SPECTROSCOPIC MEASUREMENTS OF MAGNETIC FIELDS ON SOLAR–LIKE STARS: TECHNIQUES AND RESULTS

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I. Introduction

Most efforts to measure magnetic fields on cool stars before 1980 studied polarized light, borrowing from solar physics the idea of analyzing the distinctive polarization signatures of the Zeeman components. These methods, however, proved far less successful for stars. Circular polarization (Stokes $V$) observations of solar–like stars yielded only upper limits (e.g., Babcock 1958; Preston 1967; Brown and Landstreet 1981), even for the most active stars (Vogt 1980). The lack of detections suggests that cool stars likely possess complex, solar–like field geometries with many bipolar elements; since the $V$ profile changes sign with polarity, such topologies inevitably lead to almost perfect cancellation of $V$ in the integrated signal. Broadband linear polarization (Stokes $Q$ and $U$), on the other hand, was detected (Tinbergen and Zwaan 1981), but the signals were weak and difficult to interpret (e.g., Landi Degl'Innocenti 1982). Clearly, a new approach was needed.

Work on Ap stars suggested one possible strategy. There, nearly resolved line splitting due to strong fields ($B \sim 8$ kG) could be observed directly in unpolarized (Stokes $I$) line profiles (e.g., Preston 1971). Preliminary searches for this effect in solar–type stars (Vogt 1980), however, excluded the presence of such large magnetic field strengths. Thus, with only a small magnetic broadening of lines expected, data of very high spectral resolution and high signal–to–noise were required. Although modern electronic detectors has made it possible to obtain data of this quality, a host of other factors conspire to keep things from ever becoming too easy. A major advantage of polarized light measurements – that they “see” radiation from the magnetic regions only – is lost when unpolarized line profiles are studied. Light from the entire star, both magnetic and non–magnetic regions, is included in $I$, requiring an accurate, (at least) two–component model to disentangle the different contributions. Often only a small fraction (or filling factor), $f$, of the stellar surface, is actually occupied by magnetic fields.
The history of magnetic field measurements on cool stars is thus the story of the various and increasingly sophisticated methods of extracting field information (B and f values) from (typically uncooperative) line profiles. In this article, I review the basic requirements for measuring magnetic fields on cool stars, outline main analysis methods and detail the assumptions in each of these, and finally, summarize the main results.

II. General Requirements and Difficulties

The simplest concept for analyzing magnetic fields in unpolarized light involves interpreting observed line profiles, F, using a two component model:

\[ F = (1 - f) F_q(B = 0) + f F_m(B), \]

where \( F_q \) is the profile in the quiet, non-magnetic regions and \( F_m \) is the profile the magnetic regions, which cover a fraction \( f \) of the stellar surface and have an average field strength, \( B \). Determining \( f \) and \( B \) for cool stars presents a host of difficulties, the most obvious of which is the small magnitude of the Zeeman effect. For a simple triplet, the separation of the sigma components from the central pi component is \( \Delta \lambda_B (\text{Å}) = 4.67 \times 10^{-15} g_{\text{eff}} \lambda^3 B \) (G), where \( g_{\text{eff}} \) is the effective Landé g value (a measure of a line's magnetic sensitivity). The velocity resolution \( \Delta v \) of the spectrograph/detector system used should be roughly

\[ \Delta v(\text{km s}^{-1}) \approx 2 \times \Delta v_B = 2.80 \times 10^{-7} g_{\text{eff}} \lambda(\text{Å}) B(G). \]

For smaller values of \( \Delta v \), the broadening due to \( B \) is undersampled, and it becomes increasingly difficult to separate \( f \) and \( B \) uniquely — small changes in the line width (related to \( B \)) mimic small changes in the intensity of the broadening component (related to \( f \)). More quantitatively, it can be shown (Gray 1984a; Saar 1988a) that in the optically thin line limit, models with equal \( f^{0.5} B \) yield similar profile shapes (see also Basri et al. 1990). Least-square fits to profiles show contours of equal \( \chi^2 \) defined by \( f^{0.5} B \propto \text{constant} \) (Fig. 2 of Saar and Linsky 1986b). Thus, \( f^{0.5} B \) is the best determined quantity and low resolution data are limited to measuring this product. Note the \( \lambda \) dependence of (Eq. 2), indicating the great usefulness of infrared data for magnetic measurements (e.g., Giampapa et al. 1983; Saar and Linsky 1985; Deming et al. 1988).

The filling factor, \( f \), must also be large enough for the magnetic signal to be seen above the noise. For a simple triplet, the strength of the Zeeman sigma and pi components relative to the total line strength is (for weak lines) given by the Sears relations:

\[ A_\sigma = 0.25(1 + \cos^2 \gamma) \quad \text{and} \quad A_\pi = 0.5 \sin^2 \gamma, \]

where \( \gamma \) is the angle between \( B \) and the line-of-sight. If \( S_0 \) is the strength of the line (in units of the continuum) for \( B = 0 \), the strength of the sigma components will be \( S_\sigma = f A_\sigma S_0 \). Assuming an average \( \gamma \approx 45^\circ \), the \( S/N \) needed for a \( 2\sigma \) detection of the Zeeman effect will be roughly

\[ S/N \approx 2(f A_\sigma S_0)^{-1} = 5.3(f S_0)^{-1}. \]

Note that this virtually rules out measurement of fields on the Sun—as—a-star: for \( f_0 \approx 0.01 \) and \( S_0 = 0.5 \), \( S/N \geq 1000 \), at which level inaccuracies in the model assumptions,
atmospheres, etc. will probably prevent a reliable measurement. It should be emphasized that the above S/N estimate represents the best (optically thin) case; in strong lines, saturation in the line core further reduces the ability to detect the sigma components. The most suitable lines are those of moderate strength: strong enough that \( S_0 \gg 0 \), but not so strong that magnetic information is lost in the optically thick line core.

Another constraint is set by the non–magnetic line broadening. For a high \( g_{\text{eff}} \) line (\( g_{\text{eff}} = 2.5 \)) at 6000 Å, \( \Delta v_B \) amounts to only 2.1 km s\(^{-1}\) per kilogauss, comparable to the line’s intrinsic Doppler broadening (thermal + microturbulence, \( v_D \sim 1.6 \) km s\(^{-1}\) for the Sun), and less than or equal to typical values of the granular velocities (macroturbulence, \( v_m \approx 2-4 \) km s\(^{-1}\) for cool dwarfs; Gray 1984b). This implies that it is difficult to uniquely determine the magnetic field strength for \( B \lesssim (6000/\lambda \text{ Å}) \) kG, even in the absence of rotational broadening. For \( v \sin i > v_D, \Delta v_B \), and \( v_m \), rotational smearing of the line profile increasingly reduces the relative magnetic signal at a given wavelength position in the line, requiring higher S/N to detect the effect. The S/N level needed can be estimated as follows. If we approximate all broadening agents as convolutions on the profile, and note that multiple convolutions approach smoothing by a gaussian in the limit, the combined broadening effect can be expressed as a quadratic sum. Then, as a rough guide, \( B \) and \( f \) can be reliably separated (errors \( \leq 50 \% \)) for

\[
f^{0.5} \Delta v_B \gtrsim 0.3[2v_D^2 + (\Delta v)^2 + v_m^2 + (v \sin i)^2]^{0.5},
\]

provided \( \Delta v \) is small enough to resolve \( B \) (Eq. 2) and the S/N is high enough to measure \( f \) (Eq. 4). (Here, the constant 0.3 is based partly on calculations in Saar 1988a).

Other factors are also important in measuring stellar fields. Lines used should be as free of blends as possible, to avoid confusing the blends with broadening due to fields. This becomes a severe problem for late K and M dwarfs, where molecular lines are important (see Fig. 1 of Saar 1987). Blends have complex effects on the derived \( f \) and \( B \), depending on the strength of the lines involved and relative separations (Saar 1988a). The set of lines used in the analysis should span a significant range of \( g_{\text{eff}} \), and generally at least one line should have \( g_{\text{eff}} \geq 2.0 \) for studies at optical wavelengths. It is important to note, however, that \( g_{\text{eff}} \) is actually not the optimum parameter for judging the overall Zeeman sensitivity of lines observed in unpolarized light. The \( g_{\text{eff}} \) value reflects only the intensity weighted splitting of the sigma components (i.e., transitions with \( \Delta m_J = \pm 1 \)), and thus overlooks contributions to the broadening of Stokes \( I \) due to the pi components (those with \( \Delta m_J = 0 \)). A more appropriate measure of the magnetic sensitivity for unpolarized line data, \( X_m \), has been defined by Mathys and Solanki (1989):

\[
X_m = 2A_\sigma (g_{\text{eff}}^2 + X_\sigma) + A_\pi X_\pi,
\]

which approximately describes the effect of \( B \) on line widths, \( w (\Delta w \propto X_m \lambda^2 f B^2) \). Effects of the mean shift of the sigma components from line center \( g_{\text{eff}} \), together with the dispersion of the sigma \( (X_\sigma) \) and pi \( (X_\pi) \) components about their respective centers of gravity, are included in \( X_m \). It would be useful, though, to have a parameter which is more directly comparable to \( g_{\text{eff}} \), and which also accounts for the disk–averaged nature of stellar observations. I therefore define a new magnetic parameter, \( G_{\text{eff}} \), given by

\[
G_{\text{eff}} = [\frac{2}{3} \int X_m dw]^{0.5} = [\frac{2}{3}(g_{\text{eff}}^2 + X_\sigma) + \frac{1}{3}X_\pi]^{0.5},
\]
where $d\omega$ is the stellar surface element,

$$X_\pi = (g_l - g_u)^2(3s - d^2 - 2)/10 \quad \text{and} \quad X_\sigma = (g_l - g_u)^2(8s - d^2 - 12)/80$$

(see Landi Degl'Innocenti 1985). Here, $s = J_l(J_l + 1) + J_u(J_u + 1)$ and $d = J_l(J_l + 1) - J_u(J_u + 1)$, where $g_l$, $g_u$, $J_l$ and $J_u$ are the Landé factors and angular momentum quantum numbers of the upper and lower levels, respectively. For simple triplets, $G_{\text{eff}} = (2/3)^{0.5} g_{\text{eff}}$.

I have calculated $G_{\text{eff}}$ for a large number of lines, assuming L-S coupling. Typically, the effect of the dispersions of the pi and sigma components is relatively small. Some lines, however [i.e., those with $G_{\text{eff}}/g_{\text{eff}}$ notably larger than $(2/3)^{0.5}$], show significant enhancements in broadening relative to the simple triplet case. The subset of these lines which also have large absolute $G_{\text{eff}}$ values thus comprises a new group of magnetically sensitive lines — lines which might have been overlooked using the $g_{\text{eff}}$ criterion. Indeed, in optically thick lines of this type, the redistribution of opacity out of the saturated $F_m$ line core can lead to much more significant profile and equivalent width changes than in the triplet case, greatly aiding the identification of the Zeeman effect (see Basri and Marcy 1991). Table 1 lists a selection of such lines (6000 Å $\leq \lambda \leq 9000$ Å) which are also relatively blend-free. A typical high $g_{\text{eff}}$ Zeeman triplet ($\lambda 6713$) is also listed for comparison. Several lines in Table 1 are weak in the Sun, but may be useful in cooler stars (e.g., $\lambda 6058$ in early M stars). A more complete list which spans 5000 Å $\leq \lambda \leq 23000$ Å is in preparation.

Table 1: Sample lines with large $G_{\text{eff}}$ and $G_{\text{eff}}/g_{\text{eff}}$

| ID | $\lambda$ (Å) | Transition† | $g_{\text{eff}}$ | $G_{\text{eff}}$ | $G_{\text{eff}}^{**}$ | $\chi(3/2)^{0.5}$ (mA) | $\chi^2$ (eV) | behavior in spots
<table>
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<td>V I</td>
<td>6058.2</td>
<td>$^4D_{0.5}^0$-$^2P_{1.5}$</td>
<td>2.167</td>
<td>1.938</td>
<td>2.374</td>
<td>3</td>
<td>1.04</td>
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<td>Fe I</td>
<td>6082.7</td>
<td>$^2P_{1.5}$-$^2P_{1.5}$</td>
<td>2.000</td>
<td>1.780</td>
<td>2.180</td>
<td>34</td>
<td>2.22</td>
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<td>$^2P_{1.5}$-$^2P_{1.5}$</td>
<td>2.000</td>
<td>1.780</td>
<td>2.180</td>
<td>34</td>
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<td>Fe I</td>
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<td>$^2P_{1.5}$-$^2P_{1.5}$</td>
<td>2.000</td>
<td>1.780</td>
<td>2.180</td>
<td>34</td>
<td>2.22</td>
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<td>$^2P_{1.5}$-$^2P_{1.5}$</td>
<td>2.000</td>
<td>1.780</td>
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<td>1.780</td>
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<td>1.780</td>
<td>2.180</td>
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<tr>
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<td>$^2P_{1.5}$-$^2P_{1.5}$</td>
<td>2.000</td>
<td>1.780</td>
<td>2.180</td>
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<tr>
<td>Cr I</td>
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<td>8353.1</td>
<td>$^2P_{1.5}$-$^2P_{1.5}$</td>
<td>2.000</td>
<td>1.780</td>
<td>2.180</td>
<td>34</td>
<td>2.22</td>
<td>s</td>
</tr>
</tbody>
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References: 
†Moore (1959); ‡Beckers (1969); §Moore et al. (1966)

*Simple triplet  **This scaling sets $G_{\text{eff}} = g_{\text{eff}}$ for a simple triplet

Even if all the above requirements are satisfied, magnetic field measurements will still be subject to numerous sources of systematic error. Most of these result from our ignorance...
concerning the detailed physical nature of the stellar surface. These error sources include: unknown thermodynamic differences between \( F_q \) and \( F_m \); unknown field distribution on the stellar surface; possible gradients in \( B \) with height; and the need for more complete, multicomponent models of both \( F_q \) (to account for granulation) and \( F_m \) (to include the stellar analogs of penumbrae, spots, etc.). I briefly discuss some methods of reducing or avoiding these systematic errors in the following sections.

III. Fourier Transform Methods

The first group of techniques developed to extract magnetic information from the spectra of solar–like stars employed Fourier transforms. The potential for Fourier transforms in magnetic analysis was first noted briefly by Beckers (1971). The basic strategy is to assume that observed lines profiles \( (F) \) are optically thin, simple triplets represented by

\[
F(\lambda) = (1 - f)F_0(\lambda) + f\{A_\sigma[F_0(\lambda + \Delta \lambda_B) + F_0(\lambda - \Delta \lambda_B)] + A_\sigma F_0(\lambda)\},
\]

\[
= (1 - f)F_0(\lambda) + fF_0(\lambda) * \{A_\sigma[\delta(\lambda + \Delta \lambda_B) + \delta(\lambda - \Delta \lambda_B)] + A_\sigma \delta(\lambda)\}
\]

(9)

where \( F_0(\lambda) \) is the intrinsic line profile in the absence of a magnetic field in the magnetic region, \( \delta \) is the Dirac Delta function and * stands for a convolution. Typically, \( F_0 = F_q \) is assumed, and the ratio of Fourier transforms of \( F(\lambda) \) and \( F_0(\lambda) \) is then

\[
z(s) = \frac{F'(s)}{F'_0(s)} = (1 - 2A_\sigma f) + 2A_\sigma f \cos 2\pi \Delta \lambda_B s
\]

(10)

where \( s \) is the Fourier frequency, and a prime denotes a Fourier transformed quantity. Application of Fourier transforms for analysis of solar fields was introduced by Title and Tarbell (1975) and Tarbell and Title (1976), at first concentrating on locating the zeroes of \( z(s)F'_0(s) \) and later (Tarbell and Title 1977) fitting \( z(s) \). Smith and Gray (1976) first suggested extension of the method to stars. Robinson (1980) later developed this concept more fully and made the first determinations of \( f \) and \( B \) for cool stars (Robinson et al. 1980). Later applications of the Fourier technique to stellar data have been made by Giampapa et al. (1983), Gray (1984a), Gondoin et al. (1985) and Sun et al. (1986).

Although elegant in its simplicity, the Fourier ratio method suffers in a number of respects. Both lines must be nearly blend–free to avoid spurious ripples in the Fourier ratio (Gondoin et al. 1985). Finding a suitable \( F_0 \) is difficult in stellar work (unlike the solar case, where \( F_0 \) can be obtained from a nearby quiet region). Robinson (1980) used low \( g_{\text{eff}} \) lines in the same star to approximate \( F_0 \), but this method requires that the low \( g_{\text{eff}} \) lines are nearly identical to the high \( g_{\text{eff}} \) lines in all ways (strength, \( \lambda \), excitation potential \( \chi_e \), etc.). Since this is almost never possible, the selected line must be scaled in some artificial way (see Hartmann 1987; Marcy and Bruning 1984) to better approximate \( F_0 \). Giampapa et al. (1983) obtained \( F_0 \) from the same line in a different star, chosen to have similar spectral type but much lower magnetic activity levels. While the intrinsic line properties are identical in this case, here one must artificially adjust \( F_0 \) for differences in metallicity, \( v \sin i \), and turbulence between the stars. Gray (1984a) avoided these problems by using a radiative transfer (RT) model to compute \( F_0 \). This allows use of any number of unblended
lines from various species to be averaged into a composite \( z(s) \) (at least in principle). A sample analysis of this type is shown in Figure 1.

More fundamentally, though, the basic convolution approximation itself (Eq. 9) is flawed, since in assuming the Zeeman components add linearly to the profile, it ignores RT effects (e.g., Saar 1988a). The approximation also errs in implicitly assuming that other broadening agents (e.g., \( v \sin i \)) also operate as convolutions, thereby ignoring center-to-limb variations in the \( F_0 \) profile (Bruning 1984). The significant center-to-limb variations in \( F_m \) are also glossed over, since some “average” \( \gamma \) must be used in the Sears relations. Finally, no provision for non-triplet lines is included in the analysis. All of these difficulties can potentially lead to significant systematic errors, and so the Fourier ratio method has fallen somewhat out of favor. Because of its simplicity, however, the method is still useful for obtaining rough estimates of \( f \) and \( B \) prior to a more rigorous analysis.

![Graph](image)

**Figure 1.** An example of the Fourier ration analysis for \( \epsilon \) Eri (from Gray 1984a), showing composite Fourier ratios \( z(s) \), here plotted against \( g_{\text{eff}} \) (= \( \gamma \sigma \) in Gray’s notation). Results for individual spectral lines is shown at left; the composite mean (solid) and fits (dashed) for various values of \( A_0 = 2A_e f \) are on the right. \( B = 1900 \) G and \( A_0 = 0.3 \) are derived.

**IV. Line Profile Modeling**

Marcy (1982) developed the wavelength domain of the Fourier ratio method, modeling lines directly (using Eq. 9) and adopting low \( g_{\text{eff}} \) lines as \( F_0 \). Since it is basically equivalent to the Fourier ratio method, this technique unfortunately suffers most of the same drawbacks described in §III. Nevertheless, Marcy (1984), studying a sample of 29 G and K dwarfs, successfully showed that fields were common in cool stars, and uncovered the first rough correlations between magnetic parameters, rotation and activity.

The first steps towards more realistic magnetic line models were taken by Marcy and Bruning (1984). Confronted with spectra which did not contain an optimum low \( g_{\text{eff}} \) line to use as \( F_0 \), they used LTE RT models to adjust the profile of an available line (of different \( \chi_e \)) appropriately. A more direct use of RT models was introduced by Saar and Linsky (1985; see also Saar et al. 1986a, Saar 1988a). These authors employed the analytic solution to the Unno (1956) RT equations in a Milne–Eddington atmosphere, including the full Zeeman patterns. Methods for treating blends, including direct calculation and subtraction of the
spectrum of a low activity star, were also developed. In general, they first fit a group of low $g_{\text{eff}}$ lines to determine the non–magnetic broadening ($v \sin i$, macroturbulence). These parameters are then used to model the ensemble of low and high $g_{\text{eff}}$ profiles simultaneously to determine $f$ and $B$. Saar (1988a) adopted the convolution approximation to treat rotational and macroturbulent broadening, however, fitting

$$F = [(1-f)I_q(B=0) + fI_m(B)] \ast \mathcal{V}(\lambda),$$

(11)

where $I_q$ and $I_m$ are the specific intensities in the magnetic and quiet regions at some "average" disk position, and $\mathcal{V}(\lambda)$ is the velocity convolution function (see Gray 1976). As noted in §III, this approximation is inadequate for slowly rotating stars (Bruning 1984) and worse when $B$ is large, due to significant center–to–limb profile changes (Saar 1988b; Landolfi et al. 1989). Correct treatment requires explicit integration of $I_q$ and $I_m$ over the stellar disk, first introduced by Bopp et al. (1989) and Marcy and Basri (1989). Disk–integration also eliminates the arbitrary choice of a $\tau$ in the Sears relations.

The simple Milne–Eddington atmosphere (source function linear in optical depth, all other properties constant) employed by Saar (1988a) is a reasonably good approximation for weak and intermediate strength lines, but poor for very strong lines. Basri and Marcy (1988) further improved the magnetic RT profile models by actual integration of the Unno (1956) equations through scaled solar atmospheres (Fig. 2). Their results for the strong $g_{\text{eff}} = 2.5$ Fe I 8468 Å line (Marcy and Basri 1989) differ somewhat from those of Saar (1990), who used the weaker $\lambda$ 6173 and the simpler atmosphere (Marcy and Basri generally find smaller $B$ and larger $f$). Much of the disagreement, however, may be due to different heights of formation of the two lines combined with a gradient in $B$ with height (Grossmann–Doerth and Solanki 1990) rather than the differences in modeling methods. Other recent improvements to magnetic line models include adding magneto–optical effects (Sánchez Almeida and García López 1991; Saar 1991b) and a bootstrap blend analysis using inactive stars as guides (see Saar et al. 1990; Saar 1991a).

![Image](Figure 2. Example of the improved magnetic RT profile modeling (Basri and Marcy 1988) for $\xi$ Boo A. Data for the Fe I lines 8468 Å ($g_{\text{eff}} = 2.5$) and 7748 Å ($g_{\text{eff}} = 1.1$; crosses) plus models (solid) with $B = 0$ (left) and $B = 1200$ G and $f = 0.40$ (right) are shown.)
All current modeling methods assume the atmospheres comprising $F_m$ and $F_q$ are identical, and that $\nabla B = 0$. The suggestion of a non-zero $\nabla B$ (Grossmann-Doerth and Solanki 1990) and stronger indirect (e.g., Saar et al. 1986a; Simon et al. 1985) and direct (e.g., Mathys and Solanki 1989; Donati et al. 1990) evidence for thermodynamic differences between $F_q$ and $F_m$ thus raise questions about possible systematic errors in the current set of magnetic measurements. These problems have been studied recently with a battery of realistic models. Basri et al. (1990) combined scaled solar quiet and plage atmospheres and analyzed the results with single component models (Marcy and Basri 1989). They find errors in $fB$ (the unsigned magnetic flux density) of as much as 40% can result from the standard “$F_q = F_m$” assumption. Gradients in $B$ with height were also briefly explored. As noted above, Grossmann-Doerth and Solanki (1990) have explored the effect of $\nabla B$ and height of formation on $f$ and $B$ measurements, finding that they may explain some of the differences in published results. Studies of how derived $f$ and $B$ values vary with line strength and $\chi_e$ should therefore help elucidate the differences between $F_q$ and $F_m$. Valenti (1991) has begun exploring this type of analysis. Solanki and Saar (see Saar 1991b) are developing even more realistic, two-dimensional models, in which profiles are computed by summing RT calculations made along rays passing through a self-consistent atmosphere embedded with flux tubes.

Sánchez Almeida and García López (1991) have recently explored a method of circumventing some of the uncertainties concerning heterogeneous stellar atmospheres. They concentrate on two lines (Fe I 5247 Å and 5250 Å) which are nearly identical except for $g_{eff}$. By subtracting one profile from the other, they can very nearly cancel $F_q$, leaving only a residual $\Delta F_m$ (see Eq. 1) which they model using a realistic RT code. At the moment, there appear to be some calibration problems (Saar 1991b), but eventually this method should prove helpful in exploring the nature of $F_m$.

V. Other Methods

As suggested above, one way to investigate the differences between $F_m$ and $F_q$ is to analyze lines with a wide range of strengths and $\chi_e$, thereby probing different heights of formation and sensitivities to density and temperature. Mathys and Solanki (1989) have already completed an approximate analysis of this type quite unlike the preceding detailed line models. Their method, a modification of the Stenflo and Lindegren (1977) approach, determines $f$ and $B$ from a regression analysis correlating line widths at some fractional depth, $d$, and the line area, $a$ (measured below $d$), with $\chi_e$ and $X_m$. The observed line width, $w_{obs}$, was well represented by

$$w_{obs}(d) = x_0 + x_1a(d) + x_2a(d) + x_3\chi_e w_0 + x_4\lambda^2/w_0 + x_5X_m\lambda^2/w_0,$$

where $x_0 ... x_5$ are the regression coefficients and $w_0$ is an approximate line width computed from a fit using the first three terms only. The magnetic parameters are extracted from $x_5 \propto fB^2$. Separation of $f$ and $B$ is then accomplished by comparing $x_5$ coefficients derived from $w_{obs}$ at different values of $d$.

Mathys and Solanki (1989) applied the $fB$ regression analysis to the active K2 dwarf, $\epsilon$ Eri. They derived $f$ and $B$ values comparable with other measurements, and uncovered
the first direct evidence that $F_m$ on stars is warmer than $F_q$. It is uncertain how far one can go in interpreting this result, however. While the technique yields reasonable results for spatially resolved solar regions, whether the same is true in general for disk-integrated stellar spectra, or in more heavily blended cooler stars, is unclear. Further tests and calibration against more rigorous analyses (line synthesis) are needed. The simplicity of the regression analysis, however, and the promising initial results suggest that (at least) it may be useful in estimating $f$, $B$ and approximate $F_q - F_m$ atmospheric differences.

Due to the restriction on $v \sin i$ (Eq. 5), the above analysis methods are ill-suited for the study of fields on rapidly rotating stars. Quite recently, however, new methods and instrumentation have been developed that allow us to peer over this "$v \sin i$ barrier". The most straightforward of these approaches was devised by Basri and Marcy (1991). They take advantage of the fact that for optically thick lines, Zeeman splitting (even if unresolved) will increase a line's $W_\lambda$ due to redistribution of opacity out of the saturated line core (e.g., Leroy 1962). Thus, while it is difficult to actually separate $f$ and $B$ uniquely for rapid rotators, some combination ($f0.5B$ or $fB$) may still be estimated from $\delta W_\lambda = W_\lambda(\text{observed}) - W_\lambda(\text{B = 0})$ (where the latter is derived from a detailed model). Basri and Marcy (1991) discovered an increase in $\delta W_\lambda$ versus $W_\lambda(B = 1000)/W_\lambda(B = 0)$ consistent with $f B \approx 1000$ G for the "naked" T Tauri star Tap 35 ($v \sin i \approx 17$ km s$^{-1}$). They also saw no monotonic correlation between $\delta W_\lambda$ and $g_{\text{eff}}$, undoubtedly the result of the non-triplet nature of many of their lines [i.e., $G_{\text{eff}} > (2/3)^{0.5 g_{\text{eff}}}$]. A high $G_{\text{eff}}/g_{\text{eff}}$ ratio should prove a valuable line selection criterion for this type of analysis, since the additional dispersion of line opacity reflected in $X_\pi$ and $X_\sigma$ enhances the increase in $W_\lambda$.

So far, I have focused exclusively on field measurements based on Stokes $I$, largely due to the lack of success of polarized light observations in the past (see §1). More recently, however, Kemp et al. (1987) uncovered evidence that large unipolar regions may be present on some very active stars, suggesting that at certain rotational phases, significant polarized signals may be detectable. Along these lines, Semel (1989) has proposed an analysis method employing high resolution Stokes $V$ profiles that is especially suited for measurements of rapid rotators. Called "Zeeman Doppler imaging" (ZDI), the technique relies on rapid stellar rotation to separate regions of opposite polarity in velocity. The method may thus be considered the circularly polarized analog of traditional "Doppler imaging" (e.g., Vogt 1988). If the velocity separation of the regions is sufficient ($\approx \Delta v$), the oppositely signed $V$ profiles (shifted relative to one another in wavelength) will cease to cancel each other and yield a measurable residual signal. The method thus measures the net magnetic flux in each velocity resolution element. Because the net flux is measured, $f$ is determined by assuming a reasonable $B$ value (Donati et al. take $B = B_{\text{eq}}$, the gas pressure equipartition field strength). The $V$ profiles may be used to construct Doppler-resolved magnetic distributions, $B(\Delta \lambda)$, of the longitudinal component of $B$:

$$B(\Delta \lambda) \propto \frac{F_q(\Delta \lambda)}{W_\lambda} = \frac{B}{\Delta \lambda B W_\lambda} \int_{\Delta \lambda} V(\lambda) d\lambda,$$

(13)

where $W_\lambda$ is the equivalent width of $F_q$ (Donati and Semel 1990). Eventually, a succession of detections as a function of phase may be used to build up a crude image of flux concentrations on the stellar surface. ZDI requires very high S/N ($\approx 400$) and is limited in the stellar $v \sin i$ range it can study effectively ($15$ km s$^{-1} \lesssim v \sin i \lesssim 40$ km s$^{-1}$; see Donati and
Semel 1990), thus restricting its applicability somewhat. Nevertheless, ZDI represents an exciting new way to obtain magnetic flux and spatial information on the individual active regions of rapidly rotating stars. Figure 3 shows an example of this analysis.

![Stokes I and V profiles](image)

**Figure 3.** Left: Stokes $I$ and $V$ profiles for the RS CVn star HR 1099 (top; primary and secondary components labeled) and the moon (bottom) near three high $g_{\text{eff}}$ lines. Note the clear, characteristic $V$ profile signatures (arrows) on HR 1099, and the lack thereof in the moon (as expected). Right: the magnetic distributions $B\ast (F_{\text{g}}/W_{\lambda})$ ($= F_{\ast N}$ in the notation of Donati et al. 1990) for HR 1099 at two phases. Note the 16σ detection at $\phi = 0.85$; if $B \equiv B_{\text{eq}} \approx 1000$ G, $f = 0.14$ is inferred (from Donati et al. 1990).

Once instrumentation capable of sufficient precision is available, spectroscopic observations of Stokes $Q$ and $U$ should also prove very useful in stellar magnetic research. Already, broadband linear polarization data can help in determining the spatial location of magnetic regions (e.g., Landi Degl’Innocenti 1982; Huovelin and Saar 1990, 1991), as demonstrated for $\xi$ Boo A (Saar et al. 1988). Broadband $Q$ and $U$ data can also be used to establish upper limits on $fB$ for cool stars (Saar and Huovelin 1990, 1991). A detailed discussion of these possibilities lies outside the scope of this paper.

**VI. Results So Far**

I now briefly summarize the results of magnetic analyses made to date. Due to problems with systematic errors, (§III; see also Stepien 1987) I will concentrate mostly on the results obtained by analysis methods which include RT effects (compiled by Saar 1990, 1991b).
Note that the results presented below (especially for \( f \)) must be regarded as somewhat tentative, due to several possible systematic effects in the measurements (see §II and IV).

Magnetic fields have now been surveyed throughout most of the cool part of the H–R diagram (spectral type F and later). No fields have been detected in F dwarfs (Gray 1984), non–emission M dwarfs (Saar et al. 1987) or in normal giants, bright giants, or supergiants (Marcy and Bruning 1984; Gray and Nagar 1985). The only low gravity objects detected so far are a few RS CVn binaries (Giampapa et al. 1983; Gondoin et al. 1985; Bopp et al. 1989; Donati et al. 1990) and a lone T Tauri star (Marcy and Basri 1991). Strong magnetic broadening has been observed in stars as cool as M4.5 (the spotted flare star EV Lac; Saar et al. 1987) suggesting that even stars with very deep convective zones have efficient dynamo mechanisms. A significant range in \( fB \) exists for G, K, and M dwarfs, with both non–detections and clear measurements in each class. There appears to be an upper limit to \( B \) as a function of spectral type, however, which increases with decreasing \( T_{\text{eff}} \) (Saar and Linsky 1986a; Saar 1987, 1990, 1991b). Saar (1990) interpreted this limiting field \( B_{\text{lim}} \) as the result of gas pressure \( (P_{\text{gas}}) \) confinement of the magnetic fields: \( B \leq B_{\text{lim}} \propto P_{\text{gas}}^{-0.5} \), where \( B_{\text{eq}} \) is the equipartition field (Fig. 4). Several theoretical models of magnetic flux concentration predict similar relationships (Parker 1978; Galloway and Weiss 1981). Some of the scatter below \( B = B_{\text{eq}} \) may actually be due to the effects of \( \nabla B \) on the different lines studied (§IV). There is no correlation between \( f \) and \( B \) (Saar and Linsky 1986a), which would be expected if errors in separation of the magnetic parameters dominated the results (Gray 1984a; Saar 1988a). There is also no significant correlation between \( B \) and either normalized angular velocity \( \Omega (\equiv 1/P_{\text{rot}}) \) or inverse Rossby number, \( \tau_{\text{c}} \Omega \), where \( \tau_{\text{c}} \) is the convective turnover timescale (Linsky and Saar 1987; Saar 1991b).

**Figure 4.** Left: \( B \) vs. \( B_{\text{eq}} \propto P_{\text{gas}}^{-0.5} \), with G, K, and M stars represented by circles, squares and triangles, respectively, the Sun is indicated by \( \odot \), and lower gravity stars (RS CVns, T Tauris) are filled symbols. Separate observations of the same star are connected by a dashed line and the \( B = B_{\text{eq}} \) line is also shown (solid). Right: \( fB \) vs. \( \tau_{\text{c}} \Omega \), same symbols. Both from Saar (1991b).

Some kind of dependence of the magnetic parameters on rotation is expected from both dynamo theory (e.g., Montesinos et al. 1987) and observations of stellar chromospheric
and coronal activity (e.g., Hartmann and Noyes 1987). Marcy (1984), and Gray (1985) were the first to note relationships between rotation and the magnetic parameters. Using more recent data, it is clear that the magnetic flux density, $fB$, correlates well with both $\Omega$ and $\tau_\Omega$ (Marcy and Basri 1989; Saar 1987, 1990, 1991b; Fig. 4). Both relations can be described by power laws, although the value of the exponent depends on how strongly one weights the solar point and some slowly rotating K dwarfs with lower accuracy $fB$ determinations (see Saar 1991b). In general, $fB \propto \Omega^\alpha$ or $\tau_\Omega^\alpha$, with $\alpha$ between 1 and 2. The $fB-\tau_\Omega$ relation may show a “saturated” level above $\log \tau_\Omega \approx 0.75$ (see Fig. 4).

Since $B$ is uncorrelated with rotation and $fB$ clearly is, it follows that the magnetic filling factor must lie at the heart of the rotation–activity connection. Comparison of $f$ with $\Omega$ and $\tau_\Omega$ immediately confirms this inference (Saar et al. 1987; Linsky and Saar 1987; Stepień 1988; Saar 1990, 1991b). Figure 5 shows the correlation between $f$ and $\tau_\Omega$. Consistent with the magnetic flux results, $f \propto \tau_\Omega^\alpha$, where $1 \leq \alpha \leq 2$, depending on weights assigned (Saar 1991b). Note the fairly clear definition of a “saturated” level of active region coverage for $\log \tau_\Omega \gtrsim 0.75$. (Saturation in $fB$ may have been masked due to M dwarfs with both large $B$ and $\tau_\Omega$.) The value of $\tau_\Omega$ at which $f$ reaches a maximum is roughly consistent with the observed onset of saturation in magnetic–related activity (e.g., Vilhu 1984). A saturated state also appears in some recent dynamo models (MacGregor and Brenner 1989).

![Figure 5. Left: $f$ vs. $\tau_\Omega$, same symbols as in Fig. 4. Note the apparent saturation for $\log \tau_\Omega \gtrsim 0.75$. Right: $fB$ vs. $t$, symbols as before except here, filled symbols are stars with ages based on cluster membership and open symbols are based on Li I. An exponential decay $fB \propto e^{-0.82t/(\text{Gyr})}$ is shown (solid). Both from Saar (1991b).](image)

Stellar rotation rates decrease with time due to the torque that $B$ exercises on the stellar wind (Skumanich 1972). It is therefore natural to search for similar trends in the magnetic data. Recent detections of young stars by Saar (1991a) and Basri and Marcy (1991) aid this study by extending $fB$ measurements to significantly earlier stellar ages, $t$. An exponential decay law with an e–folding time of 1.2 Gyr fits the decline of $fB$ with age best (Fig. 5). Since $B$ is uncorrelated with $t$ (Linsky and Saar 1987; Saar 1991a), this result implies that the decline in magnetic activity with time is a direct result of the
reduced surface area occupied by active regions as a star ages, spins–down, and its dynamo subsides.

I now turn to the relationships between $fB$ and “magnetic activity”, which can be used to explore the role of the fields in the heating and structure of stellar outer atmospheres. Several investigations (Marcy 1984; Saar and Schrijver 1987; Saar 1988b; Marcy and Basri 1989; Schrijver 1990; Saar 1991a) have explored correlations between magnetic and emission fluxes from stellar chromospheres, transition regions, and coronae. Schrijver (1991) gives an excellent review of the subject. Power law fits to latest data show a sequence of relations for the residual Ca II, C IV, and X–ray flux densities: $\Delta F_{\text{CaII}} \propto (fB)^{0.5}$, $F_{\text{CIV}} \propto (fB)^{0.7}$, and $F_X \propto (fB)^{1.0}$, approximately (Fig. 6). These relations are consistent with both correlations between the emission fluxes themselves, and for $fB$–flux relations seen in spatially averaged solar regions (e.g., Schrijver et al. 1989). The linear relation between $fB$ and $F_X$ may reflect a coronal heating mechanism linear in $fB$, or may simply reflect a linear proportionality between the number of photospheric flux tubes, and the number of flux tubes which reach the corona (Stepień 1988; Saar 1991a). These results are also broadly consistent with the MHD wave heating models of Musielak et al. (1990). The non–linear chromospheric relation may be due to purely geometric effects, or the interaction of geometry on the heating mechanism (see Schrijver et al. 1989). Schrijver (1990) has proposed the intermediate $F_{\text{CIV}}$ dependence is the result of a mixture of heating from chromospheric [$\propto (fB)^{0.5}$] and coronal ($\propto fB$) sources.

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{fig6.png}
\caption{$fB$ vs. $F_X$ (left) and $\Delta F_{\text{CaII}}$ (right). Symbols as in Fig. 4, except dwarfs and lower gravity stars are not distinguished. The best fit power laws, $F_X \propto (fB)^{0.92}$ and $\Delta F_{\text{CaII}} \propto (fB)^{0.56}$, are shown (solid). From Saar (1991a).
\end{figure}

One can also begin to broadly discuss the physical nature and spatial distribution of magnetic regions on stars. Most magnetic measurements have been made in the optical, where spots contribute little to the integrated line profiles. The magnetic regions detected must therefore refer to brighter elements, likely the stellar counterparts of active network and plage on the Sun. Consistent with this idea, magnetic regions on stars do appear to be hotter than quiet photosphere (Mathys and Solanki 1989; Donati et al. 1990). Just how much hotter $F_m$ is than $F_q$ is a matter of debate (Basri et al. 1990), and the answer may

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significantly effect the absolute scale of the measured $f$ values. On the other hand, quite unlike the Sun, stellar active regions sometimes form in fairly large unipolar concentrations (Donati et al. 1990). Note also that substantial magnetic flux may reside, as yet undetected, in the spots of active stars (Mullan 1984).

Rotational modulation of $fB$ can in principle be used to construct crude 1-D maps of the surface magnetic structures on a star. Longer term studies of $fB$ can be used to explore stellar dynamo activity cycles. Variability in the stellar magnetic parameters has been searched for without much success, however. Basri and Marcy (1988) and Saar et al. (1988b) saw little variation on the active K2 dwarf, ε Eri. Saar et al. (1990) detected no change in $fB$ for BD +26°730, a young spotted K dwarf, over nine rotational periods. Since the star is nearly pole-on to us, the lack of variability constrains the rate of change of the the flux level in time, and also proves the existence of substantial flux near the stellar pole (cf. Vogt 1988). No change in BD +26°730 activity levels (H$_\alpha$ or C IV) was seen between 1981 and 1988 either, despite a significant decline in brightness (presumably due to greater spot coverage during the star's advancing magnetic cycle). This led Saar et al. to suggest that stars with nearly saturated activity ($f$ was ≈ 50%) may only process plage into spots and back again over their magnetic cycles, with little net change in overall magnetic coverage or activity.

The only relatively clear magnetic variability seen to date has been seen on the active G8 dwarf, ξ Boo A. Early on, a number of detections (Marcy 1984) and non-detections (Gondoin et al. 1985) were made. Saar et al. (1988) studied ξ Boo A in a multiwavelength suite of observations ($fB$ measures, Ca II, ultraviolet emission, broadband linear polarization) and found a significant enhancement in $fB$ and related activity at one phase. Combining linear polarization (sensitive to $fB$ near the limb), Stokes $I$ magnetic measures (weighted more towards disk-center), and Ca II data, the authors constructed a crude map of the active longitudes of the star. Care must be taken to distinguish $fB$ variations from line bisector variations (Toner and Gray 1988; Bruning and Saar 1990), which may be due to enhanced convection associated with the active regions themselves (Toner and LaBonte 1990; Brandt and Solanki 1990; Saar 1991b).

To summarize, a variety of increasingly sophisticated analytical techniques have been developed to detect the presence of magnetic fields in the high resolution spectra of cool stars. Although subject to some possible systematic errors, the average field strength in active regions ($B$) and the fraction of the stellar surface occupied by these regions ($f$) have now been derived for over 30 stars. Significant correlations are seen between $B$, $f$, stellar properties (e.g., $P_{\text{rot}}$, $\Omega_\ast$, $t_e$), and non-radiative emission in the outer stellar atmospheres. I believe the future holds great promise for the continued refinement of the techniques, and more and better measurements, leading to a richer understanding of the role of magnetic fields in the structure, energy balance, and evolution of the outer envelopes and atmospheres of stars.

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