A COMPARISON BETWEEN PROGRESSIVE EXTENSION METHOD (PEM) AND ITERATIVE METHOD (IM) FOR MAGNETIC FIELD EXTRAPOLATIONS IN THE SOLAR ATMOSPHERE

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ABSTRACT

In this paper we present a comparison between two numerical methods for the extrapolation of nonlinear force-free magnetic fields, viz. (i) the Iterative Method (IM) and (ii) the Progressive Extension Method (PEM). The advantages and disadvantages of these two methods are summarized and the accuracy and numerical instability are discussed. On the basis of this investigation, we claim that the two methods do resemble each other qualitatively.

I. INTRODUCTION

It is well-known that the magnetic fields play a dominant role in all physical features which appear in the solar atmosphere; for example, the observed filamentary structures in the chromosphere seen in Hα (Martin, 1980), and coronal loops seen in UV (Cheng, et al. 1982) and X-rays (Antonucci et al. 1982; de Jager et al. 1983). All these structures in the solar atmosphere are generally considered to be aligned along the magnetic field (Zirin, 1971; Poletto, et al., 1975). Physically, these structures can be interpreted as plasma confined by the magnetic field. Hence, a detailed and quantitative analysis of these structures require a quantitative knowledge of the magnetic field in the solar atmosphere. Presently, measurements of magnetic fields are

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confined to the photospheric level; therefore, in higher levels (i.e. chromosphere and corona) the magnetic field can only be obtained through numerical extrapolation using the measured photospheric magnetic field as the source surface, as demonstrated in the early work of Schmidt (1964), Altschuler and Newkirk (1969), Nakagawa and Raadu (1972). All these early extrapolation methods are restricted to the linear approximation, which physically represents current-free field (potential field) or constant current-to-magnetic field ratio (linear force free field). It has been shown that these representations are far from realistic in describing the observed features in the solar atmosphere (Schmahl et al., 1982).

In order to improve our understanding of the physical structures of the solar atmosphere it is necessary to have quantitative knowledge of the magnetic field. Therefore, a number of extrapolation methods is developed to meet the demands. The mathematical model using a force free configuration on the basis for the extrapolation of photospheric vector magnetograms to obtain the coronal field has been given by Aly (1989) and Gary (1990). In particular, Gary (1990) presented an excellent summary and assessment on the present available extrapolation methods from a theoretical point of view. In this paper, a comparison between the progressive extension method (PEM) and iterative method (IM) is presented. The rationale for choosing these two extrapolation techniques for comparison is that they are based on observed photospheric level fields and have practical applications. A brief description of the theoretical background of these two techniques is presented in Section 2. Numerical results of direct comparison are included in Section 3. The discussion of advantages and disadvantages of these two techniques and their possible physical consequences are presented in Section 4.

II. THEORY AND TECHNIQUES

On the assumption of magnetohydrostatic equilibrium in the solar atmosphere, the mathematical model describing such an equilibrium state may be written as

$$-\nabla p + \mathbf{J} \times \mathbf{B} - \mathbf{\rho} \mathbf{g} = 0 \, ,$$

where $p$ is the hydrostatic pressure and will be represented by the equation of state,

$$p = \rho R T \, ,$$

with $\rho$ and $T$ being the mass density and temperature respectively. The other symbols have their usual meanings; $\mathbf{B}$ is the magnetic field and $\mathbf{J}$, the current density, is related to $\mathbf{B}$ by
\[ \mathbf{\dot{J}} = \nabla \times \mathbf{\dot{B}}. \]  

Finally, \( \mathbf{\dot{g}} \) is the gravitational acceleration. Physically, there are three different orders of approximation to determine the magnetic field configuration. The first and second order approximations are the current free (potential) and force-free magnetic field, respectively. Within these orders of approximation the magnetic force vanishes, and the pressure force is balanced by the gravitational force which leads to the hydrostatic equilibrium in the solar atmosphere. Under these circumstances, the mathematical model for the magnetic field configuration can be represented by

\[ \nabla \times \mathbf{\dot{B}} = \alpha \mathbf{\dot{B}}, \]  

This expression possesses three different physical meanings, which are: (i) \( \alpha = 0 \), corresponds to the current free case in which the magnetic field is potential, (ii) \( \alpha = \) constant, corresponds to the linear force-free magnetic field which implies a constant current-to-magnetic field ratio in a region and (iii) \( \alpha = \alpha(r) \), corresponds to the nonlinear force-free field which implies a non-constant current-to-magnetic field ratio in a region.

Finally, the third order of approximation is the magnetohydrostatic equilibrium in the solar atmosphere which is given by Eq. (1). If there is information on \( \mathbf{B} \) and \( p \) on the source surface, it is possible to extrapolate \( \mathbf{B} \) and \( p \) upward. Since there only are measurements of the magnetic field on the source surface (photosphere), it is not possible to extrapolate magnetohydrostatic equilibrium field-configurations at the present time.

In the meantime, we shall focus our attention on the nonlinear force-free field configuration. For the purpose of this paper, we have selected two techniques for this investigation. These two techniques are progressive extension method (PEM) (Wu et al., 1985, 1990) and iterative method (IM) (Sakurai, 1981). A brief description of these two methods is presented below:

Progressive Extension Method (PEM)

The progressive extension method is formulated as an initial-value problem (i.e., Cauchy problem) using a finite difference scheme which is similar to a Taylor expansion. A detailed description of this method is given by Wu et al. (1990). They have demonstrated the usefulness of this method, and the numerical algorithm has been verified by extrapolation of an analytical solution (Low, 1982).
Iterative Method (IM)

A number of authors (see references in Gary, 1990) have utilized an iterative method originated by Grad and Rubin (1958) to extrapolate the nonlinear force-free magnetic field from boundary data. For convenience, we simply choose the iterative method developed by Sakurai (1981) in this study. His method is based on the integral equation representation of Eq. (1), and the discretization is made by the technique of finite element method. A detailed description of this technique was given by Sakurai (1981), and we shall not repeat it here.

III. NUMERICAL RESULTS

In order to make comparison between the PEM (Progressive Extension Method) of Wu et al. (1985, 1990) and the IM (Iterative Method) of Sakurai (1981), we have chosen the vectorial magnetic field observed at Okayama Astrophysical Observatory on May 26, 1985 (Sakurai and Makita, 1986) as the boundary for extrapolation using these two methods. The observed magnetic field vector is shown in Figure 1.

![Image of magnetic field vector](image)

Figure 1. Magnetic field vector observed at Okayama Astrophysical Observatory on May 26, 1983. Solid and dotted contours show positive and negative longitudinal fields, respectively, with levels $\pm 10, 20, 50, 100, 200, 500$ G. Arrows indicate the transverse vector.
Using these observational data as a source surface, we obtained the nonlinear force-free field configuration by using the above mentioned two methods as shown in Figure 2, where Figure 2a is obtained by using the IM and Figure 2b by using PEM. In addition we have extrapolated the potential field configuration using PEM in comparison with the potential field given by Sakurai and Makita (1986), see Figure 3. From these results, observe that the deduced magnetic field configurations albeit not identical, in fact, qualitatively resemble each other to a large extent.

Figure 2. Nonlinear force-free field lines computed by (a) Iterative Method (IM) and (b) by Progressive Extension Method (PEM) using the data shown in Figure 1.

Figure 3. (a) Potential field lines computed by IM and (b) potential field lines computed by PEM using the observation given in Figure 1.
IV. DISCUSSION

Before we analyze the causes of these differences seen in the two extrapolations we review the fundamental differences between the two methods. These differences can be summarized as follows:

1. The Iterative Method (IM) specifies the value of $\alpha$ on a portion of the boundary plane (e.g. on a positive field region) and cannot assign the value of $\alpha$ on the whole boundary plane, since that would introduce an inconsistency in the extrapolation process. The values of $\alpha$ in the whole boundary plane are determined by the observed data for PEM. In this fashion, there is an electric current only along the particular field line in the IM extrapolation, while the electric current is distributed in the whole domain of calculation for the PEM extrapolation.

2. The IM type of extrapolation is convergent only for small values of $\alpha$. Physically, this implies that the electric current in the region of interest must be small. On the other hand, the PEM type of extrapolation does not have this limitation. However, the accuracy of the computed $\alpha$-value deteriorates at the points near the neutral line (i.e. $B_z \rightarrow 0$). This may cause a misrepresentation of the magnetic field configuration. The grid size of the extrapolation is controlled by the numerical stability criteria as given by Wu et al. (1990).

3. The fact that the value of $\alpha$ is assigned at one of the two foot points of a particular field line in the IM while the values of $\alpha$ are determined on the entire boundary surface in the PEM makes it difficult to match and compare the field lines for these two different methods.

On the basis of these differences of extrapolation procedures, we may understand why the magnetic field configurations obtained from the same data with these two methods are not identical. For example, Figure 2, shows some differences in magnetic field-line configurations, but the lines connecting different regions of polarities are quite similar. Note that for two regions of opposite polarities near the right center, the PEM extrapolation doesn't show any connection by field lines, while the IM type extrapolation does. However this is due simply to the fact that the field lines in this region are very low and short, and cannot be discerned in this drawing. Plots of the front view of Figure 2b, clearly indicate that the regions are connected by field lines (marked by A) as shown in Figure 4.
Figure 4. The front view of the nonlinear force-free field computed by PEM using the observation given in Figure 1. It should be noted that the field lines near the top are not accurate due to numerical procedure as discussed by Wu et al. (1990).

We further notice that the configuration of the field lines obtained by IM extrapolation is very similar to a potential field line configuration. This is because the IM requires that the value of $\alpha$ be small (i.e. slightly deviating from potential). On the other hand, the PEM extrapolation does not have this limitation. It is understood that the degree of deviation from a potential field depends on the value of $\alpha$, that is the strength of the local electric current. Therefore, the configuration of magnetic field lines is affected.

In summary, we conclude:
(i) Both methods do produce qualitatively similar results.
(ii) The accuracy of PEM has been verified by an analytical solution (Wu et al. 1990); verification of IM is still needed.
(iii) There are limitations on the value of $\alpha$ for IM, but not for PEM.
(iv) The accuracy for PEM deteriorates when the height of extrapolation exceeds one third the horizontal length, because of the propagation of the accumulated numerical errors at each level (Wu et al. 1990).

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