CENTER-TO-LIMB VARIATIONS OF CHARACTERISTICS OF SOLAR FLARE HARD X-RAY AND GAMMA-RAY EMISSION

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ABSTRACT

The observed distribution of fluxes and spectral indices of flares at X-ray and gamma-ray range are analyzed with particular attention to variation with heliocentric longitude of these distributions and their moments. It is shown how these distributions can be used to set constraints on the distribution and average values of model parameters describing the characteristics of the accelerated electrons and flare plasma. The models assumed are the nonthermal thick target models, whereby accelerated electrons with power-law spectra and Gaussian pitch angle distributions are injected at the top of a coronal loop with two kinds of converging field geometries. The basic distributions are derived from data on all flares, irrespective of their position on the sun, at low energies (hard X-rays) where the angular dependence of the parameters is negligible. Comparison of X-ray and gamma-ray spectral indices show that the spectrum of the accelerated electrons must flatten at higher energies. From comparison of observed and predicted angular variation of (1) the number of flares at different photon energy ranges (especially above 10 MeV), (2) the average gamma-ray spectral index, and (3) the gamma-ray-to-X-ray flux ratio constraints have been set on model parameters. In particular the range of the Gaussian width of the pitch angle distribution of electrons beamed along the field lines and the convergence rate of the magnetic field are delimited. Isotropic pitch angle distributions or distributions peaking perpendicular to the field lines, and field convergence rate with more than a fivefold increase in the magnetic field from corona to the photosphere can be ruled out.

Subject headings: gamma rays: bursts — particle acceleration — radiation mechanisms — Sun: flares — Sun: X-rays

1. INTRODUCTION

It is widely believed that hard X-rays and the bulk of the continuum gamma-rays, excluding the nuclear line emission between 1 and 7 MeV, observed during the impulsive phase of solar flares are the result of bremsstrahlung emission by electrons accelerated to comparable energies. The characteristics of the observed radiation, such as the spatial, spectral, and angular distributions and polarization, can therefore constrain the distribution of the properties of the emitting electrons and, with proper accounting of the transport processes, the distribution of properties of the accelerated electrons. This is the third paper in a series dealing with this problem. The transport effects relating the accelerated electrons to those emitting the radiation were discussed in Paper I (McTiernan & Petrosian 1990a). The characteristics of the emitted radiation resulting from bremsstrahlung of these electrons in a flare loop and a comparison of these with stereoscopic observations (Kane et al. 1988) were described in Paper II (McTiernan & Petrosian 1990b). In this paper we carry out a comparison of model results with observations by HXRBS and GRS instruments on board SMM.

The most widely available observation is the spatially integrated spectral distribution and its evolution throughout the duration of flare. The spatially resolved images from HXIS on SMM and Hinotori were limited to X-rays of energy < 30 keV. Exceptions to this were observations by Kane et al. (1982) of partially occulted flares which were analyzed by Brown, Hayward, & Spicer (1981) and Leach & Petrosian (1983). There have been very limited polarization measurements (see e.g., Leach, Emslie, & Petrosian 1985, and references cited therein).

Observations by near-Earth instruments of individual flares cannot provide any information about the angular distribution of flare radiations. Simultaneous observations by widely separated (on the scale of the solar system) instruments or the so-called stereoscopic observation can provide some information on this aspect of the emission. In Paper II we compared such observations from ISEE 3 and PVO satellites (Kane et al. 1988) with the model results. With the exception of two flares, these observations within their errors agree with the general nonthermal model (whereby accelerated electrons in a closed magnetic field geometry undergo thick target bremsstrahlung) with reasonable parameters. In general, the stereoscopic observations are also consistent with isotropic emission up to 1 MeV, but the uncertainties are relatively large.

Another method of obtaining information about the angular distribution is through comparison of emission characteristics of flares located at different parts of the solar disk, which present different aspects of the emission region to observation. Thus, from the study of center-to-limb (C-T-L) variations we can learn about the angular distribution of the emission. This task is made complicated because of the wide dispersion in the intrinsic parameters of flares. A large sample of flares is, therefore, needed for a statistical analysis in this kind of study.
Simple comparison of fluxes and frequency of occurrence of flares (MacKinnon & Brown 1989, 1990; Miller & Ramaty 1990) is not sufficient for quantitative results. A knowledge of the intrinsic distribution of basic model parameters and of instrumental sensitivity and threshold is needed for comparison of models with observations, a fact that has not been widely appreciated. For this task the effect of the distribution of intrinsic parameters is essential; furthermore, the appropriate parameters chosen for such a study must be selected judiciously.

For example, if an instrument gathers data only on bursts with photon fluxes (at or above certain photon energy) exceeding some threshold value, \( J_{th} \), and if the intrinsic distribution of these fluxes is a power law, then the shape of this distribution or its moments, e.g., the average value of the flux, \( \langle J \rangle \), will not show any C-T-L variation even if the intrinsic emission is highly anisotropic, but the rate of occurrence of flares will vary from center to limb according to the degree and the type of anisotropy. (Borrowing jargon from similar analyses of extragalactic sources, one could say that the observations show a number and not a luminosity C-T-L variation.) The exact inverse of this will be the case if the distribution of \( J \) is bounded at the lower end by some value \( J_{min} > J_{th} \). In this case it is the C-T-L variation of the average flux \( \langle J \rangle \) (or other moments) but not the rate of occurrence which will reflect the anisotropy of the emission (there will be a luminosity rather than a number variation). The importance of the distribution and selection effects was emphasized in an earlier paper (Petrosian 1975), in the analysis of data from \( OSO7 \) (Datlowe, Elcan, & Hudson 1974) which was limited to low-energy X-rays and showed little C-T-L variation, and more recently (Petrosian 1985) in the analysis of some very high energy (>10 MeV) gamma-ray data which show a dramatic C-T-L variation of rate of occurrence (Rieger et al. 1983; Forrest et al. 1984).

As pointed out by Petrosian (1985) and in Paper II, this strong dependence of the C-T-L variation with photon energy is what is expected from the standard thick-target nonthermal models. The gradual decrease with increasing electron energy of the importance of collisional pitch angle diffusion relative to energy loss and gradual increase of importance of relativistic beaming or anisotropy in the bremsstrahlung cross section are the two most important factors contributing to this effect. Other effects like deeper penetration into chromosphere and corona of higher energy particles also play a part. These latter effects depend on parameters such as the ambient density and magnetic field scale height in the solar atmosphere.

In addition to demonstrating presence of a strong C-T-L variation for flares with emission at photon energies >10 MeV, the GRS instrument on \( SMM \) has shown C-T-L variation of other parameters, such as the rate of occurrence of flares with >300 keV emission, the spectral index above 300 keV (Vestrand et al. 1987), and the ratio of >300 keV to >30 keV emissions (Bai 1988). Analysis of these data and their comparison with the model results presented in Paper II are the primary task of this paper. In \( \S \) 2 we briefly discuss the flare models and present general expressions relating intrinsic distributions, mean values, etc., with observed distributions of related parameters. In \( \S \) 3 we derive some basic distribution functions integrated over the full Sun for hard X-ray (30–300 keV) observations by HXRBS on \( SMM \). In \( \S \) 4 we discuss the C-T-L variations for X-rays and gamma-rays (300 keV to 1 MeV and >10 MeV) relying mainly on data published by Vestrand et al. (1987). The results are summarized in \( \S \) 5.

2. DESCRIPTION OF MODELS AND ASSUMPTIONS

The basic model parameters whose distributions or average values we would like to determine are of two kinds. The first kind describes the distribution of the accelerated electrons. The second kind describes the condition of the flare plasma, such as the density and magnetic field configurations.

2.1. Characteristics of the Electrons and Their Distributions

In general, we shall start with the simplest of assumptions and examine if these reproduce the observed averages, correlations, etc. For example, we assume that for each and every flare the flux of the accelerated electrons (with total flux \( F_T \) at energies \( E > E_0 \)) is described by

\[
F_0(E, \mu) dE d\mu = F_T(\delta_x - 1)E_0^{-4}G(\alpha)G(\beta)E^\alpha d\alpha d\beta,
\]

which has a power-law spectrum of spectral index \( \delta_x \) in energy and has a pitch angle distribution \( G(\alpha) \), which is the same at all energies. We assume a Gaussian form for \( G(\alpha) \) with a dispersion (or 1/e half-width) \( \sigma_0 \) and maximum in a direction either along (beamed, \( \delta_x = 0 \)) or perpendicular (pancake, \( \delta_x = \pi/2 \)) to the field lines:

\[
G(\alpha) = G_1 \exp[-(\alpha - \alpha_1)^2/\sigma_0^2]\sin \alpha d\alpha,
\]

where the normalization factor \( G_1 \), chosen so that \( \int_0^\infty G(\alpha) d\alpha = 1 \), is a complicated function of \( \alpha_1 \) and \( \sigma_0 \). For small values of \( \sigma_0 \), and for the special cases \( \delta_x = 0 \) and \( \delta_x = \pi/2 \), the values of \( G_1 \) are \( 2/\sigma_0 \) and \( 2/\sigma_0 \pi^{1/2} \), respectively.

First, we determine whether this type of distribution is consistent with observations and, if so, then from a statistical analysis of data on a sample of flares we try to determine the distribution of various parameters: say \( \delta_x, \alpha, \sigma_0 \). Since there is little observation with direct bearing on the distributions of \( \alpha_1 \) and \( \sigma_0 \), we shall explore these parameter spaces with the aim of determining their most probable values and ranges. However, since there are observations with direct bearing on the distributions of \( F_T \) and \( \delta_x \), we will determine these distributions directly. Thus we will start with \( \psi(F_T, \delta_x, \alpha, \sigma_0) = \psi(F_T, \delta_x, \alpha_1, \sigma_0) \rightarrow \psi(F_T, \delta_x, \alpha_1, \sigma_0) = \psi(F_T, \delta_x, \alpha_1, \sigma_0) \rightarrow \psi(F_T, \delta_x, \alpha_1, \sigma_0) \rightarrow \psi(F_T, \delta_x, \alpha_1, \sigma_0) \). As in dealing with the distribution of the particles, equation (1), here also we start with the simplest of assumptions, namely, the absence of correlation between any of these four parameters or a form

\[
\psi(F_T, \delta_x) = g_4(F_T) h_4(\delta_x).
\]

The distribution of spectral indices is normalized, \( \int h_4(\delta_x) d\delta_x = 1 \), so that \( g_4(F_T) dF_T \) is the total number of flares with electron energy flux between \( F_T \) and \( F_T + dF_T \) and with the assumed values for \( \alpha_1 \) and \( \sigma_0 \). The assumption that size and spectral index are uncorrelated is justified by the fact that the observed values of photon counts and spectral indices observed by HXRBS appear to be uncorrelated (cf. Fig. 4 of Dennis 1985 and Fig. 2 below).

2.2. Characteristics of the Plasma

We treat the parameters describing the plasma in the manner we handle \( \alpha_1 \) and \( \sigma_0 \). In the absence of compelling evidence supporting any kind of field geometry, we limit the model to a simple semicircular loop with particle injection occurring at the top of the loop. The circular portion ends at the transition region below which the field is assumed to be vertical (radial) with respect to the photosphere. The exact shape of the coronal field line and the position in the corona where the accelerated electrons are injected are unimportant in...
the treatment of spatially unresolved observations considered here. The important aspect of this geometry is the radial nature of the field lines at the footpoints. The only other quantities needed then are the variation along the field line of density and magnetic field, \( n(s) \) and \( B(s) \), where \( s \) is the distance along the field line from the injection point. In reality we require only the value of \( dB/\text{d}t \), where \( dt = \text{d} \sigma/N_0 \), \( N_0 = 5 \times 10^{-22} \text{ cm}^{-2} \) for Coulomb logarithm \( L = 20 \), so that in the corona when \( n \) is expected to be constant the only important parameter is the column depth \( N_0 = \sqrt{4 \pi} \text{ d} \sigma/N_0 \) from the top of the loop to the transition region \( s_tr \). For \( n = 10^{10} \text{ cm}^{-3} \) and \( s_tr = 2 \times 10^9 \text{ cm} \), \( N_0 = 2 \times 10^{19} \text{ cm}^{-2} \), so that all particles with energy (in units of \( m_e c^2)E > (N_0/N_0)^{1/2} \) will penetrate below the transition region and emit bremsstrahlung primarily from there.

The variation of magnetic field in the corona and chromosphere is important, especially in determining the directivity of the high energy photons. For this we shall treat two different models. In one we assume that \( \text{dln} B/\text{d}t = h_n \) is a constant and call the ratio of the field strength at transition region to that at the top the mirror ratio

\[
\frac{B_T}{B_0} = \exp \left( \frac{s_tr}{h_n} \right). \tag{4}
\]

Below the corona the density increases rapidly (density scale height \( h_n < h_b \)) and the magnetic field variation is negligible. In the second model, following Zweibel & Haber (1983) (see also MacKinnon & Brown 1989 and Miller & Ramaty 1990, or MacKinnon & Brown 1990 for yet another model of field variation), we assume \( B \propto n^2(h_b = v_n) \). Since \( n \) is constant in the corona then \( \text{dln} B/\text{d}t = 0 \) above the transition region, but because of rapid increase of the density below the transition region, effects of field variations will be felt in the chromosphere and below. In this case we define the mirror ratio as the ratio of the field line at the end point of the loop where the calculation is stopped to the field above the transition region. The end point of calculation is chosen to correspond to the stopping depth of the highest energy particles under consideration. This corresponds to a maximum column depth \( N_{max} = N_0 E_{max}^2/(E_{max} + 1) \). For an exponential atmosphere below the transition region

\[
\frac{B_T}{B_0} = \left( \frac{N_{max}}{h_n} \right)^v. \tag{5}
\]

For example, for \( E_{max} = 40 \text{ cm}^{-2} \) \(( \sim 20 \text{ MeV}) \), \( N_{max} = 2 \times 10^{24} \text{ cm}^{-2} \) so that \( B_T = 16 \) for \( n_0 = 10^{10} \text{ cm}^{-3}, h_n = 2 \times 10^8 \text{ cm} \), and \( v = 0.2 \).

2.3. Distribution of Average Values

Given the accelerated electron distribution \( F_\nu(E, \mu) \) as in equations (1) and (2), we may calculate the electron distribution throughout the loop, \( F(E, \mu, s) \), and from this determine the photon flux spectrum by integration over the source volume of the emissivity \( nF_\nu \), where \( \sigma \) is the bremsstrahlung cross section. As discussed previously and in Paper II, most of the emission of hard X-rays comes from the footpoints, where the field is perpendicular to the sun's surface. Hence, the radiation is azimuthally symmetric, depending only on the angle between the line of sight and the outward normal to the surface or the heliocentric angle \( \theta \). For a given choice of model parameters, \( \delta_x, F_T, \chi_0, \) and \( \chi_1 \), we calculate the emitted flux as a function of photon energy \( k \) and angle \( \theta \), which we denote by \( \tilde{J}(k, \theta; \delta_x, F_T, \chi_0) \).

For the power-law electron spectrum, the photon flux \( \tilde{J} \) can be approximated by a power law \( \tilde{J} \propto k^{-\gamma_x} \) over a limited range of energy, where the spectral index \( \gamma_x \) is a function of \( \theta, \delta_x \), and the \( \chi_i \)'s, but not of \( F_T \). Also, the dependence of \( \gamma_x \) on \( \delta_x \) is nearly linear (see Leach & Petrovian 1983), and we may write \( \gamma_x = \delta_x + \Gamma_x \), where \( \Gamma_x \) is a slowly varying function of \( \delta_x \); i.e., \( \delta_x = \delta_x \Gamma_x \delta_x^2 \), \( 1 \leq 1 \). In most cases we will be comparing with the flux integrated above a given energy \( k_x \) (we drop \( \chi_i \) from the arguments for simplification).

\[
J_x(k_x; \theta; \delta_x, F_T) = \int_{\delta_x}^{\infty} \tilde{J}(k, \theta; \delta_x, F_T) d\delta_x = F_T \tilde{J}(\theta; \delta_x), \tag{6}
\]

where we have used the fact that the photon flux \( J_x \) is directly proportional to the flux of accelerated electrons \( F_T \) with energies above \( E_k \) (defined here to be equal to \( k_x/m_e c^2) \). Thus, \( J_x \), the emissivity per unit electron flux, which describes the angular variation, is independent of \( F_T \) and is approximately proportional to \( (\chi_x - 1)^{-1} \).

Hence, the number of flares observed with photon flux \( J_x \), spectral index \( \gamma_x \), and at an angle \( \theta \) is given by

\[
\psi(J_x, \gamma_x, \theta) = \psi(J_x(F_T, \delta_x) \frac{dF_T}{dx} \frac{d\delta_x}{dx} \frac{d\chi_x}{dx} = \frac{g_x(F_T)^e(\delta_x)}{j_x(\theta; \delta_x)^{e(\theta; \delta_x)}} \right) \tag{7}
\]

We note that, when comparing with observations, \( \theta = 0 \) refers to a flare at the center of the solar disk and \( \theta = \pi/2 \) to a flare at the limb. The angular dependence of the distribution is determined by \( f_x, g_x, \) and \( e_x \), which depend on the model parameters such as \( \delta_x, \chi_0, \) and \( \text{dln}B/\text{d}t \).

Ideally one would like to observe the complete distribution \( \psi(J_x, \gamma_x, \theta) \) and through equation (7) obtain the intrinsic distributions \( g_x(F_T) \) and \( h_x(\delta_x) \) and the model characteristics \( j_x(\theta; \delta_x) \). Unfortunately \( \psi(J_x, \gamma_x, \theta) \) is not known, but some integrals of this distribution can be obtained from existing observations.

For example the observed distribution with angle of all flares with X-ray flux \( J_x \) irrespective of their values of \( \gamma_x \), can be compared with the expression

\[
\psi_{\gamma_x}(J_x) = \int_{\gamma_x}^{\gamma_x} d\gamma_x \psi(J_x, \gamma_x, \theta), \tag{8}
\]

where \( \gamma_{x_{\text{min}}} \) and \( \gamma_{x_{\text{max}}} \) (if different from 0 and \( \infty \), respectively) are determined by the instrument response and limitations. Similarly the spectral index and angular distribution of flares with fluxes greater than some threshold value \( J_x_{\text{thresh}} \) is given by

\[
\psi_{\gamma_x}(J_x) = \int_{J_x_{\text{thresh}}}^{\infty} dJ_x \psi(J_x, \gamma_x, \theta), \tag{9}
\]

and the distribution of fluxes and spectral index for all angles is

\[
\psi_{\gamma_x}(J_x, \gamma_x) = \int_0^{\pi/2} d\gamma_x \psi(J_x, \gamma_x, \theta). \tag{10}
\]

From these expressions we can calculate the variation of the average values of various parameters. For example, the variation of the average spectral index with angle is

\[
\bar{\gamma}_x(\theta) = \frac{\int_{\gamma_{x_{\text{min}}}^{\gamma_x_{\text{max}}}} \int_{\gamma_{x_{\text{min}}}^{\gamma_x_{\text{max}}}} \psi_{\gamma_x}(J_x, \gamma_x, \theta) dJ_x d\gamma_x}{\int_{\gamma_{x_{\text{min}}}^{\gamma_x_{\text{max}}}} \int_{\gamma_{x_{\text{min}}}^{\gamma_x_{\text{max}}}} \psi_{\gamma_x}(J_x, \gamma_x, \theta) dJ_x d\gamma_x}. \tag{11}
\]

In the next section we will compare the model values of such quantities with those obtained from observations.
3. FLUX AND SPECTRAL INDEX DISTRIBUTIONS

In this section we consider the distributions of the observed parameters independent of the position of the flares on the sun; \( \psi_d(J_x, \gamma_x) \) in equation (10). As mentioned above, at low X-ray energies the anisotropy of emission is small so that \( J_x \) is determined essentially by \( F_T \) and \( \gamma_x \) by \( \delta_x \). Therefore the distributions of observed X-ray fluxes and spectral indices from HXRBS can be used to obtain the functions \( g_d(F_T) \) and \( h_d(\delta_x) \) as outlined below.

3.1. Hard X-Ray Flux Distribution

The total distribution of X-ray fluxes is obtained by integration of the distribution \( \psi_d(J_x, \gamma_x) \) in equation (10) over spectral indices:

\[
N_d(J_x) = \int_{\gamma_{x \text{min}}}^{\gamma_{x \text{max}}} d\gamma_x \psi_d(J_x, \gamma_x).
\]  

(12)

This is to be compared with the observed frequency of flares, irrespective of their spectral indices or position on the sun. According to Dennis (1985; see also Fig. 3)

\[
N_{\text{HXRBS}}(P) \, dP \propto P^{-1.8} \, dP,
\]  

(13)

where \( P \) denotes the peak HXRBS count rate, which is related to the flux \( J_x \) at the peak of emission. In general \( P = J_x \xi_x(\gamma_x) \), where \( \xi_x(\gamma_x) \) is a function of the spectral index and the HXRBS instrument response. Since the only dependence of \( \psi_d(J_x, \gamma_x) \) on \( J_x \) arises from the dependence of \( g_d \) on \( J_x \) in equation (7), it is clear that \( g_d \) must have a power-law form:

\[
g_d(F_T) \, dF_T = CF_T^{-q} \, dF_T.
\]  

(14)

Substitution of this into equations (7), (10), and (12) yields

\[
N_{\text{HXRBS}}(P) = CP^{-q} \int_{0}^{\infty} \int_{\gamma_{x \text{min}}}^{\gamma_{x \text{max}}} d\gamma_x \, d\gamma_x \, F_T \xi_x(\gamma_x) \xi_{\text{F}}(\gamma_x)^{-1} \frac{1}{1 + \epsilon}.
\]  

(15)

The term in square brackets is independent of \( P \) or \( J_x \), and comparison of this expression with equation (13) indicates that \( q = 1.8 \). It should be noted that earlier analysis of OSO7 data (Datlowe et al. 1974) and balloon data (Lin et al. 1984) show a similar power-law distribution in the flux and not the counts; \( N(J_x) \propto J_x^{-1.9} \).

3.2. Hard X-Ray Spectral Index Distribution

Spectral indices are not known for all of the HXRBS events but only for a subset of strong bursts. The upper histogram (labeled “ALL”) in Figure 1 shows one such distribution. Assuming that this is a complete sample of bursts with \( J_x \) larger than some threshold flux \( J_{x,\text{th}} \) independent of position on the solar disk we compare it with the expected distribution obtained by integration over \( J_x \) of \( \psi_d(J_x, \gamma_x) \) from equation (10).

Using the results from the above analysis we obtain

\[
\Psi_d(\gamma_x) = \frac{CJ_{x,\text{th}}^{-q}}{(q - 1)} \int_{0}^{\infty} \int_{\gamma_{x \text{min}}}^{\gamma_{x \text{max}}} d\gamma_x \, h_d(\delta_x) \xi_x^{-1} \frac{1}{1 + \epsilon}.
\]  

(16)

All of the terms in this expression except for \( h_d(\delta_x) \) are known from model calculations so that comparison of this with the observed distribution can be used to determine the intrinsic distribution \( h_d(\delta_x) \). After some trial and error, we find that for \( a_2^2 = \infty, b_2 = 1 \), the distribution \( h_d(\delta_x) \) given by the dashed line in Figure 1 gives a \( \Psi_d(\gamma_x) \) shown as the upper solid line which is a satisfactory fit to the observed histogram.

It should be noted that the inferred \( h_d(\delta_x) \) is somewhat model dependent (the peak value of \( \delta_x \) varies by 10%-20% for wide ranges of \( z_2^2 \) and \( b_2 \)) and the uncertainties are large at high values of \( \delta_x \) and \( \gamma_x \). This is because the primary cause of the decline of the observed distribution at large values of \( \gamma_x \) is due to the \( (\gamma_x - 1)^{-1} \) dependence in \( J_x \) mentioned in connection with equation (6). Furthermore, the observed sample may not be complete down to a well-specified \( J_{x,\text{th}} \). It is more likely to be complete to a threshold value of peak photon count \( P_{\text{th}} = J_{x,\text{th}} \xi_x(\gamma_x) \). In this case it is necessary to change the variable from \( J_x \) to \( P \) in equation (16) so that, as in equation (14), we must multiply the integrand by \( \xi_x(\gamma_x)^{-1} \). In reality the dashed line in Figure 1 does not represent \( h_d(\delta_x) \), but \( h_d(\gamma_x)^{-1}h_d(\delta_x) \) with \( \gamma_x \approx \delta_x - 1 \) (cf., e.g., Leach & Petrosian 1983).

Clearly we need a more complete and better determined sample with measured spectral indices. However, since in what follows we shall be dealing with average quantities of various parameters and the relative rate of occurrence and fluxes of bursts observed by the HXRBS and GRS instruments, these effects can be ignored to the extent that the functions \( \xi_x(\gamma_x) \) of HXRBS and \( \xi_d(\gamma_x) \) of GRS are qualitatively similar. We shall ignore \( \xi_x(\gamma_x) \) and \( \xi_d(\gamma_x) \) and consider the count rate to be proportional to the total photon flux.

3.3. Distributions of GRS Events

Having derived some approximate spectral index and total flux distributions for accelerated electrons from X-ray observation, we now investigate the implication of these results for emission at higher energies. In order for a burst to be observed by the GRS, it must have a photon flux for energies \( k > k_m \) of 300 keV greater than some threshold value, \( J_{m,\text{th}} \). An event with HXRBS flux and spectral index \( J_x \) and \( \gamma_x \) will have a GRS flux (above \( k_m \)) of

\[
J(k_m) = J_x(k/m)^{\gamma_x - 1},
\]  

(17)
so the threshold will be exceeded for a given $J_x$ if

$$\gamma_x < \gamma_{\text{max}}(J_x) = 1 + \frac{\ln(J_x/J_m)}{\ln(k_m/k_x)},$$  \hspace{1cm} (18)

which decreases with decreasing $J_x$. Similarly, for a given $\gamma_x$, the X-ray photon flux must exceed

$$J_{x,\text{min}}(\gamma_x) = J_{m,\text{th}}(k_m/k_x)^{\gamma_x - 1},$$  \hspace{1cm} (19)

for GRS to observe the flare.

These relations are represented by the lines in Figure 2, which is essentially a plot of $\gamma_x$ versus peak count rate $P$, considering that $J_x \propto P$. We do not know the value of $J_{m,\text{th}}$ however, as it depends on the (unpublished) GRS background count rates and response functions. The solid line assumes $J_{m,\text{th}} = 1$ photon per cm$^2$ per 16 s interval (16 s is the averaging time for the GRS events; see Vestrand et al. 1987), and the two dashed lines are for $J_{m,\text{th}} = 2.5$ and 0.3 photons (cm$^2$ 16 s$^{-1}$), respectively. Flares lying below these lines satisfy the above inequalities and should have been detected by the GRS. As can be seen, most but not all flares satisfy this criterion. There are six events below the solid line which are not detected and eight above the line which are detected. These discrepancies may be explained by time and angular dependences or by spectral deviations from pure power law assumed above. If a flare spectrum flattens for energies above $k_m$, then it will be detected even if it lies above the line, and vice versa. Let us now compare the observed and expected distributions of X-ray spectral index $\gamma_x$ and flux $J_x$ for the observed events by both instruments.

Let $N_{\text{GRS}}(J_x)$ be the number of events with X-ray photon flux $J_x$, but which are also observed by GRS. Then

$$N_{\text{GRS}}(J_x) = C J_x^{\alpha} \int_{\gamma^{\text{min}}(J_x,J_{m,\text{th}})}^{\gamma_{\text{max}}(J_x,J_{m,\text{th}})} \frac{d\gamma_x}{1 + \epsilon}.$$

Since $\gamma_{\text{max}}$ as given by equation (18) decreases with decreasing $J_x$, the bracketed term also decreases with decreasing $J_x$, and the distribution $N_{\text{GRS}}$ falls away from the power law. This is illustrated in Figure 3, in which we plot the integral distribution $\int_{J_x}^{J_{\text{GRS}}(J_x)} dJ_x$. (Actually we plot the distribution in terms of HXRBS peak count rate $P$, as observed.) The observational results are given as a histogram. The solid and dashed lines in Figure 3 correspond to $\gamma_{\text{max}}$ for the same values of $J_{m,\text{th}}$ used in Figure 2. The plus signs give the power-law distribution of all HXRBS events, with slope $1 - q \approx -0.8$.

We can also calculate the HXRBS spectral index distribution for the events common to both instruments. This is given by an expression identical to equation (16) except with the lower limit to the flux $J_x$ replaced by $J_{x,\text{min}}$ from equation (19):

$$\Psi_{\text{GRS}}(\gamma_x) \propto \Psi_x(\gamma_x) \frac{k_x}{k_m} (q - 1)(\gamma_x - 1).$$  \hspace{1cm} (21)

This is plotted in Figure 1, where the lower histogram (labeled GRS) is the observed distribution of X-ray indices for the common events. The curve overlying it is the calculated distribution $\Psi_x(\gamma_x)$ for our standard model with isotropic injection and uniform field ($a_0 = \infty, b_T = 1$). Note that, as $\gamma_x$ increases, $\Psi_{\text{GRS}}$ decreases much more rapidly than $\Psi_x(\gamma_x)$ (curve labeled ALL), due to the factor $(k_x/k_m)^{\gamma_x - 1}$.

4. ANGULAR AND C-T-L VARIATION

Until now we have considered the observed distributions of HXRBS fluxes and spectral indices of events seen by HXRBS alone and jointly by HXRBS and GRS without any considerations of position of the flares on the Sun or angle $\theta$. We have, however, obtained from this analysis the distributions of the electron fluxes and spectral indices, which can now be used to describe the expected angular variation of the distributions or the average values of these quantities.

---

**Fig. 2.**—HXRBS spectral index $\gamma_x$ vs. peak count rate $P$, for events in 1980 with $P > 1000$ counts s$^{-1}$. The circles represent events seen only by HXRBS and crosses represent the events observed by both HXRBS and GRS. The solid line is the curve for $\gamma_{\text{max}}(P)$ for a 300 keV threshold of $J_{m,\text{th}} = 1$ photon s$^{-1}$ cm$^{-2}$ MeV$^{-1}$. The two dashed lines take into account the uncertainty in the calculation of the 300 keV flux from the observed $P$ and $\gamma_x$ and represent $J_x = 0.4$ and 2.5 photons s$^{-1}$ cm$^{-2}$ MeV$^{-1}$ for the upper and lower lines, respectively.

**Fig. 3.**—The cumulative distribution of HXRBS flares $N (> P)$ vs. peak HXRBS count rate $P$. The crosses give the distribution of all HXRBS events, which can be fitted to a power law with index $-0.8$ (see Dennis 1985). The histogram shows the distribution of flares $N_{\text{GRS}}$ ($> P$) observed by both HXRBS and GRS which deviates from the power law at low $P$. The solid and the dashed curves are model results calculated using eq. (20) and the dashed curves are model results calculated using eq. (20) and the three values of $J_{m,\text{th}}$ used in Fig. 2.
4.1. Angular Variation of Number of Events

4.1.1. Hard X-Ray Energies (30–300 keV)

The number of flares at a given angle with hard X-ray fluxes greater than the HRXBS flux threshold $J(x,\theta)$, irrespective of their spectral index is obtained from integration over $J_x$ of equation (18) or over $\gamma_x$ of equation (19):

$$N_{J(x,\theta)}(\theta) = \frac{C J_x^{1/\alpha}}{(q - 1)} \left[ \int_{\gamma_{\min}}^{\gamma_{\max}} (d\gamma_x h_x(\gamma_x)) \frac{\gamma_x^{q-1}}{1 + e} \right].$$  \hspace{2cm} (22)

The functions $j_x$ and $e$ vary slowly with $\delta_x$, and the variation with $\theta$ of $e$ is also negligible. Therefore, the angular distribution in equation (22) is roughly proportional to $J_x^{1/\alpha}$. From Leach (1984) or Paper II, $J_x$ at 30 keV varies by approximately a factor of $-3$ from $\theta = 0^\circ$ to $90^\circ$, and since $q - 1 \approx 1$ the number distribution should vary by the same factor across the disk. A careful analysis of the observations may reveal such a variation. To our knowledge no such study has been carried out for the HRXBS events. But Vestrand et al. (1987) (see their Fig. 7) show that the number of events observed by the low-energy (25–200 keV) detector associated with GRS shows some C-T-L variation. The histogram in the bottom panel of our Figure 4 shows this angular variation in three bins bounded by $\theta = 0^\circ$, $30^\circ$, $60^\circ$, and $90^\circ$, where we also show the expected variation from equation (21) for $a_\theta = \infty$ and $b_\theta = 1$ model. As evident the observed variation agrees with model calculation but is not significant to allow us to distinguish between models. If following Vestrand et al. (1987), we define the “disk” to include angles from $\theta = 0^\circ$ to $60^\circ$ and the “limb” the angles $\theta = 60^\circ$ to $90^\circ$, then for isotropic emission the ratio of the expected numbers of events $N_{\text{disk}}/N_{\text{limb}} = 2$. The observed ratio from the histogram in Figure 4 is 1.7, while that calculated for our standard model is about 1.3. There are, however, other factors which are not included in this analysis but may contribute to the isotropization of low energy emission, such as Compton backscatter or thermal emission. These factors will increase the ratio toward the isotropic value of 2.

4.1.2. Medium Energies (300 keV–1 MeV)

At higher photon energies we expect a larger deviation from uniform angular distribution because the angular variations of $j$ and $e$ increase with increasing energy. Following the procedure described above, we can calculate the expected distribution for these energies as well. The results from this are shown in the middle panel of Figure 4; the histogram is the observed number of GRS events (taken from Fig. 5 of Vestrand et al. 1987), and the dashed line is the expected distribution assuming the model with uniform field and a beam distribution with $a_\theta = 0.4$. As compared to above case there is more anisotropy, but again not sufficiently strong to constrain model parameters.

4.1.3. Ultrarelativistic Energies (>10 MeV)

As mentioned in the Introduction, there is a strong C-T-L variation for flares with this kind of emission. This is not surprising considering the large anisotropy in the radiation expected for ultrarelativistic energies. (See Petrovian 1985 or Paper II). This effect has also been analyzed by MacKinnon & Brown (1989, 1990) and Miller & Ramaty (1990) who, however, limited their discussion to the directivity of the flux at such high energies. In this regard their results and those of ours presented in Paper II are in agreement to the extent that model parameters overlap. However, as in Paper II the above authors only discuss the directivity of the flux and do not consider the effects of the dispersion in the flare distributions. Here, as in the two parts above, we include these effects properly.

The expected and observed distributions for flares with emission $>10$ MeV are shown in the top panel of Figure 4. The solid line shows the expected distribution for two beam models with $a_\theta = 0.40$, $b_\theta = 1$ and $a_\theta = 0.04$, $b_\theta = 2.5$. The histogram is the observed distribution. In spite of the fact that the data show a strong anisotropy, it is difficult to set rigid constraints on the model parameters because the number of events is very small. More data and more rigorous statistical analysis are needed.

A more rigid set of constraints can be obtained from the observed C-T-L variation of average spectral indices at energies between 300 keV and 1 MeV discussed below. The models used in Figure 4 are consistent with those below.

4.2. Angular Variation of Spectral Index

Given the above distributions, we can compare the angular variation of spectral index of flares with observations. However, there is not a sufficiently large body of data for such a comparison. Instead we calculate moments of these distributions and compare with observations the variation of these moments with $\theta$, the heliocentric longitude. As pointed out in the Introduction, one must choose the moment for this analysis with proper accounting of the distribution and the threshold of the detector. Because the distribution of fluxes extends as a power law to low fluxes without an apparent
cutoff, there will be no C-T-L variation of the average flux. But as shown in Figure 1 the spectral index distribution has finite range and therefore the average spectral index is more likely to show C-T-L variation. Since directivity of the bremsstrahlung radiation becomes pronounced at higher energies, the C-T-L variation in the hard X-ray regime is expected to be small.

To our knowledge there has been no systematic analysis of the HXRBS data of this effect. The analysis of results from the hard X-ray detector of GRS, which shows some C-T-L variation which is not statistically significant according to Vestrand (1988). We therefore consider the C-T-L variation of the average spectral index as observed by the GRS in the 300 keV to 1 MeV range, which we denote by subscript m:

\[
\bar{y}_m(\theta) = \frac{\int_{y_{\text{min}}}^{y_{\text{max}}} dJ_m y_m \psi_{y_m}(y_m, \theta)}{\int_{y_{\text{min}}}^{y_{\text{max}}} dJ_m \psi_{y_m}(y_m, \theta)}
\]

(23)

where, as in equation (9), the distribution

\[
\psi_{y_m}(y_m, \theta) = \int_{J_{m,0}}^{\infty} dJ_m \psi(J_m, y_m, \theta)
\]

(24)

In Figure 5 we show the expected variation of spectral index for the \(h_2(\theta)\) distribution derived above from the HXRBS data (Fig. 1) and for models with isotropic pitch angle injection \((\alpha_0 \to \infty)\) but with various values of field convergence \(b_T\). The small crosses show the data for individual events taken from Table 1 of Vestrand et al. (1987), and the four larger crosses (with vertical error bars) show the average values in four bins (with roughly equal numbers of events per bin) with boundary at \(\theta = 0^\circ, 35^\circ, 60^\circ, 80^\circ,\) and \(90^\circ\). Although the model results show the same trend as the observations [e.g., \(y_{\text{m}}(\text{disk}) = 0.53\), vs. the observed \(0.38 \pm 0.11\)], the absolute values of \(y_{\text{m}}\) are significantly larger at all angles. This means that the observations show spectral flattening at high energies beyond that expected from the models with simple power-law extrapolation.

This discrepancy is also evident from the limited sample of events for which we have both the HXRBS and GRS spectral indices \(y_{\text{GRS}}, y_{\text{HXRBS}}\). Figure 6 shows the variation of the difference between the two spectral indices \(\Delta y = y_{\text{GRS}} - y_{\text{HXRBS}}\) versus \(\theta\) for these few events. As is evident, most of the points are below the \(\Delta y = 0\) line, and there is no evidence for angular dependence of this difference. Similar results were obtained by Vestrand et al. (1987) using the average spectral index for the 25–200 keV range from the GRS hard X-ray detector. The model predictions are given by solid lines. We have explored a large range of possible model parameters and found that no model can give a value of \(\Delta y\) as large as observed, especially near the center of the disk.

Spectral flattening at high energies is also observed in some flares where detailed analysis of the spectrum is carried out. These evidences seem to be an indication of the existence of an intrinsic break in the electron spectrum rather than a result of the transport or radiation processes. Such a spectral hardening may be due to the presence of so-called superhot (Lin et al. 1981) thermal electrons or intrinsic to the acceleration mechanism of the nonthermal component. The photon energy for the break is not known and is most likely variable. For the purpose of illustration we shall assume that it occurs at energy \(E_{\text{m}} \approx 300\) keV, and that the electron spectral index changes from \(\delta_2\) below \(E_{\text{m}}\) to \(\delta_\text{m}\) above \(E_{\text{m}}:\n
\[
F_0(E, \varepsilon) = \left[(\delta_2 - 1)/E_0\right]F_T G(\varepsilon) \times \begin{cases} (E/E_0)^{-\delta_2} & \text{for } E_0 \leq E \leq E_{\text{m}} \\ (E_{\text{m}}/E_0)^{-\delta_\text{m}}(E/E_\text{m})^{-\delta_\text{m}} & \text{for } E_{\text{m}} \leq E < \infty \end{cases}
\]

(25)

with an average difference between \(\delta_2\) and \(\delta_\text{m}\) equal to the average value of \(\Delta \gamma \approx 1\). This also means that the distribution
\[ \psi_s (eq. [3]) \text{ is more complicated including the distributions of } \delta_s \text{ and } \delta_m : \]
\[ \psi_s (F_T, \delta_s, \delta_m) = CF_s^{-4} h(\delta_s, \delta_m), \]  
(26)
where we have again assumed a power law dependence for \( F_T \).

This distribution can be converted to a distribution of radiation parameters as before except with the added complication of two separate spectral indices: \( \psi(\delta_s, \delta_m, \gamma_x, \gamma_y, \theta) \), where \( \gamma_x \) and \( \gamma_y \) are related to \( \delta_s \) and \( \delta_m \) and \( J_1 \) is proportional to \( F_T \) and an angular dependence \( j(\theta, \delta) \) whose detail depends on the model and the energy range of photons under consideration. The evaluation of other distributions and moments of these distributions involves an extra integration over the additional parameter.

It is clear that the added degree of freedom will not affect the conclusions reached from HXRBs data, in particular about the power-law dependence of \( q_s(F_T) \) and the spectral index distribution \( h_s(\delta_s) \). Similarly, the results on the subset of HXRBs events which are also observed by GRS will be affected to an insignificant degree—the main effect being the presence of higher fluxes in the gamma-ray range. This effect, however, is independent of the angle \( \theta \) and all the other parameters determined by HXRBs observations.

The main question which now arises is what choice of \( h(\delta_s, \delta_m) \) are we to make. We could simply assume that \( \delta_s \) and \( \delta_m \) are strongly correlated with a mean value difference \( \langle \delta_f \rangle \approx 1 \), so that \( h(\delta_s, \delta_m) = h(\delta_s)h(\delta_m) \), with \( f \) a narrow function of its parameter; for example, a \( \delta \) function. The extant data are too meager to dictate a definite choice here, but what is shown in Figure 6 does not support this hypothesis. There is a considerable dispersion and no obvious correlation. Therefore, we shall assume no correlation between \( \delta_s \) and \( \delta_m \): \( h(\delta_s, \delta_m) = h(\delta_s)h(\delta_m) \), and, in the absence of any definite evidence to the contrary, we use a Gaussian form for \( h_m(\delta_m) \):
\[ h_m(\delta_m) = e^{-(\delta_m - \delta_m)^2/\Delta_m^2}. \]  
(27)

Below we shall present results as obtained from the above distribution with \( \delta_m = 0.4 \), based on the results from Figure 4 of Vestrand et al. (1987) with values of \( \delta_m \), which will depend on the model parameters \( \alpha \) and \( \beta_s \). We have carried out this same analysis for the case in which \( \delta_s \) and \( \delta_m \) are correlated \( \langle \delta(x) \rangle \rightarrow \langle \delta(x) \rangle \) and find similar results.

We can now follow the same procedure as before with equations (1) and (3) replaced by equations (25) and (26).

The gamma-ray flux for energies \( k > k_m = E_m mc^2 \) then is given by (see eq. [6])
\[ J_m = J_m(\theta; \delta_m) = F_T \frac{\delta_s - 1}{\delta_m - 1} \left( \frac{k_s}{k_m} \right)^{\delta_s - 1} J_m(\theta; \delta_m), \]  
(28)
and as in eq. (7) the distribution of observed flares by
\[ \psi(J_m, \gamma_s, \gamma_y, \theta) = \frac{C_j m(\theta; \delta_m)^{b - 1}}{J_m^b} \left[ \frac{\delta_s - 1}{\delta_m - 1} \left( \frac{k_s}{k_m} \right)^{\delta_s - 1} \right]^{b - 1} \]
\[ \times h_s(\delta_s)h_m(\delta_m) \frac{h_m(\delta_s)}{[1 + \epsilon(\theta; \delta_s)][1 + \epsilon(\theta; \delta_m)]}. \]  
(29)

We first point out that this change will not alter the results in Figure 4. The C-T-L variation of numbers of flares observed not only at X-ray energies, but at very high (>10 MeV) energies will not be drastically affected by the added feature. As we saw before (eq. [22]) most of the variation comes from the factor of \( j(\theta, \delta) \), the anisotropy of emission which is not very sensitive to the value of the spectral index \( \delta \). The other effects arising from the integration over spectral index are minor, especially since, compared to \( h_s(\delta) \), the assumed distribution \( h_m(\delta_m) \) is narrower. (Compare Figs. 4 and 6 of Vestrand et al. 1987).

The primary effect of the proposed break in the electron spectrum is to change the value of the average spectral index for photon energies greater than \( k_m \). We shall not reproduce the complicated equation which gives us the expected variation of average spectral index with angle \( \theta \). The details can be found in McTiernan (1989). The procedure is identical to that in equations (23) and (24) except with an added integration over \( \gamma_m \) or \( \delta_m \). In fact, since our assumed distribution \( h_m(\delta_m) \) is narrower than \( h_s(\delta_s) \) derived in previous section, the integration becomes simpler as we can extend the limits of integration to \( \pm \infty \) for \( \delta_m \) and integrate over electron fluxes \( F_T \) from
\[ F_T = \frac{J_m}{J_m(\theta; \delta_m)} \frac{(\delta_m - 1)}{(\delta_s - 1)} \left( \frac{k_m}{k_s} \right)^{\delta_s - 1} \]  
(30)
to infinity rather than over photon fluxes from \( J_m(\theta; \delta_m) \) to \( \infty \) as was done in equation (23). The results from this calculation are presented next.

4.3. Comparison with Observations

In Figure 7 we have plotted the calculated values for the average spectral index \( \gamma_s \) versus \( \theta \) for 20 models with magnetic

![Fig. 7.—Model results for the 0.3-1 MeV spectral index vs. heliocentric angle for constant d in B/ds models. Each set of curves gives \( \gamma_s(\theta) \) for a single value of the mirror ratio, for \( b_s = 1, 2.5, 10, \) and 25, and each is shifted upward from the last by 0.8. Each curve is labeled by the width of the injected distribution: \( \sigma_s = 0.04b, 0.40b \) for beam distributions (\( \alpha = 0 \)) and \( \sigma_s = 0.04b, 0.40b \) for pancake distributions (\( \alpha = \pi/2 \)). The isotropic distribution is labeled as 0.00. The points with error bars are the average values of the observed spectral index in the four bins of Fig. 5.](image-url)
field geometry such that \( d \ln B/ds \) is a constant. Each set of curves gives \( \gamma_m(\theta) \) for a single value of the mirror ratio, for \( b_T = 1, 2.5, 10, \) and 25, shifted upward as indicated in the legend. Each curve is labeled by the width of the injected distribution; \( \alpha_0^2 = 0.04(b) \) and 0.40(b) are beamed distributions, 0.04(p) and 0.40(p) are pancake distributions, and the isotropic injection is denoted here by 0.00. The points with error bars are the average values of the observed spectral index in the four bins of Figure 5. The value of \( \delta_m \) in equation (27) varies from model to model, and is chosen so that the calculated value of \( \gamma_m \) on the limb is equal to the observed value. These values are given in Table 1. As mentioned in connection with Figure 1, \( h(\delta, \alpha) \) changes slightly from model to model. We have included these variations in our calculations. In Figure 8 we make the same comparison for models with \( B \propto n'_v \), the "trap" models.

We can use these figures to put constraints on the values of \( b_T \) and \( \alpha_0^2 \). We shall use the uncertainties in the average values to determine the acceptable range for these parameters.

It is evident from these figures that pancake models can be rejected; the variation with angle turns out to be too large. For beamed injection, the range of values which fits the observations is similar for both field geometries for low values of mirror ratio \( b_T \). We cannot distinguish between models which have \( d \ln B/ds = \) constant or \( B \propto n' \) for \( b_T \leq 5 \). Rapidly converging field geometries \((b_T \geq 5)\) are also excluded by this comparison. In general, for a given \( b_T \) a range of \( \alpha_0^2 \) will match the observations and for a given \( \alpha_0^2 \), there is a range of \( b_T \) which fits the observations. However, there is no range of \( b_T \) for which isotropic injection fits the observations and no range of \( \alpha_0^2 \) for high convergence \( b_T \geq 5 \) that is acceptable. We require mild convergence and some beaming.

In Figure 9 we present our results for models with constant \( d \ln B/ds \) as a plot of \( \alpha_0^2 \) versus \( b_T \). The ordinate shows log \( b_T \) and the abscissa is \( \mp \log (\alpha_0^2/4) \), respectively, for the two panels: the upper panel for beam distributions and lower panel for pancake distributions (with most highly beamed or flattened distributions at the top and bottom, respectively). The two panels are joined by the isotropic distribution. Strictly speaking, this corresponds to \( \alpha_0^2 = 1 \), but in actual calculation the central line represents all values \( \alpha_0^2 > 1 \) (isotropic) for both cases. The range of values bounded by the two solid lines fits the observed variation of \( \gamma_m \). In this logarithmic plot, these lines are relatively straight and can be approximated as \( \alpha_0^2 b_T^{-0.0} = \) constant. The acceptable range at \( b_T = 1.0 \) is \( \alpha_0^2 \approx 1.0-0.4 \) so that we may write

\[
\alpha_0^2 \approx (7.0 \pm 3.0)b_T^{-0.0} ,
\]

for the best-fit models.

\[
\begin{array}{c|cccccc}
\hline
\hline
\alpha_0^2/b_T & 1.0 & 2.5 & 10.0 & 25.0 & 2.5 & 5.0 \\
\hline
0.04(b) & 2.4 & 2.8 & 3.3 & 3.4 & 3.3 & 3.7 \\
0.40(b) & 2.7 & 3.3 & 3.4 & 3.4 & 3.7 & 3.8 \\
\infty & 3.1 & 3.3 & 3.4 & 3.4 & 3.7 & 3.8 \\
0.40(p) & 3.2 & 3.4 & 3.5 & 3.5 & 3.7 & 3.8 \\
0.04(p) & 3.5 & 3.4 & 3.5 & 3.5 & 3.7 & 3.8 \\
\hline
\hline
\end{array}
\]

Note.—The distribution \( h_\alpha \) is given by eqn. (27). There are six different field structures, uniform field \((b_T = 1)\), three "precipitation" (constant \( d \ln B/ds \)), and two "trap" \((B \propto n'_v)\) models, and five values of \( \alpha_0^2 \). Beam injection is denoted by a (b), and pancake injection is denoted by a (p).

**FIG. 8.**—Same as Fig. 7 except for models with \( B \propto n'_v \) for \( b_T = 2.5 \) and 5.0

**FIG. 9.**—The results of fitting the calculated values of \( \gamma_m(\theta) \) and \( R = \int J d\omega \) to observations presented as a contour plot of \( \mp \log (\alpha_0^2/4) \) vs. \( b_T \) for the models with \( d \ln B/ds = \) constant. The upper half of the plot is for beam injection, and the lower half is for pancake injection, with isotropic injection in the middle. The area inside the solid lines gives the range of values of \( \alpha_0^2 \) and \( b_T \) which fit the observed values of \( \gamma_m \). The area inside the dashed lines gives the range which fits the observation of \( R \).
4.4. Angular Variation of Flux Ratios

An alternative way of analyzing the data is to consider the variation of ratio of X-ray flux ($k_e = 30\,\text{keV}$) to gamma-ray flux ($k_e = 1\,\text{MeV}$) as was done by Bai (1988). As shown by Bai, there is significant C-T-L variation of the ratio $R(\theta) = J_{\gamma}/J_x$ with the average value of this ratio increasing toward the limb as one would expect from the spectral flattening described above. Given the above distributions, it is straightforward to calculate the expected variation with angle $\theta$ of the average value of this ratio. The primary factor is the difference in the dependence on $\gamma(\theta; \delta \gamma)$ and $j_x(\theta; \delta \gamma)$ with different weights for different spectral indices. The integration leading to the final results is cumbersome and can be found in McTiernan (1989). We shall not reproduce these details here but present the final results.

The ratios expected for various models are shown in Figure 10. The mean of the observed value of this ratio is plotted for the same four bins used in Figure 8. The results here are very similar to those found from the analysis of spectral index data, except that a wider range of $\delta \gamma$ and $b_T$ are allowed. These ranges are shown by dashed lines on Figure 9 showing that a larger area (between the lines) is acceptable in the beam case. There exists also a small region in the lower right-hand corner which could be acceptable. But this range has perhaps an unrealistically high magnetic field convergence and, of course, requires the questionable pancake distribution. In the beam case, however, the two results are consistent.

5. DISCUSSION AND SUMMARY

We have analyzed the observed distribution of fluxes and spectral indices of X-ray and gamma-ray flares, in particular, the center-to-limb (C-T-L) variation of these distributions and their moments. We compare these observations with those expected from nonthermal models whereby accelerated electrons with power-law spectra and Gaussian pitch angle distribution are injected at the top of a coronal loop with converging field geometry. From these comparisons we constrain the value of the parameters such as spectral index, width of the pitch angle distribution, field convergence rate, etc.

The C-T-L variation arises from the anisotropy of the bremsstrahlung radiation which is determined by a combination of the field geometry and pitch angle distribution of the accelerated electrons. At low (hard X-ray) energies this variation is small and often not noticeable. Therefore, we have used the flux and spectral index distribution of all HXRBS events to determine the electron flux and spectral index distribution. Using these we have shown that the distribution of subset of the HXRBS events which are also detected by the GRS instrument, irrespective of the angular position on the Sun, is consistent with the model.

We then extend the model calculation to the gamma-ray regime where, as stressed earlier (Petrosian 1985; see also Paper II), we expect considerable emission anisotropy. This gradual increase in anisotropy manifests itself in the variation with the heliocentric longitude of rate of occurrence of flares in different energy bands, with little variation at X-ray energies but with a dramatic concentration of flares with $>10\,\text{MeV}$ emission near the solar limb. We have shown that our basic model describes this data quite well, but we were not able to constrain the model parameters significantly. This is because most of the constraint comes from consideration of flares with $>10\,\text{MeV}$ emission which are very few in number.

A more rigorous test of the models comes from consideration of C-T-L variation of the spectral indices in the gamma-ray range. The general trend of observed spectral hardening toward the limb is a direct consequence of the transport effects and radiation pattern described in Paper II. However, here we find that if we assume simple power-law electron spectra extending from 20 keV to 1 MeV and higher, then the absolute values of the observed spectral index in the gamma-ray region (GRS) are smaller (spectra are flatter) than expected from the models at all angles. We have shown that transport effects, or effects due to field geometry cannot cause such a rapid flattening of the photon spectrum if there is no corresponding flattening in the electron spectrum.

We, therefore, modify the basic model and assume electron spectra with a break at around 300 keV and repeat the analysis of C-T-L variation. We show that the C-T-L variation of the average gamma-ray spectral index is fairly sensitive to two of our model parameters. These are the width of the Gaussian pitch angle distribution, $\delta \gamma$ and the degree of convergence of the magnetic field lines which we represent by the mirror ratio $b_T$. Figure 9 shows the outcome of this analysis where from consideration of the uncertainties in the observed spectral indices we show the acceptable range of the values for these two parameters, which is bounded by the two solid lines. This range is independent of whether we use the $d \ln B/\,\,ds = \text{constant field geometry}$ or $B \propto n^\alpha$ field variation (chromospheric convergence) and clearly rules out highly converging field geometries and accelerated electrons with initial momentum mostly perpendicular to the field.

We have carried out the same analysis for the ratio of...
gamma-ray (GRS at 1 MeV) to X-ray (HXRBS above 30 keV) fluxes. Comparison of the observed C-T-L variation of the quantity with model results also requires a similar spectral flattening and sets similar constraints on the range of model parameters. These data are more obliging and allow a wider range (region between the dashed lines in Fig. 9) of values for $b_T$ and $\alpha_c$.

Our findings are important for consideration of acceleration mechanism because the mechanism must incorporate the spectral break and the pitch angle distribution dictated by these observations. More data, especially at gamma-ray energies and possibly a more thorough analysis of the existing data from HXRBS in regard to C-T-L variation, can provide further constraints on the models and the acceleration mechanism.

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