ON THE INFERENCES OF MAGNETIC FIELD VECTORS FROM STOKES PROFILES

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Received 1990 June 13; accepted 1990 October 25

ABSTRACT

We study the range of applicability of the weak-field approximation for the inference of magnetic field vectors from the profiles of the Stokes parameters. We show that, for typical solar lines in the visible, this approximation may be applied to an accuracy of \( \sim 20\% \) for field strengths as great as \( 3500{\gamma_\text{L}} \), \( \text{G} \)—where \( \gamma_\text{L} \) is the Landé factor—provided we work in the line wings. A systematic underestimate of the field strength seems to occur for strong fields—observations in several related spectral lines may allow a calibration of this effect, and so for an increase in accuracy of the inferences. The procedures have been tested using solar data obtained in two lines of Fe i, near 6300 Å, with the Stokes Polarimeter at the Mees Solar Observatory of the University of Hawaii.

Subject headings: Sun: magnetic fields — Sun: spectra — Zeeman effect

1. INTRODUCTION

In a publication by Jefferies, Lites, & Skumanich (1989, hereafter JLS) procedures were discussed which, in certain cases, allow the inference of approximate values of the magnetic-field vector directly and simply from measured profiles of the Stokes parameters \( I, Q, U, V \) across a magnetically sensitive line. The procedures were based on the so-called "weak-field" approximation (WFA) and were embodied in equations (55) and (56) of JLS. For reference these are reproduced\(^2\) below, along with the equation for determining the field azimuth \( \chi \):

\[
|v_b| \cos \gamma = -V \left( \frac{dI}{dv} \right),
\]

\[
\left( \frac{v_b \sin \gamma}{2} \right)^2 = \left( \frac{H'(a, v)}{H'(a, v)} \right) \left( Q^2 + U^2 \right)^{1/2} \left( \frac{dI}{dv} \right),
\]

\[
\tan 2\chi = U/Q.
\]

In these equations, \( \gamma \) and \( \chi \) are, respectively, the polar and azimuthal angles of the field vector in a reference frame where the vertical axis is along the line of sight (see Fig. 1 of JLS) and the axes in the \( x, y \) plane are arbitrarily established by the observer. The parameter \( v \) is the normalized line frequency \( (\Delta v/\Delta v_\text{D}) \) measured from the line center in units of the Doppler width \( \Delta v_\text{D} \); \( H', H'' \) are the derivatives, with respect to \( \nu \), of the Voigt function \( H(a, \nu) \); \( a \) is the usual damping constant; and \( v_b \) is the line splitting in units of the Doppler width, i.e.,

\[
v_b = \frac{eB_\text{LT}}{4\pi mc} \left( \frac{1}{\Delta v_\text{D}} \right) = \frac{eB_\text{LT}}{4\pi mc^2} \frac{\lambda^2}{\Delta \lambda_\text{D}}
\]

with \( B \) the field strength and \( \gamma_\text{L} \) the Landé factor for the spectral line.

In JLS, the results (1) were derived on the assumption that \( v_b < 1 \) (i.e., that the field \( B \) was "weak" in that sense) in which case the transfer equations for the Stokes parameters simplify greatly and require the conditions embodied in equations (1) for consistency. They were also based on the assumption that the line is a normal Zeeman triplet and that the magnetic field vector, the Doppler width and damping constant, and any systematic velocity flows do not vary with depth. There is, however, no restriction on the depth dependence of the LTE source function or the line strength parameter. Landi Degl'Ippocenti & Landi Degl'Ippocenti (1973) have pointed out that equation (1a) is, in fact, very robust, depending, in the WFA, only on the assumption that the longitudinal component of the field does not change with depth. To the extent that these approximations are valid, equations (1) apply at all frequencies across the line (and at all depths, too, although this is of less immediate interest). A considerable redundancy is potentially available, then, for determining the (assumed single) value of the field vector from measurements of \( I, Q, U, V \) across a line. This redundancy may be used—as was done by Ronan, Mickey, & Orrall (1987)—to combat the inevitable noise in the often weak polarization signals by using wavelength-integrated values of the Stokes parameters. In practice we can expect to find that inferences of the field vector made from data at different positions in the line do not always yield consistent results, and this will bring the assumption of depth independence into question. Indeed, the lack of the expected symmetry in the \( Q, U, V \) profiles, which is almost invariably seen in observational data, is a clear indicator that that assumption cannot be valid. More positively, such variations may open a path for inferring the depth gradient in the field vector. That question will be considered in a later study; here we continue to assume constant properties with depth.

Our motivation in this study is to pursue the limits of applicability of the weak-field approximation. The more general method developed and applied by Lites, Skumanich, and their collaborators (which involves nonlinear least-squares fitting to a special solution of the Stokes transfer equations) appears to be the only available approach for inferring strong fields in the line core. It is, however, highly demanding computationally.

\( ^1 \) Operated by the Association of Universities for Research in Astronomy, Inc. (AURA), under cooperative agreement with the National Science Foundation.

\( ^2 \) Note that the eq. (56) of JLS is incorrectly printed.
and quite impractical for on-line reduction. We might hope, then, that the WFA will turn out to be of sufficiently broad application to provide for a “quick-look” assessment as data is being gathered, as well, perhaps, as giving a good starting value for the more sophisticated Lites-Skumanich approach.

The present paper is consistent with this hope. We begin with a more detailed discussion of the conditions under which the WFA may apply and show that, under a broad range of conditions which may be commonly encountered on the Sun, simple algorithms can indeed be found for determining all three components of the vector field directly from the Stokes parameters. On the basis of the theoretical discussion we then determine the field vector for a typical situation near a sunspot using observations of the Stokes profiles of some Fe i lines obtained with the spectropolarimeter at the Mees Solar Observatory on Haleakala. The consistency found in the results of this application is reassuring enough to merit a broader study on a diverse set of data.

2. THE WEAK-FIELD APPROXIMATION

2.1. Range of Applicability of Equations (1)

To study the range of applicability of the WFA we note first that the relevant functions controlling the absorption and emission terms for the polarized components of the line radiation for a normal Zeeman triplet are3 (see JLS)

\[
\begin{align*}
 f_1(a, v, v_b) &= \frac{H(a, v + v_b) - H(a, v - v_b)}{2}, \\
 f_2(a, v, v_b) &= \frac{H(a, v + v_b) + H(a, v - v_b) - 2H(a, v)}{2}.
\end{align*}
\]

If we expand the Voigt functions about the unshifted frequency \(v\), we obtain

\[
\begin{align*}
 f_1(a, v, v_b) &= v_b H'(a, v) + \frac{v_b^2}{3!} H''(a, v) + \cdots, \\
 f_2(a, v, v_b) &= \frac{v_b^2}{2!} H'(a, v) + \frac{v_b^4}{4!} H''(a, v) + \cdots.
\end{align*}
\]

Now, as it is usually expressed, the WFA consists in neglecting all except the first term in each of the Taylor expansions (4). Clearly this will be acceptable if \(v_b\) is small enough, and this may be taken as a sufficient condition for the WFA. It is not obvious, however, that it is a necessary one; the approximation will still be valid even though \(v_b\) is relatively large provided the derivatives of the Voigt function fall off fast enough. In JLS we suggested that \(v_b = 0.5\) might be a reasonable upper limit for the sufficiency of the WFA, and for a spectral line at 6000 Å this would limit us to field strengths of \(\sim 1200/\ell \) G adopting a value of 40 mÅ for the Doppler width. This may be useful for some solar regions, and for lines with small enough Landé factors, but it is uncomfortably tight at best and makes it worthwhile to review the conditions for the WFA to see whether they will allow for a broader applicability.

The derivatives of the Voigt function will depend on the damping constant \(a\), and on the position in the line. Thus the applicability of the WFA will depend both on \(a\) and \(v\) as well as on the field strength. In a detailed study of sunspot data, Lites & Skumanich (1990) concluded that the damping constants for the Fe i lines at 6301.5 and 6302.5 Å, were \(\sim 0.5\); a similar (rather smaller) value had been obtained also by Harvey et al. (1971). Lites & Skumanich suggest that their inferred value may be influenced (too high?) by a possible depth variation of the Doppler width, which cannot be incorporated in their analysis. We shall, however, adopt their value as a standard for our illustrations.

To understand the WFA further we define the relative errors

\[
\begin{align*}
 D_1 &= \left| \frac{f_1(a, v, v_b) - v_b H'(a, v)}{f_1(a, v, v_b)} \right|, \\
 D_2 &= \left| \frac{f_2(a, v, v_b) - \frac{v_b^2}{3!} H''(a, v)}{f_2(a, v, v_b)} \right|.
\end{align*}
\]

and recognize that the WFA may be used if \(D_1\) and \(D_2\) are reasonably small (\(< 1/4\), say). Figures 1 and 2 show the variations of these relative errors across a spectral line for a range of value of \(v_b\) and a damping constant \(a = 0.5\). These figures

3 Magneto-optic terms also occur in the transfer equation; however, they do not affect the arguments given here.
suggest that the WFA will indeed apply for values of $v_b$ substantially greater than unity at least in the line wings. This conclusion is found to be not greatly influenced by the value of $a$ for $v > 4$.

Since the approximate formulae (1), relating the field vector to the observed Stokes parameters, are a direct consequence of the application of the WFA, and since, following Figures 1 and 2, the WFA appears to hold for $v_b$ as large as 1.5, at least in the line wings, we should be entitled to extend the validity range for the use of formulae (1) from the value given by the sufficiency condition $v_b < 1/2$ (or $1200/\eta_0$ G for a 6000 Å line) to magnetic fields $\sim 3500/\eta_0$ G, provided we can work beyond 3–4 Doppler widths in the line wings. This extended range of validity may be taken, in some sense, as giving a necessary condition for the WFA; if it applies then we may approximate the functions $f_1, f_2$ of equations (3) in the transfer equations for the polarized light intensities. However, radiative transfer of, and the associated coupling among, the Stokes components will modify (presumably to broaden still more) the range of parameter values over which the diagnostic relations (1) can be applied with acceptable accuracy. To clarify this we need to look at a specific solution of the transfer equation made without using the WFA and compare the results of that solution with those represented by equations (1). We are limited in the comparison we can make since the only available analytic solution is for the very restricted case of at depth variation of magnetic and atmospheric parameters. However, the exercise may lend confidence to the conclusions reached in the above analysis and provide some suggestions for improving the approximation. We discuss this issue separately for the formulae (1a) and (1b).

2.1.1. Application of Equations (1a)

We shall make use of the formal solutions to the full LTE Stokes equations for the case of a linear Planck function (and depth independence of the other relevant parameters), as given by Landolfi & Landi Degl'Innocenti (1982). On that basis we show, in Figures 3–5, the calculated values of the quantity $R_v$

$$R_v = - \frac{V}{(v_b \cos \gamma \frac{dI}{dv})},$$

as a function of $v$ for several values of the parameter $v_b$ and the inclination angle $\gamma$. In these illustrations we adopt the "standard" values $\eta_0 = 10$, $a = 0.5$ taken from Lites & Skumanich (1990). Indeed we find that $R_v$ is close to unity all across the line for $v_b \leq 0.5$; as $v_b$ increases so does the departure of $R_v$ from unity and the frequency at which it settles down to a slowly varying function of $v$. The figures suggest that analysis of the data using equation (1a) would yield results which are consistent to better than 10% in the line wings but which could be systematically in error by up to 30% for $v_b = 1.5$ and $\gamma = 0$. The accuracy of the procedure decreases with decreasing $a$ and increases with increasing line strength but in any given case remains fairly constant throughout the line wings.
2.1.2. Application of Equation (lb)

In the same way as in § 2.1.1 above, we have calculated the ratio \( R_Q \) defined as

\[
R_Q = \left( \frac{Q^2 + U^2}{F} \right)^{1/2},
\]

with

\[
F = \frac{\left( v_b \sin \gamma \right)^2}{2} \left( \frac{H''(a, v)}{H'(a, v)} \right),
\]

using the formal solutions given by Landolfi & Landi Degl’Innocenti. We show calculated values of \( R_Q \) as a function of \( \nu_b \) in Figure 6 for several values of \( \nu_b \) and \( \gamma = 45^\circ \). Essentially the same comments apply to the inference of \( \nu_b \sin \gamma \) as for the line-of-sight component, \( \nu_b \cos \gamma \), discussed in § 2.1.1 above. Note that we must take the square root of \( R_Q \) to find \( \nu_b \sin \gamma \) so that greater departures of \( R_Q \) from unity can be tolerated than for \( R_v \) for the same accuracy in the inferred field component.

2.1.3. Summary

We conclude, then, that the simple relationships (la) and (lb) can be applied (to the Fe i lines at 6300 Å, at least) to give results consistent to 10%–20% even for fields such that \( \nu_b = 1.5 \). The illustrations shown above suggest that a systematic underestimate is to be expected at larger fields; a method for at least a partial correction for this might be designed around observations of related spectral lines—e.g., lines of a common multiplet with different values of \( g_L \) and oscillator strength—and the evident requirement that they yield the same values of the field components. The illustrations in Figures 3–6 derive from a formal solution to the full Stokes equations, without appeal to the WFA, for restrictive conditions (LTE, a linear Planck function, constant radiative, and atmospheric properties). There is nothing in the development, however, that makes an evident case for the conclusion not being applicable to the more general situation obtaining for observed spectral line profiles.\(^5\)

\(^4\) Note that \( Q, U \) are both zero for \( \gamma = 0 \).

\(^5\) However, asymmetries are usually observed but have no place in this form of the theory.

Finally we note that the formulae (1) would be of little practical value if the observationally based quantities could not be estimated reliably in the line wings where the formulae are valid. To reassure ourselves on this count we show, in Figure 7, the line profile normalized to the continuum, and in Figure 8 the profiles \( Q(v), V(v) \) normalized to their maximum values. The calculated values again are taken from the formal solutions of Landolfi & Landi Degl’Innocenti, however, the same conclusion, (that \( I, V, Q \) are well measured in the line wings) applies to the Stokes profiles of the observed solar lines.

2.2. The Form of Equation (1b)

The Doppler width which enters into the definitions of \( \nu \) and \( \nu_b \) cancels out of equation (1a) which can therefore be applied directly to the observed Stokes profiles of \( V \) and \( I \). To determine the line-of-sight component for the Fe i lines at 6300 Å,
for example, we may then use the equation

\[
B \cos \gamma = \left( \frac{5.4 \times 10^4}{g_L} \right) V \frac{dI}{d\lambda} G,
\]

(9)

with the wavelength measured in angstroms.

Such a direct simplification of equation (1b) is not immediately available for the transverse field \( B \sin \gamma \), since the ratio for example, we may then use the equation

\[
\frac{B \sin \gamma}{B \cos \gamma} = \left( \frac{5.4 \times 10^4}{g_L} \right) V \frac{dI}{d\lambda} G,
\]

(10)

for several values of \( a \) and it is clear that this quantity is effectively independent of \( a \) in the line wings, having a value \( \approx -3 \). For \( a \) values in the range indicated by analysis of solar spectra (0.5), the factor \( G(a, v) \) is constant to high accuracy for all \( v \geq 2 \). Of course the value of \( F(a, v) \) of equation (8) for any given line depends on the Doppler width; however, when used in the expression (1b) the Doppler width cancels out leaving us with the result, valid for the line wings;

\[
B \sin \gamma = \frac{6.2 \times 10^4}{g_L} \left[ \frac{\Delta \lambda (Q^2 + U^2)^{1/2}}{dI/d\lambda} \right]^{1/2} G,
\]

(11)

with wavelengths again measured in angstroms.\(^6\) The quantity within square brackets will be denoted \( A_{Q} \).

In summary then, we would argue that equations (1a) and (1b) can be applied to a wide range of conditions in the Sun to infer the transverse and line-of-sight components of the magnetic field directly from the observed profiles of the Stokes parameters \( I, Q, U, V \). Since the relationship (1c) is a trivial consequence of the WFA, the full field vector may then be immediately inferred to a good accuracy using the extremely simple algorithms (9), (11), and (1c) and without appeal to the

values of the damping constant, the line strength, the Doppler width, or the depth variation of the Planck function. Furthermore, by observing the Stokes profiles in a combination of lines whose relative properties are known, and with guidance from illustrations such as those given here, it should be possible to refine these algorithms to obtain a still higher accuracy in the inference of the field vector.

2.3. The Influence of a Filling Factor

If only a part of a sampled area is occupied with magnetized regions, the analysis set out above will need some modification. Thus the \( Q, U, V \) intensities will arise only in the magnetized gas, while the intensity \( I \) will have components arising both in magnetic and nonmagnetic areas. If we were to suppose that the intensities \( I_m, I_n \) in magnetized and nonmagnetized regions had the same profile shapes, then we would have

\[
\frac{dI_0}{d\lambda} = f \frac{dI_m}{d\lambda} + (1 - f) \frac{dI_n}{d\lambda} = \frac{dI_m}{d\lambda},
\]

(12a)

where \( f \) is the fraction of the sampled area that is occupied by magnetized gas and \( I_m \) is the observed intensity. The observed values of the polarized intensities will be such that, for example,

\[
V_o = fV_m,
\]

(12b)

with equivalent expressions for \( Q_o \) and \( U_o \). Given that the WFA forms (9) and (11) apply for the magnetic regions we therefore have, for this special case when the intensity profiles are the same, the more general result

\[
fB \cos \gamma = \frac{5.4 \times 10^4}{g_L} \left( V_o \frac{dI_o}{d\lambda} \right) G,
\]

(12c)

\[
\sqrt{f} B \sin \gamma = \frac{6.2 \times 10^4}{g_L} \left[ \frac{\Delta \lambda (Q_o^2 + U_o^2)^{1/2}}{dI_o/d\lambda} \right]^{1/2} G
\]

(12d)

This result simply reflects the fact that measurements made in this way lead to values of the magnetic flux and not the field strength in the magnetic regions.

In general, of course, the line profiles \( I_m \) and \( I_n \) will not have the same slopes, and the application of equations (12c) and (12d) will introduce some error. There are two reasons for this; first the magnetic and nonmagnetic regions will not normally have the same thermodynamic parameters, and so give different profiles; second the Zeeman splitting itself will obviously change the line shapes. We are not aware of any method for addressing the first problem if the different regions are not resolvable and shall consider only the second effect here. To assess its magnitude we have again used the formal solution of Landolfi & Landi Degl’Innocenti to calculate the derivative \( (dI/d\lambda) \) for a mixture of magnetic and non-magnetic regions which are thermodynamically equivalent. In Figures 10 and 11 we plot the ratio \( R_o \) for three filling factors and for \( v_b = 0.5 \) and 1.5, respectively, and for \( \gamma = 45^\circ \).

A similar set of results for \( R_{Q_o} \) is given in Figures 12 and 13. To the extent that the values of \( R_o \) and \( R_{Q_o} \) in these figures depart from unity, so equations (12c) and (12d) are in error. At least this is the case for for the particular formal solution used here, but this should be representative of the situation for observed spectral lines. For interest we also reproduce, in Figure 14, the profile of the reduced intensity \( I^* \) for the three filling factors 0, 0.5, and 1, and \( v_b = 1.5 \).
The effect of the filling factor on the line slope is not major for the particular case considered here. Even for large values of \( v_b \) it amounts to \( \sim 10\% \) uncertainty in inference of the flux components if we know nothing about the expected value of \( f \). We must stress, however, that the assumption of thermodynamic equivalence in magnetic and nonmagnetic regions is artificial and likely to have a major impact (in any analytical scheme). However this may be amendable to improvement with multiple-line data.

3. APPLICATION TO OBSERVED PROFILES

As a first test of the procedures discussed above we have applied the results \( (9), (11), \) and \( (1c) \) to observations, taken with the Stokes Polarimeter at the Mees Solar Observatory of the University of Hawaii in the Fe I spectral lines at 6301.5 and 6302.5 Å. These spectra were acquired on 1989 October 20 in the vicinity of a fairly large sunspot. They represent one spatial position in a 12 by 15 step spatial raster covering the spot. An intensity map is shown in Figure 15 with a circle superposed to indicate the area sampled by the polarimeter. Data points were acquired at wavelength steps of 24.5 mÅ covering the range 6300.3–6303.4 Å. Plots of the \( I \) and \( V \) profiles over this wavelength range are shown in Figure 16, and for \( Q \), and \( U \) in Figure 17.

We must first delineate the wavelength intervals over which we can expect to be able to apply formulae \( (9), (11), \) and \( (1c) \) with confidence. Criteria for selecting these intervals are that (1) the relevant polarization components—\( V \), or \( Q \) and \( U \)—be well measured above the background noise; (2) the slope of the intensity be reliably measurable; in particular, that the effect of any blends be small and; (3) that the WFA should not obviously be excluded, which for our case means that we be no closer to the line center than \( \sim 2–3 \) Doppler widths. In this way, we delineate the wavelength regions shown in Table 1 for...
the application of equation (9) to determine the line-of-sight component of the field. Similarly we may delineate regions where equation (11) can be applied, these are shown in Table 2. The tables also show the ratio of the observational quantities which enter into formulae (9) and (11); here, as elsewhere in the tables, the parenthetical values refer to data which has been corrected in the way discussed below. In Table 2 the calculated value of the field azimuth $\chi$ is also given. The wavelength units used to form the intensity derivative are tabular units, each equal to 24.5 mÅ. The available points in the two tables are somewhat different because the parameters $Q$ and $U$ are not as well measured as is $V$.

The values of the ratios $V/(dI/d\lambda)$ in Table 1, and the parallel $Q$ and $U$-based parameter in Table 2 are generally consistent.

Points 55 and 57 are influenced by an apparent blend which leads to erroneous values of the intensity derivatives. The numbers in Tables 1 and 2 lead to the mean values given in Table 3 for each wing of each of the two lines in the scan.

Within the limits of our approach, the ratios of the $V$ parameters and the square root of the $Q$ and $U$ based parameter should be in the ratio of the appropriate Landé splitting factors. The azimuth angles $\chi$ should be the same and by inspection we see that they are acceptably consistent. The
6302.5 line is a Zeeman triplet and its Landé factor is 2.5; the 6301.5 line is not a triplet and the values of \(g_L\) to be used in the weak field forms (9) and (11) are different, as Landi Degl'Innocenti & Landi Degl'Innocenti (1973) have shown. From explicit formula given by these authors we find that, for the 6301.5 line, the \(g_L\) to be used in equation (9) is 1.67 while that to be used in equation (11) is 1.59. The ratios shown in Table 4 are to be compared with the theoretical ratios of 0.66 and 0.64. Some corrections to the data are suggested from examination of the observational results shown in Figures 16 and 17. To see their effect on the inferred field vector, we have smoothed (by hand) the blends around points 40 and particularly, 55, and adjusted the measured values of \(Q\) and \(U\) to correct for what appears to be a zero offset in these data. With these corrections the results in the tables are changed to the values shown in parentheses.

The numbers given in the Tables reflect the asymmetry seen in the Stokes profiles, but we defer further study of that question. If we simply take the average of all the tabular values for each line we find the inferred vector field components given in Table 5. The consistency of these values for the two lines appears to indicate that the magnetic regions effectively fill the sampled region, i.e., the filling factor is close to unity. This seems consistent with the location, at the umbral boundary, of the observation. For this particular sampled area we would therefore conclude (using the corrected data) that the whole region is characterized by a magnetic field whose vector properties are

\[ |B| = 1600 \text{ G}, \quad \gamma = 65^\circ, \quad \chi = -31^\circ. \]

### TABLE 1

| Wavelength Intervals and Estimated Values of \(V/(dI/dI)\) |
|------------------|----------------|----------------|
| Raster Point     | \(V/(dI/dI)\)  | \(g_L\) Value  |
| 42               | 0.73 (0.66)    | 1.67           |
| 43               | 0.93 (0.89)    | 1.67           |
| 44               | 0.91 (0.91)    | 1.67           |
| 45               | 0.96 (0.96)    | 1.67           |
| 46               | 1.00 (0.99)    | 1.67           |
| 54               | 0.87 (0.77)    | 1.67           |
| 55               | 1.67 (0.83)    | 1.67           |
| 56               | 0.68 (0.68)    | 1.67           |
| 57               | 0.48 (0.86)    | 1.67           |
| 58               | 0.74 (0.92)    | 1.67           |
| 81               | 1.53 (1.53)    | 1.67           |
| 82               | 1.05 (1.05)    | 1.67           |
| 83               | 1.30 (1.30)    | 1.67           |
| 96               | 1.07 (1.07)    | 1.67           |
| 97               | 1.01 (1.01)    | 1.67           |

### TABLE 2

| Wavelength Intervals and Values of Azimuth and the Ratio \(A_Q\) |
|------------------|----------------|----------------|

#### A. For \(R_Q\)

<table>
<thead>
<tr>
<th>Raster Point</th>
<th>(R_Q) Value</th>
<th>(-\chi) Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>3.4 (3.1)</td>
<td>38 (32)</td>
</tr>
<tr>
<td>44</td>
<td>3.8 (3.6)</td>
<td>33 (29)</td>
</tr>
<tr>
<td>45</td>
<td>4.3 (4.2)</td>
<td>33 (31)</td>
</tr>
<tr>
<td>46</td>
<td>3.7 (3.6)</td>
<td>34 (32)</td>
</tr>
</tbody>
</table>

#### B. For \(A_Q\)

<table>
<thead>
<tr>
<th>Raster Point</th>
<th>(A_Q) Value</th>
<th>(-\chi) Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>53</td>
<td>1.3 (1.1)</td>
<td>34 (30)</td>
</tr>
<tr>
<td>54</td>
<td>2.0 (1.7)</td>
<td>32 (28)</td>
</tr>
<tr>
<td>55</td>
<td>3.8 (1.8)</td>
<td>34 (30)</td>
</tr>
<tr>
<td>56</td>
<td>1.4 (1.4)</td>
<td>27 (17)</td>
</tr>
<tr>
<td>83</td>
<td>6.1 (6.7)</td>
<td>36 (35)</td>
</tr>
<tr>
<td>96</td>
<td>5.2 (5.7)</td>
<td>36 (34)</td>
</tr>
<tr>
<td>97</td>
<td>5.6 (6.4)</td>
<td>35 (32)</td>
</tr>
</tbody>
</table>
Averaged Values of the Inferred Parameters

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>Line 1</th>
<th>Line 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V/(dI/dx) )</td>
<td>0.91 (0.88)</td>
<td>1.29 (1.29)</td>
</tr>
<tr>
<td>( \Delta I(Q^2 + U^2)^{1/2}/dI/dx )</td>
<td>3.8 (3.6)</td>
<td>6.1 (6.7)</td>
</tr>
<tr>
<td>( \chi )</td>
<td>-34° (-31°)</td>
<td>-36° (-35°)</td>
</tr>
</tbody>
</table>

TABLE 4

Parameter Ratio for the Two Lines

<table>
<thead>
<tr>
<th>Values</th>
<th>Blue Wing</th>
<th>Red Wing</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V/(dI/dx) )</td>
<td>0.71 (0.68)</td>
<td>0.86 (0.78)</td>
</tr>
<tr>
<td>([\Delta I(Q^2 + U^2)^{1/2}/dI/dx])^{1/2})</td>
<td>0.79 (0.73)</td>
<td>0.62 (0.50)</td>
</tr>
</tbody>
</table>

TABLE 5

Inferred Field Components

<table>
<thead>
<tr>
<th>Component</th>
<th>Line 1</th>
<th>Line 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>B\cos\gamma</td>
<td>)</td>
</tr>
<tr>
<td>(</td>
<td>B\sin\gamma</td>
<td>)</td>
</tr>
<tr>
<td>( \chi ) degrees</td>
<td>-33 (-29)</td>
<td>-36 (-34)</td>
</tr>
</tbody>
</table>

In this section we have illustrated the application of the theoretical approach presented in § 2. The results appear consistent enough to lend us some confidence in their application, but a lot more study of actual data is needed. Such data should be obtained in lines with a range of splitting factors but otherwise similar. An unbiased procedure for smoothing across blends, and for removing zero offsets (preferably in the instrument itself) would also be necessary to allow a proper test of this procedure.

The authors wish to thank Egidio Landi Degl'Innocenti for a number of valuable comments, and in particular for pointing out a careless error in equation (12d): we had used the factor \( f \) rather than the correct factor \( f^{1/2} \).

REFERENCES