ABSOLUTE MAGNITUDES AND KINEMATIC PROPERTIES OF CEPHEIDS

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ABSTRACT

A maximum likelihood statistical parallax analysis of classical Cepheids has been performed to determine the relative solar motion, Oort constants, velocity ellipsoid parameters, and zero points of the PL and PLC relations. The analysis is based upon 90 proper motions drawn from the list of Karimova and Pavlovskaya and upon the analytical approach of Hawley et al. Our results give a best estimate for the mean absolute magnitude of Cepheids at log P = 0.8 of \( \langle M_r \rangle = -3.46 \pm 0.33 \) mag. This estimate for the Cepheid absolute magnitude zero point is highly stable against refinements in the mathematical technique and against additional Cepheid proper motion data of quality similar to the existing proper motions. Improvement in this value will likely come only from a marked improvement in the quality of the Cepheid proper motions.

We also determine solar motion components along the axes (x, y, z) of (−6.8, −14.3, −10.0) km s\(^{-1}\) with uncertainties less than \( \pm 2 \) km s\(^{-1}\). The velocity ellipsoid components on the same axes are (11.9, 9.9, 2.1) km s\(^{-1}\) with similar uncertainties. We obtain values for the Oort constants of \( A = 14.5 \pm 2.4 \) km s\(^{-1}\) kpc\(^{-1}\) and \( -5.9 \pm 3.3 \) km s\(^{-1}\) kpc\(^{-1}\), although we suggest that the value of \( B \) is probably too positive because of limitations in the proper motion data base.

Subject headings: stars: Cepheids — stars: luminosities — stars: stellar dynamics

1. INTRODUCTION

Cepheid variables are important standard candles for determining Galactic and extragalactic distances because they are luminous, they are easily discovered, and their luminosities are strongly correlated with pulsation period. The practical utility of Cepheids as distance indicators depends upon the accuracy of the period-luminosity relation (de Vaucouleurs 1978), and so it is not surprising that calibrating this relation has occupied researchers for nearly 80 years (Fernie 1969).

There are at present only three independent means of establishing the period-luminosity (PL) and period-luminosity-color (PLC) relations: (1) by main-sequence fitting of Galactic clusters containing Cepheids to the Hyades or Pleiades distance scales (viz., Feast & Walker 1987), (2) by the surface brightness technique (Gieren 1988; Barnes, Gieren, & Moffett 1990), and (3) by the method of statistical parallaxes (Wielen 1974; Clube & Dawe 1980; Karimova & Pavlovskaya 1981b). Both the main-sequence fitting method and the surface brightness method have included assumptions which could lead to systematic biases (see Schmidt 1984; Barnes 1980). On the other hand, the method of statistical parallaxes is a direct application of well-understood physics to the observations. The analysis models the space motions and distances of the stars to reproduce the observed radial velocities, proper motions, and apparent magnitudes. The statistical parallax method gives a distance scale which is independent of the other two scales and therefore provides an important check on the assumptions used in those methods.

We present here a statistical analysis of Cepheid motions and distances which improves upon previous efforts. It is quite similar to the analysis done on RR Lyrae variables by Hawley et al. (1986, hereafter HJBW). HJBW used a maximum likelihood model for the motions of the stars and solved for the model parameters using the numerical minimization technique of simplex optimization. Their maximum likelihood model is a more rigorous formulation of the problem than that used previously (Clube & Dawe 1980). In addition, simplex optimization solves for all the parameters simultaneously and is well suited to solving nonlinear problems of this type where there is strong interdependence among the parameters.

In the next section we discuss the sources for the data used in the analysis. In § 3 we described our analytical model and the method we used for solving it. Section 4 gives our results, and the final section discusses these results in the context of current understanding of Cepheid variables.

2. DATA

The data necessary for the statistical analysis are the Galactic location and velocity of each star in the sample. The Galactic position is determined from the celestial coordinates and an adopted distance, which is calculated from the apparent magnitude, reddening, and an absolute magnitude estimate. The three components of velocity are found from the radial velocity and the two orthogonal components of proper motion which, with the adopted distance for the star, give velocity components in the plane of the sky.

2.1. Proper Motions

The small space velocities of Cepheids relative to the Sun and their generally large distances make it difficult to measure Cepheid proper motions with precision. As a result, the availability of dependable proper motion measurements is the limiting factor in the size and quality of the statistical parallax sample. The most extensive set of Cepheid proper motions has been published by Karimova & Pavlovskaya (1981a, hereafter KP). They list proper motions and individual probable errors for 97 Galactic Cepheids. We have eliminated seven stars from
The kinematic properties of Cepheids are studied, focusing on their right ascension and declination components. A comparison is made with previous studies by Widen (1974) to assess the consistency of the KP proper motions with a previous sample used for Cepheid statistical parallax analysis.

Fig. 1—Comparison of the KP and Widen proper motion scales in right ascension.

Fig. 2—Comparison of the KP and Widen proper motion scales in declination.

Fig. 3—Comparison of the KP and Widen proper motion uncertainties in right ascension.

Four Cepheids are included from the KP list: Y Aur, RU Cam, TX Del, and YS Ari. Their proper motion components are tabulated in declination. RU Cam and TX Del are variables, while the others lack radial velocity data.

As a check on the consistency of the KP proper motions, the Widen (1974) compilation of 45 Cepheids is compared with the KP data set. The comparison shows a small systematic difference, with KP values being smaller by an average of 0.000746 ± 0.000708 (s.e.) per year. The difference in R.A. components is 0.000315 ± 0.000708 (s.e.) per year, with KP again being smaller. It is clear that there is no significant difference between the two studies.

In the maximum likelihood method, the assignment of a proper weight to each datum is crucial. As the weights are based on the uncertainties of the data, a correct understanding of the error sources is important. The probable errors given by KP are solely internal uncertainties. They do not include the uncertainty of transformation to the FK4 system, the uncertainty internal to the FK4, or the effect of the uncertainty in the precessional constant on the proper motions. These uncertainties are relatively large and important to include in the weights. Widen (1974) has discussed these uncertainties in detail, and we have followed his prescription for incorporating them, after converting KP's probable errors to mean errors.

Center-of-mass radial velocities for 48 stars of the proper motion sample have been taken from Moffett & Barnes (1987) and, for the 42 stars not listed there, from Caldwell & Coulson (1987). Moffett & Barnes show that these two data sets are in good agreement with each other, so taking velocities from both sources is justified.

2.2. Radial Velocities

Center-of-mass radial velocities for 48 stars of the proper motion sample have been taken from Moffett & Barnes (1987) and, for the 42 stars not listed there, from Caldwell & Coulson (1987). Moffett & Barnes show that these two data sets are in good agreement with each other, so taking velocities from both sources is justified.

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sets should not introduce additional errors into the results. The uncertainty for each center-of-mass radial velocity has been adopted as $\pm 1 \text{ km s}^{-1}$ following the discussion by Moffett & Barnes.

Both of these sources suggest a systematic drift in the center-of-mass Cepheid radial velocities with respect to the local standard of rest of unknown origin amounting to $-3 \text{ km s}^{-1}$. (Wielen 1974 gives a nice discussion of the possible origins of this well-known K-term.) We have removed the K-term by adjusting each velocity by $+3 \text{ km s}^{-1}$. This adjustment has a negligible effect on our results.

2.3. Additional Data

The intensity mean $\langle V \rangle$ magnitudes and $\langle B \rangle - \langle V \rangle$ colors for 62 stars in our sample were taken from Moffett & Barnes (1985), and for the 28 stars not listed there, from Schaltenbrand & Tammann (1971). The difference in the $\langle B \rangle - \langle V \rangle$ scales of Moffett & Barnes and of Schaltenbrand & Tammann, as discussed in the former reference, is insignificant in the present context.

Color excesses were adopted from Fernie (1990a) using the values $E(B-V)_{\text{clus}}$ given in his Table 1. [Only one star, VX Pup, was not included in Fernie's list. We estimated its color excess by subtracting its intrinsic $\langle B \rangle - \langle V \rangle_0$, obtained from Fernie's 1990b period-color relation, from the observed $\langle B \rangle - \langle V \rangle$ given by Moffett & Barnes 1985.] Figure 5 shows the distribution of $\langle B \rangle - \langle V \rangle_0$ colors for our sample of Cepheids. This distribution is typical for the catalogued Cepheids in our galaxy.

Apparent magnitudes were corrected for interstellar extinction using estimates of the extinction computed from

$$A_v = R E(B-V), \quad (1)$$

where

$$R = 3.15 + 0.25 \langle B \rangle - \langle V \rangle_0 + 0.05 E(B-V). \quad (2)$$

The values of $R$ ranged from 3.26 to 3.43.

The dependence of $R$ upon color index and color excess was taken from the work of Olsen (1975), modified in zero point from 3.25 to 3.15. Studies by Savage & Mathis (1979), Rieke & Lebovsky (1985), and Cardelli, Clayton, & Mathis (1989) suggest a value $R \sim 3.1$ for early-type stars of small reddening, rather than the value $R \sim 3.2$ produced by the original Olsen relation. We shifted the zero point in order to be compatible with the more recent analyses.

Pulsation periods were taken from The General Catalogue of Variable Stars (Kholopov et al. 1985), and coordinates for each star from KP. Figure 6 shows the distribution of our sample with respect to period, which is typical of the distribution for cataloged Galactic Cepheids.

Figure 7 shows the distribution of the sample projected on the Galactic plane assuming the distance scale of Caldwell &
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3.1. The Velocity Model

Our method follows that of HJBW who applied it to field RR Lyrae stars. A complete description of the procedure can be found in their paper. Briefly we construct a model that predicts the space velocity of each star based on its position in the Galaxy. The position is determined from its right ascension and declination and from its distance, which is calculated from the apparent magnitude and reddening. The parameters of the model describe the velocity components due to the reflex solar motion, differential Galactic rotation, and the random peculiar velocities of each star, which we represent by a velocity ellipsoid. The model parameters are varied until the predicted velocities match the observations with maximum likelihood.

Two of the velocity components are systematic motions. The reflex solar motion describes the apparent motion of a star that results from our own moving reference frame. There are three independent components of reflex solar motion in the model. In addition, since the stars in our sample have a radial extent of some 4 kpc in the plane of the Galaxy, their velocities are significantly affected by differential Galactic rotation. (The RR Lyrae stars studied by HJBW, on the other hand, are nearby members of the Galactic halo whose motions are not affected by differential Galactic rotation, and so it does not make a noticeable contribution to those stars' velocities, as HJBW noted in their paper.) We include differential Galactic rotation in our model by adding to the reflex solar motion a term representing a differential velocity, as suggested by HJBW.

This term depends upon the star's distance from the Sun and upon the Oort constants and introduces two additional parameters to the solution, the Oort constants \(A\) and \(B\).

Finally, each star has its own peculiar velocity relative to these systematic motions. We assume that the stars' peculiar velocities are independent and randomly distributed, and we model them with a multinormal distribution, the velocity ellipsoid. In the coordinate system in which the velocity ellipsoid is diagonal, the three orthogonal axes represent the directions along which the velocity distributions are independent random variables, and the length of each axis corresponds to the dispersion of the Gaussian velocity distribution in that direction. In general, the velocity ellipsoid has six independent parameters which collectively specify the directions of the axes and the dispersions along the axes.

The adopted statistical model therefore incorporates 13 parameters: the three components of the reflex solar motion, the Oort constants \(A\) and \(B\), the zero point in absolute magnitude and its cosmic dispersion, and the six parameters of the velocity ellipsoid. These are the parameters that are to be determined in the maximum likelihood calculation.

The coordinate system of the velocity model is the Galactocentric cylindrical coordinate system \((\pi, \theta, z)\), where \(\pi\) is positive toward the Galactic center \((l = 0^\circ, b = 0^\circ)\), \(\theta\) is positive toward the direction of Galactic rotation \((l = 90^\circ, b = 0^\circ)\), and \(z\) is positive toward the north Galactic pole \((b = +90^\circ)\).

3.2. The Likelihood Calculation

We define the likelihood of obtaining all the observations simultaneously as

\[
L = \prod \text{prob}(v),
\]

where \(v\) is the velocity residual vector, that is, the observed velocity minus the velocity expected from the model. The product is taken over all of the stars in the sample. Assuming the velocity residuals follow a Gaussian distribution with zero mean and a covariance tensor \(M\), we have

\[
\text{prob}(v) = (2\pi)^{-3/2}|M|^{-1/2}\exp\left(-0.5v^\top M^{-1}v\right),
\]

where \(|M|\) denotes the determinant of \(M\) and \(v^\top\) is the transpose of the vector \(v\). The quantities \(M\) and \(v\) are calculated from the 13 parameters of the model as discussed in HJBW.
The quantity we wish to maximize is the logarithmic likelihood function

\[ \ln L = \Sigma \ln \text{prob (r)}, \]

\[ \ln L = -0.5 \Sigma (\ln |M| + v' M^{-1} v) + \text{constant}. \]

To maximize this likelihood, we use the geometric minimization technique known as simplex optimization, which is described by HJBW. This technique varies the parameters of the model until an error function which is associated with those parameters has been minimized. We define our error function \( S \) to be the negative of twice the logarithmic likelihood

\[ S = -2 \ln L = \Sigma (\ln |M| + v' M^{-1} v), \]

where we have dropped the constant. Minimizing \( S \) will maximize the likelihood as desired.

The uncertainty in the final value of a parameter is found by computing the derivative numerically. To find the uncertainty in parameter \( X_i \), we fix \( X_i \) at the value \( X_{i0} + d_i \), where \( X_{i0} \) is the value of \( X_i \) at the converged solution and \( d_i \) is a small, usually 1%, deviation from \( X_{i0} \). We then allow the other parameters to converge to a new solution having an error \( S \). If the error at the true solution is \( S_0 \), the variance in \( X_i \) is taken to be

\[ s_i^2 = d_i^2/(S - S_0). \]

### 3.3. Tests of the Model

Before applying the model to the actual data, we performed several tests on synthetic data, generated with parameters similar to those expected for the Cepheids. These tests showed that we were able to consistently recover nine of the 13 parameters in our model. However, we were not successful in determining four of the parameters, specifically, the cosmic dispersion and the direction cosines of the three axes of the velocity ellipsoid.

In the first test solutions the cosmic dispersion in absolute magnitude tended to converge either to zero or to an unreasonably large value. When we computed trial solutions with the cosmic dispersion held fixed at a reasonable value, the values found for the other parameters were unchanged. We interpret this as evidence that the uncertainties in the data preclude determination of the cosmic dispersion and that its value has no effect on the other parameters. We note that HJBW were also unable to determine the cosmic dispersion in absolute magnitude for the RR Lyraes and that they also found its exact value to make little difference in the resulting values of the other parameters. Therefore when we computed the models for the actual data, we held the cosmic dispersion fixed at a reasonable value.

The direction cosines of the velocity ellipsoid axes never converged in our test solutions, even after all other parameters had converged. However, when the direction cosines were forced to agree with the true axes of the test sample, the resulting values of the other parameters were unchanged. We believe that the inability of the method to determine the direction cosines stems from the relative dispersions along the \( \pi \) and \( \theta \) axes. These dispersions are nearly the same for Cepheids, making the velocity ellipsoid appear as an oblate spheroid. This causes the \( \pi \) and \( \theta \) axes to become indeterminate. Therefore, when we ran solutions on the actual data, we forced the axes of the velocity ellipsoid to be the Galactocentric cylindrical axes, a choice which is not unreasonable since the dispersions should be along these axes from simple dynamical arguments for a steady state, axisymmetric system.

In Table 1 we present the results for two sets of synthetic data. In each test we assigned known values to the 13 parameters and computed exact apparent magnitudes, radial velocities and proper motions. We then added random errors, drawn from Gaussian distributions of known dispersions, to the exact values to obtain the final test data.

The two tests given in Table 1 differed in the way in which the uncertainties were assigned. Let us define the error in each datum as the amount by which the exact value was perturbed for the test and the uncertainty of the datum as the estimate of this error which is used to assign weights in the maximum likelihood calculation. We can set the uncertainty equal to the error, because we know the error in the test case, or we can estimate the uncertainty by some appropriate means, for example by setting the uncertainty equal to the average value of the errors for that particular quantity (e.g., proper motion). In the first test shown in Table 1 (col. [2]), the uncertainty assigned to each datum was set equal to the adopted error in

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Input Value</th>
<th>Recovered Value (exact weights)</th>
<th>Recovered Value (mean weights)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflex solar motion components:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_t )</td>
<td>-10.0 km s(^{-1})</td>
<td>-9.24 ± 1.47</td>
<td>-9.20 ± 1.63 km s(^{-1})</td>
</tr>
<tr>
<td>Theta</td>
<td>-10.0</td>
<td>-9.79 ± 1.17</td>
<td>-8.32 ± 1.54</td>
</tr>
<tr>
<td>Z</td>
<td>-1.0</td>
<td>-0.15 ± 1.15</td>
<td>-1.83 ± 2.01</td>
</tr>
<tr>
<td>Oort constants:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A )</td>
<td>15.0 km s(^{-1}) kpc(^{-1})</td>
<td>15.57 ± 1.99</td>
<td>13.28 ± 2.00 km s(^{-1}) kpc(^{-1})</td>
</tr>
<tr>
<td>( B )</td>
<td>-10.0</td>
<td>-10.52 ± 1.86</td>
<td>-12.66 ± 2.15</td>
</tr>
<tr>
<td>Velocity ellipsoid dispersions:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_t )</td>
<td>10.0 km s(^{-1})</td>
<td>10.44 ± 1.26</td>
<td>11.43 ± 1.22 km s(^{-1})</td>
</tr>
<tr>
<td>Theta</td>
<td>9.0</td>
<td>7.93 ± 1.03</td>
<td>10.38 ± 1.12</td>
</tr>
<tr>
<td>Z</td>
<td>6.0</td>
<td>4.64 ± 2.47</td>
<td>6.02 ± 3.19</td>
</tr>
<tr>
<td>PLC relation:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c )</td>
<td>-3.69 mag</td>
<td>-3.61 ± 0.24</td>
<td>-3.82 ± 0.32 mag</td>
</tr>
<tr>
<td>( \sigma_M )</td>
<td>0.20</td>
<td>held constant</td>
<td>held constant</td>
</tr>
</tbody>
</table>

\(^a n = 82.\)
the same datum. In the second data set (col. [3]), the uncertainty assigned to each datum was set equal to the average of the errors. In both cases the parameters found by the program are, within the uncertainties, in agreement with the parameters used to generate the test sample. Table 1 shows that our formulation of the maximum likelihood model properly recovers the known values of the nine parameters that were allowed to vary.

4. RESULTS

Table 2 gives our results for the kinematic and absolute magnitude parameters based on the sample of 90 stars from the KP proper motion catalog. Columns [2] and [3], respectively, give the solutions using the assumed PL and PLC coefficients from CC, that is,

\[
\langle M_v \rangle = -2.78 \log P - 0.8 + \beta, \\
\langle M_o \rangle = -3.53 \log P - 0.8 + 2.13 \\
	\times [(\langle B \rangle - \langle V \rangle) - 0.65] + c. 
\]

Uncertainties for all parameters were found as described in §3.2.

To check on the effects of our assumptions on the solutions, we ran five test models. The results of the tests are shown in Table 3 (PL) and Table 4 (PLC). For ease of comparison with our principal results, we repeat the latter as model 1 in Tables 3 and 4.

First, we compared solutions using the KP and Wielen (1974) proper motion samples by carrying out the analysis on each set using only the stars in common. These solutions are shown as model 2 (Wielen) and model 3 (KP). Recall that the KP proper motion uncertainties seem to be somewhat smaller than the Wielen uncertainties (Fig. 3-4). We noted in §2.2 that there is some indication that the KP proper motion uncertainties may be systematically underestimated. Because of the importance of weights to the maximum likelihood solution, it is possible that such a difference in the uncertainties could lead to different results between solutions using the KP data and the Wielen data. As models 1, 2, and 3 show, this is not the case. Whether we use the full KP data set, the Wielen data set, or the KP data restricted to the same stars as included in Wielen's work, all parameters of the analyses except Oort's constant B agree within 2 σ. We conclude that any underestimation of the uncertainties in the KP data does not significantly affect our results, with the possible exception of Oort's constant B.

When we restrict the sample to the 42 stars in common between Wielen and KP, we are restricting the analysis to the nearer stars with the higher quality proper motions. Because Oort's constant B is strongly dependent upon the proper motions, it is the parameter most likely to be affected by this restriction. The full KP data set gives \( B = -5.7 \pm 3.3 \text{ km s}^{-1} \text{ kpc}^{-1} \), whereas Wielen's data give \(-10.3 \text{ km s}^{-1} \text{ kpc}^{-1} \), and the restricted sample of KP data gives \(-14.9 \text{ km s}^{-1} \text{ kpc}^{-1} \).

TABLE 3

Tests of the Assumptions in the Cepheid PL Relation Results

<table>
<thead>
<tr>
<th>MODEL</th>
<th>( n ) (number)</th>
<th>( \Pi )</th>
<th>( \Theta ) (km s(^{-1}))</th>
<th>( Z )</th>
<th>( \Pi )</th>
<th>( \Theta ) (km s(^{-1}))</th>
<th>( Z )</th>
<th>( A ) (km s(^{-1}) kpc(^{-1}))</th>
<th>( B ) (mag)</th>
<th>( \sigma_M ) (mag)</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90</td>
<td>-6.72</td>
<td>-14.43</td>
<td>-10.00</td>
<td>11.91</td>
<td>9.91</td>
<td>1.85</td>
<td>14.65</td>
<td>-5.89</td>
<td>-3.50</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>42</td>
<td>-6.33</td>
<td>-10.77</td>
<td>-9.99</td>
<td>10.98</td>
<td>8.02</td>
<td>3.13</td>
<td>14.47</td>
<td>-10.71</td>
<td>-3.53</td>
<td>0.25</td>
</tr>
<tr>
<td>3</td>
<td>42</td>
<td>-5.88</td>
<td>-11.07</td>
<td>-7.82</td>
<td>10.88</td>
<td>8.39</td>
<td>1.23</td>
<td>17.13</td>
<td>-15.22</td>
<td>-3.04</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
<td>-6.77</td>
<td>-14.15</td>
<td>-10.02</td>
<td>11.89</td>
<td>9.95</td>
<td>2.13</td>
<td>14.41</td>
<td>-6.11</td>
<td>-3.50</td>
<td>0.15</td>
</tr>
<tr>
<td>5</td>
<td>90</td>
<td>-6.76</td>
<td>-14.43</td>
<td>-9.89</td>
<td>11.89</td>
<td>9.92</td>
<td>1.87</td>
<td>14.35</td>
<td>-6.05</td>
<td>-3.49</td>
<td>0.25</td>
</tr>
<tr>
<td>6</td>
<td>44</td>
<td>-8.14</td>
<td>-14.03</td>
<td>-7.23</td>
<td>13.86</td>
<td>9.46</td>
<td>0.00</td>
<td>14.43</td>
<td>-6.10</td>
<td>-3.49</td>
<td>0.25</td>
</tr>
</tbody>
</table>
(We did not compute uncertainties in the latter two analyses, but they are probably about ±5 km s\(^{-1}\) kpc\(^{-1}\).) The more negative values cannot be said to be significantly different from −5.7 km s\(^{-1}\) kpc\(^{-1}\), because of the large uncertainties; yet, knowing that they are based on the higher quality proper motions, and suspecting that the KP uncertainties may be underestimated, which may be important for the more distant stars, we are inclined to believe that the Cepheids have B closer to −10 or −15 km s\(^{-1}\) kpc\(^{-1}\) than to −6 km s\(^{-1}\) kpc\(^{-1}\).

Next, we wanted to determine how sensitive the solutions were to our choices of cosmic dispersion, \(\sigma_M\). According to CC, the cosmic dispersion of the PL relation is in the range 0.19−0.23 mag. We ran two solutions for the PL zero point on the KP data: one with \(\sigma_M\) held fixed at 0.15 mag (model 4) and one with \(\sigma_M = 0.25\) mag (model 1). There is essentially no difference between the two solutions as can be seen in Table 3. Adding a color term to the PL relation reduces its dispersion, so for the PLC relation we estimated \(\sigma_M\) to be in the range 0.01−0.10 mag. When we solved for the zero point of the PLC relation with \(\sigma_M\) held constant at 0.01 mag (model 4) and 0.10 mag (model 1), the two solutions were again the same within the uncertainties (Table 4). Clearly the specific value of \(\sigma_M\) is not important to the solutions for either the PL or the PLC zero point, as we had already expected from our tests on the synthetic data and from the HJBW results.

We also varied the assumed coefficients of the PL and PLC relations to see how much uncertainty these choices introduced. For the PL relation, CC give the value of the log P coefficient as −2.78 ± 0.11. We found that changing this coefficient by 0.10 did not significantly change the other parameters (compare models 1 and 5 in Table 3). Feast & Walker (1987) suggest that the PLC relations given by CC (eq. [12]) and by Martin, Warren, & Feast (1979), viz.,

\[
\langle M_\odot \rangle = -3.80(\log P - 0.8) + 2.70[(\langle B \rangle - \langle V \rangle)_{\odot} - 0.65] + c,
\]

represent the two extremes of this relation. Using equation (13) in our model gave no significant change in any of the parameters (see model 5 in Table 4). Thus the specific choice of coefficients in the PLC relation does not materially affect our results for the zero point in the relation.

According to Szabados (1990) the frequency of Cepheids in binary systems is at least 50%, so it is possible that unrecognized binaries could have an effect on our results through the velocities and the apparent magnitudes. However, a solution using 44 stars of the KP data which excluded all binaries listed by Szabados did not differ significantly from results using the entire sample (compare models 1 and 6). The presence of known binaries in the sample has apparently not affected our result.

5. DISCUSSION

Our statistical analysis of Cepheid kinematics yields nine parameters each for the assumed PL relation and for the assumed PLC relation: the zero point in absolute magnitude, the three components of the reflex solar motion, the Oort constants A and B, and the three dispersions of the velocity ellipsoid. It is clear from Table 2 that both the PL and PLC analyses give essentially the same results for these parameters. Therefore we have simply averaged those values, and in Table 5 we adopt these means as our best estimates of the various parameters. Table 5 also shows the most recent previous statistical parallax results for these same quantities.

5.1. Absolute Magnitudes from Statistical Parallaxes

Because of the way equations (3) and (4) are formulated, the zero points in absolute magnitude which we determined are simply the absolute magnitude of a Cepheid with log \(P = 0.8\), for the PL relation, and the absolute magnitude of a Cepheid with log \(P = 0.8\) and \(\langle B \rangle - \langle V \rangle)_{\odot} = 0.65\) mag, for the PLC relation. On the ridge lines of the Cepheid PL and PLC relations these two absolute magnitudes are essentially the same. It is therefore satisfying that we obtain essentially the same answer from both analyses, as seen in Table 2. At the level of precision obtained here, we are justified in simply averaging the two values as we have done in Table 5, obtaining \(\langle M_\odot \rangle_{0.8} = -3.46 \pm 0.33\) mag as our best estimate.

Kurimova & Pavlovskaya (1981b) did a statistical parallax solution on the 38 nearest stars in the KP data set as well as for a larger sample of 75 stars. For their initial distance scale they adopted the PL relation of Fernie (1967),

\[
\langle M_\odot \rangle = -0.38(\log P - 0.8)^2 - 1.89(\log P - 0.8) - 3.75.
\]

(14)

They arrived at a correction to this distance scale of −0.11 ± 0.21 mag for the 38 star sample and −0.33 ± 0.16 mag for the larger sample. These two results imply zero points of \(\langle M_\odot \rangle_{0.8} = -3.86 \pm 0.21\) mag and \(\langle M_\odot \rangle_{0.8} = -4.08 \pm 0.16\) mag, respectively. KP prefer the former value.

However, we suspect a flaw in their analysis. Cepheid proper motions should decrease with distance from the Sun, assuming the tangential velocities remain roughly constant; but, since the proper motions are so small, at about 1 kpc from the Sun the observational errors become comparable to the actual proper motions of the Cepheids. At all greater distances the
errors dominate, and the proper motions, instead of decreasing with distance, fluctuate at the level of the observational errors. Karimova & Pavlovskaya attempt to compensate for this effect by adjusting the proper motions for the influences of random errors by Eddington's (1940) method. (They adjusted the radial velocities and distances for random errors as well, but these corrections are negligible.)

Our first concern with the KP correction process is that Eddington specifically states that his method should not be applied to data which would then be used for a statistical analysis, because it greatly overcorrects the observational errors. Second, the correction coefficient they determine for the declination proper motions is physically impossible, as they applied to data which would then be used for a statistical analysis, because it greatly overcorrects the observational errors. This reasoning is supported by our own analysis of the KP data. When we analyzed the KP data for the set of 42 stars in common with Wielen, which is essentially the same set as the 38 star sample used by Karimova & Pavlovskaya (1981b), we find \( <M_o> \),\(_{0.8} \) = -3.46 ± 0.33 mag. This is markedly different from their result for the absolute magnitude zero point to well within the errors. This is extremely encouraging. These three analyses used progressively more sophisticated mathematical treatments on the same data, yet find the same result. Clearly, the choice of mathematical treatment does not matter.

Furthermore, when the data set is more than doubled to include all currently available Cepheid proper motions, the result again does not change. Our best estimate of \( <M_o> \),\(_{0.8} \) = -3.46 ± 0.33 mag, based on 90 proper motions, is essentially the same as found by all three studies for the smaller data set. We interpret this to indicate that the additional proper motions in the KP data set add very little information to the statistical parallax because of their large uncertainties.

We conclude that our best estimate for the Cepheid absolute magnitude zero point is stable against refinements in the mathematical technique and is also stable against additional Cepheid proper motion data, unless those data have markedly lower uncertainties than the existing proper motions. For further improvement of the Cepheid PL relation using statistical parallaxes, we require more data, rather than more high quality proper motion data than have hitherto been available.

### 5.2. Comparison with Other Cepheid Absolute Magnitude Scales

At the outset of this paper we noted that there are only three independent techniques available for determining the absolute magnitudes of Cepheid variables: cluster fitting, the surface brightness technique, and statistical parallaxes. It is of interest to compare our result with the other two methods. Schmidt (1991) has recently summarized zero point determinations for the Cepheid PL relation, to which the reader is also referred.

There are two cluster-based distance scales in common use: that of Feast & Walker (1987) and Caldwell & Coulson (1987). Both of these fit the main sequences of clusters containing Cepheids to a Pleiades distance modulus of 5.57 ± 0.08 mag.

---

**Table 5**

<table>
<thead>
<tr>
<th></th>
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<tr>
<td>( n )</td>
<td>90</td>
<td>38</td>
<td>45</td>
<td>45</td>
<td>188</td>
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<td>( \Pi )</td>
<td>-6.8 ± 1.6</td>
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<td>-9.0 ± 2</td>
<td>-11 ± 2</td>
<td>-8.1 ± 1.4</td>
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<tr>
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<td>...</td>
<td>-13.8 ± 2.1</td>
<td>-12 ± 2</td>
<td>-12.8 ± 2.1</td>
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<tr>
<td>( Z )</td>
<td>-10.0 ± 2.0</td>
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<td>-10.0 ± 2.1</td>
<td>-10 ± 2</td>
<td>-7.0 fixed</td>
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<td>Oort constants (km s(^{-1}) kpc(^{-1}))</td>
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<tr>
<td>( A )</td>
<td>14.5 ± 2.3</td>
<td>...</td>
<td>12 ± 2</td>
<td>...</td>
<td>14.6 ± 1.7</td>
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<tr>
<td>( B )</td>
<td>-5.7 ± 3.3</td>
<td>...</td>
<td>-18 ± 2</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>( A - B )</td>
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<td>21.3 ± 5.8</td>
<td>30</td>
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<td>Velocity ellipsoid dispersions (km s(^{-1}))</td>
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<tr>
<td>( \Pi )</td>
<td>11.9 ± 1.2</td>
<td>13.3 ± 1.2</td>
<td>9.8 ± 1.6</td>
<td>8 ± 2</td>
<td>...</td>
</tr>
<tr>
<td>( \Theta )</td>
<td>9.9 ± 1.0</td>
<td>12.5 ± 1.0</td>
<td>8.1 ± 1.3</td>
<td>7 ± 2</td>
<td>...</td>
</tr>
<tr>
<td>( Z )</td>
<td>2.1 ± 1.8</td>
<td>10.3 ± 0.9</td>
<td>6.4 ± 2.8</td>
<td>5 ± 2</td>
<td>...</td>
</tr>
<tr>
<td>Absolute magnitude (mag) ( &lt;M_o&gt; ),(_{0.8} )</td>
<td>-3.46 ± 0.33</td>
<td>-3.86 ± 0.21</td>
<td>-3.43 ± 0.3</td>
<td>-3.38 ± 0.4</td>
<td>...</td>
</tr>
</tbody>
</table>

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The kinematic parameters determined here are consistent with those expected for an extreme disk population group. Our results generally confirm and improve those of Wielen (1974), Clube & Dawe (1980), and Karimova & Pavlovskaya (1981b), as can be seen from Table 5.

The reflex solar motion components we obtained are consistent with those expected for a disk population corotating with the local standard of rest. We found the reflex solar motion to be \( V_r = -6.8 \pm 1.7 \) km s\(^{-1}\), \( V_t = -14.3 \pm 1.3 \) km s\(^{-1}\), and \( V_z = -10.0 \pm 2.0 \) km s\(^{-1}\). Our results are essentially the same as those obtained by Clube & Dawe (1980) in their reanalysis of the Wielen (1974) data, although our uncertainties are slightly smaller. We also are in agreement with the values \( V_r = -8.1 \pm 1.4 \) km s\(^{-1}\) and \( V_z = -12.8 \pm 1.2 \) km s\(^{-1}\) determined by Caldwell & Coulson (1987) from 188 Cepheid radial velocities (also included in Table 5).

For Oort's constants we obtained the values \( A = 14.5 \pm 2.4 \) km s\(^{-1}\) kpc\(^{-1}\) and \( B = -5.7 \pm 3.3 \) km s\(^{-1}\) kpc\(^{-1}\). Our value of \( A \) is in excellent agreement with the value of \( 14.6 \pm 1.7 \) km s\(^{-1}\) kpc\(^{-1}\) determined by Caldwell & Coulson (1987). The result for Oort's constant \( B \), however, is considerably more positive than the expected value of \(-11 \pm 3 \) km s\(^{-1}\) kpc\(^{-1}\) (Mihalas & Binney 1981). As we discussed in § 4, our result for \( B \) is suspect because it is unstable to changes in the proper motion sample. When we analyzed the Wielen sample of 42 stars, we obtained \( B = -10.3 \) km s\(^{-1}\) kpc\(^{-1}\), and for the KP data on those same 42 stars, \(-14.9 \) km s\(^{-1}\) kpc\(^{-1}\). Clube & Dawe (1980) studied a sample of only 17 stars from Wielen's (1974) set which had the most accurate proper motions and found \( B = -18 \pm 2 \) km s\(^{-1}\) kpc\(^{-1}\) (and \( A = 12 \pm 2 \) km s\(^{-1}\) kpc\(^{-1}\)). The limited quality and quantity of the proper motion sample for Cepheids preclude a reliable determination of \( B \), even with the better mathematical techniques now available. The only solution is to obtain a higher quality sample of accurate proper motions on which to do the statistical analysis.

The dispersions of the velocity ellipsoid are similar to those determined for Cepheids in other analyses. We obtained \( 11.9 \pm 1.2 \) km s\(^{-1}\), \( 9.9 \pm 1.0 \) km s\(^{-1}\), and \( 2.1 \pm 1.8 \) km s\(^{-1}\), in \( \pi, \theta, \) and \( z \), respectively. These do not differ significantly from

### 5.3. Kinematic Parameters

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### Table 6

**Summary of Independent Cepheid Distance Scales**

<table>
<thead>
<tr>
<th>Source</th>
<th>( \langle M_0 \rangle_{0.8} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical parallax:</td>
<td></td>
</tr>
<tr>
<td>This work</td>
<td>(-3.46 \pm 0.33 ) mag</td>
</tr>
<tr>
<td>Cluster fitting methods:</td>
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</tr>
<tr>
<td>Binaries</td>
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<td>Cluster Cepheids</td>
<td>(-3.57 )</td>
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<tr>
<td>Surface brightness methods:</td>
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<tr>
<td>Theoretical</td>
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</tr>
<tr>
<td>Empirical</td>
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</tr>
<tr>
<td>Source</td>
<td>Evans 1991a, b</td>
</tr>
<tr>
<td>Source</td>
<td>Feast &amp; Walker 1987</td>
</tr>
<tr>
<td>Source</td>
<td>Caldwell &amp; Coulson 1987</td>
</tr>
<tr>
<td>Source</td>
<td>Hindsley &amp; Bell 1989</td>
</tr>
<tr>
<td>Source</td>
<td>Barnes et al. 1990</td>
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</tbody>
</table>

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the results of other researchers as shown in Table 5, although the uncertainties are smaller in our analysis.

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(HJBW)