DEPTH AND LATITUDE DEPENDENCE OF THE SOLAR INTERNAL ANGULAR VELOCITY

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ABSTRACT

We present estimates of the solar internal angular velocity as functions of radius and heliographic latitude over the outer portion of the solar interior that stretches between 0.6 and 0.95\(R_\odot\). We obtained these estimates by first analyzing the frequency splittings of solar intermediate-degree \((3 \leq l \leq 170)\) p-mode oscillations which were observed with the 60 foot (18 m) tower telescope of the Mount Wilson Observatory during 16 days of 1984 July and August. We then combined these splittings with those from other published studies in order to obtain composite rotational frequency splittings as a function of degree, \(l\), for the solar equatorial plane, for middle latitudes, and for the solar rotation axis. These composite splittings were then converted into radial profiles of the angular velocity by assuming that the composite frequency splitting within each bin was a measure of the internal rotational velocity at a radial distance which was located midway between the surface and the innermost turning points of the p-modes contained within that bin. Finally, these composite inferred rotational profiles were then compared with the results of formal inversions of two of the sets of observed frequency splittings. Along the equatorial plane, our results suggest that the angular velocity first increases slightly with increasing depth below the photosphere (i.e., \(\Omega / \sigma \approx 66 \text{ Hz}/\text{R}_\odot\) for \(0.6\text{R}_\odot < r \leq 0.95\text{R}_\odot\)) until it reaches a maximum in the outer portion of the convection zone. The equatorial angular velocity then appears to decrease with increasing depth (i.e., \(\Omega / \sigma \approx 66 \text{ Hz}/\text{R}_\odot\) for \(0.6\text{R}_\odot < r < 0.95\text{R}_\odot\)) until it reaches a value that is at least 3% below the surface gas rotation rate in the radiative zone somewhere below the base of the convection zone. Our results also suggest that the latitudinal differential rotation which is visible in the photosphere persists over most or all of the convection zone. Finally, the comparisons of our composite angular velocity profiles with those obtained by the formal inversions suggest that this latitudinal differential rotation might disappear completely below the convection zone, although not all of the published splittings are in agreement on this point.

Subject headings: Sun: interior — Sun: oscillations — Sun: rotation

I. INTRODUCTION

The first attempt to determine the internal rotation of the Sun from the frequency splittings of acoustic oscillations employed high-degree p-modes which probed only the outer few percent of the equatorial zone of the Sun (Deubner, Ulrich, and Rhodes 1979). Frequency shifts were observed between the prograde and retrograde traveling sectoral modes, which implied that the rotation rate increases with depth. Similar observations from Kitt Peak failed to confirm such a radial gradient (Rhodes, Harvey, and Duvall 1983); however, in both cases, the observations which were available spanned only a few days.

Claverie et al. (1981) employed low-degree \((1 \leq l \leq 3)\) p-modes and found evidence that the rotation of the solar core is substantially higher than that of the photosphere, although their results were called into question one year later on the grounds that they had found more peaks in their power spectra for the \(l = 1, 2,\) and 3 modes than simple estimates of their spatial sensitivity would have suggested (Gough 1982). More recently, Jefferyes et al. (1988a) claim to have evidence from more extensive whole-disk velocity measurements for a rapidly rotating solar core as well as a modulation of their signal due to the rotation of active regions across the solar disk.

Several recent studies have obtained estimates of the solar internal angular velocity from measurements of the frequency splittings of intermediate-degree \((4 \leq l \leq 98)\) p-mode oscillations (see, e.g., Duvall and Harvey 1984; Hill et al. 1984; Brown 1985; Hill and Caudell 1985; Duvall, Harvey, and Pomerantz 1986; Hill, Rabaey, and Rosenwald 1986; Libbrecht 1986; Brown and Morrow 1987; Rhodes et al. 1987, 1988; Jefferyes et al. 1988b; Tomczyk 1988; Libbrecht 1989). All but four of these studies (the three of H. Hill and his colleagues and that of Duval and Harvey) obtained estimates of the equatorial frequency splittings which were either near or slightly below the photospheric rotation rate of magnetic features. Duvall and Harvey obtained such values for all but the deepest interior. For degrees, \(l\), of 1 and 2, Duvall and Harvey obtained larger splittings which were subsequently interpreted by Duvall et al. (1984) as providing weak evidence for a rapidly rotating solar core similar to that found earlier by Claverie et al. The Hill et al. (1984), the Hill and Caudell (1985), and the
Hill, Rabaei, and Rosenwald (1986) studies, on the other hand, showed evidence for an internal rotation rate which was substantially higher than the surface rate throughout most of the solar interior.

For higher degree modes, the number of studies is rather limited. For degrees between 80 and 150, Hill, Rust, and Appourchaux (1988) inverted the frequency splittings of 79 prograde and retrograde sectoral harmonics and found weak evidence for a decrease in the sectoral rotation rate with increasing depth below the photosphere for depths between 3% and 9% of the solar radius. At even higher degrees (80 \( \leq l \leq 995 \)), Hill et al. (1988) inverted the frequency splittings of 709 sectoral modes and found evidence for a rapid rise of about 30 nHz in the subphotospheric rotation rate over the outer 0.4% of the Sun's radius. This rise was found to occur at shallower depths than was a similar increase found by Deubner, Ulrich, and Rhodes (1979), who found the peak in their subphotospheric rotational profile to occur at a depth of 2.5% of the solar radius. In both the Deubner, Ulrich, and Rhodes (1979) and Hill et al. (1988) studies, the rotational profiles appeared to decrease inwardly after reaching their respective peak values. That is, the only discrepancies between these two studies concerned the depth at which the peak in the sectoral rotational profile was located.

In addition to studying the internal angular velocity as a function of radius, several authors (e.g., Brown 1985; Duvall, Harvey, and Pomerantz 1986; Hill et al. 1986; Libbrecht 1986; Brown and Morrow 1987; Rhodes et al. 1987; Jefferies et al. 1988b; Morrow 1988; Tomczyk 1988; Tomczyk et al. 1988; Libbrecht 1989) analyzed the p-mode splittings in such a way that they could also probe the latitudinal structure of the angular velocity. In the first of these studies, Brown found the latitudinal differential rotation inward of the convection zone to be so much smaller than at the surface that he concluded that the radiativezone shows constant rotation on spheres. Duvall, Harvey, and Pomerantz, on the other hand, concluded that the surface latitudinal differential rotation persists at least throughout most of the convection zone and possibly over the outer 0.4% of the Sun's radius. More recently, Brown and Morrow (1987) presented rotational splitting results which fell between those of Brown and those of Duvall, Harvey, and Pomerantz. Nevertheless, Brown and Morrow indicated that they believed their results were still consistent with an absence of latitudinal differential rotation within the radiative zone. The results of the Hill et al. (1986) study were quite different from those of the other studies in terms of the radial behavior of the differential rotation. Hill and his colleagues found the differential rotation to decrease rapidly in magnitude below the solar surface.

Working independently of all the other authors, we developed a dedicated helioseismology observing station which is located at the 60 foot (18 m) solar tower telescope of the Mount Wilson Observatory. One of the design goals for this system was the measurement of the internal rotation of the Sun with the use of solar p-mode oscillations. In this paper, we report the first p-mode splittings which we obtained from Mount Wilson, and we compare these splittings with those from several of the previously published studies as well as those from several of the more recent studies which we have referred to above. We also demonstrate that our measured frequency splittings agree quite well with the composite frequency splittings which we have obtained from our comparisons. Next, we show that our splittings suggest that the angular velocity in the equatorial plane is a function of depth below the solar photosphere. Finally, we indicate also that the comparisons of these composite frequency splitting profiles with those obtained from several formal inversions of various sets of the frequency splittings suggest that the latitudinal differential rotation pattern visible at the surface persists at least throughout the solar convection zone, although it may disappear in the radiative zone somewhere beneath the base of the convection zone.

II. OBSERVATIONS AND ANALYSIS

a) Mount Wilson Filtergrams, Dopplergrams, and Power Spectra

Using the Mount Wilson Observatory 60 foot (18 m) solar tower, we were able to obtain a pair of full-disk solar filtergrams every 40 s for up to 11.4 hr per day on 90 different days between 1984 June 9 and September 9. Each pair of filtergrams consisted of one image obtained in the " red " wings of the two Na D lines followed 5 s later by a second image obtained in the " blue" wings of the same lines. At the rate of one pair of filtergrams every 40 s, we were able to obtain up to 1024 pairs of filtergrams on each separate day.

The filtergrams were obtained with a two-cell version of the Cacciani magneto-optical filter (MOF) and a 244 \( \times \) 248 pixel CID camera system. The Cacciani MOF has been described in a series of articles, including Agnelli, Cacciani, and Fofi (1975), Cacciani and Fofi (1978), Cacciani, Fortini, and Torelli (1980), Cacciani et al. (1981), Cacciani (1981), and Cacciani and Rhodes (1984). The installation of the MOF at the Mount Wilson 60 foot tower was described by Rhodes et al. (1986, 1987). The CID camera and the data acquisition system to which it was interfaced were both described by Rhodes et al. (1981, 1983, 1984, and 1986).

The filtergrams which were employed in the current study were obtained during a 16 day interval which extended from 1984 July 29 through August 13. The relationship of this observing run to our entire 1984 observing campaign was given in Figure 3 of Rhodes et al. (1986). All the 1984 observations were carried out by Tomczyk, and the 16 day interval described here comprises a portion of the data which is included in his Ph.D. dissertation (Tomczyk 1988).

Due to some minor difficulties which we encountered in our initial processing of the data tapes from 1984 August 9 and 10, the data from those two days were not included here. Rather, the time series which we employed in the work described below consisted of 12,131 pairs of filtergrams which were obtained on the remaining 14 days of the interval. These raw filtergram pairs were converted later into a series of calibrated full-disk Dopplergrams through a series of steps which were outlined in Rhodes et al. (1986). In that paper, both spatial and temporal calibration methods were described. In the current study, the spatial calibration method was employed rather than the temporal method. This spatial calibration method was described in more detail recently by Tomczyk (1988).

Once the 12,131 calibrated Dopplergrams had been computed, they were spatially filtered in two different ways, depending upon the degree of the spherical harmonic. For 0 \( \leq l \leq 89 \), they were spatially filtered with all of the 2l + 1 azimuthal order \( m \) spherical harmonics for each degree, while for 90 \( \leq l \leq 200 \), only the \( m = l \) and \( m = -l \) sectoral harmonics were computed. For \( l \leq 89 \), the spatial filtering process was carried out in part by first interpolating the radial velocity at each pixel onto a uniform grid in heliographic longitude and
The evenly spaced longitude-sine latitude arrays which resulted were then filtered on a CSPI Mini-MAP array processor by computing a Fourier transform of the arrays in the longitudinal direction and by then computing a Legendre transform in sine latitude. As was pointed out by Brown (1985), this procedure is only slightly less effective in isolating the desired spherical harmonics than is a truly optimal weighting technique. The $2l+1$ time series which resulted for each $l$ value were then cleaned to remove outlying points. The cleaned time series which resulted were then interspersed with zeros wherever actual Dopplergrams were unavailable. These 16 day time series were finally converted (without apodization) into $2l+1$ power spectra for each value of $l$ with the use of one-dimensional fast Fourier transforms in the array processor.

In contrast to the spatial filtering process which we employed for $l \leq 89$, we did not interpolate our Dopplergrams onto evenly spaced longitude-sine latitude arrays for the computation of the sectoral harmonics above $l = 89$. Instead, we carried out the spatial filtering for these sectoral harmonics by directly evaluating the two-dimensional double integrals of the Dopplergrams with the sectoral harmonic arrays. By employing this alternative filtering method, we did not have to use the "nearest neighbor" approach. The influence of employing a "true" approach on the precision of isolating the rotationally induced frequency splittings is shown at the bottom, while the zonal harmonic spectrum is shown at the top. Hence, as Brown and Morrow (1987) have pointed out, the correlation technique which we employed thus includes some information from these sidelobes in the estimates of the frequency splittings. If only the temporal sidelobes were involved, this contamination would not cause any difficulty, because the sidelobe "ridges" are parallel to and symmetrically displaced about the actual ridges. The difficulties arise because of the leakage of spatial sidelobes from adjacent-degree $m$-$v$ diagrams into the diagram of interest. In this case, as Brown and Morrow (1987) have pointed out, the spatial sidelobe ridges have slightly different slopes than do the "true" $p$-mode ridges in each $m$-$v$ diagram. Because of these different slopes, the sidelobe ridges affect the cross-correlation results in a systematic fashion. Specifically, they act to make us overestimate the $m$-$v$ slopes of these low $l$ ridges. However, as Brown and Morrow also pointed out, these effects are only expected to be noticeable below $l = 10$.

We next represented the frequency variation with $m$ for a given $l$ value as the following polynomial series:

$$v(n, l, m) - \bar{v}(n, l) = L \sum_{i=0}^{\infty} a_i P_i^l (m/L),$$

where $\bar{v}(n, l)$ is the average over $m$ of the frequencies of order $n$ and degree $l$, the $P_i$ are the Legendre polynomials of order $i$, and $L = (\lfloor l + 1 \rfloor)!/2$. This expansion is exactly that employed by Duvall, Harvey, and Pomerantz (1986), and it was carried out with software which was also adapted directly from their codes. These authors also noted that, expressed in terms of equation (1), the surface differential rotation as measured by the motion of magnetic features (Snodgrass 1983) yields even-indexed coefficients equal to zero and odd-indexed coefficients $a_1 = 442.8$, $a_3 = 21.7$, and $a_5 = -2.5$ (nHz sidereal). Here we note that, even in the case that the internal rotational velocity is not identical to the surface rotational velocity at all depths, any rotational velocity profile that preserves latitudinal symmetry about the equatorial plane at all depths would imply that the even-indexed coefficients would both be equal to zero.

For the purposes of assessing the significance of deviations from these coefficients, we have computed the normalized $m$-$v$ scatter about the three mean values using the scatter values published by Snodgrass (1983). The rms values are $\pm 0.77$, $\pm 0.84$, and $\pm 1.01$ nHz, respectively.

The results of applying this analysis to the power spectra which resulted from our 16 day observing run are illustrated in Figure 2, where we have plotted the one standard deviation error brackets for the three odd expansion coefficients for $3 \leq l \leq 89$. 

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Fig. 1.—Three m-v power spectra corresponding to degrees 60 (top), 45 (middle), and 30 (bottom). These three spectra were obtained from a 9.5 day portion of the 16 day Mount Wilson data set which was described in the text. For all three spectra, the observed power is plotted as a set of grey levels as a function of temporal frequency v (increasing horizontally), and azimuthal order m (increasing vertically). Also visible in all three cases are the various spatial and temporal sidelobes which surround each of the true modal peaks. The s-shaped nature of each frequency curve is what is represented by the Legendre polynomial expansion described in the text. The overall slope of each curve is a measure of the equatorial rotational velocity. The much higher rotational velocity described by the SCLERA group would result in a much larger tilt to the curves in all three parts of this figure than is observed.

Rhodes et al. (see 351, 689)
The top panel of Figure 2 shows the variation of the $a_1$ coefficient with degree. Also shown as the horizontal dashed line is the surface magnetic value of $a_1$, converted to the synodic rate of 411 (±0.8) nHz. The most striking feature of $a_1$ is its systematic increase with increasing degree. For the 10 highest degrees shown, $a_1$ lies above the surface magnetic value, by 6.5 nHz, while below $l = 10$, $a_1$ lies below that rate by 12.4 nHz. Thus, the total spread of our values of $a_1$ was 18.9 nHz. The smaller range of $a_1$ of 7 nHz between $l = 20$ and 89 is similar to the range of 3.8 nHz found by Duvall, Harvey, and Pomerantz; however, the systematic variation with $l$ shown here by our values of $a_1$ was not seen by those authors. In synodic terms, their results ranged from 414.4 nHz for $l = 20$–29, to 410.6 nHz for $l = 50$–59, and to 414.3 nHz for $l = 80$–89.

In comparison with the more recent results of Brown and Morrow (1987), we note that between $l = 15$ and 30 their $a_4$ values were only 2 nHz less than ours. However, between $l = 76$ and 83, their results were 10 nHz less than ours. Thus, at low $l$, our $a_4$ values fell between those of Brown and Morrow and those of Duvall, Harvey, and Pomerantz, while at the higher $l$ values our $a_4$ results fell above both other sets of values.

The middle panel of Figure 2 gives the variation of the $a_3$ coefficient with degree. Again, the value of $a_3$ which corresponds to the rotation of surface magnetic features is shown as the horizontal dashed line at 21.7 (±0.8) nHz. As was the case for $a_1$, our results are similar to those of Duvall, Harvey, and Pomerantz. Between $l = 20$ and 89, the Duvall, Harvey, and Pomerantz results for $a_3$ averaged 20.9 nHz, whereas our average value of $a_3$ over the same range in $l$ was 17.8 nHz. This difference may be significant, but we believe that additional data will have to be analyzed before we will be able to say so with confidence.

Brown and Morrow's results for $a_3$ fell within our 1σ error bars from $l = 15$ to $l = 62$. Above $l = 62$, their $a_3$ values were 1–3σ higher than ours; hence, Brown and Morrow saw more variation with $l$ in $a_3$ than did we. Possibly, some subtle differences in our data reduction techniques might have shifted some of the variation with $l$ which we saw in $a_1$ into the larger variation of $a_3$ in the Brown and Morrow study.

In the bottom panel of Figure 2 are our values of $a_5$. To within the scatter of the 1σ error bars, our results appear to be identical to the surface magnetic feature rate of $-2.5$ (±1.0) nHz. Specifically, over the range $l = 20$–89, our average value of $a_5$ was $-2.8$ nHz, while the Duvall, Harvey, and Pomerantz results averaged $-4.4$ nHz. By comparison, Brown and Morrow's values for $a_5$ averaged $-3.4$ nHz between $l = 15$ and $l = 83$. Thus, their $a_5$ values fell between ours and those of Duvall, Harvey, and Pomerantz.

In summary, our $a_1$ results show more systematic variation with $l$ than do either the Duvall, Harvey, and Pomerantz or Brown and Morrow results; our $a_3$ values are smaller than their results and the surface rate, and our $a_5$ values show less variation with $l$ than do the Brown and Morrow results; lastly, our $a_5$ results are slightly closer to the surface value than are either the Duvall, Harvey, and Pomerantz or the Brown and Morrow results.

c) $a_2$ and $a_4$ Coefficient Results

In addition to computing the odd $a_i$ coefficients, we also computed the even coefficients $a_2$ and $a_4$. Averaged from $l = 89$ down to $l = 3$, our values of $a_2$ and $a_4$ equaled 0.7 ± 0.5 and 2.1 ± 0.6 nHz, respectively, where these uncertainties are the standard errors of the mean values. In comparison with other published results, our average value for $a_2$ is one-half the value of $1.5 ± 0.4$ nHz found by Brown and Morrow (1987), and it is also considerably smaller than the corresponding value of 8 nHz found by Duvall, Harvey, and Pomerantz (1986). Furthermore, the size of the standard error for our mean value of $a_2$ suggests that this value is not statistically different from zero. Turning to $a_4$, our average value of 2.1 nHz is somewhat smaller than the value of 2.9 nHz obtained by
Duvall, Harvey and Pomerantz (1986); however, it is actually larger than the value of $1.7 \pm 0.3$ nHz found by Brown and Morrow (1987). Hence, the three studies give closer agreement on the average value of $a_1$ than they do on the value of $a_2$. Nevertheless, we believe that additional independent observations will be required before we can conclude that either $a_2$ or $a_4$ is nonzero.

**d) Dependence of Equatorial Frequency Splittings upon Spherical Harmonic Degree**

As soon as we saw the systematic variation of $a_1$, we wanted to determine whether or not these variations in the individual Legendre expansion coefficients were also evident in a plot of an estimate of the internal angular velocity as measured by the sectoral harmonics, which (except for the difference between $l$ and $m$) is simply obtained by summing $a_1$, $a_3$, and $a_5$. The results of computing this simple sum, which corresponds to an average of the angular velocity that is centered about the solar equatorial plane, are illustrated here in Figure 3 for all of the degrees between 3 and 89. In this figure, the values of $a_1 + a_3 + a_5$ are given in sidereal, rather than synodic, units, and the values themselves are plotted rather than the $\pm 1$ a error bars. In this figure, the surface equatorial magnetic feature frequency splitting is given by the horizontal dashed line at 462 (±1.5) nHz. With the exception of the fluctuating behavior of the curve between $l = 3$ and 15, which may simply be a result of random scatter in the results, Figure 3 shows evidence for a systematic decrease in $a_1 + a_3 + a_5$ with decreasing degree from $l = 89$ downward.

In order to study the fluctuation in $a_1 + a_3 + a_5$ between $l = 3$ and 15, we calculated the estimated errors in $a_1 + a_3 + a_5$, based on the standard deviations of the fits to the individual $a_i$ coefficients. We found that these predicted standard deviations increased from 8.2 nHz at $l = 19$ to 17.3 nHz at $l = 10$. We also found that the value of the sum at $l = 15$ was roughly three standard deviations below the corresponding values at $l = 16$ and 17 and those at $l = 11$ and 12. Similarly, the standard deviations below $l = 10$ ranged from 12.4 nHz at $l = 8$ to 114 nHz at $l = 3$. The three points at $l = 6, 7, and 8$ were each three standard deviations above the points at $l = 5$ and $l = 9$. All other values differed by approximately 1 a or less from their neighbors. Due to the fact that we employed observations extending over only 14 days and due to the increasing sizes of the errors between $l = 19$ and $l = 3$, we believe that it is premature to attach any significance to the apparent fluctuation in $a_1 + a_3 + a_5$ at this time.

Instead, in order to compare our values for the sum of $a_1$, $a_3$, and $a_5$ with the published results of Duvall, Harvey, and Pomerantz (1986), we next averaged with equal weights the sums shown in Figure 3 in bins which were 10 degrees wide. The results of this binning analysis are shown as the X's in Figure 4. As a function of $l$, these binned sums show a smooth decrease with decreasing $l$.

Because intermediate-degree $p$-modes of degree 89 are sensitive to the internal solar rotation at a finite depth below the photosphere, we have also included in Figure 4 the frequency splittings between the prograde and retrograde sectoral harmonics which we obtained directly from our sectoral power spectra for $90 \leq l \leq 170$. These sectoral frequency splittings are shown as the boxes in Figure 4. In order to compute these sectoral harmonic splittings, we first grouped all of the 400 sectoral power spectra (i.e., one spectrum for both $m = l$ and $m = -l$ for each degree, $l$, between one and 200) into a two-dimensional $l - v$ array. We then cross-correlated rectangular areas in the prograde and retrograde portions of the $l - v$ plane to obtain the total frequency differences. These rectangular areas were each 10 degrees wide in $l$ by 1400 $\mu$Hz high. They included all of the $p$-mode peaks which fell between the frequencies of 2200 and 3600 $\mu$Hz and which fell within the particular range in $v$. After the cross-correlation analyses had estimated the prograde minus retrograde frequency differences, these differences were each divided by $2l$, where the $l$ corresponded to the midpoint of each 10 degree wide bin. This division converted the splittings into normalized estimates of the solar angular velocity, which are the eight values plotted in Figure 4. The sectoral frequency splittings which resulted are exactly equivalent to the sums of the odd Legendre expansion coefficients, except that they show more scatter, because they were each obtained from 10 pairs of power spectra, while the $a_1 + a_3 + a_5$ values were each obtained from all of the 2$l + 1$ spectra for each of the 10 degrees within each bin.

Subsequent to the completion of Figure 4, we have realized that we should have divided the sectoral frequency differences by 2$l$ rather than 2$l$, because equation (1) employs $l$ rather than $l$. Renormalization of the eight sectoral angular velocity estimates in this fashion would decrease the point at $l = 100$ from 467 to 465 nHz. It would also decrease the estimate at $l = 170$ from 456 to 455 nHz and the estimates between $l = 170$ and $l = 100$ by proportionate amounts.

The highest degree sectoral harmonics which we computed from our observations corresponded to $l = 200$. Because of an increase in the background noise level in the $l - v$ plane between $l = 170$ and $l = 200$, we limited our cross-correlation...
analyses to \( l = 170 \). As is shown in Figure 4, the sectoral splittings for the \( l = 150-170 \) bins appear to fall between the rotation rates of the photospheric gas and of magnetic tracers as observed in the photosphere. Then, as the degree of the sectoral harmonics decreases from 170 to 100, the corrected frequency splittings appear to increase from 455 nHz to 465 nHz, a value which is two standard deviations above the surface magnetic feature rotation rate of 462 (± 1.5) nHz.

Continuing to the left in Figure 4, we come next to the frequency splittings, which we computed from the Legendre polynomial fits to the frequency difference curves. As was pointed out above, this portion of Figure 4 shows a systematic decrease in the splittings with decreasing degree. The binned splittings decrease from a value of 463.7 nHz for the bin corresponding to \( l = 80-89 \) down to a value of 441.9 nHz for the \( l = 3-9 \) bin. This latter value is significantly below the surface gas rate of 451.7 (± 0.3) nHz (Snodgrass 1984).

e) Comparison of Equatorial Frequency Splittings With Results From Other Studies

Because the trend illustrated by the crosses in Figure 4 was so smooth, we were interested in seeing how representative it was of the variation in equatorial frequency splittings from other published studies. The results of this comparison are shown in Figure 5. In Figure 5, our binned sums of \( a_1 \), \( a_3 \), and \( a_5 \) are again shown as the X’s, except that here they have been connected by a solid line. The results of the Duvall, Harvey, and Pomerantz South Pole study are shown as the connected

![Figure 4](image-url)

**Fig. 4.—** Binned equatorial frequency splittings computed from Legendre polynomial expansions, \( a_1 + a_3 + a_5 \) (shown as the X’s), and from sectoral harmonics (shown as the squares). Both sets of frequency splittings correspond to averages of the 16 day Mount Wilson coefficients over 10 degree wide bins in degree, \( l \), and are plotted at the degree corresponding to the middle of each bin. The corresponding equatorial angular velocities of the photospheric gas (Snodgrass 1984) and of magnetic tracers visible in the photosphere (Snodgrass 1983) are indicated at the right. The observed points appear to increase with decreasing degree between \( l = 170 \) and \( l = 100 \) and then appear to decrease with decreasing degree below that point. (As is pointed out in the text, a renormalization of the sectoral splittings will decrease them by 2.5 nHz at \( l = 90 \) and by 1.3 nHz at \( l = 170 \)).

![Figure 5](image-url)

**Fig. 5.—** Binned values of the equatorial frequency splittings vs. the degree corresponding to the center of each bin from several published studies. The triangles and the open squares were obtained from studies of sectoral harmonics alone. All other curves were obtained from Legendre polynomial expansion fits to the frequency shifts as functions of azimuthal order, \( m \). The legend relates the symbols to the various locations where the observations were obtained; it also gives the epoch of each observation and lists the authors of each study. The X’s and the open squares were repeated from Fig. 4.
circles, while those from Brown and Morrow (1987) are shown as the connected dots. The two sums of \( a_1 \) and \( a_3 \), which were computed by Libbrecht (1987), are shown as the two pluses at the left. The sectoral harmonic frequency splittings for degrees 3 through 100, which Duvall and Harvey (1984) computed from observations which they had previously obtained at Kitt Peak, are shown as the connected triangles.

The results of two very recent studies are also included in Figure 5. The first of these studies is that of Libbrecht (1989), who analyzed frequency splittings which were computed from a 100 day data set which had been obtained at the Big Bear Solar Observatory in mid-1986. We evaluated Libbrecht's (1989) tabulated coefficients at a frequency of 3 mHz before we binned them in \( l \). The binned values which resulted from this analysis of Libbrecht's splitting coefficients are shown in Figure 5 as the connected diamonds. The second recent study which is included in Figure 5 is that of Jefferies et al. (1988b), who obtained observations from the South Pole in late 1987. The binned values of the sum of \( a_1 + a_3 + a_5 \) which Jefferies and his colleagues obtained are shown here as the inverted triangles. With the sole exception of the 1982 South Polar results of Duvall, Harvey, and Pomerantz (1986) for \( l = 20-29 \) and \( l = 40-49 \), all the other curves shown in Figure 5 illustrate the clear decrease in the equatorial frequency splitting as a function of decreasing degree below \( l = 90 \), which our results alone showed in Figure 4.

Above \( l = 90 \), we have repeated the sectoral harmonic splittings from Figure 4 as the set of open squares at the right of Figure 5. Also, the three filled squares between \( l = 90 \) and \( l = 120 \) were obtained by binning the highest degree results published by Tomczyk (1988). For these degrees, Tomczyk's results were obtained from the same 16 day subset of the 1984 Mount Wilson observations which we used to generate the sectoral harmonic frequency splittings from Figure 4 (although he utilized observations from all 16 days and not just from 14 of the 16 as we had done). Since the two different sets of squares are not independent of each other, the scatter between them indicates that the sectoral harmonic splittings have to be viewed with caution until a complete Legendre polynomial expansion can be carried out above \( l = 120 \).

Because the \( a_1 + a_3 + a_5 \) sum is equal to the frequency splitting of the sectoral harmonics, and because the latitudinal extent of these harmonics increases with decreasing degree, we were aware that the trend of decreasing splitting shown below \( l = 90 \) in Figure 6 could have been caused by the sampling of increasing amounts of the latitudinal differential rotation as the degree was decreased. In order to obtain a preliminary estimate of the magnitude of this effect, we assumed that the surface differential rotation was constant with depth and calculated an estimate of the average frequency splitting due to rotation as sampled over the range of solar latitudes extending between the half-amplitude points of the sectoral harmonics. When we estimated the frequency errors introduced into our binned values of \( a_1 + a_3 + a_5 \), we found them to be less than 2 nHz for all but the \( l = 3-9 \) bin. For this one bin, our simple analysis suggested that the varying latitudinal sampling resulted in a 4.5 nHz depression of the observed splitting. However, since the actual angular velocity is not constant with depth as we have illustrated above, our simplified analysis most likely resulted in an overestimate of this effect.

f) Latitudinal Dependence of the Observed Frequency Splittings

Because of the differing claims described earlier regarding the extent to which the latitudinal differential rotation pattern seen in the photosphere persists into the solar interior, we also wished to convert our measurements of the \( a_1 \), \( a_3 \), \( a_5 \) expansion coefficients into estimates of the internal angular velocity profile at two nonzero heliographic latitudes. To do so, we first noted that the zonal harmonic modes (i.e., those with \( m = 0 \)) have their largest amplitudes near the solar poles, in contrast to the sectoral harmonics, which, as we have already pointed out, reach their largest amplitudes in the equatorial plane. Next, we recalled that Brown (1985) employed \( -dv/dm \) evaluated at \( m = 0 \) to measure the rotation rate away from the

![Figure 6](https://example.com/figure6.png)

**Figure 6.** Binned values of the so-called zonal or midlatitude linear combination of the odd \( a_n \) coefficients (i.e., \( a_1 - 3/2 a_3 + 15/8 a_5 \)). As with Fig. 5, 10 degree bins were employed. Note that different symbols are employed in Figs. 5 and 6 for the Mount Wilson and Big Bear results.
Since the tesseral \((m \neq 0)\) harmonics of differing values of \(m/l\) vary in how rapidly their amplitudes change with latitude, each tesseral harmonic effectively corresponds to a different range of latitudes over which the frequency splitting is sampled. Furthermore, since the odd \(a_1\) coefficients were determined from least-squares fits to the entire set of \(2l + 1\) frequencies at each degree and not just from a single tesseral harmonic alone, each solution to equation (2) above included information from a group of tesseral harmonics. Since each of the tesseral harmonics which were employed at every degree covered a different extent in latitude, the frequency splittings which resulted from equation (2) corresponded to averages which were computed for ranges of latitudes stretching between the equatorial plane and somewhat different limiting latitudes for each different degree.

The results of evaluating equation (2) using our binned \(a_1, a_3, a_5\) coefficients are illustrated here in Figure 6 as the X's. These values all fall well below the \(a_1 + a_3 + a_5\) values, which were shown in Figure 5. They all cluster between 395 and 422 nHz and average 412 ± 6 nHz, where the error indicated is the standard deviation of our nine binned values. By comparison, when we substituted the odd \(a_1\) coefficients for the surface magnetic features into the above linear combination, we found 405.6 ± 2.4 nHz, where the error here is the normalized rms scatter computed from the data published by Snodgrass (1983).

We also computed the same linear combination using the \(a_i\) coefficients corresponding to the photospheric gas (Snodgrass 1984) and obtained 396.3 nHz.

We repeated the computation of the above linear combination using the published values of several other studies. Figure 6 includes the results of these computations for the 1982 South Polar observations of Duvall, Harvey, and Pomerantz (1986, shown as the open circles), the values published by Tomczyk from his analysis of the 1984 Mount Wilson observations (1988, shown as the plus signs), the 1984 Sacramento Peak observations of Brown and Morrow (1987, shown as the dots), the 1986 Big Bear observations of Libbrecht (1989, shown as the open squares), and the 1987 South Polar data of Jefferies et al. (1988b, shown as the open triangles). In contrast to Figure 5, which included values of \(a_1 + a_3 + a_5\) from 1984 Kitt Peak observations of Duvall and Harvey (1984), Figure 6 does not include any corresponding values, because the Duvall and Harvey results were obtained from sectoral harmonic power spectra rather than from Legendre polynomial expansions of the frequency splittings of different tesseral harmonics. Consequently, the linear combination of equation (2) could not be formed from the Duvall and Harvey values.

An average of the Duvall, Harvey, and Pomerantz (1986) values shown in Figure 6 yielded 405 ± 5 nHz. In addition, inspection of Figure 6 shows that all of the published studies agree with each other to within 20 nHz, or 5%. Comparison of Figures 5 and 6 shows that, in spite of this scatter, the midlatitude values all fall well below the corresponding equatorial values.

A final point to be made concerning Figure 6 is that below \(l = 50\), all the studies show evidence for an increase in the midlatitude frequency splittings with decreasing degree, with the sole exception being our Mount Wilson results for the \(l = 3-9\) bin. Since no other results have been published for such low \(l\) values, it is too early for us to say whether the Mount Wilson \(l = 3-9\) bin values are in error or whether the trend shown in Figure 6 between \(l = 10\) and \(l = 50\) does not extend to the lowest degrees.

The third linear combination we computed was equal to \(a_1 - 4a_3 + 8a_5\). This is a so-called polar combination, which would literally be the angular velocity at a heliographic latitude of 90° if the angular velocity were constant with depth. The results of computing this combination from our binned \(a_1, a_3, a_5\) coefficients are illustrated here in Figure 7, again as

![Figure 7](image_url)
the X's. Our values ranged from 318 to 379 nHz and averaged 351 ± 13 nHz. As can be seen in Figure 7, they all fell well below the "midlatitude" values which were shown in Figure 6. By comparison, when we substituted the odd \( a_l \) values corresponding to the rotation of magnetic features at the surface into this linear combination, we got a value of 336 ± 8.8 nHz. (Here again, the error was computed from the rms scatter values published by Snodgrass 1983.) When we computed the same linear combination using the coefficients for the gas rotation, the result was 316.5 nHz.

We repeated our computation of this "polar" linear combination using the published expansion coefficients from the same set of additional studies which were employed in the computation of the "midlatitude" combination shown in Figure 6. The results of these computations are shown in Figure 7, where we have employed the same symbols for the different studies which we used in Figure 6. Inspection of Figure 7 shows that all of the computed values of the "polar" combination fall systematically below the corresponding "midlatitude" points of Figure 6. For example, when we used the Duvall, Harvey, and Pomerantz (1986) values in the evaluation of this expression, we obtained values ranging from 291 to 341 nHz, with a mean value of 327 ± 19 nHz.

As was the case with the "midlatitude" results of Figure 6, Figure 7 shows that between \( l = 10 \) and 50, there is evidence for a possible increase in the "polar" splittings with decreasing degree. Again, the only points which do not agree with this trend are the Mount Wilson values for the \( l = 3 \)-9 bin. Again, it is premature to say whether this trend continues below \( l = 10 \) or whether the Mount Wilson values are correct and there is no trend. Finally, close inspection of Figures 6 and 7 shows no evidence for any credible temporal variations in the "midlatitude" or "polar" frequency splittings.

g) Creation of Composite Linear Combinations

Because the comparisons shown above in Figures 5, 6, and 7 did not show evidence for systematic temporal variations in any of the three linear combinations, but showed instead that systematic variations between the results of the different authors are more significant, we attempted next to minimize the effects of the different analysis techniques employed by those authors. We attempted to do this by combining the binned values from all of the binned values from all of the studies shown in Figures 6 and 7 into three composite linear combinations.

For those \( l \) values where there were values from Mount Wilson from both our 16 day run and from Tomczyk's (1988) 35 day study, we employed the latter values in our composite calculations. Also, we combined the results from the different studies in an unweighted manner. We did so because all the values except those computed from the two South Polar runs (i.e., those of Duvall, Harvey, and Pomerantz 1986 and Jeffries et al. 1988b), and from the 1986 Big Bear campaign of Libbrecht (1989), came from studies which ranged in duration between 14 and 35 days. Hence, we believed that, with the exception of these three cases, any differences in the sizes of the internal error bars from one study to the next were most likely due to systematic differences in the analyses rather than to true differences in the inherent accuracy of the different sets of points. Given the agreement between the 16 day 1984 Mount Wilson and the 100 day 1986 Big Bear results, we believe that in all but a few cases, the use of a weighting algorithm in the computation of the composite mean values and standard deviations would have altered the results by no more than 1–3 nHz at the most.

The composite, binned linear combinations which resulted from the unweighted combination of the various studies shown in Figures 5 through 7 in the fashion described above are given in Figure 8. In this figure, the composite equatorial, mid-latitude, and polar linear combinations are shown as functions of degree, \( l \). Also plotted as the error bars in Figure 8 are the standard errors of the three composite mean values located within each \( l \) bin.

In Figure 8, we note again the systematic decrease in the composite equatorial frequency splittings with decreasing degree below \( l = 95 \). We note also that the latitudinal differential rotation is roughly independent of degree between \( l = 110 \) and \( l = 40 \). Below \( l = 40 \), however, all three curves begin to converge. The only points which do not suggest a continued diminishing of the latitudinal differential rotation with decreasing degree are the midlatitude and polar points for \( l \leq 10 \). As we saw in Figures 6 and 7, these points came solely from our Mount Wilson studies. We will have to await the publication of additional Legendre expansion coefficients for degrees less than 120 before we will be able to say whether the differential rotation truly does continue to converge as the degree decreases below 10.

III. INTERPRETATION

a) Estimation of Radial Variations of the Internal Angular Velocity

The three composite linear combinations of the odd \( a_l \) coefficients which are shown in Figure 8 illustrate the \( l \) dependence.
of the frequency splittings in each of three latitude zones. In order to infer the radial profile of the Sun's internal angular velocity within those latitude zones, we analyzed several aspects of the depth sensitivity of p-modes to solar rotation. First, we employed the radial distances of the innermost turning points of p-modes of frequency = 3 mHz, which we had obtained from a standard solar model provided to us by Gough (1986). These turning point radii are close to the inner boundary points below which the p-modes cannot convey any information about the rotation. Hence, they provide upper bounds to the possible subphotospheric depth sensitivity of the corresponding modes. Furthermore, since these turning points vary in a nonlinear fashion with the degree, \( l \), of the modes, we believed that a simple mapping of the frequency splittings of Figure 8 onto these radii might at least yield a smooth estimate of the relative variation in the internal angular velocity profile.

When we employed these inner turning point radii to map our binned values of \( a_1 + a_3 + a_5 \), we found that the values fell along a straight line when plotted as a function of the turning point radius. The correlation coefficient of a linear regression fit to this straight line equaled 0.998. While the linear trend shown by that straight line was unmistakable, the relatively large errors associated with the innermost two bins made it unclear how deeply the linear trend persisted.

Because we were well aware that the innermost turning points of the p-mode eigenfunctions are overestimates of the depths below the photosphere at which those modes are sensitive to the internal rotation of the Sun, we also analyzed our binned \( a_1 + a_3 + a_5 \) values as a function of the so-called effective depths, which were first described by Ulrich, Rhodes, and Deubner (1979). While these "effective depths" were later criticized severely by Gough (1978), we believe that they are probably closer approximations to the actual subphotospheric depths at which the various p-modes are effectively sensitive to rotation than are the innermost turning points.

By subtracting the "effective depths" of the modes from one solar radius, we were able to replot our equatorial frequency splittings as a function of fractional solar radius instead of subphotospheric depth. When we did so, we found a much steeper variation in the equatorial angular velocity with radial distance than we had found when we had employed the turning points.

Because of the wide spread between the "turning point" and "effective depth" curves, we also examined the depth dependence which resulted from simply employing the midpoints between the surface and the inner turning points (i.e., we estimated the subphotospheric depth of a given p mode for rotational splitting purposes as being located halfway between the surface and the innermost turning point).

Next, we calculated the midpoints for those p-modes which were closest in frequency to 3 mHz and which had degrees located within a given bin. Once we had computed the midpoints of the modes which corresponded to each \( l \) bin, we simply averaged those midpoints to obtain an average midpoint for each bin. The result of this averaging process was a set of average modal midpoints for all of the \( l \) bins which were shown in Figure 8. Employing our assumption that these modal midpoints represented the depths at which the solar angular velocity was being sampled by those modes, we replotted the three composite linear combinations of Figure 8 as functions of the modal midpoints instead. The assumed radial profiles of the internal angular velocity which resulted from this process are shown in Figure 9.

Inspection of Figure 9 shows that the systematic decrease in the equatorial angular velocity with decreasing degree below \( l = 95 \) which was shown in Figure 8 actually appears to correspond to a positive (i.e., \( \partial \Omega / \partial r > 0 \)) radial gradient. Between radii of 0.6 and 0.95\( R_\odot \), the composite equatorial angular velocity profile increases from 400 to 463 nHz. This corresponds to a radial gradient of roughly 66 nHz/\( R_\odot \) over this portion of the solar interior. The relatively small size of the standard errors around each of the composite values of \( a_1 + a_3 + a_5 \) indicates that this radial gradient is statistically significant. For comparison purposes, the values of the corresponding equatorial frequency splittings which are obtained from the rotation of the photospheric gas (Snodgrass 1984) and from the rotation of magnetic patterns in the photosphere (Snodgrass 1983) are shown near the top of the right-hand axis of Figure 9.

Inward of a radius of 0.9\( R_\odot \), the radial profiles at middle and polar latitudes are somewhat less well defined. At both the middle latitudes and at the pole, the angular velocity appears to increase toward the equatorial angular velocity as the radius
decreases between 0.9 and $0.7R_\odot$. In particular, the difference in angular velocity between the equator and the pole decreases from a value of 120 nHz, which is 26% of the equatorial velocity, at $r = 0.95R_\odot$, to roughly two-thirds of that value, or 80 nHz, which is roughly 18% of the corresponding equatorial velocity at the base of the convection zone (at $r \approx 0.7 R_\odot$).

Inward of the base of the convection zone, however, neither the midlatitude nor the polar composite profiles continue the trend toward increasing angular velocity with decreasing radius. Rather, at both latitudes, the composite profiles show a decrease in the estimated angular velocity. As was mentioned earlier, these two data points both correspond to the $l = 3$–$9$ bin and were obtained solely from our own 1984 Mount Wilson observations, because no other published studies include $a_3$ or $a_9$ estimates for that bin.

Confirmation of the trend showing the convergence of the angular velocity at all three latitudes inward of the base of the convection zone will have to await future studies which should obtain more accurate values of the $a_l$ coefficients for $l < 10$.

b) Comparison of Composite Radial Profiles with Results of Inversions of Observed Frequency Splittings

Because we were well aware that the simple mapping procedure which we had employed to estimate the radial dependence of the angular velocity at each of the latitude zones could be criticized as being an oversimplification of a complicated situation at best and of offering no real information of the radial behavior at worst, we also compared our composite radial profiles shown in Figure 9 with the results of several different inversions of observed $p$-mode frequency splittings. First, we inverted the splittings obtained from our 16 day 1984 Mount Wilson frequency splitting data set. The Mount Wilson frequency splittings which went into this first inversion were $n$-averaged splittings corresponding to $3 \leq l \leq 89$. (By $n$-averaged splittings, we mean that they were computed with the iterative cross correlation techniques described earlier in § IIIb).

The results of a two-dimensional $(r, \theta)$ inversion computed with an iterative variation of the spectral expansion method on a 6 point radial grid are compared with our three composite radial profiles in Figure 10. The details of the method employed in this inversion are given elsewhere (Korzennik et al. 1988). In Figure 10, we see that the positive radial gradient along the equator described above is also seen in the inversion as the curve indicated by the plusses. The dip at $r/R_\odot = 0.83$ and the rise at $r/R_\odot = 0.75$ in the equatorial profile of this inversion are only separated by 1.3 times the formal uncertainties of the inversion, and hence the difference between them cannot be considered to be significant. Second, we also see a dip in the mid- and pole latitude inverted profiles (given by the triangles for a latitude of 45° and by the circles for 90°, respectively), although the location of the dip is shifted inward relative to that shown in our composite profiles. Both of these dips at $r/R_\odot = 0.75$ in the midlatitude and polar profiles from this inversion fall below the adjacent inversion points by roughly twice the formal uncertainties from the inversion; however, the comparisons between the composite profiles and the inverted profiles shown in Figures 11 and 12 suggest that the actual minima in the midlatitude and polar angular velocity profiles lie outside of $r/R_\odot = 0.8$.

In order to compare our composite profiles with the results of additional inversions which did not include any of our Mount Wilson observations, we also made comparisons with two independent inversions of Libbrecht's 100 day 1986 Big Bear splittings. These two comparisons are shown here in Figures 11 and 12. In Figure 11, we compare our three composite profiles with a piecewise constant constrained least-squares inversion, which was obtained from splittings which were not averaged in $n$ but which covered the range of $10 \leq l \leq 60$. This inversion was computed by Dziembowski, Goode, and Libbrecht (1989).

Figure 11 shows the positive radial gradient along the equator which we have referred to above. It also shows a "bump" inward of $r = 0.8R_\odot$, which is followed by a "dip" below $r = 0.7R_\odot$. There is a possibility that such an oscillation in the equatorial angular velocity is real, even though our high-smoothed composite profile does not show such an oscillation along the equator.

For latitudes of both 45° and 90°, the inversion profiles shown in Figure 11 agree remarkably well with our composite...
profile lies slightly above the composite profile, in agreement with the inversion shown in Figure 11. On the other hand, for the polar case the inversion profile lies slightly above our composite profile, in contrast to the inversion profile shown in Figure 11, which fell slightly below our composite profile. Hence, along the polar axis the last two inversions simply bracket our composite radial profile.

IV. DISCUSSION

The comparisons between our radial profiles of the composite frequency splittings and the different inversions shown here are surprisingly good. They serve to indicate several key features of the current status of internal rotation studies. First, the systematic discrepancies between the results of the various observational groups are small enough to allow a new, general picture of the Sun’s internal rotation to emerge. That is, these discrepancies are not so large that they prevent the development of such a picture as a function of radius and latitude. Second, a positive radial gradient in the equatorial angular velocity which extends over most of the convection zone profiles except for the innermost bins and for the polar profile near the surface, where our composite profile is systematically above the inverted profile. It is important to note that the innermost and outermost points of the inversions shown in both Figures 11 and 12 should be viewed with caution, because those two inversions were computed from a frequency splitting data set which covered a more restricted range in $l$ than did the data sets from which our composite curves were computed. In addition, Figure 11 does show the apparent convergence of the angular velocity below the convection zone, which we also discussed above.

Figure 12 shows the results of an inversion of Libbrecht’s splittings which was computed by Christensen-Dalsgaard, and Schou (1988) using the optimal averaging kernel method. As with Figure 11, the three different profiles computed from this inversion also agree remarkably well with our composite profiles. The positive gradient which we described above along the equator is seen here again. At a latitude of 45°, the inversion

![Image](image-url)
appears now to be well substantiated. This is in contrast to Morrow, who has argued that the angular velocity is constant along the equator throughout the convection zone (Morrow 1988). Third, the negative latitudinal gradient (i.e., the latitudinal differential rotation) which is seen at the solar surface has been found to persist over much of the convection zone. Fourth, this latitudinal gradient appears to decrease in magnitude inwardly over the innermost two-thirds of the convection zone and may even disappear altogether below its base.

For the radii between 0.9$R_\odot$ and the photosphere, our sectoral harmonic results are likewise too imprecise at the moment to provide anything more than a hint of an oppositely signed gradient. This oppositely signed gradient might be consistent with those found by Deubner, Ulrich, and Rhodes (1979) or by Hill et al. (1988); however, our experiment was not designed to concentrate on the higher degree sectoral harmonics, and so we did not sample the shallow subsurface layers well enough to make such a comparison on a quantitative basis. In any event, our sectoral harmonic splittings are considerably larger than the values of 432–444 nHz found for comparable degrees (and hence for comparable depths) by Hill, Rust, and Appourchaux (1988).

There is a crucial need for future observational studies of the lowest degree ($1 \leq \ell \leq 9$) frequency splittings. Such studies of these splittings will be necessary in order to confirm the current suggestion from the inversions that the latitudinal gradient does indeed disappear in the radiative interior. Until observations of frequency splittings are obtained which have a much smaller intrinsic observational uncertainty, the simple mapping approach that we have adopted herein should continue to provide useful information concerning the Sun's internal rotation.

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