NON–LTE ANALYSIS OF THE OIfPe/WN9 STAR HDE 269227 (R84)

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ABSTRACT

A spectral analysis of the OIfPe/WN9 star HD 269227 (R84) is presented. The equations of radiative transfer are solved for a spherically expanding atmosphere. A non–LTE model is used to predict the spectrum of hydrogen and helium. In almost all cases, the calculated and observed line profiles agree within observational error. The stellar parameters determined for R84 are \( L = 10^{5.7} \, L_\odot \), \( T_{\text{eff}} = 28,500 \, \text{K} \), \( R_* = 30 \, R_\odot \), \( M = 10^{-6} \, M_\odot \), \( \dot{M} = 400 \, \text{km s}^{-1} \), and \( \gamma = 0.63 \) by mass. Since the blueshift of the helium line P Cygni absorptions depends upon the line strength, the observed helium lines allow the velocity law to be probed from the terminal velocity of the wind down to approximately 35 km s\(^{-1}\). Thus, we can empirically determine the shape of the velocity law.

The above stellar parameters are used in a subsequent hydrodynamic calculation assuming that the stellar wind is driven by radiation pressure. This model reproduces both the observed mass-loss rate and the terminal wind velocity. A comparison of computed velocity law with the law determined by the spectroscopic analysis reveals good agreement, which strongly supports the argument that the stellar wind is radiation-driven. The observational opportunity to investigate the wind’s velocity law makes OIfPe/WN9 stars ideally suited objects of testing predictions of radiation-driven wind theory.

From its stellar parameters R84 can be identified as a star which has lost about half its initial mass. A comparison of the stellar parameters with those calculated from evolutionary models reveals that R84 is probably a post–red supergiant and that it experienced violent mass loss during its red supergiant phase. R84’s close spectroscopic similarity to S Doradus variables at minimum visual brightness suggests that these stars have already experienced a phase of violent mass loss in their past during which they lost a significant fraction of their initial mass. Whether or not this enhanced mass loss occurred during the red supergiant phase cannot be determined.

Subject headings: radiative transfer — stars: abundances — stars: atmospheres — stars: individual (HDE 269227) — stars: winds — stars: Wolf-Rayet

1. INTRODUCTION

A large number of luminous early-type stars with emission-line characteristics have been known in the Large Magellanic Cloud (LMC) since the surveys by, e.g., Feast, Thackeray, & Wesselink (1960), and by Bohannan & Epps (1974). Walborn (1977, 1982) and Bohannan (1979) drew attention to an interesting subset of emission-line stars in the LMC: the class of OIfPe/WN9 stars. The spectral morphology of these stars is intermediate between Of and WN7-A stars (Bohannan 1990). At present, 10 OIfPe/WN9 stars are known in the LMC (Bohannan & Walborn 1989).

One of the striking characteristics of the OIfPe/WN9 spectra is the presence of nebular lines such as [N ii] \( \lambda 6548, 6583 \) (see Stahl & Wolf 1986). These lines can be understood in terms of a model in which shells are periodically ejected by the underlying star. In several cases, shells that had been ejected at previous outbursts could be detected by direct imaging. Stahl (1987) investigated the physical parameters of these resolved shells. The ejection of such shells can significantly influence the evolution of these stars and is possibly related to the S Doradus-type outbursts occurring in luminous blue variables (LBVs). Further evidence for these suggestions has been given by Stahl (1986), who emphasized the strong resemblance of the spectra of OIfPe/WN9 stars and the LBV AG Carinae during minimum. Another example is HDE 269858 ( = R127), which is included in Walborn’s (1982) original list of OIfPe/WN9 stars. Around 1983, this star displayed a major outburst (Stahl et al. 1983), then went through a spectral phase characterized by a

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The evolved character of the Ofpe/WN9 stars is further illustrated by an overabundance of CNO-processed matter detected in several stars. Abundance analyses of the circumstellar nebulae hint at nitrogen overabundances by large factors (see Walborn 1989).

In view of the importance of Ofpe/WN9 stars for the evolution of massive stars, the need for reliable stellar parameters for these stars is obvious. However, no detailed atmospheric analysis has been published for Ofpe/WN9 stars to date. The intimate connection between the photosphere and the outflowing matter in these stars requires the use of state-of-the-art model atmospheres which account for the photosphere and the stellar wind self-consistently. Hamann & Schmutz (1987) presented calculations adequate for Wolf-Rayet stars using non-LTE model atmospheres in spherical geometry. Since the physical conditions in Ofpe/WN9 stars are virtually identical to those in weak-lined Wolf-Rayet stars of the nitrogen sequence, we apply these models to Ofpe/WN9 stars.


The spectrum of R84 indicates also the presence of a red supergiant. It is unknown whether the Ofpe/WN9 and the red supergiant are companions of a binary system or just on the same line of sight. The analysis of the Ofpe/WN9 star is not affected by the red supergiant if only the blue spectrum is analyzed, since the red supergiant is so cool and not luminous enough to contribute to the spectrum shortward of about 5000 Å. The name RM 2-54 actually refers to the red supergiant present in the spectrum and not to the blue star we are studying here. In the following we are referring only to the Ofpe/WN9 star when we use the name R84.

The photometric history of R84 has been studied by Rebeirot et al. (1983) and Stahl et al. (1984). No major photometric variation of R84 has been observed since about 1975, although moderate spectroscopic variability in certain wind lines has been detected by Stahl et al. (1985).

In § 2 we present new observations and summarize earlier data. A description of the computer codes used for interpreting these observations is given in § 3. The spectroscopic analysis of R84 is presented in § 4, and in § 5 we explain the mass-loss rate of R84 by means of radiation-driven wind theory. The evolutionary status of R84 is discussed in § 6.

2. OBSERVATIONAL DATA

Three blue high-resolution spectra of R84 are available from the literature. The earliest is included in an atlas of high-dispersion spectrograms of peculiar emission-line stars in the Magellanic Clouds (Stahl et al. 1985). This spectrum was obtained on 1982 January 13 with the coudé spectrograph at the ESO 1.5 m telescope recorded on a photographic plate. The dispersion was 20 Å mm⁻¹, resolution 0.4 Å, and free spectral range 3600–4900 Å. The star was reobserved on 1985 January 2 with the identical equipment (Stahl 1986). A third spectrum was obtained on 1984 August 31 with CASPEC at the ESO 3.6 m telescope (Wolf, Stahl, & Seifert 1987). This spectrum was recorded on a CCD and has a resolution of 0.2 Å. The CASPEC observation clearly has the best quality of the three.

In order to choose a spectrum for analysis we have to consider more than just the quality of the spectrum. Unfortunately, the high-quality CASPEC spectrum is not well suited for two reasons: first, the He II λ4686 line—which is crucial for the spectral diagnostic—is at the very end of the last spectral order of the echelle spectrogram, and therefore its strength has to be regarded as unreliable because of the uncertain correction of the echelle ripple; second, and more important, no photometric measurements have been published close in time to the CASPEC observation. Since the strengths of spectral lines of R84 are time-variable (Stahl et al. 1985) the atmospheric conditions of this star have to be regarded as variable as well. Stellar parameters can be derived only if an absolute magnitude is known (see § 3), and consequently there is not enough information to analyze R84 for 1984 August.

For the other two epochs all important pieces of information needed for an analysis are available. We will concentrate on the spectrum obtained in 1982 January, mainly because it is more complete in terms of (published) wavelength coverage than the 1985 observation. The observed line-profile tracings shown in the present paper are those of 1982 January 13 digitized from Figure 15 of Stahl et al. (1985). The 1982 observation has an additional advantage over the one from 1985 because three are UV observations by the IUE satellite before and after the optical observation date, which allows one to interpolate the UV flux in time rather than extrapolate.

Photometric spectra have the reputation of being unreliable with respect to their intensity calibration. Fortunately, we are able to judge the quality of the 1985 spectrum by a comparison with a medium-resolution observation (1.5 Å resolution) obtained with a linear detector only 2 days earlier (Bohannan & Walborn 1989). Agreement between the two spectra is within 10% if one allows for differences in the continuum level chosen by the observers. Since the 1982 and 1985 spectra are obtained with identical equipment, we may assume that the 1982 observation is also reliable within 10%. The line strengths of the 1984 (CASPEC) high-resolution spectrum agree almost perfectly with the medium-resolution one (within the noise of the observations, i.e., within 5%) with the exception of He II λ4686. This line is about 30% weaker in the CASPEC spectrum, a difference which might not be real (see above). Thus, R84 probably did not vary significantly between 1984 August and 1985 January. The line strengths of the 1982 January spectrum are weaker than those observed in 1984 and 1985. We believe that these differences are real.

Photometric UBV observations of R84 obtained practically simultaneously with the 1982 January 13 high-resolution observation have been published by Stahl et al. (1984). Between 1981 December 28 and 1982 January 11 they report 14 observations for which the B — V values did not vary by more than the observational errors of about 0.02 mag. The U — B variations were slightly larger, about 0.05 mag, but the authors called these “probably small variations,” i.e., it is not clear whether there were real variations. Details of the technical data of these observations can be found in their paper. In Table 1 we list the
average of their flux measurements together with RIJKL observations closest in time.

In addition to the observations in the optical wavelength range, we have collected all UV spectrophotograms obtained by IUE from the IUE archive, using the NASA Regional Data Facilities in Boulder. There are 12 low-resolution observations taken between 1978 and 1982. Shore & Sanduleak (1984) give a brief description of the IUE observations in 1981, and Stahl et al. (1984) compare two LWR images obtained in 1978 and 1982. Stahl et al. (1984) found a 40% flux difference between the images LWR 2630 (1978) and LWR 14548 (1982). To read the IUE observations, we used a retrieval procedure that includes a correction for time degradation of the LWR camera. We find that the differences between individual LWR observations are drastically reduced when the correction is applied. However, by inspecting the images, we find they can be divided into two sets: observations obtained between 1978 and 1979 and observations obtained between 1980 and 1982. Each of the two sets contains six images (three SWP and three LWR); fluxes of images within a set agree within observational errors. Between the two sets there is a slight disagreement which reaches a maximum of about 15% at 2000 Å. This value is close to the reliability of IUE observations but might be real because it is confirmed by all images.

In Figures 1 and 2 we show the mean of the 1980/1982 images obtained with the SWP and LWR cameras. The individual images have been weighted by their exposure time, and contaminated data have been excluded. The overlap between the means of the SWP and LWR spectra is excellent. In particular, the flux drop at about 1990 Å is observed by both cameras and therefore can be regarded as real and not due to a mismatch of the camera calibrations. A high-resolution SWP spectrum was obtained by one of us (C. L.) on 1984 August 9 (SWP 23655). The reader is referred to the paper by Wolf et al. (1987) for a more detailed description of the high-resolution observation.

In Figure 3 we display spectrophotometric observations at CTIO by P. Massey between 1981 November 19 and 23 using the SIT-Vidicon detector on the Cassegrain spectrograph at the 1.5 m telescope. The data were made available by Torres-Dodgen & Massey (1988) through the Astronomical Data Center. The spectrum is combined from observations in two wavelength ranges, a red and a blue, with an overlap at Hβ. The instrumental resolution as measured from the FWHM of spectral lines is ~25 Å in the red spectral region (4000–6970 Å) and ~12 Å in the blue (3410–4930 Å), respectively. The spectrum shown is identical to the one used by Torres-Dodgen & Massey (1988) to measure colors and magnitudes of R84. More information about the reduction procedure is given in their paper.

Also indicated in Figure 3 are the fluxes obtained from the photometric observations in 1982 January (Table 1) and the fluxes resulting from narrow-band photometric observation published by Smith (1968) and recalibrated with the formulae given by Schmutz & Vacca (1991). If we convolve the spectrophotometric observation with the filter functions B2 and V (Buser & Kurucz 1978, Table 1) we find B = 12.25 mag and V = 11.96 mag. These numbers have to be compared with the photometric measurements obtained only 40 days later, B = 12.05 mag and V = 11.96 mag; i.e., R84 has brightened by 0.20 in B in only 40 days! The difference is even larger if the contribution of the emission lines to the broad-band filters is

### Table 1: Observed Fluxes in log F_x of R84

<table>
<thead>
<tr>
<th></th>
<th>1982 January</th>
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<th>1983 August</th>
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<tr>
<td></td>
<td>U</td>
<td>B</td>
<td>V</td>
</tr>
<tr>
<td>R</td>
<td>-12.82</td>
<td>-13.00</td>
<td>-13.21</td>
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<tr>
<td>J</td>
<td>-13.64</td>
<td>-14.22</td>
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<tr>
<td>H</td>
<td>-13.64</td>
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* These values are calculated from the photometric observations of Stahl et al. 1984, converted to fluxes with the calibration of Bessell 1979 for UBVRI and of Wamsteker 1979 for JHKL. Units: ergs s^{-1} cm^{-2} Å^{-1}.

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removed: The lines of the spectrophotometric observation contribute 0.09 mag to the flux observed in the broad-band B filter; however, only 0.06 mag is contributed by the weaker lines observed in the 1982 January high-resolution observation. This yields a difference in continuum fluxes of 0.23 mag at 4400 Å. There are no significant flux variations in V and R between the two epochs. The agreement in R can be understood because this filter is dominated by the red supergiant included in the aperture; that in V is puzzling, since the contribution of R84 to the V flux is still significant and one would expect V to vary if R84 varies. The B magnitude reflects essentially the uncontaminated flux of R84 (see Fig. 4).

The sudden change in brightness of R84 is in strong contrast to the photometric stability during 3 years from 1981 December to 1984 December (Stahl et al. 1984; Stahl 1986) and to the nonvariability in the UV from 1980 to 1982. The IUE images were taken before and after the spectrophotometric observation in 1981 November. The IUE image obtained closest in time is LWR 11048, which was exposed 1981 September 28. Thus, if the optical observation in 1981 November is correct, R84 switched from a state which was stable for years to another state and back to the former one within 100 days. This seems to be very unlikely, and one might conclude that the blue spectrophotometric data are faulty. However, there are facts which argue against rejection of the 1981 November observation. First, the spectrophotometric data agree with Smith's (1968) ubv values obtained at least 14 years earlier! If there is something wrong with the spectrophotometric data, an accidental agreement with an old observation seems to be unlikely as well. Also, there is no plausible origin of such errors in the spectrophotometric data. In 1981 November the observations were made with a large aperture (10'), and the continuum was found to be low and the lines strong. In 1982 January the continuum was high and the lines were weak, but the high-resolution spectrum was obtained with a much narrower aperture than used for the spectrophotometric observation. Furthermore, a quantitative comparison of the line strengths does not support an error in the continuum level (either too much background subtracted in the blue in 1981 November or light contributed in 1982 January). The equivalent widths observed in 1981 November are a factor of 2 (hydrogen lines) to 4 (He II λ4686) stronger than those of 1982 January. If these values are corrected for a 0.23 mag continuum difference, there still remains a significant difference between the observed line strengths.

We are not able to draw any conclusion from the contradictory observational data presented here. However, in order to define the continuum energy distribution of R84 which corresponds to the analyzed situation in 1982 January, the date of the first high-resolution observation, it is certainly consistent to adopt the fluxes of the broad-band UBV photometric data (given in Table 1) together with the 1980/1982 UV data.

3. Method

3.1. Model Atmosphere

The helium and hydrogen line profiles presented in this paper are calculated from a semiempirical model atmosphere. It is assumed ad hoc that the star has a strong stellar wind. We will see in § 5 that our assumption is consistent with the results from a radiatively driven wind. In the manner described by Hamann & Schmutz (1987), the velocity field of the expanding atmosphere can be specified in terms of three parameters: \( g_{\text{rel}}, \beta, \) and \( V_\infty. \) The first parameter defines the low-velocity regime

\[
V(r) = V_{\text{min}} \exp \left[ \frac{(r - R_g)}{H} \right],
\]

for \( r < r_c \).

where \( H \) is a constant scale height, \( H = kT_\text{eff} (\mu m_\text{H}) \). Despite the similarity of equation (1) to the solution of the equation of hydrostatic equilibrium in plane-parallel geometry, we deliberately name the parameter \( g_{\text{rel}} \) and not \( g_{\text{eff}} \) because we find in § 4.4 that the observable influence of this parameter is on the velocity law above the sonic point. In other words, the parameter \( g_{\text{rel}} \) is not related to the stellar gravity, and equation (1) has to be viewed as nothing but a parameterized description of the low-velocity law. The remaining two parameters, \( V_\infty \) and \( \beta \), define the supersonic regime

\[
V(r) = V_\infty (1 - r_g/r)^\beta.
\]

for \( r > r_c \).
The radius at which the two regimes are connected, $r_c$, and the radius $r_0$, in equation (2) result from the condition that the velocity law is a differentiable function. The velocity at the inner boundary, $V_{\text{inner}}$, has no influence on the results provided that it is chosen small enough that the resulting Rosseland optical depth at the inner boundary is larger than about 10. Once the velocity law is defined, the specification of the mass-loss rate, $\dot{M}$, allows one to calculate the density structure via the continuity equation. Then, for a given luminosity $L$ and stellar radius $R_*$, the temperature structure is obtained in the manner described by Wessolowski, Schmutz, & Hamann (1988) by solving the gray LTE problem in spherical geometry, simplified by means of a generalized Eddington approximation. This approximation has been shown to be adequate (Hamann & Wessolowski 1990) as long as only those transitions are investigated for which collisional rates are unimportant. This is the case for the helium and hydrogen lines we are examining in this paper.

With the physical structure of the atmosphere specified, we obtain a simultaneous solution of the equations of statistical equilibrium and of the radiation transfer equations by means of an iteration technique with approximate $A$ operators (Hamann 1986, 1987). Hydrogen is represented by 11 levels, and the model atom of helium consists of 28 levels. From a total of 226 bound-bound transitions between these levels, all allowed ones are treated explicitly in the comoving frame, i.e., 45 hydrogen lines, 45 lines of ionized helium, and 54 lines of neutral helium. The line radiation field of the remaining 82 He lines, intercombination or forbidden transitions is approximated by the code of Abbott (1982). We solve for a steady state (Hamann & Wessolowski 1990) as long as only those transitions are investigated for which collisional rates are unimportant. This is the case for the helium and hydrogen lines we are examining in this paper.

As far as is known today, the most severe deficiencies of our models are that they do not allow for metal bound-free continua and do not include line blanketing. This gives rise to systematic errors of basically unknown magnitude. However, recent estimates of these effects (Hamann & Wessolowski 1990; Schmutz 1990) suggest that the net error in the bolometric magnitude derived with the present method is probably less than 1 mag.

### 3.2. The Hydrodynamic Model Calculations

Our hydrodynamics code and its principal assumptions have been discussed in detail by Leitherer et al. (1989); it is based on the code of Abbott (1982). We solve for a steady state solution of the equation of motion as formulated by Castor, Abbott, & Klein (1975) and by Abbott (1980). The list of atomic transitions was compiled by Abbott (1982). The contribution to the acceleration of the flow by each spectral line is calculated including the finite disk correction term (Pauldrach et al. 1985; Friend & Abbott 1986). The ionization balance is described by a modified recombination theory,

$$\frac{N_{j+1}N_e}{N_j} = \zeta W \left( \frac{T_e}{T_{\text{ion}}} \right)^{1/2} \left( \frac{N_{j+1}N_e}{N_j} \right)^*.$$  \hspace{1cm} (3)

(Details can be found in Abbott & Lucy 1985.) We treat the ionization temperature, $T_{\text{ion}}$, as a free parameter and determine its value empirically by comparing the predicted ratios of UV line strengths of strategic ions with observations, assuming that the ionization equilibrium of all ions can be characterized by a single value of $T_{\text{ion}}$.

### 3.3. Static Plane-parallel Model Atmosphere

In § 4.3 we discuss absorption troughs of UV resonance lines. To demonstrate that the formation of these features does not depend critically on an expanding atmosphere, we use a static plane-parallel model atmosphere to predict "pure" photospheric line profiles. The computer code is that published by Hubeny (1989). The atmospheric structure is calculated with the population of five hydrogen and three helium levels allowed to deviate from their LTE values, and seven hydrogen line transitions are included explicitly. Once the atmospheric structure is specified, the theoretical spectrum is calculated assuming the LTE source function for metal lines, with the exception of resonance lines of the lithium and sodium isoelectronic sequence (as, e.g., Si iv $\lambda\lambda1394, 1403$, for which the source function is given by the expression following from the second-order escape probability theory (Hummer & Rybicki 1982); the corresponding escape probability is evaluated through the kernel function $K_4$ using the numerical procedure developed by Hummer (1981). The intrinsic line profiles have the form of a Voigt function and account for the effects of natural, Stark, van der Waals, as well as thermal Doppler broadening. Further details of the adopted spectrum synthesis procedure are given in Hubeny, Steff, & Harmanc (1985) and Hubeny, Harmanc, & Steff (1986).

### 4. Spectroscopic Analysis

The aim of our analysis is to derive the stellar parameters of R84. To achieve this goal, we adjust the free parameters of our model atmosphere until we get satisfactory agreement of the model predictions with the observed spectrum. The model parameters that are directly related to those of the stellar photosphere are "stellar" temperature $T_*$, radius at the inner model boundary $R_*$, and helium abundance $N(\text{He})$. In addition, there are a few parameters that we cannot relate to the properties of the stellar photosphere but which are necessary in order to specify a calculation completely: the mass-loss rate $\dot{M}$, the terminal velocity of the wind $V_c$, the wind-velocity law ($\beta$, $\beta_{\text{osc}}$), and the "microturbulence" which determines the intrinsic line profile.

For practical reasons it is evidently impossible to construct a multidimensional fit diagram where all parameters can be read off. Instead, we examine (and reexamine) one parameter after the other, or, if appropriate, we construct a two-dimensional fit diagram and determine two parameters at the same time. The latter possibility always includes the temperature as one of the two parameters. A parameter value is varied until the theoretically predicted feature most sensitive to the particular parameter agrees with the observation. Of course, sometimes after fitting one parameter a readjustment of previously determined ones is required. The fit procedure is terminated successfully after all free parameters have been adjusted. A total of 31 models have been calculated. We refer to the model that agrees best with the observations in all predicted features as the "final model."

Our fit procedure has the obvious disadvantage that we do not know the global performance of our parameter space.

* Subsequently, we use the terms effective temperature or "stellar" temperature to denote $T_*$, which is defined by the relation $L = 4\pi R_*^2 \sigma T_*^4$, for given luminosity and radius at the inner boundary, $R_*$.  

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Therefore, the determination of the internal errors is very difficult, and it might be that we underestimate them in some cases. In the following subsections we discuss each of the free parameters.

4.1. Stellar Radius

Radiation always emerges from a range of optical depths, and, hence, for a star with an extended atmosphere, the radiation is emitted from different radii. Therefore, the definition of a photospheric radius has to remain somewhat arbitrary. Given a definition for the photosphere as the depth at which \( \tau \) has a certain value, such as 1 or \( \frac{1}{2} \), there is the additional problem that the radius of the photosphere is wavelength-dependent.

In the case of R84 we find the effects of the extended atmospheres not to be extreme, i.e., physically reasonable definitions, such as the location of the thermalization depth, result in radii only a few percent larger than the inner boundary of the model atmosphere. Therefore, the simple definition of the stellar radius by the radius at the inner boundary of the model atmosphere, \( R_\odot \), is sufficient.

The stellar radius cannot be determined spectroscopically but can be found from a comparison of the observed monochromatic luminosity with the predicted flux of the model calculation. To calculate the observed luminosity, we have to know the distance and the interstellar reddening. R84 is situated in the LMC, and for its distance modulus we adopt \( y_0 \) = 18.3 mag, from a determination of the distance to SN 1987A by Schmutz et al. (1990). This distance to the LMC is in agreement with other, reliable estimates (Fest 1988). We determine the amount of interstellar extinction by dereddening the observed flux distribution until it fits the slope of the continuum predicted by the final model. The best agreement is shown in Figure 4, where the observations are corrected for a reddening \( E_{B-V} = 0.17 \) mag using the LMC reddening law as determined by Fitzpatrick (1986). Although our procedure formally allows a determination of better than 0.02 mag, it is obvious that the derived reddening depends on our interpretation of how much line blanketing lowers the true UV continuum level, and also on the adopted form of the reddening law. Therefore, assuming an error of 0.05 mag is probably more realistic. There is additional uncertainty introduced by the fact that the IUE spectrum was not obtained simultaneously with the optical spectrum. The theoretical flux distribution refers to the atmospheric condition of R84 in 1982 January, and this energy distribution might not be the correct reference for dereddening the observed spectrum. There are indications that R84 was variable in the UV, although probably not during 1980/1982 (see § 2).

We derive the stellar radius from the flux observed in the \( B \) filter. (Note that \( V \), \( R \) and measurements in the IR are contaminated by the red supergiant.) Our preference for the \( B \) filter over the spectrophotometric observation (see § 2) is the main difference in the results obtained in this work compared with the preliminary results reported by Schmutz et al. (1989b). In the latter publication we derived the reddening and the radius using the observed spectrophotometric flux, and correspondingly the reddening, radius, and luminosity reported here are slightly larger than those of Schmutz et al. (1989b).

Correcting the flux value given in Table 1 for reddening \( \Delta A_V = 0.7 \) mag, line contamination (0.06 mag), and distance, we find \( L_B = 4.5 \times 10^{34} \) ergs \( s^{-1} \) \( \AA^{-1} \), or \( M_B = -6.87 \) mag. Our final model predicts a monochromatic flux of \( \mathcal{F}_1 = 8.06 \times 10^8 \) ergs \( cm^{-2} s^{-1} \) \( \AA^{-1} \) at 4400 \( \AA \) for our choice of the inner radius of the model atmosphere. Thus, we find \( R_\odot = 30 \) \( R_\odot \). Since there is no significant Balmer jump predicted \( \Delta \lambda = 0.05 \) mag in absorption), we may apply the same procedure to the flux observed in the \( U \) filter. This calculation also yields \( 30 \) \( R_\odot \).

We adopt \( R_\odot = 30 \pm 6 \) \( R_\odot \), where the error includes the uncertainty in distance and reddening and an estimate of the influences of the uncertainties of other model parameters.

All reasonable definitions of the radius refer to the region where the emergent radiation is created. However, if the atmosphere of a hot star is extended, the mean radius where the observed photons are last scattered is much larger than the mean radius of creation. This is due to the fact that electron scattering is the dominant contributor to the opacity. For R84 we find a relatively steep density gradient at the photosphere, and correspondingly the difference between the two radii is not dramatic, but still significant. If we define the mean radius scattering, \( R_{2/3} \), by the location where the Rosseland optical depth equals \( \frac{2}{3} \), the final model predicts this radius to be larger by 16% than \( R_\odot \); hence \( R_{2/3} = 35 \) \( R_\odot \).

4.2. Effective Temperature

Since the effective temperature is defined relative to a radius, the same remarks as for the radius apply to the "stellar" temperature \( T_\star \): any definition is somewhat arbitrary. Hence we define the effective temperature by

\[
T_{\text{eff}} = T_\star = \left( \frac{L}{4\pi R_\star^2 \sigma} \right)^{1/4}.
\]

Again, in the case of R84, physically more reasonable definitions would result in temperatures only a few percent cooler.

Following Davidson (1987), we also introduce \( T_0 \) because its value is descriptive of the energy distribution of the radiation field. The value \( T_0 \) is the temperature of a Planck field with the same average photon energy as the predicted emergent radiation:

\[
T_0 = \frac{15\Gamma(3)\xi(3)}{\pi^4 k} \int_0^\infty \mathcal{F}_\nu dv \int_0^a \left( \frac{\mathcal{F}_\nu}{\hbar \nu} \right) dv,
\]

where the numerical value of the product of the \( \Gamma \)-function and the Riemann \( \zeta \)-function is \( \Gamma(3)\xi(3) = 2.404 \). The final model is calculated with \( T_0 = 28,500 \) K and yields \( T_\odot = 31,200 \) K. Since it is the radiation field that determines the ionization balance, the difference of the few thousand degrees between the effective temperature and the characteristic temperature of the radiation field explains why R84 with its temperature of an early B star displays O star features, i.e., strong He II lines. In fact, Wolf et al. (1987) found the spectral characteristics of R84 to correspond to an O9.5 Ia star which has, according to Voels et al. (1989), an effective temperature of 30,000 K.

In Figure 5 we illustrate the extreme sensitivity of the He II \( \lambda 4686 \) line to the "stellar" temperature. The line profiles shown by the solid and dashed lines result from models with effective temperatures that differ by only 300 K! The reason for this sensitivity is that the physical conditions in the atmosphere are such that in the photosphere helium is just at the threshold of being doubly ionized; the ions of He++ and He+++ are about equally abundant. Not far out in the wind He++ becomes the dominant stage. Small changes in effective temperature shift this ionization balance and consequently make the strength of the He II recombination lines very sensitive temperature indicators. Monitoring He II \( \lambda 4686 \) is probably...
the most sensitive way of finding variabilities in the stellar parameters of Ofpe/WN9 stars. Another consequence of helium being doubly ionized only close to the photosphere is that the width of He II λ4686 is predicted to be much narrower than that of He I or H lines—in agreement with the observations—indicating that the He II line is formed only in the accelerated part of the wind.

The model results shown in Figure 5 indicate that by considering only the temperature we can determine the effective temperature to better than 100 K. However, He II λ4686 is also sensitive to other model parameters, and including their uncertainties, we find $T_e = 28,500 \pm 500$ K. The systematic errors of our model calculations are certainly much larger than the given precision of 500 K for the "stellar" temperature. The final model yields a bolometric correction of $BC = -2.78$ mag.

Combining the derived effective temperature with the value found for the radius we get for the luminosity of R84, $L = 10^{5.7} \pm 0.3 L_\odot$ ($M_{bol} = -9.6 \pm 0.8$ mag). The error estimate includes an assumed 10% systematic error of the effective temperature and 20% uncertainty of the radius.

### 4.3. Terminal Velocity of the Wind

If we assume the wind velocity law to approach monotonically a terminal value—as predicted by the radiative hydrodynamic calculations of § 5—the determination of the final velocity is in principle straightforward, since it can be measured in the observed spectrum without the need for any model calculations. However, in practice the major problem is to find a line suitable for the velocity measurement. The best lines to choose from are those with a P Cygni profile. The blueshifted absorption allows a relatively easy measurement of the velocity. An obvious requirement is that the blue edge of the P Cygni line profile should be free of blends. In the UV this requirement is rarely met. On the other hand, in the optical wavelength region, where blending is no problem, the lines are often not strong enough to show blueshifted absorptions out to the maximum wind velocity. A line can also be too strong, in which case the blue edge of the P Cygni profile is found to be rounded and additionally blueshifted owing to the intrinsic line profile.

For R84 the difficulty of measuring the terminal velocity is obvious. From UV P Cygni profiles Wolf et al. (1987) derive a terminal velocity of the order of 900 km s$^{-1}$, but this value is in direct conflict with the value obtained from the optical lines, which show absorptions that are shifted by an amount corresponding to only about half this velocity. The question is, which lines indicate the terminal velocity of the wind: are the UV profiles misleading, or are the optical transitions too weak to show the terminal velocity?

We answer this question in two steps. We first show that at least one line in the optical wavelength region is strong enough to yield the true terminal velocity; second, we demonstrate that the absorption troughs of the UV lines are not formed by the expansion of the wind.

#### 4.3.1. The He I λ3888 Profile Indicates the True Terminal Velocity

Since we know exactly the velocity law of our model, we also know unambiguously whether or not the calculated line profiles are influenced by the regions where the wind has reached its terminal velocity. When we inspect the optical spectrum published by Stahl et al. (1985), He I λ3888 looks most promising for a determination of the terminal velocity, because this line has a sharp and quite strong blueshifted P Cygni absorption. From the line profile predicted by the final model we determine that the deepest point of the absorption is blueshifted by 89% of the adopted terminal velocity. The blue edge of the absorption, which we define here to be where the profile crosses 90% of the continuum intensity, indicates a velocity larger than the terminal velocity by 1 Doppler width of the assumed intrinsic line profile. Therefore, there is no doubt that this line is a good indicator of the true terminal velocity. This statement holds as long as the adopted velocity law does not deviate too seriously from the real one. Of course, in principle it could be that the real velocity law increases further beyond the region traced by the He I line. Such a "second acceleration zone" was one of the possibilities considered by Hamann, Schmutz, & Wessolowski (1988) in order to explain the inconsistency between the terminal velocity determined from optical line fits and that found from the UV line profiles in the case of the WN5 star HD 50896.

The accuracy with which we are able to determine the terminal velocity is illustrated in Figure 6. The theoretical He I λ3888 lines resulting from two models with terminal velocities of 400 and 500 km s$^{-1}$, respectively, are compared with the observed profile. We conclude from this figure that the terminal velocity of R84 is 400 km s$^{-1}$ with an uncertainty of 50 km s$^{-1}$.

The two models also differ in parameters other than the terminal velocity. We compare them because both are able to reproduce equally well most of the other optical helium and hydrogen line profiles. This sentence implies that most optical lines are not strong enough to allow clearly observable features to be formed in the region where the wind has reached its terminal velocity. This is demonstrated in Figure 7, where we compare the observed He I λ4471 line with theoretical profiles resulting from the same two models as those in Figure 6. Although the profile calculated with a terminal wind velocity of 400 km s$^{-1}$ matches the observation slightly better than the one calculated with 500 km s$^{-1}$, it is much less evident than in the comparison of Figure 6 which of the two velocities is correct, especially when considering the noise of the observa-
The deepest point of the P Cygni absorption of the He i λ4471 line is shifted only by 66% of the terminal velocity. Thus, because this line is formed at lower velocities, the differences between the two models are reduced by roughly a factor of 2, making it more difficult to select one of the two models as the better one.

Another example of the insensitivity of most lines to the terminal wind velocity is the He n λ4686 line. The profile represented in Figure 5 by the dotted line can hardly be distinguished from the profile of the final model (solid line). Yet the two profiles result from the two models that yield the profiles of Figures 6 and 7, i.e., from models which have $V_w = 500 \text{ km s}^{-1}$ and $V_w = 400 \text{ km s}^{-1}$.

4.3.2. The Broad Absorption Troughs of UV Lines Are Photospheric

The next step in establishing the terminal velocity is to show that the broadening of UV resonance lines is not due to the expansion velocity of the wind. To prove this, we calculate theoretical profiles with a static, plane-parallel non-LTE model atmosphere. Our line of argument is that if a static atmosphere predicts a profile as broad as observed, it follows that to first order these UV absorption troughs are not influenced by the velocity structure of the stellar wind.

The atmospheric structure is calculated for an effective temperature of 28,700 K, a gravity of log $g = 3.0$, and a helium abundance of 0.3 by number. With the exception of the gravity, these numbers agree with the stellar parameters for R84 as derived in this section. With regard to the results of § 5, a gravity of log $g \approx 2.8$ would be more appropriate. However, a static solution with such a low gravity does not exist, in agreement with the fact that the star has a strong stellar wind. But the difference in gravity between the adopted value and what we think would be a better one has no influence on the predicted line profiles, since it is natural broadening (not pressure broadening) that is responsible for the broad wings.

We have checked the insensitivity of the profiles to gravity by calculating models with log $g = 2.9$ and log $g = 3.5$. The resulting Si iv λλ1394, 1403 profiles are shown in Figure 8. They are calculated assuming a silicon abundance of $\frac{1}{3}$ solar. Our approximate treatment of the line formation is certainly not very accurate, especially for the line core, but it nevertheless suffices to demonstrate that a broad photospheric absorption profile is predicted that agrees basically with that observed in the spectrum of R84. Correspondingly, for the C iv resonance lines it can also be shown that the observed absorptions are of photospheric origin.

It is interesting to note that R84 is not unique in this respect. Inspecting spectra of early B main-sequence stars, we find absorptions of the Si iv resonance lines similar to those of R84. The effective temperature of R84 is comparable to the temperatures of B0.5 V or B1 V stars (we compare with main-sequence stars in order to avoid wind contamination). In Figure 9 we show the observed Si iv resonance lines of the B0.5 V star HD 55857. This star has absorption profiles which are practically identical to the absorption troughs of R84. We picked this particular star from the list of stars examined by
Sekiguchi & Anderson (1987), and we would like to stress that HD 55857 does not have exceptionally strong Si iv absorptions. There are stars, e.g., HD 212571 (B1 V), with equivalent widths almost as large as the example shown in Figure 8. Thus, broad Si iv absorptions are normal for B0.5 or B1 stars (see also Hamann 1981).

As already noted by Wolf et al. (1987), there is a significant difference between R84 and the majority of late O stars, in that the spectrum of R84 exhibits strong Al iii λλ1855, 1862 P Cygni profiles. (Wolf et al. 1987 showed that the photospheric UV absorption lines of R84 correspond approximately to type O9.5.) Normal O stars have only weak Al iii λλ1855, 1862 absorption lines (Walborn, Nichols-Bohlin, & Panek 1985), because the photospheres of O stars are too hot for Al$^{+2}$ to have a significant population, the main ionization stage being Al$^{+3}$. The same is true for the photospheric regions of R84. But because of the special ionization conditions in stars with strong winds (see Schmutz & Hamann 1986), He$^{+2}$ recombines eventually to He$^+$ at some distance out in the wind (in the case of R84, quite close to the photosphere), and this also allows Al$^{+2}$ levels to be stronger populated. For these reasons the Al iii resonance line profiles do not have underlying broad photospheric absorption troughs, and we expect them to indicate the true terminal velocity of the wind. In fact, the blue edges of the Al iii profiles are blueshifted by 400 km s$^{-1}$ (Wolf et al. 1987), in perfect agreement with the result we obtain from fitting the absorption. A comparison with calculated line profiles resulting from models with different velocity gradients around the sonic point (see Fig. 22) demonstrates that this line is sensitive to the velocity law close to the photosphere. The model with a shallow law (log $g_{\text{max}} = 1$; dashed profile) predicts this line in absorption, while the final model with a steeper gradient (log $g_{\text{max}} = 2.3$; solid line) yields this line in absorption. Note that the observed profile is blended by emissions of N iv multiplets (12) and (13) and Si iv (2). The continuum chosen for the normalization might be too high.

Since the continuum is formed deeper than the absorption of He ii λ4541, we find that the photospheric expansion velocity is less than 35 km s$^{-1}$. The adopted structure of the final model has the thermalization depth at a radius where the expansion velocity is about 10 km s$^{-1}$. As already mentioned in § 4.1, because of the dominance of electron scattering, optical depths of order unity in the continuum are at larger radii and correspondingly at larger expansion velocities than the values at the thermalization depth. In the final model the expansion velocity has already reached a value of 60 km s$^{-1}$ at the radius $R_{\text{2/3}}$, where the Rosseland optical depth is $\frac{1}{3}$.

Our value for photospheric expansion velocity is considerably smaller than the blueshift of 250 km s$^{-1}$ measured by Wolf et al. (1987) from UV absorption lines. Since these lines are stronger than He ii λ4541, we expect them to be formed farther out in the wind. Thus, the large blueshift of the UV absorption lines is additional observational evidence for a step velocity gradient at the base of the wind.

He ii λ4541 is the only line from which we are able to constrain the parameter $g_{\text{max}}$. Only weak lines are sensitive to this parameter, because they are formed predominantly in the region of small expansion velocities. But if the lines are too weak, they disappear in the noise of the observation. This is the case, e.g., for He ii λ4200 and higher He ii 4–n transitions.

For the parameter $\beta$, which determines the form of the velocity law at supersonic velocities, we have assumed a value of $\beta = 1$. Since the entire velocity law is probed by the P Cygni absorptions of helium lines of different strengths, it is possible in principle to determine empirically the form of the velocity law. In practice, however, we are limited by the noise of the observation. Since we find good agreement between the predicted and observed line profiles by assuming a law with $\beta = 1$, we conclude that the true velocity law has to be close to the assumed form. We compare the adopted velocity law with a theoretically predicted one in § 5.

From theoretical expectations—different velocity gradients give rise to different escape probabilities of line photons—and
from practical experience as well (see Schmutz, Hamann, & Wessolowski 1988), we know that varying the parameter $\beta$ somewhat affects the derived mass-loss rate and to some extent the stellar temperature also. The magnitude of its influence could be determined only by calculating models with variations of this parameter, but since we expect other influences, such as the ignored metal bound-free opacities or line blanketing, to be the source of larger systematic errors, we feel that the further exploration of this parameter is not justified.

4.5. Mass-Loss Rate

As pointed out by Schmutz (1988), the basic parameters that determine the equivalent widths of hydrogen and helium wind emission lines are stellar temperature and "wind density." The latter parameter is a combination of mass-loss rate, terminal wind velocity, and stellar radius: $M/(V_w R^2)$. As we have determined the velocity structure above, it is the value of the ratio $M/R^2$ that we determine by fitting the line strengths. (Because the radius cannot be determined by spectroscopic analysis, we do not yet insert its value here.) More exactly, since the strengths of the He i lines depend mainly on the fractional mass-loss rate of the corresponding element, we are actually determining separate "wind densities" for each element.

In Figure 11 we illustrate the difference in the line strength of H$\beta$ resulting from a difference of 0.1 dex in the ratio $M/R^2$. From a comparison of these line profiles with the observed one, we conclude that the precision of the derived ratio is 0.03 dex if only this parameter is considered. We estimate the uncertainties of the other parameters to increase the internal error to about 0.06 dex, and, finally, if we want to give a mass-loss rate, we have to add 0.12 dex introduced from the uncertainty of the radius. We find that the mass-loss rate of R84 on 1982 January was $M = 10^{4.61 \pm 0.18} M_\odot\, yr^{-1}$.

4.6. Helium Abundance

The relative strengths of He i lines are best reproduced with a helium abundance of $N(\text{He}) = 0.30$ by number. As mentioned above, a determination of the mass-loss rate also includes implicitly a determination of the abundance. We therefore obtain from the same estimates as above the precision of the abundance determination. However, the influences of the uncertainties of some parameters cancel in the case of the abundance, e.g., those from the radius or the velocity, and we estimate the uncertainty to be of the order of 0.5. For a fixed helium mass-loss rate the difference between the two theoretical H$\beta$ profiles shown in Figure 11 illustrates as well a difference between $N(\text{He}) = 0.7$ and $N(\text{He}) = 0.75$, as well as a difference in total mass-loss rate.

4.7. Intrinsic Line Profile

We assume that the intrinsic line profile in the expanding model atmosphere has a depth-independent Gaussian shape corresponding to a Doppler velocity $V_0 = 40$ km s$^{-1}$. Test calculations have shown that varying this width does not significantly affect the results of the analysis. The effects are subtle and are visible only in the absorptions of P Cygni profiles and in the He ii $\lambda4541$ absorption. Because of the limited resolution of the observations, we are not able to comment on the presence of microturbulence. Calculations with intrinsic profile widths narrower than the assumed one match the observations equally well. (All theoretical profiles shown in this paper are convolved with the resolution of the observation.) However, we can exclude profile widths considerably broader than those we have assumed. Calculations with $V_0 = 10$ km s$^{-1}$ clearly yield profiles that are broader than observed. This is true for both the low-velocity region, which is traced by He ii $\lambda4541$, and the high-velocity region probed by the P Cygni absorption of He i $\lambda3888$.

4.8. Redundant Line-Profile Fits

The analysis of R84 is based on the detailed study of four line profiles only: H$\beta$, He ii $\lambda4541$, 4686, and He i $\lambda3888$. But our model calculations predict several other lines in the observed wavelength region. An important test of the quality of a model is the comparison of these additional lines with the observations. In this comparison, unexpected line blends or improper spectral recording can be revealed. In Figures 12–14 we show fits to three Balmer lines, and in Figures 15–17 three comparisons with He i lines. In all cases we find excellent agreement of predicted with observed profiles. It is worthwhile to recall that there is no free parameter left that allows us to manipulate the theoretical lines shown in Figures 12–17.

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the final model is determined, the information obtained from these fits is redundant. Of particular interest are the absorptions in the P Cygni profiles. In all analyses of Wolf-Rayet stars to date (e.g., Hillier 1987; Hamann et al. 1988) the calculations have always predicted absorptions to be much too strong. The reason for the better fits in the case of R84 is not clear. It might be that this is an indication of a better agreement of the adopted velocity law with reality.

4.9. Radial Velocity of the Ofpe/WN9 Star

As mentioned in § 1, there is a red supergiant star contributing to the spectrum of R84. If the two stars are not just by chance on the same line of sight but rather are companions of a binary system, this would provide the opportunity to determine spectroscopically the masses of the two stars. Therefore, the radial velocity of the Wolf-Rayet star is a quantity of special interest. Wolf et al. (1987) determined the radial velocity of the red supergiant to be $251 \pm 4$ km s$^{-1}$. They also measured the radial velocity of forbidden emission lines and found $250$ km s$^{-1}$. These lines are thought to originate in a circumstellar envelope ejected by R84. Therefore, the envelope’s radial velocity is associated either with R84 or with the system. Stahl & Wolf (1986) found (circumstellar?) absorptions of the sodium lines shifted by $256$ km s$^{-1}$.

With theoretical line profiles at hand, we are able to determine the radial velocity of R84 by adjusting each theoretical profile in wavelength to its observed position. We estimate that we can derive the radial velocity from an individual line within 20–40 km s$^{-1}$, depending on the quality of the fit. Using fits to 11 lines, we find a radial velocity of $259 \pm 35$ km s$^{-1}$. The large standard deviation of the measured values is due not only to the errors of the individual measurements but also to a nonuniform wavelength scale in the figure published by Stahl et al. (1985). When we realized this deficiency, we tried to improve the data by scanning a copy obtained from the original negative kindly provided by O. Stahl. Unfortunately, the distortion is also present on this copy. Thus, we find no indication of orbital motions, although with a rather large uncertainty of 10 km s$^{-1}$. A more stringent constraint was found by Moffat (1989), who concludes from a constant radial velocity within $5$ km s$^{-1}$ of 11 observations that the star is likely to be single.

The conclusion that R84 and the red supergiant are not companions of a binary system is supported by astrographic
images obtained by Prévot-Burnichon et al. (1981). They find R84 to have a complex image with one or two visual companions closer than 3" and fainter by about 1 mag in V. Such a magnitude difference fits approximately the ratio of R84 to the red supergiant at V (see Fig. 4). There is also a remark by Walborn (1977) that the "star is located in a Trapezium-like system." The fact that the stars are resolved at the distance of the LMC excludes binarity.

4.10. Summary of the Derived Stellar Parameters

The following list summarizes the stellar parameters determined above. The given values refer to the condition of R84 in 1982 January. For the definitions of the parameters and our estimates of the internal errors the reader is referred to the corresponding subsections.

\[
\begin{align*}
\log (L/L_\odot) &= 5.7 , \\
T_0 &= 31,200 \text{ K} , \\
T_e &= 28,500 \text{ K} , \\
R_e &= 30 R_\odot , \\
R(\tau_e = 2/3) &= 35 R_\odot , \\
\log [\dot{M}/(M_\odot \text{ yr}^{-1})] &= -4.61 , \\
V_e &= 400 \text{ km s}^{-1} , \\
X &= 0.37 \text{ (by mass)} , \\
Y &= 0.63 \text{ (by mass)} .
\end{align*}
\]

In addition to the results given by the above numbers, we have been able to determine an approximate form of the velocity law (§ 4.4).

5. A RADIATIVELY DRIVEN WIND MODEL FOR R84

On the basis of its spectral morphology, R84 appears to be intermediate between the Of stars and the WN7-A stars. Stellar winds from OB stars can be modeled quantitatively along the line of the theory of radiatively driven winds formulated by Castor, Abbott, & Klien (1975, hereafter CAK). It has not yet been demonstrated that the expansion of Wolf-Rayet atmospheres can also be understood in terms of radiation pressure. The momentum of stellar winds from Wolf-Rayet stars exceeds the photon momentum of the radiation field, and it is not clear how W-R stars so efficiently supply mechanical momentum to the flow.

We can estimate the amount of momentum transferred from the photons to the gas in the case of R84. Abbott (1980) and Panagia & Macchetto (1982) showed that the radiative momentum is used (1) to remove wind material from the star and (2) to support the stellar wind against the gravitational well of the mass-losing star. The efficiency \( \epsilon \) of the momentum transfer can be written as

\[
\epsilon = \frac{M V_e}{L/c} + \frac{1 - \Gamma_e}{\Gamma_e} \tau_e ,
\]

where \( \Gamma_e \) is the luminosity of the star in terms of the classical Eddington luminosity and \( \tau_e \) is the total electron scattering optical depth in the wind. The first term in equation (6) represents the rate of momentum which is ejected by the wind. The second term reflects the rate at which momentum is consumed to support the envelope against gravity. Using the parameters derived above, we find \( M V_e/(L/c) \approx 0.9 \). The fraction of momentum consumed to support the envelope against gravity depends on the stellar mass and the ionization equilibrium. This fraction is \( \approx 0.9 \) for \( M = 20 M_\odot \) and \( \approx 0.0 \) for \( M = 10 M_\odot \), respectively, if \( \tau_e = 1 \) is adopted for the inner boundary of the stellar wind (\( \tau_e \approx 0.9 \) at the sonic point of the final model). Therefore, the total efficiency of the momentum transfer turns out to be \( \epsilon < 2 \), and it is reasonable to invoke radiation pressure as the driving mechanism for the stellar wind of R84.

5.1. Hydrodynamic Calculations

The stellar parameters derived from our non–LTE analysis are input for the radiation hydrodynamics calculations. Because of the core-halo assumption of the hydrodynamics calculations, we take the lower boundary condition of the wind at \( \tau_e = 1 \) where the continuum becomes transparent. The input parameters for the hydrodynamic calculations are

\[
\begin{align*}
\log (L/L_\odot) &= 5.7 , \\
T_{1''} &= 26,900 \text{ K} , \\
\log (R_1/R_\odot) &= 1.51 , \\
Y &= 0.63 \text{ (by mass)} .
\end{align*}
\]

The energy distribution of the input radiation field is given by the flux at \( \tau_R = 1 \) of the final non–LTE model atmosphere.
Fig. 18.—Line optical depths as calculated by the hydrodynamical model with the Sobolev approximation at the radius \( r = 1.15R_1 \), where \( R_1 \) is the inner boundary of the hydrodynamical model. The ionization equilibrium is obtained with eq. (3) and \( T_{\text{ion}} = 18,000 \, \text{K} \). The predicted strong lines are observed in the IUE high-resolution spectrogram (see Wolf et al. 1987).

This distribution is practically identical with the emergent flux shown in Figure 4, except that it has no IR excess. For all elements heavier than helium (except C, N, and O) solar abundances are adopted and reduced by a factor of 3 to take into account the heavy-element abundances of the Large Magellanic Cloud. The element abundances of carbon, nitrogen, and oxygen have been taken from the evolutionary calculations of Maeder & Meynet (1988, Table IV, time step 21) assuming that R84 is a massive star in its post–red supergiant phase, i.e., its C and O are converted to N, and the CNO abundance reduced by a factor of 3: we assume \( Z(^{12}\text{C}) = 0.00005, Z(^{14}\text{N}) = 0.0041, Z(^{16}\text{O}) = 0.00015 \) by mass. To calculate the radiation force on the spectral lines, we must know the line opacities. In equation (3) we have with \( T_{\text{ion}} \) a free parameter which influences these line opacities. We adjust this parameter until we find a reasonable agreement between predicted and observed line strengths. Figure 18 shows the line strengths in the wind at \( r = 1.15R_1 \). The ionization temperature is \( T_{\text{ion}} = 18,000 \, \text{K} \). The calculated line blanketing is in agreement with what is observed in the IUE low-dispersion spectrum (see Figs. 1 and 4); notice that the blanketing around \( 1900 \, \text{Å} \) (and longward) due to numerous Fe iii lines causes a significant flux depression in the IUE spectrum. We are aware that there is an even larger observed flux depression around \( 1600 \, \text{Å} \), which we attribute to photospheric blanketing by Fe iv which does not oppose the wind acceleration, analogous to the Si iv absorption troughs discussed in § 4.3.

The IUE SWP high-dispersion spectrum (see Wolf et al. 1987, Fig. 4) can be used to compare theoretical and observed line strength ratios of individual lines. The strongest features in our synthetic spectrum are labeled with their identifications and multiplet numbers. They are also the strongest features in the observed UV spectrum (see Fig. 4 of Wolf et al.). Note that our model predicts no significant optical depth in the wind for Si iv (1). As shown previously in § 4.3 for the Si iv (1) multiplet, the observed, strong absorption trough of C iv (1) can also be explained by a purely photospheric profile. On the other hand, Si iv (1) has large optical depth in the wind. This is also in agreement with observations: the IUE spectrum shows Si iv emissions, but there are none for C iv. The emissions clearly indicate that Si iv is present in the wind. (The corresponding wind absorption is blueshifted by \( \sim 400 \, \text{km} \, \text{s}^{-1} \) only, and therefore it is masked by the broader photospheric profile; see § 4.3.)

Figures 19 and 20 illustrate the sensitivity of the optical depths to the ionization temperature \( T_{\text{ion}} \) in the range from 16,000 to 20,000 K. The spectral features are denoted by open (16,000 K) and filled (20,000 K) squares. Figure 19 covers the wavelength region \( 1500 \, \text{Å} < \lambda < 1700 \, \text{Å} \), and Figure 20 shows the region \( 1800 \, \text{Å} < \lambda < 2000 \, \text{Å} \). The dotted lines connect four sets of temperature-sensitive lines calculated with the two ionization temperatures. The range of ionization temperatures producing line strength with an overall agreement with observations is 16,000 K < \( T_{\text{ion}} < 20,000 \, \text{K} \). Above \( \sim 20,000 \, \text{K} \), Al iii (1), which is a prominent feature in the observed spectrum, becomes very weak because of the decrease of Al ii / Al iii +...
with increasing $T_{\text{ion}}$. In contrast, many triply ionized lines of iron and C IV (1) are stronger than the doubly ionized iron lines around 1900 Å, which is also not observed. The lower limit of $T_{\text{ion}}$ is around 16,000 K. Below this value, Al III (1), Fe III (34), and many other doubly ionized iron lines increase in strength dramatically, and strongly saturated wind lines should be observed, which is not the case.

5.2. Stellar Mass

With all input parameters for the hydrodynamic models known but one, the stellar mass $M$, we computed a series of wind models for different masses. As a result of our calculations we predict values of two quantities that can be compared with observations: the mass-loss rate and the terminal velocity of the wind. Although the line strengths resulting from models with $T_{\text{ion}}$ above 20,000 K and below 16,000 K are not in agreement with the observations, we included a larger range of this parameter in our series to study the stellar wind properties associated with these values of $T_{\text{ion}}$.

The observed low terminal velocity of the wind hints at a low stellar mass. If the stellar wind of R84 is radiatively driven, a tight correlation between $V_\infty$ and the surface escape velocity (and thus the mass) is predicted. The derived value for the terminal velocity of 400 km s$^{-1}$ is much lower than that typically observed in O stars. Howarth & Prinja (1989) and Groenewegen, Lamers, & Pauldrach (1989) both found $V_\infty \approx 2000$ km s$^{-1}$ in late O-type stars. The difference in the terminal velocities of R84 and other O stars can be understood in terms of a radiatively driven wind if the mass of R84 is lower than the average mass of stars in the same region of the H-R diagram.

![Figure 21](image_url)

Figure 21.—Terminal wind velocities and mass-loss rates predicted from hydrodynamical calculations. A total of 19 models are represented in this graph. The results represented with the same symbol belong to models with the same stellar mass $M = 10, 12, 15 M_\odot$. There are six models for each stellar mass, corresponding to different assumptions for the ionization temperature $T_{\text{ion}} = 18,000$ K (label 3), 16,000 K (label 3), 14,000 K (label 4), 12,000 K (label 5), and 10,000 K (label 6). The observed ionization in the wind of R84 (see Figs. 18–20 and text) corresponds to $T_{\text{ion}} \approx 18,000$ K (filled symbols). All but one of the hydrodynamic solutions shown here are obtained with the calculated radiation force reduced by 35% in order to account for the effect of line overlap and multiple scattering. The model represented by the cross (label 2a) has been obtained without the reduction. The observed terminal velocity and mass-loss rate, indicated by a star and error bars, are reproduced with a model of 10 $M_\odot$. This result, however, is subject to systematic errors. The actual mass of R84 is estimated to be $M = 20 M_\odot$ (see text).

Figure 21 gives the results of the radiatively driven wind models in the $(\log M, V_\infty)$-plane. A series of models with different parameters $M$ and $T_{\text{ion}}$ have been computed. The gross behavior of $M$ and $V_\infty$ as a function of $M$ and $T_{\text{ion}}$ can be understood as follows. Models with the same $T_{\text{ion}}$ and lower masses have higher $M$ and lower $V_\infty$. If $M$ decreases, the radius where the gravitational acceleration balances radiative acceleration moves closer to the photosphere and $M$ increases. A lower mass also produces a lower surface escape velocity at the photosphere and $V_\infty$ decreases. Models with the same mass and lower $T_{\text{ion}}$ generally have higher mass-loss rates and higher terminal velocities. A lower $T_{\text{ion}}$ means lower ionization stages. Strong lines of lower ionization stages typically are at longer wavelengths, which are more favorably situated to transfer momentum from the radiation field to the flow. The higher terminal velocities result from an increase in the number of strong lines, which is reflected by the increase of the parameter $\alpha$ in the formulation of a CAK theory, with lower $T_{\text{ion}}$ (see Leitherer et al. 1989). As a word of caution, these plausibility arguments may not hold in every case (cf. Fig. 21), and a detailed study of the excitation and ionization conditions in the wind must be performed. Note especially that a change in the mass will also influence the excitation and ionization in the wind due to the change in $M$ and, thus, in the wind density.

Figure 21 demonstrates that $M$ and $V_\infty$ predicted by the line-driven wind model are in reasonable agreement with the mass-loss rate determined by the non–LTE analysis if $M \approx 10 M_\odot$. Note that the lower mass limit cannot be extrapolated linearly from Figure 21. With the mass 10 $M_\odot$ the model is already very close to the Eddington limit, and thus there is no solution with a mass of 9 $M_\odot$. The resulting terminal velocity is not very sensitive to the stellar mass but is more sensitive to the value of $T_{\text{ion}}$. We find that the range 18,000 K < $T_{\text{ion}}$ < 20,000 K, which we prefer from our considerations in § 5.1, also correctly reproduces the terminal velocity. This result is an independent check of our assumptions regarding the parameter $T_{\text{ion}}$.

A stellar mass of 10 $M_\odot$ for R84 is surprisingly low, and we suspect that this result is strongly influenced by systematic errors. The reason for our skepticism will be discussed in § 6. We think we can identify some of the systematic errors.

First, as discussed in Leitherer et al. (1989), we reduce the force multiplier by 35% to account for the combined effects of multiple scattering and line overlap in the wind. Our assumption of the force reduction follows from the results of Abbott (1987), who found a larger line overlap effect for dense winds of LBV-type stars than Puls (1987) found for “normal” O stars. This factor is obviously somewhat arbitrary, and to investigate its effect we have calculated one model ($M = 12 M_\odot$, $T_{\text{ion}} = 20,000$ K; denoted by a cross in Fig. 21) without reducing the calculated line force. We find that our assumption incorporates a potential systematic error of up to 30%.

Second, we have assumed a metal abundance of $1/2$ solar. Since there are numerous optically thin lines (see Figs. 19 and 20), an increase of the metal abundance would increase the line force, potentially by a large factor. Recent investigations of the metallicity in the LMC indicate that the iron and nickel abundances are depleted by less than a factor of 2 rather than by the canonical factor of 3 (e.g., Russell & Dopita 1990).

Third, we have to recall that we assumed a core-halo approximation for the hydrodynamic calculations. That means we have an artificial separation between wind and photosphere which does not exist in reality. The largest systematic error
from this approximation does not arise from the density structure—which is actually quite core-halo like—but from the ionization equilibrium. In the hydrodynamic calculation we assume a ionization equilibrium which is observed in the outer wind. But one of the most important results of non-LTE calculations that treat the photosphere and wind consistently and simultaneously is that the ionization equilibrium changes not abruptly but gradually (Schmutz & Hamann 1986). The way the hydrodynamic calculations are set up in the models presented here, the critical point is in the $\text{He}^+$ zone. There, the ratio of radiation force on electrons to gravity is $\Gamma_e = 0.8$. However, we know that helium is fully ionized deeper in the photosphere and consequently there is a higher electron opacity. In fact, if $M = 10 M_\odot$, radiation pressure on electrons would already exceed gravity in the $\text{He}^{++}$ zone. If the critical point were located in the $\text{He}^{++}$ zone, we would gain another 30% in radiation pressure. In addition, there also might be a considerable contribution to the radiation pressure in the photosphere from bound-free continua. (At the location of the critical point in our model the radiation pressure on the H and He continua contributes only a tenth of the force on free electrons and therefore can be neglected.) Thus we find that there are several, possibly large, systematic errors in our mass estimate. To assume an error of a factor of 2 seems reasonable.

So far, the only other attempt to apply radiation-driven wind theory to Wolf-Rayet stars without adding additional forces (e.g., Poe, Friend, & Cassinelli 1989) was by Pauldrach et al. (1985), who modeled the stellar wind of V444 Cygni. However, in order to reproduce the observed mass-loss rate and the terminal velocity, they had to assume a luminosity that was too high compared with that derived from the models of Schmutz et al. (1989a), or to the value implied theoretically by the Wolf-Rayet mass (Langer 1989). We also note that Pauldrach et al. (1985) did not include the effects of line overlap, thus allowing radially streaming photons to impart their momentum more than once. In addition, in an unpublished attempt to construct a unified model for V444 Cygni, we calculated the helium emission lines emerging from a model based on the density and temperature structure of the best solution of Pauldrach et al. (1985). We found that the predicted emission lines are much fainter than observed and—the most severe inconsistency—that helium in the wind is completely ionized ($\text{He}^++$), and thus the model does not predict any observable $\text{He}\,\text{i}$ lines. This prediction is in conflict with observations of rather strong $\text{He}\,\text{i}$ lines (e.g., Schmutz et al. 1989a).

The reason to discuss here the paper by Pauldrach et al. (1985) is that we note that our results suffer from exactly the same deficiency as theirs: both calculations need an unreasonably high $L/M$ ratio, and, interestingly, both need a ratio that is about a factor of 2 too large.

### 5.3. Velocity Law

The shape of the theoretical velocity law is very insensitive to the stellar mass. Even if the derived stellar mass is wrong by a large factor, the hydrodynamic predictions of the wind velocity law are still reliable. It is therefore reasonable to compare the theoretical velocity law with that used for the non-LTE analysis. In Figure 22 a model series with $M = 10 M_\odot$ and $T_{\text{eff}} = 16,000, 18,000, \text{and } 20,000 \text{K}$ is plotted. We note that the velocity structure predicted by the radiation-driven wind theory is quite similar to the velocity law we find empirically in the non-LTE analysis. The hydrodynamic solution produces a slightly steeper velocity law close to the stellar photosphere ($\beta = 0.8$ at the radius of the critical point $r_c$) and more gradual farther out.

The similarity between the empirical velocity law with $\beta = 1$ and the hydrodynamic solutions is even more evident if we scale the hydrodynamic solutions to the observed terminal velocity. In Figure 23 we have scaled the predictions of the theoretical velocity laws as shown in Fig. 22, but scaled to a terminal velocity of 400 km s$^{-1}$ (solid lines). All three models predict an almost identical form of the velocity law. The long-dashed line indicates the assumed law of the final model. The short-dashed curve gives the velocity law as determined empirically by the non-LTE analysis. The parameterized law (eq. [2]) uses $\beta = 1$, which is a straight line in this representation.
three models shown in Figure 22 to $V_\infty = 400$ km s$^{-1}$. Also shown are two different laws which we have assumed for the non-LTE calculations. Both are obtained with $\beta = 1$, but one has a steep photosphere log $g_\infty = 2.5$ and a terminal velocity of $V_\infty = 400$ km s$^{-1}$ (the final model) and the other has log $g_\infty = 1.0$ and $V_\infty = 500$ km s$^{-1}$. The latter model is of interest because it is also very successful in reproducing most of the observed profiles. From the comparison shown in Figure 23 we now understand why: in the regions where most lines are formed, its velocity law agrees as well with the hydrodynamic solution as that of the final model. Only far out in the wind where the velocity has reached its terminal value, and deep in the photosphere where the assumed law is determined from equation (1), do we find significant deviations. The regions where we find these differences are exactly the regions traced by the P Cygni absorption of He I $\lambda$3888 and by the He II $\lambda$4541 absorption line. It is the better agreement of these two lines with the observations that distinguishes the final model from the other solution.

An interesting property of Ofpe/WN9 stars is that these stars have P Cygni lines of different strength, and, correspondingly, blueshifts of their absorptions such that the whole velocity law can be traced empirically. With observations of better quality than the profiles shown here, a careful fit to the observed profiles would allow us to map the velocity law empirically in greater detail than simply adjusting two velocity-law parameters. Most interesting is the fact that there are He II absorptions from the 4–n series that are formed so deep in the photosphere that they probe the region around the critical point of the solution of the equation of motion, defined by the singularity and regularity conditions (Abbott 1980, eqs. [18] and [19]). The profiles of our final model predict the minima of the He II $\lambda$25411, $\lambda$4541 absorptions to be blueshifted by 64 and 35 km s$^{-1}$. The hydrodynamic solution predicts the critical point to be at the location where wind expands with $V_\infty = 54$ km s$^{-1}$. Thus, by observing Ofpe/WN9 stars, we have the unique opportunity to test the predictions of radiation-driven wind theory.

6. EVOLUTIONARY STATUS OF R84

With the stellar parameters of R84 determined by the non-LTE analysis, we may associate its parameters with a particular stage of stellar evolution. In principle, with its luminosity of $L = 10^{5.7} L_\odot$ and its temperature of $T_\star = 28,500$ K, R84 could be in any one of four different evolutionary phases: on its main-sequence evolution from blue to red, in the core-contraction phase, on its first crossing of the H-R diagram after the end of core hydrogen burning, or on its second crossing of the H-R diagram from red to blue after the red supergiant phase. The latter possibility can be distinguished clearly from the other three on the basis of the prediction of extreme non-solar abundances. In particular, Maeder (1990) calculates a surface hydrogen abundance $X$ between 0.3 and 0.4, for a star which started its evolution at $M_{\text{ini}} = 25$ or $40 M_\odot$ and $Z = 0.02$ or 0.005, with the temperature of R84, and which is in the red to blue evolutionary phase. These values compare very favorably with the hydrogen abundance of $X = 0.37$ found for R84. Thus, we conclude that R84 is in a post-red-supergiant phase on its evolution from red to blue, and its initial mass was $M_{\text{ini}} \approx 30-40 M_\odot$.

The idea that at least some Wolf-Rayet stars are in a post-supergiant phase is widely accepted; that this is also the case of Ofpe/WN9 stars is not yet common knowledge, although it has previously been suggested (e.g., by Lortet & Testor 1988). Our conclusion is further supported by the presence of strong nitrogen lines and the absence of carbon and oxygen features, which hints at a high ratio of nitrogen to carbon and oxygen. This is expected for a star in the proposed evolutionary stage: basically the combined initial C and O should be converted to nitrogen.

Further evidence that R84 has been a red supergiant before its current phase comes from its luminosity-to-mass ratio. The low terminal velocity and the high mass-loss rate strongly argue for an unusually high value of this ratio. This is also expected from the stellar evolution models: stars with R84’s luminosity have lost nearly half of their initial mass when they cross the H-R diagram from red to blue.

The reason for our disbelief in the stellar mass derived in §5.2 is based on the ratio of initial mass to present mass: a value of $10 M_\odot$ would imply that the star has already lost two-thirds to three-fourths of its initial mass. If this were true we would expect from the results of evolutionary models that there is no hydrogen left in the atmosphere. But obviously R84’s atmosphere still contains hydrogen.

A relatively reliable estimate of the present stellar mass of R84 can be obtained from the mass-luminosity relation of helium stars (Maeder 1983; Langer 1989). Since R84 still contains a lot of hydrogen in its atmosphere, the relation is not really applicable to R84. But comparing the mass and luminosity entries given by Maeder (1990) (Tables 18 and 19, time steps 25 and 26 and 37 and 38, respectively) with Langer’s (1989) equation (18), we find that a star in the evolutionary phase of R84 has a mass-luminosity relation very close to that of helium stars. Thus, for $L = 10^{5.5} L_\odot$, we obtain $M \approx 20 M_\odot$.

Several Wolf-Rayet stars are members of associations. Schild & Maeder (1984) and Humphreys, Nichols, & Massey (1985) tried to use these links to derive information about the ages and initial masses of these stars. Their results are probably valid in a statistical sense. However, for individual stars, extreme caution is necessary because the inherent assumption of coeval evolution may result in serious systematic errors. In particular, Schild (1987) concluded from the membership of R84 in the association LH 39 (Lucke and Hodge 1970) that its stellar age is approximately 8 Myr and its initial mass is only about 25 $M_\odot$. But he also remarks: “There is probably a considerable age spread among the association members. . .”

Since the age and initial mass proposed by Schild (1987) are significantly lower than our estimate obtained from its luminosity, we reinvestigate the known members of LH 39. In Figure 24 we place all members of LH 39 for which Schild (1987) gives a spectral type in a H-R diagram. The error bars result from the assumption that the classification is uncertain by one spectral type and by one luminosity class. For a given spectral type we calculate effective temperature, reddening, and luminosity on the basis of the stellar type calibration for effective temperature, bolometric correction, and intrinsic color given by Schmidt-Kaler (1982). The observed visual magnitudes and colors are taken from Lucke (1972). We use a distance modulus to the LMC of 18.3 mag. Overplotted on the stellar positions in the H-R diagram are isochrones from the evolutionary models of Maeder & Meynet (1987). By inspecting Figure 24 we deduce that most of the stars of LH 39 fit an isochrone with an age of about 12 Myr. If we assume that all stars are systematically misclassified by one type too late, i.e., by the length of the error bars, it would be possible to fit an
Fig. 24.—H-R diagram for members of the association LH 39. Overplotted are isochrones resulting from stellar evolution calculations of Maeder & Meynet (1987). Most of the association members fit to an age of about 12 Myr, including the red supergiant present in the spectrum of R84. We find that R84 and probably LH 39-1 are only 5–6 Myr old and therefore much younger than most of the stars of LH 39.

Isochrone with an age of 8 Myr. But we had to stretch the limits even more in order to include the brightest association member, LH 39-1, and the Ofpe/WN9 star R84. We need to invoke an isochrone of about 6 Myr to include these two stars. (In the post-red supergiant phase 6 Myr old stars have a luminosity of about \( L = 10^{5.6} L_\odot \).) Such a large systematic mis-calibration of the association stars seems to be unlikely even when we allow that the spectral type calibration in the LMC could be somewhat different from that given by Schmidt-Kaler (1982). It seems to be more rational to abandon the assumption of coeval evolution. An age of about 12 Myr for most association stars would also fit nicely to the observed luminosity of the red supergiant that is included in the spectrum of R84. According to Maeder (1990), a star with an initial mass of 15 \( M_\odot \) reaches the red supergiant stage after about 14 Myr and has then a luminosity of \( 10^{4.9} L_\odot \). The M2 Ia star has \( L = 10^{4.86} L_\odot \) (McGregor, Hillier, & Hyland 1988). The luminosity of this star has a small uncertainty because basically all of its emitted energy is observed; the authors estimate an error of about 0.1 dex. A quick glance at Figure 4 reveals that the luminosity of the red supergiant has to be much less than that of R84. Thus, we conclude that most of the stars of LH 39 have an age of about 12 Myr, but there are also some considerably younger stars present in this group: R84 and also at least one association star, LH 39-1. Further proof that there are stars with ages other than the bulk of the stars in the same area is the presence of LH 39-20 and LH 39-22, two A supergiants. From their luminosities we know that these stars are certainly older than 20 Myr. Therefore, we do not think it is possible to derive an age for R84 from its membership in LH 39, or to conclude from the presence of a less luminous red supergiant that R84 is in a post-red supergiant evolutionary phase (although we come to the same conclusion with different arguments).

7. CONCLUSIONS

On the basis of semiempirical model calculations, we are able to reproduce observed hydrogen and helium line profiles of the Ofpe/WN9 star R84 to high accuracy. The line fits are within the noise of the observations of all 11 lines we have analyzed. According to Wolf et al. (1987), the spectral type of R84 is O9.5. Using the physical parameter–spectral type calibration of Howarth & Prinja (1989), we find that R84's parameters are an extrapolation of the luminosity class sequence V–III–I of late O stars: its effective temperature is somewhat lower than that of a O9.5 I star, its luminosity is higher, its radius larger, and its gravity lower (\( M \approx 20 M_\odot \) implies \( \log g = 2.8 \)). We may therefore assign to R84 a luminosity class Ia+. Stars of this luminosity class are commonly termed "hypergiants," a class which has not yet been defined for O stars. This luminosity class should not be confused with Walborn's (1971) definition of \( f^+ \), which designates stars with \( N \) in 224633, 4640, He \( n \) 24686, and Si \( iv \) 224089, 4116 in emission. In the spectrum of R84 all these lines are in emission, and thus a complete classification of R84 would be O9.5 Ia+f. The obvious analogy of R84 to the known later type hypergiants is that the spectral characteristics are "more extreme" than those of a class Ia star. However, as noted by de Jager (1984), the observed \( \log T_{\text{eff}} \) and \( \log g \) of hypergiants appear to be linearly related (de Jager's eq. [12]). Inserting the values of R84, we find that R84 fits de Jager's (1984) relation. Since the luminosity of R84 is basically determined by the mass of its helium core (see § 6), we realize what this linear relation actually is: these stars are approaching the mass-luminosity relation of bare helium cores! The mass-luminosity relation of bare helium cores poses an upper limit on the luminosity-to-mass ratio that a star may reach during its evolution (see Fig. 2 of Maeder 1983). This boundary in the mass-luminosity diagram can be transformed into a limit in the effective temperature–gravity diagram:

\[
\log g = 4 \log T_{\text{eff}} - 14.38.
\]

This equation is derived using a linear form of the mass-luminosity relation:

\[
\log (L/L_\odot) = 3.77 + 1.49 \log (M/M_\odot)
\]
(Maeder 1983; see Langer 1989 for a quadratic form). Inserting the masses and effective temperatures of the stars given by de Jager (1984), we basically reproduce all the observed gravities of the hypergiants and LBVs, including that of ν Car. The only exception is ρ Cas, for which a much smaller mass would be required than given by de Jager (1984). The reason why the stars in de Jager’s sample appear to follow a unique relation (with the exception of ρ Car) is that most of them have similar masses and that the relation depends only weakly on the mass. Thus, hypergiants have a luminosity production that is nearly equivalent to that of a Wolf-Rayet star, i.e., the fraction of hydrogen-rich material which is still left on their stellar surfaces is too small to produce a measurable deviation from the mass-luminosity relation of bare helium cores.

The theoretical effective temperature–gravity relation not only reproduces de Jager’s (1984) empirical boundary in the \( \log(T_e) - \log(g) \)-plane but also explains why there are hypergiants (and LBVs) with luminosities that are not close to the upper luminosity limit of the H-R diagram (Humphreys & Davidson 1979). Obviously, the “stability limit” is not directly connected with the absolute value of the luminosity but with the luminosity-to-mass ratio approaching the value of bare helium cores. The indirect connection to the luminosity would be that only massive hot hypergiants, say \( M > 10 M_\odot \), develop strong winds.

Our ad hoc assumption of the non–LTE analysis is that R84 loses mass at a high rate. With subsequent hydrodynamic calculations that use as input the stellar parameters determined by the preceding analysis, we are able to justify this assumption: we can reproduce the observed mass-loss rate and the terminal velocity assuming the wind is driven by radiation pressure. Although our hydrodynamic wind models are rather crude (the treatment of excitation and ionization in the wind, the core-halo approximation, the assumption of stationarity), they are useful for a qualitative study of the wind properties of R84. The unusually high mass-loss rate (for an O star of R84’s luminosity) and the exceptionally low terminal velocity of the wind (compared with that of O normal stars) are a consequence of the stellar mass of R84, which is lower than is typical of stars in the same region of the H-R diagram. Previous higher estimates of the terminal velocity can be explained by incorrect interpretations of broadend photospheric absorption troughs of UV resonance lines as Doppler-shifted absorption features.

The spectral analysis yields a high helium abundance: \( Y = 0.63 \) by mass. Comparing this result with the predictions of evolutionary calculations, and taking into account R84’s luminosity, we conclude that the star is in a post–red supergiant phase on its way from red to blue. Its age is about 5–6 Myr, and its initial mass was 30–40 \( M_\odot \). The strong spectroscopic similarity of R84 to the S Doradus variables AG Car, HD 269852 (Stahl 1986), and R127 (Walborn 1977) in their minimum phases suggests that S Doradus variables in general have high luminosity-to-mass ratios, i.e., these stars are in an evolutionary phase after they have experienced a phase of violent mass loss. Whether or not this enhanced mass loss occurred for all of them during the red supergiant phase cannot be determined, because the stellar temperature during the enhanced mass loss does not influence the spectroscopic appearance in the following phases. Even for R84 the only evidence that the star was a red supergiant comes from evolutionary calculations.

Our interpretation challenges the standard ideas about the evolutionary stage of S Doradus variables. They are thought to experience an enhanced mass-loss rate as they approach the true Eddington limit when they evolve from blue to red. Their temperature variations would be caused by a variable mass loss, which in turn causes a “pseudophotosphere” to be variable in radius (e.g., Lamers & Fitzpatrick 1988). This idea might still be correct for objects like ν Car, but we do not think it is correct for S Doradus variables. Model calculations have demonstrated that the variations in effective temperature cannot be caused by variations in the mass-loss rates, but that the mass-loss rate variations can be explained by radius and thus temperature variations of these stars (Leitherer et al. 1989). This theoretical result has now been proved observationally: the S Doradus variable R110 is observed to have a large radius, i.e., to be cool, without showing signs of strong mass loss (Stahl et al. 1990; Leitherer & Langer 1991). Thus, R110’s radius is large without a stellar wind, which would be necessary for the standard explanation.

With our interpretation of the evolutionary stage of S Doradus variables as stars which have already lost a significant amount of their mass, even stars with luminosities well below the observed upper luminosity limit for red stars (Fitzpatrick & Garmady 1990), e.g., R71, are close to the true Eddington limit. The low effective gravity compared with that of “normal” stars with the same luminosity explains their high mass-loss rates and their low terminal wind velocities. However, their variability is not caused directly by the low effective surface gravity but has its origin in the stellar interior. In this context it is interesting to note the evolutionary tracks of stars that evolve from red to blue after the red-supergiant phase show temperature loops (Maeder 1990, Figs. 1–4).

Although the analysis of R84 helped to understand some aspects of the S Doradus variables, there is a difference between the S Doradus variables and the Ofpe/WN9 stars: the strong variability of the former and the relative stability of the latter. There is no obvious spectroscopic feature that could hint at the origin of the difference. Maybe there is no difference, and all of them are sometimes variable and sometimes relatively stable. This is a possibility which might be resolved by future observations.

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