SEISMOLOGY FOR THE FINE STRUCTURE IN THE SUN’S OSCILLATIONS VARYING WITH ITS ACTIVITY CYCLE

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Received 1990 September 11; accepted 1991 January 30

ABSTRACT

We inverted the symmetric part of the fine structure in the 1986 and 1988 solar oscillation data of Libbrecht and Woodard to find statistically significant evidence for a steady megagauss toroidal field at the bottom of the convective envelope. We confirm the sizable amplitude of a cycle-dependent near-surface perturbation which we argue has its origin in the fibril field.

Subject headings: Sun: interior — Sun: magnetic fields — Sun: oscillations

1. INTRODUCTION

Solar oscillation data for the fine structure in the Sun’s acoustic mode spectrum are available spanning the time from the previous solar maximum until now. These data have been used by Kuhn (1988) to argue for a solar cycle dependence in the symmetric part of the fine structure. Subsequent observational evidence has strengthened this argument (Jefferies et al. 1988; Libbrecht & Woodard 1990). Libbrecht & Woodard (1990) have shown that a dominant near-surface perturbation is the proximate cause of the cycle dependence. Further, they demonstrated that the perturbation has a latitudinal dependence correlated with that of surface magnetic activity.

It is our purpose to develop the helioseismic implications of the results of Libbrecht & Woodard (1990). In particular, we show that one can learn about the interior of the Sun from the symmetric part of the fine structure in the oscillation data even though the near-surface perturbation may dominate. We demonstrate that the residual signal, after accounting for centrifugal distortion and the near-surface perturbation, is consistent with a persistent megagauss quadrupole toroidal field centered near the base of the convection zone. This reinforces our earlier seismic evidence for such a field (Dziembowski & Goode 1989). Finally, we attribute the near-surface perturbation to a fibril field and inspect the nature of that field.

2. INVERSE PROBLEM FOR THE SYMMETRIC PART OF THE FINE STRUCTURE

The fine structure in helioseismic data is usually described by

\[ v_{n,l,m} - v_{n,l,0} = L \sum_{i=1}^{N} a_{i,n,l} P_i \left( \frac{m}{L} \right), \]

where \( v \) is the frequency of the oscillation and \( n, l, \) and \( m \) are the radial and angular descriptors of the oscillation. \( P \) is a Legendre polynomial, and \( L = l \) or \( [l + 1]^{1/2} \) depending on the choice of the observer in the data reduction. \( N \) is usually equal to 5 or 6, depending on the data set. The antisymmetric \( a \)'s are straightforwardly attributed to the linear effect of rotation. The rotation rate is determined by inversion of these \( a \)'s, see, e.g., Brown et al. (1989). The symmetric \( a \)'s are another matter. They may arise from various sources which have their symmetry axis defined by that for rotation. In particular, quadratic effects of rotation, like centrifugal distortion contribute to the even \( a \)'s. Beyond this, however, we must assume an origin for the distorting perturbation(s) and then, if possible, pose an inverse problem for the perturbing agent(s).

A global toroidal field is a logical candidate, after all, the shear of differential rotation on even a weak poloidal field could generate a sizable toroidal field. And we know from helioseismology that surface-like differential rotation persists throughout the solar convective zone and, perhaps, somewhat beneath. For a review of solar internal rotation from helioseismology, see Goode et al. (1991). The region just beneath the base of the convection zone would be a sensible place to find such a field. That is, the field could still be fed by differential rotation while not being interfered with by convection. With this in mind, Dziembowski & Goode (1989) assumed a global toroidal field was such an agent and formulated and solved the inverse problem for the field. They used an earlier version of the 1986 data (Libbrecht 1989). Here we generalize their approach to include the effect of the near-surface perturbation treated by Libbrecht & Woodard (1990).

To describe the effect of the near-surface perturbation, \( \Delta L(\xi) \), without specifying here its physical nature, we start with the general expression

\[ \Delta \omega = \int \xi^* \cdot \Delta L(\xi) \rho \, d^3r, \]

where \( \omega = 2\pi v \) and \( \rho \) is the local density. To simplify the argument here, we ignore the perturbation of the outer boundary condition. This simplification will be justified \textit{a posteriori}. The \( n, l, \) and \( m \) labels on \( \omega \) and \( \xi \) have been suppressed. The quantity \( \xi \) is the normal coordinate of the oscillation,

\[ \xi_{n,l,m} = [r y_n,\xi + r z_n,\xi \mathbf{V}_H] Y_l^m(\theta, \phi). \]
From here on in the integral in the denominator of equation (2) will be denoted by $I_\nu$. The mode inertia—$I$—is proportional to the mode mass used by Libbrecht & Woodard (1990). In the region of the near surface, the quantity $\xi$ is essentially radial, and $y_\nu(r)$ is closely $l$-independent. Owing to this property of the eigenfunction, the radial part of the integral in the numerator of equation (2) is also $l$-independent. Assuming that $\Delta L$ has its angular dependence described by the Legendre polynomial $P_{2s}(\cos \theta)$, then the angular part of this integral, $Q_{l,t,m}$, as shown by Gough (1988) and Goode & Kuhn (1990) takes a particularly simple form for $L^2 > s^2$,

$$Q_{l,t,m} = (-1)^t \frac{(2s-l-1)!!}{2s!!} \frac{p_{2s}(m/L)}{I_L}.$$  \tag{4}

For the accurate data of Libbrecht & Woodard (1990), which range from $l=5-60$, equation (4) is a usable approximation down to $l=5$. For us equation (4) is extremely accurate for $l < 10$. The $l < 10$ data are relatively noisier, and therefore our results are insensitive to the inaccuracies in equation (4). In the absence of effects beyond the near-surface perturbation, equations (1), (2), and (4) imply that

$$a_{2j} \propto \frac{\gamma_j(r)}{IL}.$$  \tag{5}

We remark that the $l$-dependence enters through the denominator. The form of $\gamma_j(r)$ depends on the way the eigenfunctions are normalized. In our calculations we used $\gamma_j = 1$ at the photosphere. With equation (5) we can include the near-surface effect in the inverse problem for the toroidal magnetic field.

Dziembowski & Goode (1989) and Gough & Thompson (1990) showed that one can pose an inverse problem for $\beta_k(r)$ which describes the toroidal field, $B_{\phi r}$, where

$$B_{\phi r}^2 = 4\pi p \sin^2 \theta \sum_{k=1}^{\infty} \beta_k(r) \cos^{2k-2} \theta,$$  \tag{6}

and where $p$ is the local gas pressure. Using the inverse equation of Dziembowski & Goode (1989) and extending it to include the near-surface perturbation developed here, we have

$$a_{2j,d} = \sum_{k \neq j} \int_0^R A_{k,j,d} \beta_k(r) dr + (a_{2j,d})_{rot} + \frac{I_4 y_\nu(r)}{I_d L_d},$$  \tag{7}

where $a_{2j,d}$ is the symmetric part of the fine structure in the $d$th multiplet in the data. The factor $I_4$, which is $I$ for the $n=15$, $l=20$ mode was introduced to make $\gamma$'s comparable in magnitude to the $\alpha$'s of Libbrecht & Woodard (1990). The ratio of $I/I_4$ depends primarily on the frequency, $v$, and is ~40 at 2 mHz and ~1 above 3 mHz. The toroidal field kernel $A_{k,j,d}$ is defined in equations (26) and (45) of Dziembowski & Goode (1989). The $(a_{2j,d})_{rot}$ term represents the quadratic order effect of rotation as calculated by Dziembowski & Goode (1991). This effect must be removed from the data before an inversion can be performed to determine $\beta_k(r)$ along with a simultaneous polynomial fit for $y_\nu(r)$.

We first performed regularized least-square inversions on equation (7) because we could obtain continuous functions for $\beta$ and $\gamma$. We employed the data of Libbrecht & Woodard (1990, and personal communication) for $l$-values between 5 and 60. We solved the hierarchical equations using $a_{2d}$ data to determine $\beta_3$ and $\gamma_3$. Then we used $a_{2d}$ data and $\beta_3$ to solve for $\beta_5$ and $\gamma_5$, etc. We chose a first derivative constraint in $\beta$ so that the regularized inversions would be sensitive to a field confined to a narrow region in radius.

The results for the $\gamma$'s are shown in Figure 1. These $\gamma$'s translate to $\alpha$'s which are quite consistent with the $\alpha$'s of Libbrecht & Woodard (1990)—being well within the error bars. The largest difference occurs for the $\gamma_3$'s (alternatively their $ax_3$). If we ignored the quadratic effect of rotation, as Libbrecht & Woodard (1990) did, even this small difference would go away. In fact, the trend in the $ax_3$'s and their mean value of $ax_3$ from their 1986 data reflects the quadratic effect of rotation. Near 3 mHz, where $I_4$ is defined, the $\gamma$'s are directly comparable to the $\alpha$'s in Figure 3 of Libbrecht & Woodard (1990). We see from Figure 1 that the $\gamma$'s from the 1986 data are consistent with zero. The summer of 1986 was the time of the most recent activity minimum. The $\gamma$'s in 1988, a time of considerably more surface activity, are statistically quite distinct from zero. The relatively weak $v$-dependence in the $\gamma$'s indicates that the perturbing agent must reside close to the photosphere. That is, if the perturbation resided in the outer atmosphere $\gamma$ would be a sharply increasing function of $v$ reflecting sharply increasing eigenfunction amplitudes. If the perturbation resided below the
photosphere, then \( \gamma \) would have an oscillatory behavior with \( v \). Thus, we confirm the conclusion of Libbrecht & Woodard (1990) about the location of the perturbation. The fact that \( \gamma \) is not a steep function of \( v \) justifies our neglect of the perturbation of the boundary condition.

The \( \beta_1 \) and \( \beta_3 \) functions from the inversions are everywhere consistent with zero from both data sets. Also, from both data sets \( \beta_2 \) is only significant near the base of the convection zone.

The calculated field is roughly a megagauss. In the future, one expects \( a_6, a_{10}, \) etc., data. Including such data would have some effect on the \( \beta^2 \) calculated here, but none on the \( \gamma \)’s. We emphasize, however, that even though the \( a_6 \)’s are particularly large in the 1988 data they have no consequential effect on our calculated \( \beta^2 \)’s. We note that regularization tends to oversmooth so that the megagauss result is only indicative of something significant. To go further, we performed Backus & Gilbert (1970) inversions on equation (7), assuming the \( \gamma \) functions shown in Figure 1 describe the near-surface perturbation here as well. In practice, this means replacing the third term on the right-hand side of equation (7) by the results for the near-surface perturbation from the regularized inversions and then performing the Backus & Gilbert (1970) inversions. In this way we calculated \( \beta_3 \) and \( \beta_2 \) and determined the resolution width of the data as a function of radius. The results are shown in Figure 2. Because the \( a_n \) data are relatively noisy, we could form reasonable kernels only between 0.6 and 0.7 of the solar radius, \( R \). The \( \beta_3 \) values are consistent with zero, although there is a roughly constant offset between them. Since the effect of the near-surface perturbation in the \( a_n \) data in 1988 is much more significant than elsewhere, it could be that there is some residual left that we are seeing as an offset. Still, \( \beta_3 \) is the most difficult case, and our results are consistent with zero. Thus, in the Backus & Gilbert (1970) inversion for \( \beta_3 \), we did not include \( \beta_2 \). For \( \beta_2 \), there is a \( \sim 3-4 \sigma \) effect near the base of the convection zone—in particular for the two statistically independent points at 0.7\( R \) and 0.75\( R \) from each data set. We point out that in this critical region there is a slight and not significant offset of the results from the 1988 data set with respect to those from the 1986 data set. This offset might also be due to a residual of the surface perturbation in the 1988 data after our removal of the surface effect. However, regularized inversions suggest that the difference is actually due to ignoring the small effect of \( \beta_3 \) in the inversion for \( \beta_2 \). The results of the Backus & Gilbert (1970) inversions for the 1986 and 1988 data are each consistent with a \( 2 \pm 1 \) MG quadrupole toroidal field centered near the base of the convection zone. The statistically significant part of the field determined here is very much the same as that determined by Dziembowski & Goode (1989) using an earlier version of the 1986 data (Libbrecht 1989). In the earlier calculation we did not account for the near-surface perturbation. As we have seen, that perturbation was relatively small at the activity minimum.

3. THE CAUSE OF THE NEAR SURFACE EFFECT

For a nonradial perturbation, \( \Delta L \), all quantities needed to evaluate it follow from the perturbed momentum equation; thus our operator can always be expressed in terms of the perturbing force. Thus, phenomena affecting heat transport alone such as thermal shadowing are not germane here. That is, thermal shadowing effects splitting through the dynamical effect of the meridional flows it induces. In the absence of such nonradial forces there would not be an asphericity in temperature despite an asphericity in heat conduction. Therefore, to determine the cause of the near surface effect in the symmetric splittings, we have to consider possible nonradial forces acting in the vicinity of the photosphere.

We do not know the behavior of the rotation rate in the subphotospheric layers well enough to rule out spatial and temporal variations at the level of, say, 20 nHz. We found, however, that even if such a rise were most suitably located, it could not account for the order of the observed \( \gamma \)’s. In particular, the upper limit for \( \gamma_3 \) would be 1–3 nHz, Dziembowski & Goode (1991). The inertial forces due to meridional flow are certainly of secondary importance here. Thus, we are left with the Lorentz force as the only strong candidate.

It seems most likely that the fibril structure of the magnetic field persists to at least some depth below the photosphere, and therefore we consider this type of field here. The field affects the oscillations directly as well as through a modification of the Reynolds stress due to convective motion. We will limit ourselves to the former effect as it would appear to be dominant because the direct effect of the magnetic field is expected to be larger by a factor on the order of the ratio of the gas pressure to the turbulent pressure. Zweibel & Däppen (1989) have developed a mean field formalism to treat the effect of the fibrils on the oscillation frequencies. In their formalism, the field was described by its mean intensity in the flux tube, \( b \), the filling factor, \( f \), and the mean product, \( \langle e, e_i \rangle \), of the unit vectors directed along the fibrils. Without loss of generality we treat \( b \) as a constant, incorporating spatial changes of the field inten-
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We ignore here the spherically symmetric part as it does not affect the \( \gamma \)-coefficients. Inserting this formula in equations (III.16) and (III.17) of Zweibel & Däppen (1989) and making approximations consistent with those which led to equation (5), we arrive at the following simple expression for the frequency perturbation, valid if its source lies in the outer propagation zone and above,

\[
\Delta \omega = \frac{b^2}{8 \pi \omega l} \sum_{s=1}^{\infty} \bar{Q}_{s,m} \times \int \left\{ \frac{d}{dr} \left[ \left( \frac{1}{2} - 1 + \eta \right) \frac{\partial}{\partial r} \frac{\partial}{\partial r} \right] - \frac{\omega^2 y^2}{g} \frac{d}{dr} (\eta \Lambda) \right\} r^4 dr ,
\]

where \( \eta = \langle e^2 \rangle \), \( \Gamma \) is the adiabatic exponent, and \( g \) is gravitational acceleration. We further simplify the expression by setting \( \eta = 1 \) and after some manipulation involving the use of the unperturbed equation for oscillations, we obtain the following expression for the \( \gamma \)-coefficients occurring in equation (7),

\[
\gamma_s = \frac{(-1)^{s-1}}{(2s-1)!!} \frac{\nu b^2}{8 \pi l} \int \left[ -y^2 \left( \frac{\partial}{\partial r} \right)^2 + \frac{1}{2} \frac{\partial^2}{\partial r^2} \right] \frac{\partial x_s}{\partial r} r^4 dr + \frac{dy_s}{dr} \left( \frac{\partial x_s}{\partial r} + \frac{1}{2} \frac{\partial x_s}{\partial r} \right) r^4 dr ,
\]

Equation (10) poses the inverse problem for \( \chi_s(r) \) from the splitting data. It would be premature to attempt to solve this inverse problem in view of the uncertainties in both the data and the theory. In particular, for the theory we do not know the applicability of the mean field theory to sunspots. However, we believe that with equation (10) we can check whether or not the fibril fields, as observed in the Sun’s photosphere, can account for the order of magnitude of the measured \( \gamma \)'s. We calculated \( \gamma_1 \) for three ad hoc \( \chi(r) \)-profiles assuming \( b = 10^3 \) G. The results are shown in Figure 3. The corresponding \( \gamma_2 \)- and \( \gamma_3 \)-functions can be calculated using equation (10) in correspondence with the \( \gamma_1 \)-functions of Figure 3. For each of the three profiles, the dominant part of the perturbation occurs on its rising slope. This is so because as one can see in equation (10) the rapidly increasing pressure/density appears in each denominator. In going from the first profile to the third one, the perturbation goes from being dominated by the upper photospheric region to the subphotospheric region. The absolute values of \( \gamma \) from the 1988 data are of order 100. We see that we can easily account for this magnitude with \( \chi \) having the reasonable order of 10^{-3} - 10^{-2} in the atmosphere, or somewhat larger if the field is subphotospheric. Taking the results of the \( \gamma \)-calculations in Figure 3 literally one would conclude, after comparison with Figure 1, that \( \chi_1 < 0, \chi_2 < 0 \) and \( \chi_3 > 0 \). Note that even though \( f \) in equation (8) is positively defined, it is not precluded that individual \( \chi \)-values are negative. However, the sign of the calculated \( \gamma \)'s are not certain because the four terms in equation (10) are comparable in magnitude with two being positive and two being negative. Despite quantitative uncertainties, we believe that our results support the view that the near-surface perturbation is due to the local fibril field. This is so because the order of magnitude of our \( \chi \)-values inferred from the \( \gamma \)'s agree with photospheric observations of the filling factor.

4. MEGAGAUSS FIELD NEAR THE BASE OF THE CONVECTION ZONE

We have assumed that the symmetric part of the fine structure in Libbrecht & Woodard’s (1990) oscillation data is due to the combined effects of the near-surface perturbation, the quadratic effect of rotation, and a buried toroidal field. Solutions to the inverse problem for the field yield near surface effects quite comparable to those of Libbrecht & Woodard (1990) from both their 1986 and 1988 data sets. Further, the inversions for both sets reveal a statistically significant steady megagauss quadrupole toroidal field centered near the base of the convection zone. This field has the geometry and approximate location that one would expect for a field generated by a dynamo action. However, it differs in two ways from the one favored by dynamo theorists. First, it is about two orders of magnitude too intense. Second, it appears to be steady. In fact, variability of a megagauss field on a time scale of years is excluded by energy considerations. We remark that the megagauss field seems to lie partly in the convection zone. Such a field would strongly modify the nature of convection near the base of that zone.

We note that just because the field is statistically significant does not mean, per se, that the field really exists. The most likely scenario for misinterpretation here is that equation (5)
does not sufficiently describe the near-surface perturbation. However, it is difficult to imagine a surface region perturbation, with similar effects at and away from solar minimum that would cause the persistent feature that we report as a toroidal field. It is also difficult to imagine a nonmagnetic source of perturbation residing at the interface region. One should, however, contemplate fields with more complicated geometries than we considered.

We thank Ken Libbrecht and Martin Woodard for use of their data prior to publication and Ryszard Sienkiewicz for the solar model used by us. P. R. G. is partly supported by AFOSR 89-0048. Part of this work was done during our one month visit to the Institute of Theoretical Physics at the University of California at Santa Barbara which is supported by NSF PHY82-17853.

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