SHORT-TERM CHANGES IN SOLAR OSCILLATION FREQUENCIES AND SOLAR ACTIVITY

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ABSTRACT

We show that the frequencies of solar p-mode oscillations change significantly over periods as short as 1 month. These changes correlate significantly with variations in the strength of surface solar activity as measured by the average, over the Sun's visible surface, of the magnitude of the line-of-sight magnetic field component from magnetograms. The frequency and mean magnetic variations are found to obey a linear relationship. We find that the mean frequency shift at any time depends on the history of solar activity over an interval of, at most, several months prior to the measurement and conclude that the dominant mechanism of the frequency shift is correlated with surface magnetic activity.

Subject headings: Sun: activity — Sun: oscillations

1. INTRODUCTION

Evidence that the frequencies of solar oscillations change over the solar activity cycle has been available for about half a decade (Woodard & Noyes 1985). Dynamic oscillation modes of slowly rotating stars can be specified by three indices \( n, l, \) and \( m \), respectively, the radial order, spherical harmonic degree, and azimuthal order of the mode. The approximate spherical symmetry of a slowly rotating star implies that the mode frequency \( v_{nlm} \) depends mainly on \( n \) and \( l \) and only weakly on \( m \). For solar p-mode oscillations, changes have been observed both in the multiplet frequencies, \( v_{nl} = \langle v_{nlm} \rangle \), averaged over \( m \), and in the even-index “splitting” coefficients \( \alpha \) of the expansion

\[
v_{nlm} = v_{nl} + \sum_{i} \alpha(n, l) P_i(m/L),
\]  

where \( L = [l(l + 1)]^{1/2} \) and \( P_i \) is the Legendre polynomial of order \( i \). The dependence of the \( v_{nl} \) and \( \alpha(n, l) \) variations upon \( n \) and \( l \), or equivalently upon \( v = v_{nl} \) and \( l \), provides information about the radial dependence of changes in solar structure responsible for the frequency shifts. The relative strengths of the different even-index coefficients, on the other hand, allow the latitude dependence of the perturbation to be determined (the \( v_{nl} \) can be regarded as an \( i = 0 \) term in eq. [1]). The possibility that oscillation frequency shifts might permit detection of changes deep in the solar interior and thereby provide clues to the mechanism of the sunspot cycle accounts for much of the interest in p-mode variations. By examining the \( \alpha_2 \) and \( \alpha_4 \) coefficients obtained from various oscillation data sets taken between 1981 and 1988, Kuhn (1988) showed that the relevant perturbation is strongest in magnetically active latitudes. More recently Libbrecht & Woodard (1990), comparing long strings of solar oscillation data taken in 1986 and 1988 at Big Bear Solar Observatory (BBSO), were able to infer that the changes which produce frequency shifts occur mainly within a few pressure scale heights of the solar photosphere. The basis for this inference is the fact that the frequency shift of a mode is observed to be inversely proportional to the so-called “mode mass,” which is a steeply increasing function of mode frequency.

In this Letter we describe an analysis of p-mode frequency changes occurring within individual seasons of BBSO data. The primary motivation for this undertaking was to see whether there might be two components of the frequency shift, one of which is proportional to the total level of surface magnetic activity at any given time, the other a more gradual component, perhaps related to activity well below the solar surface. For example, if the gradual component were dominant we would expect that on short time scales the ratio of mode frequency fluctuations to solar activity variations to be much smaller than the ratio measured over long time scales. We therefore obtained and analyzed solar magnetograms and Ca K-line data to estimate the overall strength of surface solar activity and have compared the resulting activity indices with the \( p \)-mode data by means of regression and correlation analysis as described below.

2. DATA, ANALYSIS, AND RESULTS

2.1. Oscillation Data

The BBSO helioseismology data and their reduction and analysis have been described in detail elsewhere (Libbrecht & Zirin 1986). Analysis of the 1986 and 1988 data sets has provided us with, among other things, very accurate determinations of the frequencies and frequency splittings of solar p-modes (Libbrecht, Woodard, & Kaufman 1990; Libbrecht 1989) and has led to a clearer picture of how solar activity affects the p-mode frequencies as noted above. Briefly, the data consist of \( 240 \times 192 \) pixel full-disk solar Doppler images obtained at a rate of one per minute. A typical observing season lasts 4–6...
months during which ~65,000 images are obtained. For the earlier determinations of $p$-mode frequencies a single $l \sim v$
power spectrum was computed for each season of data. In this work we divided each season into contiguous, nonoverlapping
subintervals of length 215 minutes $\approx 22.8$ days and computed
an $l \sim v$ spectrum for each subinterval. To reduce time and
labour, spectra were computed only over the range $18 \leq l \leq 62$
of spherical harmonic degree.

The $p$-mode frequency shifts were obtained both by modeling
the power spectra and by cross correlating spectra from
different epochs. We describe only the results based on model-
ing, since the results from cross correlation are essentially the
same. A nonlinear, least-squares peak-fitting algorithm, similar
to that described in Libbrecht, Woodard, & Kaufman (1990),
was used to estimate the frequency $\nu_i$ of a $p$-mode multiplet of
radial order $n$ and degree $l$. The model includes false peaks
caused by nightly data gaps and by the invisibility of half the
solar surface. Only modes in the frequency range $2.0 < \nu < 3.8$
mHz were studied. Although the fitting routine provides a
formal estimate of the error in $\nu_i$, we thought it safer to estimate
this error from the scatter of measurements within a
given season. Accordingly, the error in $\nu_i$ for a given epoch is
the sample standard deviation of $\nu_i$ for all the epochs of that
season and therefore contains a contribution from real fre-
quency changes.

Figure 1 shows frequency shifts for all the epochs analyzed
so far. To produce this figure the $p$-modes were divided into
three 0.6 mHz–wide frequency intervals centered on 2.3, 2.9,
and 3.5 mHz. Each of the three curves is the unweighted
average frequency of all the multiplets in a given frequency
interval, except that the curves have been arbitrarily zeroed in
frequency for the first epoch of data, in accordance with our
focus on time variations. From Figure 1 we see that in 1988
and 1989, which were years of considerable activity, significant
variations occurred over periods as short as our time
resolution (22.8 days). The amplitude of the variations
increases markedly with mode frequency as found earlier by
Libbrecht & Woodard (1990), both on long time scales (years)
and on short time scales (months). The strong frequency
dependence of the short-term variations implies that they, like
the long-term variations, result from changes close to the
photosphere.

The observations are roughly consistent with a time-
dependent mode frequency $\nu_i(t)$ of the form $\nu_i + \Delta \nu_i(t)$, in
which the frequency shift, produced by solar activity, is given by

$$\Delta \nu_i(t) = \Delta \nu(t) \Delta \nu_i,$$

where $\phi_{al}$ is a time-independent function. In what follows we
take $\phi_{al}$ to be inversely proportional to the mode mass. In
doing so we ignore an observed departure from the reciprocal
mass dependence for frequencies exceeding $\approx 3.5$ mHz (as sug-
gested by Fig. 2 in Libbrecht & Woodard 1990, and confirmed
by the analysis of the 1989 BBSO data). This discrepancy is not
a serious problem for the present selection of modes. We normal-
ize $\phi_{al}$ to unity for $l = 0, n = n^*$ such that $\nu_{n^*} \approx 3$ mHz.
With this convention, the parameter $\Delta \nu(t)$ is interpreted as the fre-
quency shift of a fiducial, $l = 0$ mode of $\approx 3$ mHz frequency
and serves as an arbitrary standard of the overall strength of solar
activity at epoch $t$.

Apart from an irrelevant offset, an unbiased estimate of $\Delta \nu$
is provided by a weighted sum of the measured frequencies
having the form

$$\Delta \nu(t) = \sum \frac{\phi_{al}}{\sigma^2_{al}} \nu_i(t) \left/ \sum \frac{\phi^2_{al}}{\sigma^2_{al}} \right..$$

To maximize the signal-to-noise ratio we used all the available
modes in computing expression (3) and chose $\sigma_{al}$ to be the
standard error, averaged over all observing seasons, of $\nu_i$. The
resulting mean frequency shift is plotted in Figure 2 for each
epoch.

2.2. Solar Activity Indices

The fact that solar magnetic fields can, in principle, produce
$p$-mode frequency shifts spurred us to seek a suitable measure
of the overall level of magnetic activity for comparison with the
oscillation data. To this end we obtained essentially all the
daily Kitt Peak full-disk magnetograms from March of 1986 to
implies that the mode frequency shift is proportional to the
average over the Sun’s surface of the square of the field
strength. We note that if the magnetic field is concentrated in
thin flux tubes all having the same field strength, then the mean
absolute field is proportional to the mean square field. We used
the average of the absolute value of the field obtained from

![Figure 1](image1.png)

**Fig. 1.** Unweighted mean frequency shifts for $p$-modes in the 2.0–2.6, 2.6–
3.2, and 3.2–3.8 mHz frequency ranges. The mean frequency shift increases
with frequency range. Errors shown are single standard deviations.

![Figure 2](image2.png)

**Fig. 2.**—Optimally weighted average $\Delta \nu$, defined by eq. (3), over all the
observed modes of the frequency shift (solid curve). Errors are single standard
deviations. Magnetic field index $B$ (dashed curve). The frequency and magnetic
scales have been drawn in accordance with the linear regression analysis of the
full data sets described in § 2. Thus the two observed variations, as plotted,
agree in a least-squares sense.
magnetograms as an overall activity index. This index is certainly not ideal, since only the line-of-sight field is observed and only over the visible surface. In performing the average a given patch of the solar surface was weighted by the true area of the patch. Our averages do not differ substantially, however, from the averages supplied to us directly by J. Harvey (1990) in which magnetogram pixels were weighted equally. The daily averages were collected into 22.8 day bins corresponding to the time periods over which the \( p \)-mode frequencies were calculated and are plotted in Figure 2 with the frequency shifts.

We also compared the frequency shift data to another solar activity proxy. The Sacramento Peak CaK index (Keil & Gilliam 1990) was averaged in time bins to match the frequency shift data.

2.3. Relationship between Frequency Shifts and Solar Activity

Based on Figure 2, the magnetic and frequency shift variations appear to be correlated within individual seasons of high solar activity and the ratio of the two variations is comparable on all time scales. Table 1 shows the correlation between frequency shifts and mean magnetic field, and between frequency and CaK index. Since we cannot guarantee either a linear trend or normal distributions for these variables we favor a nonparametric test for association. In this case we used a simple rank correlation statistic. The Spearman coefficient, \( r_s \), and its two-sided significance, \( P_s \), (see Gibbons 1976) were evaluated individually for each season of data pairs and collectively using all 22 data pairs (six epochs in 1986, eight each in 1988 and 1989). To quantify the relative variation in frequency and magnetic field we tested for a linear relationship of the form

\[
\Delta v = a B + b,
\]

where \( B \) is the field strength index. The slope, \( a \), obtained from a standard linear regression analysis using all 22 data epochs is given in Table 1 with the standard deviation determined from the scatter of the residuals of the fit. The data, together with the best-fit regression line, are plotted in Figure 3, which shows that the data as a whole are consistent with the assumed linear relationship. Figure 3 shows errors in frequency only, since we lack a reliable estimate of error in field index. We see that the frequency errors account for very little of the scatter about the fit. Similarly, we find that in performing the regression analysis on separate seasons of data that the residual scatter is not dominated by frequency error. An error in \( B \) of 0.7 G would account for the residual scatter of the fits. We comment on the nature of the field error in § 3. Within individual seasons the field index variations are not large compared to 0.7 G. Thus, for the seasonal fits, we are not justified in using \( B \) as the independent variable in a linear regression, because its scatter significantly biases the fit. Instead we parameterized the seasonal fits in the form

\[
B = c \Delta v + d,
\]

where \( c = 1/a \). With \( \Delta v \) as the independent variable the bias in \( a \) is estimated to be negligible in 1988 and 1989. The 1986 season provides no slope information, since the frequencies did not vary significantly then. In Table 1, we report the seasonal values of the slope \( a = 1/c \) and its standard deviation inferred from the fit defined by equation (5).

3. Discussion

The correlation between \( B \) and \( \Delta v \) is formally significant in data for both 1988 and 1989. In 1988, the source of the correlation is evidently the linear trend in both data sets (Fig. 2), whereas in 1989 the source of the correlation must be variations over periods even shorter than the observing season. The 2 years taken together provide evidence of a correlation on a timescale of a few months. The absence of a correlation in 1986 is consistent with the lack of significant frequency variations in that year.

The Ca K data are also strongly correlated with the frequency shift observations in the full data set but none of the seasonal data subsets show a significant association. The significant long-term correlation results from the apparent solar-cycle-scale variations in both data sets.

The measured slopes \( a \) from Table 1 tell us that the sensitivity of the \( p \)-mode frequencies to variations in mean magnetic field strength over a number of years is, to within 30\% or so, the same as the sensitivity to variations over just a few months. These results place limits on contributions to the frequency shift that might arise from long-term (e.g., thermal) readjust-
ment of the subphotospheric layers to the presence of magnetically related activity. Our data suggest that the dominant readjustments occur in a matter of months, since we would otherwise expect the proportionality factor between frequency shift and field strength to depend on the time interval over which we observe. One could similarly entertain the possibility that part of the frequency shift is caused by a gradual, thermal restructuring of the subphotospheric layers, driven by a mechanism acting well below the surface, which on a time scale of months is uncorrelated with surface activity. If this gradual component of the frequency shift exists, it is too weak to be seen in our data.

As we noted in § 2, a 0.7 G error in $B$ can explain the residual scatter in the fits of frequency shift and magnetic field. Potential sources of error in magnetogram data include changes in the weather (e.g., variable seeing) at the observing site and incomplete sampling. An example of the latter is that magnetic data are not available for the invisible half of the Sun's surface, whereas the oscillations sample its surface uniformly. Even without measurement errors, our choice of magnetic index is probably less than ideal. For example, the contribution to the frequency shift of strong (e.g., sunspot) fields is expected to saturate at kilogauss field strengths (Goldreich et al. 1991).

In summary, we have found that the frequencies of solar $p$-modes change in response to solar activity not only on a time scale of several years, but over periods as short as our 22.8 day time resolution. Using the mean absolute value of the line-of-sight component of the solar magnetic field as an overall activity index, we find that the sensitivity of the mode frequencies to changes in activity is the same for changes occurring both over a few months and over a few years. Our data are, in fact, consistent with the hypothesis that the mean frequency shift is a linear function solely of the mean field strength index $B$, indicating that on all time scales the bulk of the frequency shift is correlated with magnetic fields near the solar surface.

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REFERENCES

Gibbons, J. D. 1976, Nonparametric Methods for Quantitative Analysis (Columbus, OH: American Sciences)
Harvey, J. 1990, personal communication
Keil, S. L., & Gilliam, L. B. 1990, personal communication