values of radius or equatorial velocity.

Key words: δ Scuti stars—nonradial pulsations

1. Introduction

Observations of rapidly rotating, pulsating stars with high spectral and time resolution have revealed a new class of variable stars, the "line-profile variables". Their absorption lines change shape cyclically with time. At low spectral resolution there are changes in line asymmetry, while at high resolution subfeatures (or bumps) travel through the rotationally broadened line profiles. The bumps appear to be related to features on or near the surface of the star. Rotational broadening creates a one-dimensional map of the stellar surface in each line profile, an effect dubbed "Doppler Imaging" by Vogt & Penrod (1983).

The traveling-bump phenomenon has been observed among rapid rotators in at least two instability regions of the Hertzsprung-Russell diagram. Both the early-type OB variables and the δ Scuti variables exhibit similar line-profile variations. The first detection of the moving-bump phenomenon was made for the O9.5 V star ζ Ophiuchi by Walker, Yang & Fahlman (1979). Soon after, Walker et al. (1982) discovered similar variations in a second star, α Virginis (B1.5 IV). Traveling surface oscillations, better known as nonradial pulsations (NRPs), are favored as the source of the line-profile variations although other models which invoke temperature variations on the stellar surface (spots) or occultations of the surface by blobs of material have also been explored (see Vogt & Penrod 1982).

During a project to measure precise radial velocities of δ Scuti stars, Yang & Walker (1986) found that the star α¹ Eridani (F2 II–III) displayed features moving through its line profiles. Walker, Yang & Fahlman (1987) followed up this discovery with observations of four rapidly rotating δ Scuti stars (α¹ Eri, κ² Bootis, υ Ursae Majoris, and 21 Monocerotis), all of which showed regular progressions of traveling subfeatures. Because of the extremely high quality of the spectra, these four stars are ideal candidates for testing the theories which have been suggested to
explain the variations. In this paper we demonstrate how we have used an NRP model to construct theoretical line profiles which mimic the variations for one of these stars, \( \kappa^2 \) Boo, and thereby infer the modes, periods, and amplitudes of its oscillations. \( \kappa^2 \) Boo is a rapidly rotating \( \delta \) Scuti star \( (v \sin i = 115 \text{ km s}^{-1}) \) of spectral type A8 IV with an apparent magnitude of \( V = 4.54 \). Breger (1979) lists the photometric period as \( P_{\text{phot}} = 0.066 \text{ day} \) with an amplitude \( \Delta m = 0.03 \text{ magnitude} \); however, a 16-day beat period has also been reported (e.g., Desikachary, Parthasarathy & Kameswara Rao 1971). Few spectroscopic investigations have been performed on this star.

2. Observations

Details of the 2.5-hour time series obtained at CFHT in 1987 of \( \kappa^2 \) Boo are given by Walker et al. (1987). The spectra were taken with the f/8.2 camera of the coudé spectrograph using an RL1872F/30 EG&G Reticon detector (Walker, Johnson & Yang 1985) at a dispersion of 0.035 Å per pixel. The exposure times were typically 15 minutes with signal-to-noise ratio per pixel of \(- 450\).

Figure 1 illustrates the time series of observations obtained for \( \kappa^2 \) Boo. Each spectrum has been convolved with a Gaussian to smooth the data. The spectra are positioned vertically according to the median time of observation. Fractions of a Julian day with respect to JD2446838 are listed on the right-hand side.

Residual spectra were generated by subtracting the mean spectrum for the series from each individual spectrum. The resulting plot, presented in Figure 2, dramatically illustrates the presence of traveling bumps within the line profiles, even in the weaker lines. The absolute mean deviation of the variations was also generated for the time series. We prefer this as a measure of the amplitudes of the variations in the lines because it does not unduly weight the center of the line as would the variance. Figure 3 illustrates the mean deviation and the mean spectrum and highlights the localization of the variation within the lines.

The chosen spectral region is rich with absorption features. Line identifications were performed from comparisons with the spectrum of Procyon (Griffin & Griffin 1979). A Mg ii-Fe i blend around \( \lambda 4482 \) dominates the spectrum. Although many of the lines suffer from blending, four absorption features relatively free from blends show the behavior of the NRP bumps most clearly. These are: \( \lambda 4476.061 \) (Fe i doublet), \( \lambda 4501.278 \) (Ti ii), \( \lambda 4508.289 \) (Fe ii), and \( \lambda 4515.342 \) (Fe ii).
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what by introducing temperature variations as well as geometrical variations.

Provided the ratio of the oscillation frequency to the rotation frequency is large (σosc/Ω ≫ 1), the period of oscillation, Posc (in days) of a rotating star is related to the apparent period, Δt (in days) by (Lee & Saio 1990)

\[
\frac{1}{P_{osc}} = \frac{1}{\Delta t} + \left[0.01976\right] \frac{mv^2 \sin \iota}{R \sin \mu} (1 - C),
\]

where m is the mode, v sin i is in km s⁻¹, R is the radius in solar units, and C is the Coriolis correction term. The above formula assumes constant rotational velocity within the region of interest (i.e., the outer envelope) of the star.

We developed a FORTRAN computer program to simulate absorption line profiles of a rotating, nonradially pulsating star. Our model at present is strictly geometrical and the pulsations of the star are described by spherical harmonics. Theoretical line profiles were first calculated by Osaki (1971) for comparison with β Cephei stars and numerous studies since have implemented this technique. Theoretical line profiles were used by Campos & Smith (1980) and Smith (1982) to determine the low-degree modes among a selection of δ Scuti stars. More recent results can be found in Kambe & Osaki (1988) and Lee & Saio (1990). Our program produces equivalent results to those published by Kambe & Osaki (1988).

The model is relatively simple. The unperturbed star is represented by a rotating (oblate) spheroid. The surface is divided into many segments specified by lines of latitude and longitude. Spherical harmonics are imposed as a perturbation of this geometry and the positions and velocities of the segments are monitored as the pulsation proceeds with time. The velocity along the line of sight of each segment can be translated into a Doppler shift. The flux from each segment is described by the same intrinsic line profile (for a star without temperature or pressure variations) weighted by the effective area of the segment, by limb darkening, and by gravity darkening. A standard limb-darkening law is used, with I(μ) ∝ (1 - β (1 - μ)) where μ = cos θc, θc being the angle between the line of sight and the line normal to the surface of the star and β is the limb-darkening coefficient. A Von Zeipel gravity darkening law is assumed. Profiles from all segments of the visible disk are Doppler shifted in wavelength and summed to yield the stellar line profile. The model star is specified by its mass, radius, v sin i, inclination, limb-darkening coefficient, and gravity-darkening coefficient. Each mode of pulsation is specified by the mode numbers (ℓ, m), wave speed, velocity amplitudes, and phase.

Synthetic line profiles (or entire spectral regions) are generated from model atmospheres to represent the intrinsic line profile which is required as input to the program. In this way, the effects of intrinsic linewidth on the resolution (and line blending) are directly included in the analysis. Matching the line strengths of the synthetic spectra to those in the observed spectra also provides

3. Profile Modeling

Surface nonradial pulsations generate a velocity field which can be described in terms of spherical coordinates (r, θ, φ) at time t as

\[
\dot{V} = \left[V_{osc} kV_{osc} \frac{\partial}{\partial \theta} kV_{osc} \frac{1}{\sin \theta} \frac{\partial}{\partial \phi}\right] Y_{lm}^m(\theta, \phi) e^{i\omega t},
\]

where \(V_{osc}\) is the velocity amplitude of the oscillations in the radial direction, k is the ratio of the horizontal to radial-velocity amplitude, \(\sigma\) is frequency of oscillation, and \(Y_{lm}^m(\theta, \phi)\) are spherical harmonics with degree ℓ and m = -ℓ, -ℓ + 1, ..., ℓ.

Theoretically the value of k is given by (Ledoux 1951)

\[
k = \frac{74.44}{P_{osc}} \frac{M R^2}{R^4},
\]

where \(P_{osc}\) is the period of oscillation in the rotating frame and \(M R\) and \(R\) are the mass and radius in solar units. This relation is a direct result of the boundary conditions on the pressure at the surface of the star. The value of k is related to the pulsation constant \(Q = P_{osc} (\rho/\rho_0)\) by (Unno et al. 1989)

\[
k = \left(\frac{Q}{0.116}\right)^2
\]

and, in theory, can be used to distinguish between p-modes (high-frequency) k ≈ 0.1 or g-modes (low-frequency) k ≈ 1.0. There has been a long-standing problem with k for early-type line-profile variables. The value necessary to match the observations is always very much smaller than that predicted theoretically. However, Lee & Saio (1990) claim to have alleviated this problem somewhat by introducing temperature variations as well as geometrical variations.
The intrinsic spectra are generated from LTE model atmospheres tailored to the known or estimated $T_{\text{eff}}$ and log $g$ for the star. A detailed description of the model-atmosphere program we have used, TLUSTY, is given by Hubeny (1988). A complementary program, SYNspec (Hubeny, unpublished), was employed to construct synthetic spectra for the wavelength region 4460 Å–4520 Å using the Kurucz-Peytremann line list (Kurucz & Peytremann 1975) updated based on data provided by Martin, Fuhr & Wiesse (1988) and Fuhr, Martin & Wiesse (1988).

The intrinsic width of the line, $v_b$, and $v \sin i$ largely determine the resolution on the surface of the star. The resolution improves with greater rotational velocity and intrinsically narrow lines. Thus, the ratio $v_b/v \sin i$ is a measure of the resolution of the surface features. While the amplitude of the bumps in the profiles is a reflection of the velocity amplitude of the waves on the surface of the star, the amplitude also depends on the intrinsic resolution. By using synthetic spectra to specify the intrinsic linewidth the effects of resolution are automatically taken into account and the velocity amplitude determined.

Instrumental resolution also influences the amplitude of the variations. However, this effect is easily approximated by convolving the model profile generated by the NRP program with a Gaussian profile to represent smoothing introduced by the instrumental resolution. Further smoothing incurred by time averaging will be negligible provided the exposure times are short relative to the apparent period of variation.

4. Line-Profile Variations

4.1 Stellar Model

The projected rotational velocity ($v \sin i$) of κ² Boo was estimated by matching the width of the observed mean line profiles with theoretical rotationally broadened profiles generated with the NRP program (without any NRP modes). We find $v \sin i = 115 \pm 5$ km s$^{-1}$. The uncertainty is due primarily to line blending in the wings of the lines. Our value differs from the published values (e.g., the Bright Star Catalog gives $v \sin i = 139$ km s$^{-1}$) but is probably more reliable because of the high resolution and high S/N of the observations.

We have used empirical calibrations of $ubvy$ photometry to estimate the mass and radius of the star. Breger (1990) relates the $T_{\text{eff}}$, log $g$, and absolute magnitude $M_v$ of a typical δ Scuti star (with $T_{\text{eff}} = 7750$ and log $g = 3.75$) to the Strömgren indices $\beta$ and $c_1$ by the following equations:

$$\log T_{\text{eff}} = 0.5242 \beta - 0.0027 c_1 + 2.4347, \quad (5)$$

$$\log g = 4.9187 \beta - 2.6298 c_1 - 7.3982, \quad (6)$$

$$M_v = -9 c_1 + 15 \beta - 32.22. \quad (7)$$

Provided the characteristics ($T_{\text{eff}}$, log $g$) of κ² Boo are not too different from those of the typical star for which the above relations were derived, the application of equations (5), (6), and (7) to κ² Boo should be valid. The values of $\beta$ and $c_1$ were taken from Breger (1979). Uncertainties in $T_{\text{eff}}$, log $g$, and $M_v$ were estimated by assigning reasonable photometric errors to $\beta$ and $c_1$ ($\pm 0.02$). From these results, the luminosity $L$, radius $R$, and mass $M$ are derived. The results are listed in Table 1 (along with Breger’s (1979) photometric indices.)

4.2 Mode Analysis

A Fourier transform of variations in the residuals will display a peak at the frequency corresponding to the interval between the bumps, $\Delta t$, at all positions within the profile. If a sufficiently long series is obtained, one will also be sensitive to multiple periods and their beating. This approach was adopted by Gies & Kullavanijaya (1988). In the work here, the profiles of λ4508 were sampled at 10 km s$^{-1}$ intervals between $-50$ to $+50$ km s$^{-1}$ and Fourier amplitude spectra were obtained with a periodogram routine for unequally spaced time series (Matthews & Wehlau 1985). The mean amplitude spectrum for κ² Boo is displayed in Figure 4. The dashed line is the spectral window assuming a single period of $\Delta t = 0.045$ day. The limited number of spectra in each series, the limited frequency resolution, and the possible complication of additional modes add to the uncertainty in this quantity.

For small velocity perturbations, the positions of the subfeatures as they move through the profiles can be described by

$$V = (v \sin i) \sin (2\pi (t - t_0)/P), \quad (8)$$

where the period of the curve, $P$, is the time it takes for a wave crest to encircle the star relative to the observer. For a given line (e.g., λ4508), the positions of the features were measured and the paths traced out by the features were superimposed using as the mean interval between features the value of $\Delta t$ obtained from the

<table>
<thead>
<tr>
<th>Stellar Characteristics from $ubvy$ Photometry</th>
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<tr>
<td>$b - y$</td>
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<tr>
<td>$\beta$</td>
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<tr>
<td>$c_1$</td>
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<tr>
<td>$T_{\text{eff}}$ (K)</td>
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<tr>
<td>$\log g$ (cm/s$^2$)</td>
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<td>$M_v$ (mag.)</td>
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<td>$M/M_\odot$</td>
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<tr>
<td>3.1 ± 0.3</td>
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<tr>
<td>2.0 ± 0.4</td>
</tr>
</tbody>
</table>

$^a$ calculated from Equation 7 and tabulated value of $M_v$.
Fig. 4—The Fourier amplitude spectrum of the line-profile variations (solid) indicates an apparent period of variation, Δt. A sinusoidal window function (dashed) generated at the same period is illustrated for comparison.

Fourier analysis. A least-squares fitting package, OPDATA, was then used to obtain the period, \( P = 0.52 \pm 0.02 \) day, and amplitude, \( v \sin i = 109 \pm 3 \) km s\(^{-1}\), of the resulting curve (Fig. 5). The agreement of this amplitude with the rotational width of the line profiles requires that the source of the variations be confined to the surface of the star. The \( m \)-value corresponding to a sectorial mode is given by

\[
|m| = \frac{P}{\Delta t}
\]

and was determined to be \( |m| = 11.6 \pm 1.4 \) from this analysis. Although this technique works well if there is a single mode dominating the line-profile variations, in the presence of multiple modes of nearly equal amplitude the pattern can become very complicated.

Fig. 5—By fixing \( \Delta t \), the period, mode, and \( v \sin i \) of the variations were determined from a fit to the superposition of the curves traced out by the subfeatures as they travel through the absorption profiles. The agreement of \( v \sin i \) with the value derived from profile modeling implies that the variations originate at the surface of the star.

4.3 NRP Model

An iterative procedure is followed to reproduce the observed line-profile variations with the theoretical model. Initial estimates of the pulsation constants from the previous section were adopted. The waves are assumed to be prograde and sectorial (\( \ell = -m \)). An inclination of 90° is assumed. The value of \( k \) is determined from equation (2) and an initial guess is provided for the velocity amplitudes. The program then generates a series of output profiles which can be compared with the observations. The goodness of fit is judged by eye with the model residuals superimposed on the observations. Parameters are adjusted and new profiles are generated until a satisfactory match is achieved.

Figure 6 illustrates the model time series of residuals (dashed) superimposed on the observations (solid) of \( \lambda 4508 \). The mean Fourier amplitude spectrum for the model is compared with the observations in Figure 7. The amplitude of the variations in both cases has been normalized to the depth of the line. We find that the best fit under these criteria is achieved with a model for which

![Figure 6](image)

**Velocity (km/s)**

Fig. 6—The time series of residuals generated by the model (dashed) are compared with the observations (solid) for the \( \lambda 4508 \) line.
Eq. (10). The radial-velocity curve for $\kappa^2$ Boo is shown in Figure 9. A period of $P_{\text{RV}} = 0.071 \pm 0.003$ day with an amplitude $K = 0.86 \pm 0.08$ km s$^{-1}$ is determined. This period is not significantly different from the time scale for the photometric variations, $P_{\text{phot}} = 0.066$ day (Breger 1979).

The velocity-to-light ratio $(2K/\Delta m \sim 55)$ is an indication of the mode of pulsation (Yang 1990). Empirically, $2k/\Delta m \sim 100$ km s$^{-1}$ mag$^{-1}$ is expected for radial modes and $2k/\Delta m \sim 50$ km s$^{-1}$ mag$^{-1}$ is expected for nonradial modes. For $\kappa^2$ Boo, the ratio indicates a nonradial rather than a radial mode of pulsation, most likely $\ell = 1$ or $\ell = 2$.

6. Discussion

Although the star is rapidly rotating, all effects of the Coriolis force greater than first order were neglected in the calculation of the oscillation period of $\kappa^2$ Boo (eq. (4)). This approximation is justified if the ratio of the pulsation frequency, $\nu_{\text{osc}}$, to the apparent oscillation frequency, $\Omega$, is large, as would be the case for high-degree $p$-modes. Our results for $\kappa^2$ Boo indicate $\nu_{\text{osc}}/\Omega = 19$ (at $i = 90^\circ$) implying that our simplification is justified and that $p$-mode oscillations are responsible for the high-degree variations.
In order to reproduce the line-profile variations with an NRP model demanding minimal velocity excursions we adopted an inclination of $90^\circ$. However, the inclination is highly uncertain. If we assume that the observed upper limit of $200 \, \text{km s}^{-1}$ on the distribution of $v \sin i$ for δ Scuti stars (Wolff 1983) represents a maximum in their rotational velocities, then a lower limit for the inclination of $\kappa^2$ Boo is $i_{\text{min}} = 35^\circ$. Table 2 illustrates how changes in inclination affect the derived values of $\kappa$, $P_{\text{osc}}$, and $V_{\text{amp}}$. While the $p$-mode nature of the oscillations is unaffected by the choice of $i$ there is an unavoidable uncertainty in $P_{\text{osc}}$.

The concept of a “superperiod” was introduced by Smith (1985) for line-profile variables. It is defined as $mP_{\text{osc}} = \text{constant}$ for all $\ell = -m$ modes of oscillation and represents the time required for a wave to encircle the star once. This concept has been applied to the observations of multiperiodic OB stars. If such a restriction exists, then it must be linked to the excitation mechanism driving the pulsations. The problem with the rapidly rotating OB stars is that the rotation frequency dominates the apparent oscillation frequencies. Under these circumstances, it is difficult to measure the wave speeds.

The oscillation frequencies of the δ Scuti stars can be very much greater than the rotation frequencies, making them much easier to distinguish. Our results for $\kappa^2$ Boo indicated that both the high ($\ell = 12$) and the low ($\ell = 1$, or 2) degree modes pulsate with similar periods ($\sim 0.071$ day) suggesting that perhaps the modes are excited in resonance.

NRP is by far the most widely accepted explanation for line-profile variations. An unfortunate aspect of the NRP theory is that it is very difficult to rule out with certainty due to the flexibility introduced by the wave speed. The “spoke” model, in which elongated blobs of material occult the surface of the star to produce line-profile variations, suffers from the same sort of flexibility as the NRP model but requires more stringent limits on the inclination of the star. In this case, the rate of variability can be adjusted by changing the distance from the star at which the obscuring material orbits. (The greater the distance the larger the inclination limit.) On the other hand, the “spot” models which invoke temperature and brightness differences require that the features responsible for the variations be fixed to the surface of the star and travel at exactly the rotation rate, so that the observed period of the bumps in the line profiles is the rotation period of the star. This restriction makes the spot model easiest to test.

If one assumes that the spots are distributed about the equatorial region and that the period of the traveling bumps is equal to the rotation period, then the equatorial velocity, $V_\alpha$ (in $\text{km s}^{-1}$) and inclination, $i$, can be derived. Applying this model to $\kappa^2$ Boo, we find that an inclination of only $23^\circ$ and an equatorial velocity of about $300 \, \text{km s}^{-1}$ are required to fit the observations. This velocity is well above the maximum value ($\sim 200 \, \text{km s}^{-1}$) expected for δ Scuti stars. In order to produce the large amplitudes of the bumps in the line profiles, very elongated spots stretching nearly from pole to pole with large brightness contrasts must be invoked. Such a fixed pattern so extensive in latitude is unlikely to persist on the surface of a rotating star in the presence of differential rotation. These arguments safely eliminate the starspot model as the explanation of rapid line-profile variations at least in the case of $\kappa^2$ Boo.

Time-series observations of $\sim 1-2$ hours have revealed much about the nature of high- and low-degree NRP in rapidly rotating δ Scuti stars. Stars with a single high-degree mode of oscillation can be effectively modeled with even short time series. However, if the star pulsates in more than one high-degree mode it becomes more difficult to identify the periods of the oscillations from the line-profile variations. A long time series ($\sim 10$ hr) obtained over one or more nights would significantly improve the resolution of the frequencies in the Fourier spectrum of the variations. With observations of this length, the apparent relationship between the pulsation periods could be confirmed. At any rate, a complete spectrum of oscillation frequencies would be an important step toward understanding the excitation mechanism and the internal structure of δ Scuti stars through the application of stellar seismology (Däppen 1990).

Many questions remain to be answered. Is there a connection between the high- and low-degree oscillations? Could many of the apparently photometrically “constant” stars in the δ Scuti strip have high-degree oscillations? Does rotation play a role in exciting or determining the modes of oscillation? Are the modes stable over long time scales? Continued investigations of rapidly rotating stars in the δ Scuti strip promise to greatly improve our understanding of the nonradial oscillations.

The authors would like to thank Dr. Jaymie Matthews for providing the Fourier periodogram program and for his many helpful suggestions. We also thank Stevenson Yang for his assistance throughout the project and Phil Bennett for making available the OPDATA reduction package. This work was supported by grants from the Natural Sciences and Engineering Council of Canada.

### Table 2

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<th>$i$ (deg.)</th>
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<th>$k$</th>
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