MODELING MESOGRA NULES AND EXPLODERS ON THE SOLAR SURFACE

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ABSTRACT

Correlation tracking of high-resolution observations of solar granulation reveals a horizontal flow field containing both the supergranulation pattern and a much smaller pattern which corresponds to mesogranulation. Exploding granules (exploders) occur preferentially near the centers of mesogranules. This raises the question: are mesogranules just collections of exploders? Radial outflows in exploders and mesogranules can be modeled by superposing Gaussian source functions. This model is used to explore the relationship between mesogranules and exploders. Although we demonstrate that there is a mathematical equivalence between mesogranules and exploders distributed normally about the mesogranule centers, our results indicate that the observed mesogranular velocity pattern is not consistent with a flow pattern generated by exploders dropped randomly on the solar surface. Detailed comparisons with observations suggest that the averaged mesogranular velocity is produced by a combination of a persistent outflow from a source together with exploders distributed randomly about its center. Similar analysis also shows supergranules are not the result of random occurrences of mesogranules.

Subject headings: convection — Sun: atmospheric motions — Sun: granulation — Sun: magnetic fields

1. INTRODUCTION

Recent observations have clarified the nature of horizontal motions at the solar surface (Spruit, Nordlund, & Title 1990). Excellent time series of the photospheric granulation have been obtained from space with the Solar Optical Universal Polarimeter on Spacelab 2 and from the ground at Pic du Midi Observatory, at the Swedish Solar Observatory on La Palma, and at Sacramento Peak Observatory. Techniques for digitally processing these results, developed at the Lockheed Palo Alto Research Laboratory (LAPRL), make it possible to filter out the Sun's oscillatory modes (the "5 minute oscillations") and to remove seeing distortions by "destretching" the images. After such processing surface flow patterns become visible and the evolution of granules can be followed. Correlation tracking of high-resolution observations of solar granulation reveals a horizontal flow field containing both the supergranulation pattern and a much smaller pattern which corresponds to mesogranulation.

a supergranule. They drift systematically toward the boundary with velocities of about \(0.03\) minute^{-1} (0.4 km s^{-1}). At any instant there are perhaps 20 mesogranules within the supergranule. Of these one, on average, disappears every 30 minutes to be replaced elsewhere, suggesting a lifetime of about 3 hr for the mesogranules. It appears that this lifetime is set by the rate of advection toward the supergranule boundary rather than being an intrinsic property of the mesogranule. A typical granule has a diameter of \(1.5\) (1 Mm) but larger features called exploding granules, which were earlier thought to be relatively rare (Rösch & Hugon 1959; Carlier et al. 1968; Mehltretter 1978), are now known to be common (Title et al. 1989). These granules (which we shall refer to as exploders) expand to form a ringlike structure with a dark core before splitting into several fragments; their maximum diameter is about 4", and they have a total lifetime of 10–20 minutes. During the phase of rapid expansion, lasting for about 5–10 minutes, the radial velocity of \(0.1–0.2\) minute^{-1} (1.2–2.4 km s^{-1}) sweeps aside those granules that are in the vicinity of the exploder. Thus exploders form within a region about 3.5" in radius at a rate of 10–1 hr^{-1} in such a way that the area is swept clear every 20 minutes. Title et al. (1989) also established that exploders occur preferentially where the divergence of the time-averaged flow is positive. So there exists a close association between exploders and mesogranules. The white-light movies described by Frank et al. (1989) and Muller et al. (1990, 1991) show that small bright granules appear spasmodically near the center of a mesogranular source, move outward as they develop, and then explode. This process is repeated many times during the lifetime of a recognizably mesogranule. If these individual events are averaged either spatially or temporally they appear as a weak radial outflow on the scale of a
Kinematic modeling provides an excellent technique for answering these questions. Simon & Weiss (1989, hereafter Paper I) showed that individual mesogranules can be represented by axisymmetric Gaussian sources. Using data obtained from Spacelab 2, they calculated the divergence of the observed velocity field and matched sources to individual features with positive divergence. The derived velocity field was then compared with the original flow by computing their effects on passive test particles (corks) which are transported horizontally by the flow. These cork patterns matched the behavior of magnetic fields measured at Big Bear Solar Observatory (Simon et al. 1988). Having demonstrated in Paper I that mesogranular velocities can be satisfactorily represented by axisymmetric sources, we use the same technique here to model both mesogranules and exploders and to explore the relationship between them. Of course, this approach evades the fundamental problem of explaining the dynamical origin of velocities observed in the solar photosphere. But it does allow us to calculate how a flow field evolves for different configurations of exploders, mesogranules and supergranules. Moreover, at this time it is not possible to carry out numerical experiments on fully compressible three-dimensional convection in volumes large enough to investigate interactions on such scales.

Our kinematic model is described in § 2, where we show that a single axisymmetric source centered at the origin is exactly equivalent mathematically to suitably scaled smaller sources distributed randomly with a normal distribution about the origin. In § 3 this equivalence is demonstrated numerically. We also present illustrative results for sources distributed on a regular lattice and discuss the statistical analysis of cork patterns in terms of their fractal dimensions. Mesogranules and exploders are compared in § 4. We first contrast the cork patterns produced by randomly distributed mesogranules with those generated by an equivalent distribution of exploders and by an equal mixture of both. Then we consider two other possibilities: First, that the position of each new exploder is determined not by a fixed mesogranule but by that of its predecessor, so that each sequence of exploders follows a random walk; second, that each generation of exploders is deposited randomly over the whole domain. In § 5 we go on to explore the analogous relationship between supergranules and mesogranules. Finally, in § 6 we provide answers to the questions that prompted this investigation. There our model calculations are related both to earlier observations and to new results obtained at the Pic du Midi.

2. THE KINEMATIC MODEL

We begin by outlining the simple kinematic description of horizontal flows at the solar surface. The basic assumption is that any flow can be satisfactorily represented by a superposition of axisymmetric sources. Each source contributes a radial outflow \( u = (v, 0) \), referred to plane polar coordinates with an origin at the center of the source, and \( v = (v/r) \), where \( r \) is the radial distance from the origin. Since this flow is rotational we can introduce a potential \( \phi(r) \) such that \( u = -\nabla \phi \) and \( v = -d\phi/dr \). It was shown in Paper I that individual mesogranular sources were accurately represented by Gaussian sources with

\[
\phi(r) = \frac{1}{4} V R e^{-v(r)/R^2}, \quad v(r) = (Vr/R) e^{-v(r)/R^2}.
\]

Here \( V \) and \( R \) are measures of the size and strength of an individual source: the radial velocity \( v(r) \) is zero at the origin, rises to a maximum value \( v_{\text{max}} = 0.429 V \) at \( r = R/2 \), and then falls monotonically to zero as \( r \to \infty \). Moreover, a regularly tesselated cellular pattern can be generated by distributing sources on an appropriate regular lattice.

In Paper I we also demonstrated that the mesogranular velocity field observed on Spacelab 2 could be modeled by a suitable distribution of sources with different values of \( R \) and \( V \) chosen to match the observations. For a typical mesogranular source it was found that \( R \approx 3' \) and \( V \approx 0.05 \text{ minute}^{-1} \), corresponding to \( v_{\text{max}} \approx 0.26 \text{ km s}^{-1} \). The correlation tracking procedure has been calibrated more precisely by Title et al. (1989) who pointed out that the velocities are systematically underestimated if the Gaussian window is too wide. Hence we shall adopt the value \( V = 0.1 \text{ minute}^{-1} \), corresponding to \( v_{\text{max}} \approx 0.5 \text{ km s}^{-1} \), for mesogranules in this paper.

2.1. Randomly Distributed Sources

We investigate now the properties of a random distribution of sources centered on a fixed point. Suppose that individual sources (exploders), described by the Gaussian potential of equation (1) with \( R \) and \( V \) fixed, are distributed normally about the origin (the center of a mesogranule) with a probability distribution function

\[
\psi(r_0) = \frac{1}{\pi \rho^2} e^{-(r_0/\rho)^2},
\]

where the exploder is centered on the point \( r_0 = (x_0, y_0) \). Then the expected velocity potential is given by

\[
\Phi(r) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi(r_0) \delta(r - r_0) dx_0 dy_0.
\]

This integral can readily be evaluated by completing squares in the exponents, and we find that

\[
\Phi(r) = \frac{1}{2} \tilde{V} \tilde{R} e^{-(r/\tilde{R})^2},
\]

where \( \tilde{R}^2 = R^2 + \rho^2 \) and \( \tilde{V} = (R/\tilde{R})^3 V \).

Thus Gaussian sources distributed normally about a center are equivalent to a single Gaussian source with a larger radius \( \tilde{R} \) and a velocity reduced by the factor \( (R/\tilde{R})^3 \). [Note that in the limit as \( \rho \to 0 \) eq. (3) for \( \Phi(r) \) reduces to the distribution for a single source, while in the opposite limit as \( R \to 0 \) and \( \phi(r) \to \delta(x)\delta(y) \) we recover \( \Phi(r) = (\pi \rho^2)^{-1} \exp \left( - (r/\rho)^2 \right) \), as expected.]

Let us apply this result to individual exploders distributed randomly with a normal distribution about the origin. Suppose that each source survives for an interval \( \Delta t \) (the effective lifetime of an exploder) and that a new source is created after an interval \( t_0 \). Then there are on average \( N = \Delta t / t_0 \) sources at any instant and the mean velocity, averaged over a time interval \( T \) such that \( \Delta t < T, t_0 < T \), is given by

\[
\bar{V} = N(R/\tilde{R})^3 V
\]

from equation (5). For exploders the available data suggest that the effective lifetime \( \Delta t = 5-10 \text{ minutes} \) and \( t_0 \approx 10 \text{ minutes} \).
minutes (so that both $\Delta t$ and $t_0$ are much smaller than the observed lifetime of a mesogranule). We shall therefore assume that $N = 1$. Taking $R = 0.1$ minute$^{-1}$, $R = 3''$ for the mesogranule, with $R = 1'5.5$ for the exploders, we find that $V = 0'8$ minute$^{-1}$ and from equation (5) that $p = 2'6$; therefore the maximum velocity in an exploding granule is given by $v_{\text{max}} = 4$ km s$^{-1}$. (Note, however, that because of its Gaussian shape from eq. [1], the velocity drops quickly to 2 km s$^{-1}$ by $r = 2''$.) Because the ratio of $V$ to $v_{\text{max}}$ depends on the third power of the size ratio of mesogranules to exploders, the value obtained for the exploder velocity is very sensitive to this ratio. For example, a $2''$ exploder radius would imply that $v_{\text{max}} = 1.8$ km s$^{-1}$, which is much closer to observed values.

It is worth pointing out that the same approach can be used to estimate the effects of smoothing caused, for example, by correlation tracking with a finite Gaussian window. If the Gaussian window has a full width at half-maximum $W$, this corresponds to multiplying by a further factor

$$
\Psi(r_0) = (\pi\sigma^2)^{-1} e^{-(r_0^2)/\sigma^2}, \quad \tilde{\sigma}^2 = (4 \ln 2)^{-1} W^2 = 0.361 W^2.
$$

After integrating, we obtain another Gaussian with parameters $\tilde{R}$ and $\tilde{V}$ such that

$$
\tilde{R}^2 = R^2 + \tilde{\sigma}^2,
$$

$$
\tilde{V} = \left(\frac{\tilde{R}}{R} \right)^3 \tilde{V} = (1 + (\tilde{\sigma}/R))^3 \tilde{V} = [1 + 0.361 (W/R^2)]^{-3/2} \tilde{V}.
$$

This result shows how the apparent mesogranular velocity is reduced by smoothing or bad seeing. For example, a tracking window the size of the mesogranules reduces the measured horizontal velocity by almost 40%.

2.2. Random Walks

In contrast to the hypothesis that exploders occur randomly with respect to a fixed point is the hypothesis that there are families of exploders, and that within each family the position of a new exploder is near that of its predecessor so that exploders walk away from their initial positions. In this case there is a sequence of sources with position vectors $r_i = (x_i, y_i)$, $i = 0, 1, 2, \ldots, n$, starting at the origin with $r_0 = (0, 0)$, such that each offspring is distributed normally about its parent with a probability density

$$
\psi(s) = \frac{1}{\pi \sigma^2} e^{-(s/\sigma)^2},
$$

for a source distant $s$ from its parent. Then the expected mean square displacement $\delta(s^2) = \rho^2$. After $n$ generations, this random walk therefore leads to an expected total mean square displacement

$$
\rho^2 = \delta(r_{\text{tot}}^2) = np^2.
$$

If the initial distribution of exploders has a characteristic spacing $d$, then some memory of their original distribution will persist until $\delta^2/p^2$ generations have elapsed.

3. ILLUSTRATIVE RESULTS

We compare velocity fields generated by different arrangements of axisymmetric sources by computing their effects on passive test particles (corks). We carry out our calculations in a region 60" square, but in order to minimize edge effects, we display only the central 40" x 40" of the region. The corks are initially deposited on a square lattice with 40 nodes in each direction so that there are 676 corks within the central region. After a short time-interval $\Delta t$ (typically 1 or 2 minutes) the $i$th cork is displaced by $\delta r_i = U_i \Delta t$, where $U_i$ is the resultant velocity produced at the $i$th cork's location by sources within the entire region. This process is repeated after each successive interval $\Delta t$. Corks that are carried outside the 60" square region are ignored. After a suitable interval (ranging from 30 minutes to 1 day) has elapsed, we display the new positions of the corks. These displays are used as diagnostics for the motion. In order to compare various models we permit ourselves to follow corks for times that may exceed the actual lifetime of mesogranules on the Sun.

3.1. Steady Flows and Random Explosions

First, we demonstrate via the evolution of the cork patterns the equivalence, already established in the previous section, between a single mesogranular source and a Gaussian distribution of exploders. Figure 1a shows the effect of a single source, marked by a small square at the center of the region; for this illustrative calculation we assume a velocity $V_m = 0.3$ minute$^{-1}$ and radius $R_m = 6''$. (We use the subscripts $g$, $m$, and $s$ to denote exploding granules, mesogranules, and supergranules, respectively.) The circle indicates the radial distance $r_{\text{max}} = R_m (2^2/9)$ where the outward velocity has its maximum value. The cork positions are computed with a time step $\Delta t = 1$ minute and are displayed at 30 minute intervals. Corks are swept away from the central region within the first 30 minutes. In 90 minutes a disk about 10" in radius has been cleared, but thereafter the pattern scarcely changes. (The apparent deviations from circular symmetry result from the particular regular pattern chosen for the initial cork distribution.)

This outflow can be represented by randomly distributed exploders, as shown in Figures 1b and 1c, which illustrate displacements produced by successive exploders distributed so as to mimic a single source. The exploders have a radius $R_g = 3''$, so their velocity $V_g = 2'4$ minute$^{-1}$, and the distribution has a radius $p = 5'2$. Their positions are indicated by small circles which again show where $v(r)$ is a maximum. The time step $\Delta t = 0.1$ minute, so satisfying the accuracy requirement $\Delta t \ll R_g v_{\text{max}}$. In order to generate a symmetric pattern, it is necessary to have many short-lived exploders, so that each has time to produce only a slight displacement of the corks. Figure 1b shows the patterns created with $\Delta t = 0.1$ minute: even an exploder far from the center, like that at $t = 90$ minutes, produces only a slight effect. Comparing Figures 1a and 1b we see that distributed exploders are indeed almost equivalent to a single source. If the lifetimes of the exploders are too long, however, the patterns generated remain asymmetric for a considerable time. Figure 1c shows results for exploders with $\Delta t = 10$ minutes. After 12 explosions, the pattern still retains the asymmetries produced by a few eccentric exploders, though the area cleared is not significantly different from that of the corresponding regions in Figures 1a and 1b.

3.2. Tessellated Patterns

If identical sources are distributed in two dimensions, it is always possible to construct a corresponding tessellation of the
Fig. 1.—Illustration of the basic model. (a) (Top row) A single axisymmetric source (mesogranule) located at (20', 20') (open square) displaces test particles (corks), shown as small filled squares, which are initially distributed on a uniform grid. The ring marks the radial distance from the source center at which maximum radial velocity occurs. (b) (Middle row) The steady source of (a) is replaced by a series of exploding granules normally distributed about the center of the grid. The exploders have radii one-half that of the mesogranule of (a) and 8 times the velocity. The exploders occur successively at intervals of 0.1 minute. (c) (Bottom row) Like (b) but with exploder lifetimes $\Delta t = 10$ minutes. Both (b) and (c) will eventually approach the symmetric pattern of (a).

plane. For sources on a rectangular or hexagonal lattice this tesselation is obvious (cf. Paper I). More generally, we construct lines joining nearest neighbors, which form interlocking triangles (Delaunay triangles). Then the perpendicular bisectors of their sides give a network enclosing cells around each source, with junctions (corners) at the centroids of the triangles, thereby tesselating the plane. Such tesselations (Voronoi tesselations) are also of interest in the context of galaxy formation (Icke & van de Weygaert 1987).

We shall compare the cork patterns produced by mesogranules distributed on a square lattice (with spacing $d = 10'$) with the patterns generated by a corresponding distribution of exploders. Figure 2a shows the locations of the 16 mesogranules that lie within the central region. From now on we shall assume that all mesogranules have a radius $R_m = 3'$ and a velocity $V_m = 0.1\text{ minute}^{-1}$ ($v_{\text{max}} \approx 0.5 \text{ km s}^{-1}$), in agreement with the observations, while the corresponding exploders have $R_g = 1.5', V_g = 0.8\text{ minute}^{-1}$ ($v_{\text{max}} \approx 4 \text{ km s}^{-1}$). In the figures that follow, the centers of all mesogranules are indicated by squares, but the circle of radius $r_{\text{max}}$ is present only if the mesogranule is active ($V_m \neq 0$). In this section each exploder is indicated by a smaller circle of radius $0.707R_g$. All cork displacements are computed after a time step $\delta t = 2$ minutes.

The mesogranular flow (cf. eq. [1]) in Figure 2a rapidly sweeps the corks away from cell centers. The small Gaussian tail of this velocity field then slowly carries them toward the nodes. Although there are hints of a linear structure in this evolving pattern at times around 2–3 hr, the corks continuously move into regions that progressively contract. After 10 hr, almost all the corks lie within 1' of a node in the pattern.

By contrast, the exploders in Figure 2b, with a lifetime $\Delta t = 10$ minutes, produce an irregular pattern. Once again, corks are swept away from cell centers, but now there is a clear linear structure which persists for times from 2 to 5 hr. Although most of the corks are at the corners, a substantial number remain along the edges of the cells for several hours. The reason is obvious: corks in the linear structures are affected only by occasional exploders that occur far from mesogranule centers. The effect of such an exploder is primarily to distort the structure rather than to move particles along it. It is only in the unlikely event of two granules exploding simultaneously on either side of a boundary that corks will be driven along it. Hence cell boundaries last longer in Figure 2b while nodes form rapidly in Figure 2a.

We should note that for simplicity of computation, we create each new pattern of exploders at precisely equal time intervals, at $t = \Delta t, 2\Delta t, \ldots$. On the real Sun, we would expect new exploders to appear not only at random locations, but also randomly in time, with lifetimes normally distributed about a mean value $\Delta t$. 

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3.3. Statistical Analysis of Cork Patterns

It is useful to strengthen this qualitative geometrical discussion by developing a more quantitative description which can be applied to observational and synthetic data. One procedure is to calculate a characteristic fractal dimension of the cork pattern (Spruit et al. 1990). Suppose there are $M$ corks with position vectors $c_i$, $i = 1, 2, 3, \ldots, M$, within the region; then we compute the distances $d_{ij} = |c_i - c_j|$, $1 \leq i < j \leq M$, between all distinct pairs of corks that lie within the region. Let $m(l)$ be the number of pairs with $d_{ij} < l$; then the quantity

$$C(l) = \frac{2m(l)}{M(M - 1)}$$

Fig. 2.—(a) (Upper six images) Time series showing the evolution of corks in a flow field produced by identical mesogranules uniformly distributed on a square grid. (b) (Lower six images) Now each mesogranule is replaced by a series of exploders normally distributed about each of the mesogranular sites. Each exploder lives for 10 minutes and is then replaced by a new one.
is an estimate of the correlation integral (Schuster 1988), and its logarithmic slope
\[
\nu(l) = \frac{d \log C}{d \log l}
\]
(12)
is an estimate of the correlation dimension. If \(\nu(l)\) has a constant value over a wide range of \(l\) we refer to that value as the fractal dimension of the pattern. It is easy to verify that if the typical separation between closest neighbors \(\delta < l < D\), where \(D\) is the scale of the region, then a uniformly filled area, a linear structure and a localized point singularity have integral dimensions 2, 1, 0, respectively. More generally, we expect cork patterns to have dimensions \(\nu(l)\) which depend on the value of the separation \(l\).

To illustrate the usefulness of this procedure and the interpretation of its results, we consider first a relatively simple velocity field generated by three large sources (modeling supergranules with radius \(R_s = 12\)° and velocity \(V_s = 0.1\) minute \(^{-1}\)) dropped randomly on the region as shown in Figure 3a. The mean spacing between their centers (indicated by large hollow squares) is 28°, and the large circles, as before, denote the radial distance where the outward velocity is greatest. This flow will be considered further in § 5. Here we note only the most striking features of the cork patterns in Figure 3a. After 4 hr, the centers of the cells are mainly cleared, and by 8 hr the corks are concentrated in the neighborhood of the network. At 12 hr there is a striking linear pattern (corresponding to the Voronoi tessellation) with a high concentration of corks at the node. Thereafter the network weakens as all corks are either swept into the corner or carried out of the region until, after 3 days (not shown in Fig. 3a), all remaining corks lie within less than 1° of the node.

In Figure 3b we show the corresponding behavior of the correlation integral \(C(l)\) and the fractal dimension \(\nu(l)\). At \(t = 0\) the corks are uniformly distributed with a minimum spacing \(\delta = 15°\), but there is only a narrow range (5° \(\leq l \leq 10°\)) where \(\nu(l) \approx 2\). At the lower end of this range \(\nu\) changes discontinuously, owing to the regular spacing of the corks at \(l/\delta = 1, 2^{1/2}, 2, \ldots\). At the upper end, the slope falls owing to the finite size of the region. Thus the presence of a perfectly regular two-dimensional distribution of corks is not at all obvious from the fractal dimension \(\nu(l)\). To obtain a uniform logarithmic slope in \(C(l)\) over a convincing range of \(l\) values would require many more corks. In general, we need 40° corks in the central region to demonstrate a fractal dimension \(\nu\) (cf. Smith 1988). Here we would have to set \(M = 4 \times 10^4\) to give \(\nu \approx 2\) over a decade in \(l\); computing requirements would then become prohibitive.

As time increases, corks are brought closer together and \(C\) becomes nonzero for smaller and smaller values of \(l\). At \(t = 4\) hr in Figure 3a one can see a broad band of corks (width about 6°) along the network. Hence \(\nu \approx 1\) when \(l \approx 25°\); i.e., on a scale comparable with the spacing between supergranule centers. On a smaller scale, with \(l \approx 2°\), the two-dimensional band structure becomes apparent and \(\nu \approx 2\). These tendencies persist when \(t = 8\) hr, though the build-up of corks at the junction in the network reduces the value of \(\nu\) around \(l = 10°\). The striking linear pattern at \(t = 12\) hr in Figure 3a scarcely shows up in Figure 3b. As \(l\) decreases there is an initial increase in \(\nu\) at \(l \approx 25°\), corresponding to the cell spacing, followed by a drop to \(\nu \approx 0.5\) at \(l \approx 10°\); the linear structure is dominated by the massive concentration of particles at the junction. Thereafter there is a slow increase in \(\nu\) until the fine structure is resolved around \(l = 0.4°\), where \(\nu \approx 2\). At later times the correlation dimension gradually drops to zero as the remaining particles congregate first into three small groupings, and finally in the neighborhood of the single junction point between the cells. By \(t = 3\) days (not shown in Fig. 3b), the remaining corks are all gathered into a region of diameter 0.3° and \(\nu(l) \approx 0\) for
$l > 0.4$. This analysis shows how the variation of $v$ with $l$ in Figure 3b can, with care, be related to properties of the visual patterns in Figure 3a. It also reveals the weaknesses of this procedure, for the results can easily be misinterpreted.

Now we return to the regular grid of Figure 2. Corresponding values of $C$ and $v$ are displayed in Figure 4. At $t = 0$ the behavior of $C(l)$ and $v(l)$ is, of course, identical with that shown in Figure 3b. As $t$ increases, the minimum spacing between corks decreases, slowly for the mesogranular flow, but rapidly for the exploders. For large $l$ there is a persistent peak in $v$ at $l = d = 10^\circ$, with $v \approx 2$, corresponding to the spacing between cell centers, with subsidiary peaks at $l = 14^\circ$, $20^\circ$, ... . With a steady mesogranular flow, the peaks are sharp; their widths are limited by binning into only 60 equal logarithmic intervals over a range of $10^6$ in $l$. With random exploders the peaks are naturally broader. The mesogranular pattern in Figures 2a and 4a shows a gradual evolution from an almost two-dimensional structure at $t = 1$ hr to a configuration with isolated two-dimensional patches. The diameters of these patches gradually decrease, so the effect becomes apparent after 10 hr, when $v(l)$ drops to zero at $l \approx 5^\circ$ and then rises towards $v = 2$ for $0.1 < l < 1.0$. (Owing to the cruciform structures in Fig. 2a, the peak value of $v$ is only about 1.) The persistent erratic behavior at very low $l$ is due to the regular compression at the nodes which locally preserves the initial grid structure.

The random exploders in Figure 2b produce a qualitatively different effect. Although the patches are initially similar to those in Figure 2a, so that the curves for $v(l)$ at $t = 1$ hr in Figures 4a and 4b are almost identical for $l > 2^\circ$, they rapidly develop into elongated linear structures. This shows up clearly in Figure 4b, where the curves for $v(l)$ are fairly flat for $0.2 < l < 5^\circ$ and relax toward $v = 1$ as time increases. If the run is pursued for $t > 10$ hr, the dip at $l < 5^\circ$ develops as corks gradually accumulate at junctions in the network, but this occurs far more slowly than with the steady flow of Figures 2a and 4a.

4. MESOGRANULES AND EXPLODERS

We now attempt to model the observed distribution of mesogranules in the solar photosphere and to compare the effects of steady motion with those produced by random distribution of exploders. In each case we shall first study the cork patterns visually and then consider their statistical properties, as represented by the fractal dimension $v(l)$. We begin by distributing 36 mesogranules randomly over the entire region, subject to the constraint that the distance between adjacent centers should not be less than $2R_m$. The total number distributed is equal to that in Figure 2a, but only 14 lie completely within the central region. Figure 5a shows the location of these mesogranules. There is some clumping just below the center, with a compensatory void around the strip $\{x = 27^\circ, 20^\circ < y < 40^\circ\}$. Similar nonuniformities appear in observational data; cf. Figure 6 of Paper I. For the purposes of this section each mesogranule is regarded as permanent.

The evolution of the cork pattern under the action of these steady mesogranular sources is displayed in Figure 5a. The cells are cleared after 2 hr, and a network develops around the sources, forming a Voronoi tessellation, over a period of about 5 hr. The nature of this network depends critically on the ratio of the local spacing $d$ to the source radius $R_m$. At center left, where $d \approx 4R_m$, there is an almost perfect hexagonal pattern after 10 hr. A little higher, where $d \approx 3R_m$, all corks are concentrated at the junction, while at the bottom right, where $d \approx 2R_m$, only a few survive after 5 hr. At the top right, on the other hand, where $d > 6R_m$, the initial cork distribution is virtually undisturbed. These distinctions may to some extent be an artificial consequence of our Gaussian source model (cf. Fig. 2 of Paper I), but it seems clear that the observed structures are sensitive to the spacing and packing, as well as the flow velocity, of the sources. However, such distribution effects should be much less dominant on the Sun, where mesogranules are advected toward the supergranule boundaries.

4.1. Mesogranules Represented by Distributed Exploders

We now suppose that each mesogranule is replaced by a sequence of random exploders, distributed normally about the center of the mesogranule. Each exploder lives for 10 minutes and is then replaced. Figure 5b shows the evolution of the cork field, with the current exploders represented by circles of radius $0.6r_m = 0.42R_m$. (Previously, in Figs. 1 and 2b, the circles had radius $r_m$. In this and following figures, we use smaller circles...
to give the plots a "cleaner" appearance.) The overall cork pattern is very similar to that in Figure 5a. As in the Figure 2a versus Figure 2b comparison, the exploders again empty their immediate neighborhoods more rapidly (since $V_g = SV_m$) producing a clear but ragged network after 2 hr. Where the cell centers are close together, the exploders leave only a few point-like concentrations, but the undisturbed regions are almost identical with those in Figure 5a.

The statistical properties of the patterns in Figure 5 are contrasted in Figure 6. The results for mesogranules in Figure 6a show a slow transition from two-dimensional to one-dimensional behavior over about 6 hr, though there is still a
peak corresponding to the average mesogranular spacing of about 10". It is only for \( t \approx 1 \) day (not illustrated here) that \( v(t) \) gradually dips toward zero. The corresponding curves for explorers are markedly different for times up to 2 hr, with a rapid spread to smaller values of \( l \) while the mean value of \( v \) hovers around 1.5. At \( t = 4 \) hr the two curves are very similar for \( l > 2" \), but \( v \approx 1 \) for explorers at smaller \( l \). By \( t = 10 \) hr the curves in Figures 6a and 6b do not significantly differ. This confirms the visual impression from Figure 5 that the effects of mesogranules and explorers cannot readily be distinguished.

We shall show later in Figure 9 (§ 5) the effect of combining mesogranule and exploder flows.

4.2. Exploders on a Random Walk

We represent this process by supposing that each generation survives for a finite time and is followed by a new generation with positions distributed normally about those of the previous generation, as outlined in § 2.2. Thus explorers are no longer tied to mesogranule centers as they were in § 4.1, above. Instead, successive members of a sequence follow a random walk and stagger slowly away from their initial position. We implement this process by starting each sequence from one of the mesogranule sites introduced above and assuming that the position of a new exploder is distributed randomly about that of its predecessor with the same normal distribution as before. In addition we impose the constraint that the distance between any pair of explorers should not be less than \( 2R_g \).

Figure 7a shows a realization of this procedure with the corresponding cork patterns. The initial sites are marked by squares with a mean separation \( l \approx 10" \), and each generation is replaced after 10 minutes. At \( t = 2 \) hr, the explorers have moved away from their original positions. We recall from equation (10) that the mean square displacement after \( n \) generations is given by

\[
\bar{r}^2 = np^2 = n(R_m^2 - R_g^2) = \frac{1}{2}nR_m^2.
\]  

(13)

So the number of generations that must elapse for an rms displacement \( \bar{r} = pR_m \) is given by \( n = \frac{\bar{r}^2}{p^2} \). For \( \bar{r} = 9" \) (\( p = 3 \)), this gives \( n = 12 \), corresponding to a time of 2 hr; for \( \bar{r} = 20" \) (\( p \approx 7 \)), \( n \approx 60 \), giving \( t = 10 \) hr. Thus it takes about 2 hr before the explorers cease to remember their initial positions, as can be verified from Figure 7a. On the other hand, the initial distribution is itself nonuniform, with clumping on a scale larger than the mean spacing \( d \). In our model calculations this scale is the half-width (20") of the central region; in a bigger domain, with many more explorers present, fluctuations on larger scales would be proportionately less. So the characteristic time for clumps and voids to change is about 10 hr. Figure 7a is consistent with this time scale, for the last portion of the empty region at the upper right has disappeared when 12 hr have elapsed.

Comparing the cork patterns in Figure 7a with those in Figure 5b we see that the randomly shifting explorers affect most of the corks within 2 hr. Initially the pattern is more linear, but by 12 hr most of the corks have been confined to pointlike regions. Some of the cell boundaries, once formed, are unaffected from \( t = 2 \) hr to \( t = 12 \) hr, but others are shifted as the exploder positions change, so that much of the small-scale structure can no longer be recognized after more than 2 hr have elapsed. The overall statistical properties, shown in Figure 8a, are similar to those in Figure 6b, though the formation of a linear pattern after 2 hr with \( v(t) \approx 1 \), and the subsequent transition towards a more pointlike structure with \( v(t) \approx 0 \) after 8 hr, for \( 0.05 < l < 2" \), are much clearer and occur more quickly with a random walk.

4.3. Randomly Distributed Exploders

Finally we consider the effects of redistributing each new generation of explorers randomly over the whole region, again with a minimum spacing \( 2R_g \). Now there is no correlation between the positions of successive generations of explorers, as is clear from Figure 7b. A linear network develops rapidly, and no regions remain undisturbed for longer than 2 hr. Although some individual features may persist, the overall pattern changes significantly in an hour (after only six exploder generations). After 10 hr have elapsed, most of the corks are confined to isolated concentrations or to short linear features. The variation of \( v \) with \( l \) in Figure 8b is distinctive. We see that \( v \) drops quickly to 1, and then relaxes towards zero as the
Pattern becomes more and more point-like for $0.05 < l < 2^\circ$ when $t \geq 8$ hr. At the same time, a broad peak with $v \approx 2$ develops around the mean spacing ($10^\circ$) of the randomly distributed exploders. These curves are clearly different from those in Figures 6b and 5a.

There are three ways of distinguishing among the exploder results in Figures 5b, 7a, and 7b. First, we can look at linear structures, which persist for up to 10 hr in Figure 5b, but only 5 hr in Figure 7. These distinctions are borne out by the statistical results in Figures 6b and 8. Next we can focus on cellular patterns, which persist throughout Figure 5b, where exploders are tied to mesogranules. In Figure 7a, where the

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**Fig. 7.** (a) (Upper six images) Starting from the same mesogranular sites of Fig. 5a, successive exploders stagger across the solar surface in a random walk progression. Note the clumping of nearby exploders and the corresponding voids. (b) (Lower six images) We drop exploders on the solar surface in a completely random manner and repeat this procedure every 10 minutes. The resulting pattern changes significantly over a 2 hr interval and contains almost no memory of its previous structure. Linear network patterns seen on the Sun are almost nonexistent in this illustration.
exploder pattern alters completely after 2 hr, the network changes on the same time scale. The randomly dropped exploders in Figure 7b also change most of the network before 2 hr have passed. Third, we can inspect the distribution of exploders. In Figure 5b the exploders are tied to mesogranules which are widely spaced. There are some persistent voids, but the probability that two exploders lie close together is small: of about 550 pairs in Figure 5b only four are separated by less than 4". The initial distribution in Figure 7a is similar, but random walks lead to the development of clumps and voids which persist for several hours. Thus the overall pattern has a large scale, shown by the broader hump in v, extending to 20" in Figure 8a. In Figure 7b the instantaneous distributions of exploders show clumps and voids like those in Figure 7a, but they change from one generation to the next without allowing large-scale network structures to develop.

5. SUPERGRANULES

On the Sun, mesogranules have a finite lifetime and drift with the underlying supergranular velocity. So we should also explore the relationship between supergranules, mesogranules, and exploders. The effect on passive particles of large-scale supergranular motion has already been shown in Figure 3. But the pattern is distorted by mesogranules and exploding granules. To demonstrate this process we can superpose supergranules and mesogranules drifting in the velocity field of the supergranules. The simplest model is a scaled-up version of that in Figure 5a, to which we now add three supergranules, allowing the slowly moving mesogranules to survive indefinitely. What happens is that within a few hours the mesogranules are either carried out of the region or into the network. Eventually they either escape or collect in the corner. In this process they also drive out the corks so that all of them disappear from the region. Thus the mesogranules, which at first appear to enhance the formation of the network pattern, in the end destroy it. So this model is unrealistic and unsatisfactory.

We therefore constructed a more elaborate model. The supergranules remain fixed (in the same positions as in Fig. 3a). Mesogranules are initially deposited randomly over the entire region as before and allowed to drift with the local supergranular velocity. If a mesogranule (1) escapes from the region, (2) approaches too close (<2Rg) to another mesogranule, or (3) becomes "trapped" and stationary in the network, then that mesogranule is removed. It is replaced by a new mesogranule in a position distributed randomly with a normal distribution about the center of the nearest supergranule. This procedure preserves a more uniform distribution of mesogranules, and is consistent with the most recent Pic du Midi observations of mesogranular evolution. For times shorter than 2 hr, the resulting cork patterns resemble those obtained with the supergranular velocity alone; thereafter, there is a qualitative difference in their overall appearance. Particles still accumulate in the network and near the junction, but mesogranules destroy the simple Voronoi pattern in Figure 3a and make it much less regular. Next we replaced each mesogranule with random exploders distributed normally about its center and drifting with the local supergranular velocity. Although mesogranule centers are in the same positions as before, the resulting network is more ragged.

We demonstrated earlier in Figure 5 that the effects of mesogranules and exploders are visually hard to distinguish. We shall argue in § 6 that the observations imply that both types of sources contribute significantly to the surface motion. Hence we show in Figure 9 results obtained with equal contributions from mesogranules and exploders. We set \( V_{m} = 0.05 \) minute\(^{-1} \) and \( V_{e} = 0.4 \) minute\(^{-1} \) to give the same combined effect as before; the peak velocity, \( v_{\text{max}} \approx 2 \) km s\(^{-1} \), for the exploders is still consistent with the observations. The centers of individual exploders drift in the combined velocity field of the mesogranules and supergranules; thus a cork at the center of an exploding granule remains there as they both drift together. The number of corks is replenished by generating new corks near the centers of mesogranules to compensate for those that escape from the region and to keep the total number approximately constant. These corks drift gradually toward the network under the combined velocities of supergranules, mesogranules and exploders.
Fig. 9.—(a) (Upper six images) In this time series we have added three fixed supergranules to the flow pattern. Each supergranule (large open square) has an axisymmetric horizontal velocity field on which the mesogranules (small open squares) now drift. Whenever mesogranules stagnate in the supergranular network, or drift too close to each other or completely out of the image area, we replace them by new mesogranules normally distributed about the supergranule centers. Exploders (small circles) are distributed around mesogranule centers, and corks are driven by the supergranular velocity and an equal mixture of mesogranular and exploder flow fields. As time proceeds, corks either drift out of the viewing area or collect in an irregular supergranular network. To keep the number of corks in the region approximately constant, we generate two corks each minute. These patterns provide our best representation of observed solar magnetic structure. (b) (Bottom row) Statistical analysis of the cork evolution, showing the transition from a linear toward a pointlike structure.

Figure 9a shows the rapid formation, within 2 hr, of linear structures enclosing mesogranules as they move towards the supergranule boundaries. After 4 hr these structures accumulate within the broad bands of Figure 3a. The supergranular network becomes more apparent after 10 hr, but its fine structure at any instant of time is controlled by the current arrangement of mesogranules and exploders (which, of course, changes after the interval Δt = 10 minutes). In their general appearance these cork patterns resemble those generated using measured velocities from the Pic du Midi data (Muller et al. 1991). We consider that Figure 9 provides our best model representation of the magnetic network on the Sun.

The statistical properties of these cork patterns are displayed in Figure 9b. Replacing corks that escape from the region has only a minor effect on ν(l) for times up to 12 hr. Comparing Figure 9 with Figure 3 we see that the addition of mesogranules or exploders brings corks rapidly together, so that the curves for C(l) in Figure 9b extend down to l = 0′02 after only 4 hr. At t = 4 hr, there is only a broad network in Figure 3a leading to ν ≈ 2 around l = 5′–10′ in Figure 3b. By contrast the networks in Figure 9a are already linear by t = 4 hr so ν ≈ 1 in Figure 9b on scales around l = 0′5–5′. After 12 hr, there is a significant accumulation of corks near the network junction, and ν gradually drops toward zero, although the constant generation of new corks maintains the linear appearance in the network, which tends to keep ν close to 1. In Figure 9b we can also discern two peaks at l ≈ 25′ and l ≈ 10′, corresponding to the spacings of supergranules and mesogranules, respectively.

6. DISCUSSION

We now apply our results to issues raised by observations of convection in the solar photosphere. The first question, which prompted this investigation, is: Are mesogranules just exploders? That is, are outflows on a mesogranular scale just the time-averaged effect of recurrent exploding granules? We have shown that mesogranular velocities can indeed be produced by a suitable distribution of exploding granules. Our
model calculations with fixed mesogranules (Fig. 5a) and with random exploders centered on mesogranular sites (Fig. 5b) generate cork patterns that are essentially equivalent. Indeed, the differences between these two figures are probably too slight for current observations to distinguish between them.

However, we can rule out the hypothesis that mesogranules simply appear as a result of averaging over randomly exploding granules. Mesogranules observed from Spacelab 2 survived for at least the 27 minute length of the available time series, while velocity patterns produced in our model by exploders dropped randomly over the entire region (Fig. 7b) become uncorrelated after a single generation (10 minutes).

We have also examined a mechanism that produces new exploders in the vicinity of their predecessors—families of exploders. Then the sources follow a random walk, producing a mesoscale velocity pattern that changes slowly over several hours (Fig. 7a)—on a time scale comparable with the estimated lifetime of mesogranules. The cork patterns in Figure 7a differ significantly from those in Figure 5. The former have larger, more ephemeral structures with a shorter lived and more fragmented network pattern, but it is not clear that observational data could decide between them during the first 2–3 hr of cork motions. A better approach is to study the distributions of mesogranules and exploders. The initial distribution in Figure 5a is random and therefore nonuniform, with some mesogranules close together at the bottom of the region and an empty space at the top. Because of our constraint that mesogranules must have centers no closer together than 2R_m, they remain sufficiently well separated so that there is no significant clumping of exploders in Figure 5b. By contrast, randomly displaced exploders tend to form clumps and corresponding voids which persist for several hours: this is apparent if the exploder positions in Figure 7a are compared with the squares that mark their starting points, i.e., the mesogranule centers in Figure 5a. The randomly dropped exploders in Fig. 7b are similarly clumped, but the pattern does not survive from one generation to the next.) Observations show a more uniform distribution of mesogranular-scale motion which is not compatible with the untethered exploders of Figure 7a (or those of Fig. 7b).

It is clear from the 27 minute duration Spacelab 2 observations (Title et al. 1989) that the time-averaged velocities of exploding granules do provide a substantial contribution to the mesogranular velocity field. To determine whether there is any underlying large-scale flow we turn to the 3 hr long Pic du Midi observations (Frank et al. 1989; Muller et al. 1990, 1991), which show sources that survive for several hours (perhaps up to half a day) and linear structures, preserved in the corresponding cork patterns, which differ significantly from the random patterns in Figure 7b. The fact that an individual mesogranule survives for several hours, probably until it is swept into the supergranular network, means that it is more than a collection of exploders with lifetimes of 10 minutes. Further, these observations show that individual exploders begin as small bright granules which appear near the center of a mesogranule and swell as they move outward. This process occurs repeatedly and indicates the presence of a systematic mesogranular outflow. So we infer that the mesogranular velocity is made up of roughly equal contributions from a long-lived circulation and from the time-averaged effects of exploders distributed about the center of the mesogranule, as in Figure 9.

Thus our calculations, together with the observations, especially those from the Pic du Midi, lead us firmly to the conclusion that mesogranules and exploders are distinct but closely related phenomena. The observed evolution of the granulation pattern apparently results from a combination of exploders which occur randomly around the centers of mesogranules, a systematic mesogranular outflow, and general advection by a large-scale flow directed toward the supergranule boundaries.

Another question concerns the connection between mesogranules and supergranules. The distinction between them is not simply one of size. Supergranules are easily recognizable as the largest structures in any region on the solar surface (there is still no convincing observational evidence for the existence of "giant cells," which have been discussed in the literature for many years), with lifetimes of at least a day and a one-to-one correspondence between horizontal velocities and the magnetic network. In a magnetically quiet region supergranules have a typical diameter of about 40'" but their scale is reduced as the mean local magnetic flux density increases. For instance, the Spacelab 2 results show two features with diameters 15'–20' (labeled 1 and 2 by Simon et al. 1988) at the edge of a strong field region containing several pores. These cells were rimmed by magnetic structures which lasted for many hours, and they should probably be regarded as small supergranules rather than large mesogranules. At the other extreme, long-lived sunspots are partially surrounded by large annular moat cells, with diameters up to 80'. These cells can also be regarded as supergranules of a special type.

One can ask whether mesogranules bear the same relationship to supergranules as exploders do to mesogranules. The close correspondence between network and velocity field immediately shows that supergranules cannot be produced by randomly walking sources or by a random distribution of ephemeral mesogranules (cf. Figs. 7a and 7b). This leaves the possibility that mesogranules centered on a fixed point might collectively produce a supergranular velocity field. Suppose that the supergranule contains N mesogranules at any instant with lifetimes much shorter than that of the supergranule. Then the mesogranular velocity would have to be such that

\[ V_s \approx N(V_m/R_m)^3 V_m. \]

Taking N = 25 for a typical supergranule with R_s = 15", R_m = 3" we have \( V_m = 5V_s \), implying a peak velocity of about 2.5 km s\(^{-1}\) in mesogranules. This is significantly greater than the observed velocities (0.5–1.0 km s\(^{-1}\)); such a large disparity firmly rules out the possibility that mesogranules randomly distributed about a fixed center might produce the observed velocity field of the supergranule. So we should regard mesogranules as independent features drifting with the local supergranular velocity, as suggested by the Pic du Midi observations.

We conclude then that Figure 9a, which combines (1) supergranules, (2) mesogranules appearing within and moving in the velocity field of the supergranules, and in addition (3) exploders occurring within and drifting with the flow field of the mesogranules, produces our best estimate of the horizontal velocity pattern at the solar surface. Our simple model with axisymmetric sources has already clarified the relationships among supergranules, mesogranules, and exploders. The next stage will be to make a detailed statistical comparison between...
model predictions and precise observational data. The best available data set is the Pic du Midi run, currently being processed; in the future, results from the Swedish Solar Observatory at La Palma will also be available. In analyzing these data, we need to record the locations of exploders and mesogranules and investigate their relation to each other. The relation of mesogranules to supergranules should similarly be investigated. Then we can calculate the fractal dimension and temporal autocorrelation of cork patterns generated by the observed velocities and compare them with those produced by model flows.

Finally, we emphasize that all our discussion so far has only been concerned with kinematic descriptions of motion at the solar surface. Granules and supergranules are actually surface manifestations of turbulent convection in the compressible and radiating plasma beneath the photosphere. The scale of granules is largely determined by the fact that very few of them are larger than $3'$. Their average properties are familiar because of long observations but recent analysis has shown that these average properties provide a poor description of individual granules. Supergranules have a much better defined size of between 30" and 40" in the quiet Sun. (In regions of significant magnetic field, this size can drop to one-half or less of the quiet Sun value.) They seem to be less complex, perhaps because they evolve much more slowly and because their motion is detected only through advection of the solar atmosphere. Mesogranules are another aspect of solar convection. Their scale is not well defined, partly because of exploders and partly because it depends on the position of the structure relative to supergranule boundaries. So it may be a mistake to assign a unique scale to each of these three phenomena, especially if too much significance is attached to differences between these scales. Rather we should consider the convective processes that lead to their appearance. To understand the physics of these processes we must explore the underlying three-dimensional dynamics (Spruit et al. 1990). Various advances—in numerical and experimental techniques and in bifurcation theory—have increased our understanding of nonlinear convection. In particular, there has been substantial progress in modeling three-dimensional compressible convection (Chan & Sofia 1989; Stein & Nordlund 1989; Nordlund & Stein 1990a, b; Weiss 1990; Toomre et al. 1990; Malagoli, Cattaneo, & Brummell 1990; Cattaneo et al. 1991). Although we shall comment only briefly on dynamical behavior here, this topic is discussed in more detail by Simon & Weiss (1991).

Our results suggest that there is a weak mesogranular circulation which controls the incidence of exploding granules. We postulate that there are aperiodic oscillations (involving fluctuations in temperature and velocity) about a steady circulation which interact with the overlying granulation to produce bursts of exploding granules. The exploders generate horizontal outflows which in turn enhance the circulation and so help to maintain these symbiotic oscillations. This picture implies that the persistent outflow and the recurrent exploders are intimately coupled.

Laboratory studies (Zocchi, Moses, & Libchaber 1990) may provide a clue to the relationship between supergranules and mesogranules. In these experiments, as in some numerical simulations, there is a long-lived cellular network outlined by prominent sinking sheets or plumes; we suggest this corresponds to supergranules. Within these cells there is a weaker time-dependent pattern formed by sheets breaking away from the unstable thermal boundary layer: these would correspond to mesogranules. If so, mesogranules could be ascribed to secondary instabilities of supergranules (stimulated by exploders) without invoking any additional mechanism.

That leaves the problem of explaining the different scales of granules and of supergranules. Granulation presumably originates in an unstable boundary layer near the photosphere, where the superadiabatic gradient is large and the scale height is small. But there is still no satisfactory explanation for the well-defined scale of supergranular convection, or for the apparent lack of structure on any larger scale.

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