Nonradial wave propagation in line-driven stellar winds is analyzed including both finite disk effects and the line-drag effect of scattered line radiation. Within the local (WKB) analysis the results apply to wavelengths both longer and shorter than the Sobolev length. The finite disk causes short-wavelength waves with lateral velocity polarization to be unstable in the idealized case of pure absorption; however, the growth rates are smaller than for radially polarized waves, and the instability is damped by a relatively small amount of scattering. Hence, in realistic stellar winds, where the driving is primarily by scattering lines, perturbations with an arbitrary mixture of lateral and radial polarizations at the wind base should quickly become nearly radially polarized farther out in the wind. The implications of these results are discussed, both for interpretation of observational signatures of wind structure and for theoretical calculations aimed at modeling the nonlinear evolution of wind instabilities.

Subject headings: radiative transfer — stars: winds

I. INTRODUCTION

In previous papers of this series (Owocki and Rybicki 1984, 1985, 1986, hereafter Papers I, II, and III), we investigated the nature of instabilities in line-driven stellar winds caused by the velocity-sensitive nature of the line profile. These papers verified and extended earlier results that showed such winds to be violently unstable to perturbations in velocity with short spatial wavelength. In Paper I we derived the full dependence of the instability on perturbation wavelength (the "bridging law") for the case of pure absorption, showing how the results of MacGregor, Hartmann, and Raymond (1979) and Carlberg (1980) could be reconciled with those of Abbott (1980). In Paper II we extended the bridging-law analysis to include the line-drag effect of Lucy (1984) and finite disk effects. In Paper III we showed in the pure absorption case that the instability was of the advective type. (For other references to earlier work and for discussions of other types of instabilities in winds see Papers I—III and the review by Rybicki 1987.)

The analysis of Papers I—III were strictly one-dimensional, in that not only was the background flow assumed to be spherically symmetric and in the radial direction, but the small-amplitude perturbations considered were also assumed to be radial in both the perturbed velocity and the direction of propagation. The latter assumptions seem particularly artificial and restrictive because, even if one can reasonably assume that such stellar winds are spherically symmetric in either a large-scale or a time-averaged sense, spatial variations are in general likely to have a horizontal as well as a radial component.

Nonradial perturbations in radiation-driven stellar winds have been considered by a number of workers. Abbott (1980) found the dispersion relation for nonradial waves, but his analysis applied only to the long-wavelength limit, where the waves are stable. Carlberg (1980) considered nonradial waves in the short-wavelength limit and showed that the purely radial perturbations have the largest growth rates and eventually dominate the instability. Kahn (1981) also considered nonradial perturbations, and claimed to have found a strong horizontal instability, but his treatment of the radiative transfer by means of a "quasi-monochromatic" method seems suspect, given the strong frequency dependence of the radiation field in the vicinity of a spectral line. Martens (1985) formulated the full nonradial problem in general terms, but did not carry the analysis through to any final results.

All previous treatments of the nonradial instabilities neglect two important effects, the line-drag effect of Lucy (1984) and (except for Kahn 1981) the effect of a finite stellar disk. From the purely radial perturbation analyses (Lucy 1984; Paper II) these effects were shown to have the following consequences: For a point star, where the incident radiation is confined to a vanishingly small solid angle, the line-drag effect reduces the instability rates moderately (to 80%). Near the stellar surface, where the incident radiation occupies a nearly hemispherical solid angle, the growth rates may be reduced more drastically, approaching zero for the case of no limb darkening.

Since these two effects have such importance in the purely radial case, it is desirable to find out what consequences they have for nonradial perturbations. It is the purpose of this paper to generalize previous nonradial analyses, retaining the basic assumptions of linear perturbations in a spherically symmetric background flow, but now to consider the perturbations to be fully three-dimensional, with both the perturbed velocity and the propagation wavevector having arbitrary orientations. The analysis will include the effect of a nonzero stellar disk, since so much of the growth of the instabilities takes place near the stellar surface. In
order to study the effect of line scattering, we treat a line with a scattering albedo as a parameter, so the two limiting cases of pure absorption and pure scattering, as well as intermediate cases, can be treated in a unified fashion.

The practical motivation in carrying out this more general analysis is to consider how the three-dimensional nature of wind variations can affect both the interpretation of observational evidence for wind structure and the theoretical modeling of the nonlinear evolution of instabilities that can give rise to such structure. From an observational point of view, there are several lines of evidence indicating that hot-star winds are highly structured, perhaps consisting of a highly clumped flow with a nonmonotonic velocity field and embedded with strong shocks. These include observations of nonthermal radio emission (Abbott, Bieging, and Churchwell 1981, 1984; White 1985), enhanced infrared emission (Abbott, Telesco, and Wolff 1984), UV lines from superionized ions (Rogerson and Lamers 1975), and soft X-ray emission (Seward et al. 1979; Cassinelli and Swank 1983). Such observational evidence is ambiguous, and may be explainable in terms of hot coronae (Cassinelli and Olsen 1979; Waldron 1984; Clark, Minato, and Mi 1988) or non–LTE effects (Pauldrach 1987); however, one apparently does need inhomogeneous density and velocity structures to explain the black profiles from saturated UV lines (Lucy 1982) and the variable narrow absorption features in unsaturated UV lines (Lamers, Gathier, and Snow 1982; Henrichs 1984; Prinja, Howarth, and Henrichs 1987; Prinja and Howarth 1988). Although none of these observations is spatially resolved, some are nonetheless quite sensitive to the scale of the presumed structure. For example, while it is possible to obtain steady narrow absorption features from a scattered collection of clumps having coherence in velocity, variable narrow absorption features require horizontal coherence either of the clumps themselves or of some large-scale triggering phenomenon. A principal question of interest, then, is the degree to which the horizontal coupling of the flow is sufficient to force a large horizontal scale for velocity variations.

From a theoretical point of view, a relevant question is how strong a role horizontal variations play in the formation of flow structure from the nonlinear evolution of wind instabilities. Recently, we have carried out one-dimensional numerical simulations of this nonlinear evolution (Owocki, Castor, and Rybicki 1988, hereafter OCR); the results indicate that even small-amplitude perturbations in the radial velocity near the wind base can result in a highly structured wind, with relatively slow, dense shells separated from regions of high-speed, rarefied flow by strong shocks. The restriction to one dimension is made for obvious reasons of conceptual simplicity and computational tractability, but it does require a rather unrealistic form for the assumed base perturbations, e.g., radial sound waves applied coherently around the star.

Ultimately, of course, one would like to extend these nonlinear simulations to two or three dimensions. However, such simulations seem to be outside the capabilities of current computers, and one is led first to carry out a much simpler, analytic study of the line of evolution of perturbations with a general three-dimensional character. The hope is that this will delineate which, if any, horizontal effects are likely to be important, and thus provide a framework for both interpreting and extending the existing one-dimensional simulations. Examples of some of the specific questions to be addressed are: What are the growth and propagation characteristics of horizontal velocity perturbations? What effects can couple variations horizontally? And what, if any, are the preferred horizontal scale lengths?

The remainder of this paper is organized as follows: We first develop (§ II) the basic equations for studying three-dimensional perturbations. Next we extend (§ III) the radial perturbation analysis of Paper II to the case of perturbations with arbitrary directions of polarization and propagation, and thereby derive expressions for the tensor $T_{ij}$ describing the variation of the perturbed radiative acceleration with perturbed velocity. This tensor is then applied (§ IV) to a three-dimensional stability-dispersion analysis, with particular focus on the long-wavelength (§ IVa) and short-wavelength (§ IVb) limits. Finally, the concluding section (§ V) discusses the results of this analysis in the context of understanding the three-dimensional structure of hot-star winds.

II. EQUATIONS FOR THREE-DIMENSIONAL PERTURBATION ANALYSIS

The hydrodynamical state of the wind is specified by the density $\rho$, the pressure $P$, and the velocity $v_i$ as functions of the spatial Cartesian coordinates $r_i$ and time $t$. (In our tensor notation, indices run from 1 to 3.) In order to simplify the analysis, we assume a barotropic gas law $P = P(\rho)$ to replace the energy equation. Then we need only the mass and momentum equations,

$$ \frac{\partial \rho}{\partial t} + v_j \frac{\partial \rho}{\partial r_j} + \rho \frac{\partial v_i}{\partial r_i} = 0, $$

$$ \frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial r_j} = - \frac{1}{\rho} \frac{\partial P}{\partial r_i} \frac{GM r_i}{r^3} + g_i, $$

where $M$ is the stellar mass, $G$ is the gravitational constant, and $g_i$ is the radiative acceleration.

To investigate the linear stability of a solution, the hydrodynamical variables are first expressed as a sum of an unperturbed part (with subscript zero) and an infinitesimal perturbed part (preceded by a $\delta$). For example, $\rho = \rho_0 + \delta \rho$, $v_i = v_{i0} + \delta v_i$, and $g_i = g_{i0} + \delta g_i$. When these expressions are substituted in equations (1) and (2), neglecting second-order and higher terms in the perturbations, the equations for the unperturbed steady state are found:

$$ v_{i0} \frac{\partial \rho_0}{\partial r_j} + \rho_0 \frac{\partial v_{i0}}{\partial r_j} = 0, $$

$$ v_{i0} \frac{\partial v_{i0}}{\partial r_j} = - \frac{1}{\rho_0} \frac{\partial P_0}{\partial r_i} \frac{GM r_i}{r^3} + g_{i0}. $$

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Similarly, the equations for the perturbations are

\[
\begin{align*}
\frac{\partial \delta \rho}{\partial t} + v_\rho \frac{\partial \delta \rho}{\partial r_j} + \delta v_j \frac{\partial \rho_0}{\partial r_j} + \rho_0 \frac{\partial \delta v_j}{\partial r_j} + \rho \frac{\partial \delta \rho}{\partial r_j} &= 0, \\
\frac{\partial \delta v_i}{\partial t} + v_\rho \frac{\partial \delta v_i}{\partial x_j} + \delta v_j \frac{\partial v_\rho}{\partial x_j} &= \frac{a^2}{\rho_0} \frac{\partial \delta \rho}{\partial x_i} - \frac{\partial^2 \delta \rho}{\partial x_i \partial r_j} + \delta \phi_i,
\end{align*}
\]

where the sound speed \( a \) is given by \( a^2 = (\partial P/\partial \rho)_0 \).

As in Papers I and II, we make the WKB assumption that the solutions of interest are those in which the perturbations vary on a scale small compared with typical scale lengths of the unperturbed solution. The terms involving gradients of the unperturbed quantities can then be dropped, and the coefficients in the equations are considered to be locally constant. Also within the WKB approximation, a locally spherically symmetric flow and a locally plane-parallel flow are identical to lowest order, so that the Cartesian coordinates used here are adequate and convenient. (Compare the treatment in Paper II, where spherical coordinates were introduced, and where the sphericity effects were subsequently shown to be negligible in WKB order.)

Having made the WKB approximation, we may now assume that all perturbation quantities have spatial and temporal variations given by the plane wave \( \exp \{ ik_j x_j - \omega t \} \), where \( \omega \) is the angular frequency and \( k_i \) is the wavevector of the wave. Substitution in equations (5) and (6) leads to the algebraic equations,

\[
\begin{align*}
-i \omega \delta \rho + i \rho_0 k_j \delta v_j &= 0, \\
-i \omega \delta v_i + \frac{a^2}{\rho_0} k_i \delta \rho - \delta \phi_i &= 0,
\end{align*}
\]

in a frame of reference comoving with the mean flow.

III. THE PERTURBED RADIATIVE ACCELERATION TENSOR

In order to complete the specification of these equations, the perturbation in the radiative acceleration \( \delta \phi_i \) must be expressed in terms of the other hydrodynamical variables in the problem. Since our main concern is the instability due to velocity shifts in the line, we focus solely on the dependence of the radiative acceleration perturbation \( \delta \phi_i \) on the velocity perturbation \( \delta v_j \). (See the review by Rybicki 1987 references to work on other types of radiative instabilities.) As we shall see below, we may write this dependence in terms of a tensor \( T_{ij} \), where

\[
\delta \phi_i = T_{ij} \delta v_j.
\]

Martens (1985) formally introduced this tensor (denoting it by \( A \) topped with two arrows) but did not give explicit forms for it. In this section we derive expressions for \( T_{ij} \) and these will then be applied in the next section (§ IV) to a three-dimensional stability-dispersion analysis.

Let us first derive an expression for the contribution of a single, isolated line to the perturbed radiative acceleration. (The collective effect of a large ensemble of lines will be discussed briefly at the end of this section, with further details in the Appendix.) Our approach is to generalize the analysis of Paper II, relaxing the assumption of radial perturbations made there, but otherwise keeping the remaining assumptions, such as that the background flow is radially outward and spherically symmetric. Except for denoting perturbed quantities with a prefix \( \delta \) instead of with a subscript 1, our notation will follow that of Paper II unless otherwise indicated.

We begin by writing the three-dimensional form of the comoving frame equation for line transfer of radiation along an arbitrary direction defined by the unit vector \( n_j \):

\[
n_j \frac{\partial I}{\partial r_j} - Q \frac{\partial I}{\partial x} = -\chi \phi(x)(I - S),
\]

where

\[
Q = \frac{1}{v_{th}} \frac{\partial \nu_j}{\partial r_j} n_j n_i.
\]

The symbols have the same meaning as in Paper II: \( I, S \), and \( \chi \) are the specific intensity, source function, and profile function, and \( v_{th}, x, \) and \( \chi \) are the thermal speed, the frequency displacement from line center, and the line strength, which is related through the density to the line opacity \( \kappa = \chi/\rho \). If we choose the \( r_3 \)-axis to be directed radially outward, with the \( r_1 \) and \( r_2 \)-axes in the transverse plane, then in spherical polar coordinates the vector \( n \) has components \( n_1 = \sin \theta \cos \phi, n_2 = \sin \theta \sin \phi, \) and \( n_3 = \cos \theta = \mu \). As in Paper II, we use the background flow transfer solution to derive a simple form for the perturbed transfer equation (cf. Paper II, eq. [29]; equations from that paper will henceforth be cited as eq. [II-29], and so on):

\[
\begin{align*}
in_j k_j \delta I - Q_0 \frac{\partial \delta I}{\partial x} - \delta Q \frac{\partial I_0}{\partial x} &= -x_0 \phi(x)(\delta I - S),
\end{align*}
\]

where

\[
Q_0 = \frac{v_0}{r_{th}} (1 + \sigma \mu^2), \quad \delta Q = \frac{1}{v_{th}} in_j k_j \delta v_j,
\]

\[
\]

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with \( \sigma \equiv (d \ln v_0/d \ln r) - 1 \). The solution for the frequency-averaged perturbed intensity is given by

\[
\delta I = \delta S A + \int D - (1 - \epsilon) \frac{\beta_s}{\beta} \frac{\delta Q}{Q_0} B
\]

(14)

(cf. eq. [II-34]), where \( I_s \) is a constant specifying the magnitude of the star's core intensity near the line frequency \( \nu \) and \( D \) is a function specifying its extent and variation in angle over the stellar disk. Note that we now account for partial scattering with the factor \((1 - \epsilon)\) (cf. eq. [II-54]), where \( \epsilon \) is the line photon destruction probability. The angle-averaged escape probability \( \beta \) and the core penetration probability \( \beta_c \) are defined by (see eqs. [II-24] and [II-25])

\[
\frac{1}{\tau} \left( 1 - e^{-\tau} \right), \quad \beta_c = \left\langle D, \mu \frac{1 - e^{-\tau}}{\tau} \right\rangle,
\]

(15)

where the angle-dependent Sobolev optical depth is \( \tau \equiv x_0/Q_0 \). The quantities \( A \) and \( B \) are still defined by equations (II-35) and (II-36) if we make the simple substitution \( \mu K_{\mu} \rightarrow n_j k_j/Q_0 \), i.e.,

\[
A = \tau \int_{-\infty}^{\infty} dx \phi(x) \exp \left[ i(n_j k_j/Q_0)x - \tau \Phi(x) \right] \int_{-\infty}^{\infty} dx' \phi(x') \exp \left[ -i(n_j k_j/Q_0)x' + \tau \Phi(x') \right],
\]

\[
B = \tau \int_{-\infty}^{\infty} dx \phi(x) \exp \left[ i(n_j k_j/Q_0)x - \tau \Phi(x) \right] \int_{-\infty}^{\infty} dx' \phi(x') \exp \left[ -i(n_j k_j/Q_0)x' \right],
\]

where

\[
\Phi(x) \equiv \int_{-\infty}^{\infty} \phi(x') dx'.
\]

We next obtain the perturbed line flux, \( \delta H_i \), by taking the first moment of the perturbed intensity \( \delta I \) with respect to the direction \( n_i \) (cf. eq. [II-39]):

\[
\delta H_i = I_s \left[ \left( D - (1 - \epsilon) \frac{\beta_s}{\beta} \right) n_i + (1 - \epsilon) \frac{\left\langle n_j A \right\rangle}{1 - \left\langle A \right\rangle} \frac{\delta Q}{Q_0} B \right].
\]

(17)

Note that the angle brackets here now denote averaging over the full variation in solid angle. The contribution that such a line makes to the perturbed radiative acceleration is

\[
\delta g_i = \left( \frac{4\pi k v L_{\text{th}}}{c^2} \right) \delta H_i
\]

(18)

(see eq. [II-7]). Thus \( \delta g_i \) is indeed expressed in the form (9), where

\[
T_{ij}^{\delta} = \left( \frac{4\pi k v L_{\text{th}}}{c^2} \right) \left[ \left( D - (1 - \epsilon) \frac{\beta_s}{\beta} \right) n_i + (1 - \epsilon) n_j \frac{\left\langle n_j k_i B \right\rangle}{Q_0} n_j \right].
\]

(19)

The superscript \( L \) here emphasizes that this result applies to a single line with characteristic properties like \( \kappa, \nu, \) and \( \epsilon \).

The term in equation (19) with \( \beta_s/\beta \) represents a "line-drag" effect of the mean scattered radiation (cf. Lucy 1984), and the term with

\[
\eta_i = \frac{\left\langle n_j A \right\rangle}{1 - \left\langle A \right\rangle}
\]

(20)

represents the effect of perturbations in that scattered radiation field. It is important to distinguish this line-drag effect, which is of order \( \nu/v_{th} \), from the much weaker Thomson drag effect, which is of order \( \nu/c \) (Rybicki 1987). The line-drag term can have important effects on the growth rate of instabilities, but the results of Paper II suggest that the perturbed scattering term plays a relatively minor role; it generally has little effect on growth rates and vanishes entirely in both the long- and short-wavelength limits that are the focus of our analysis in § IV. For simplicity, we shall thus henceforth ignore this perturbed scattering term and set \( \eta_i = 0 \). Further details to justify this simplification are given in the Appendix and in the relevant discussions for each of these two limits (see §§ IVa and IVb).

Equation (19) gives the desired three-dimensional generalization for the contribution of a single, isolated line to the perturbed radiative acceleration in terms of the tensor \( T_{ij}^{\delta} \). In a radiatively driven stellar wind, there are actually a large number of both optically thin and optically thick lines that contribute to the mean radiative acceleration (Abbott 1982). However, as discussed in greater detail in the Appendix, the total perturbed acceleration is dominated by the contribution of relatively strong lines with \( \tau \gg 1 \). This suggests that the tensor \( T_{ij} \) for an ensemble of lines will have a form given approximately by that of equation (19) in the optically thick limit \( \tau \gg 1 \). Equations (15) show that, in this limit, the scattering line-drag term may be written

\[
\frac{\beta_s}{\beta} \approx \frac{\left\langle D/\tau \right\rangle}{\left\langle 1/\tau \right\rangle} = \frac{\left\langle D \delta Q \right\rangle}{\left\langle Q_0 \right\rangle}.
\]

(21)
In addition, for such strong lines we may use the "exponential shadowing approximation" (see eq. [II-A3]) to obtain

$$B \approx \frac{Q_0}{
abla \phi(1 + \frac{k_d}{x_n Q_0})},$$  

(22)

where \(x_n \approx \phi(x_n) r_n/2\) is an order-unity, slowly varying function of \(\mu\).

Using these approximations, we can thus write the perturbed radiative acceleration tensor as (cf. eq. [A2])

$$T_{ij} = C \left[ \frac{D - (1 - \epsilon) \langle DQ_0 \rangle}{
\langle Q_0 \rangle} \right] \frac{in_i k_i}{1 + \frac{k_d}{Q_0} n_i n_j},$$

(23)

where \(\epsilon\) now represents a suitable average of the destruction probability for the optically thick lines, and we have, for simplicity, set the order-unity factor \(2x_n\) equal to 1. The constant \(C\) can be written in several alternative ways (cf. Papers I and II):

$$C = \frac{4\pi F_\star}{\rho_0 c^2} = \frac{\nu_{\text{th}} \langle D\mu Q_0 \rangle}{\langle Q_0 \rangle} = \frac{4}{1 - \mu^2} \frac{\omega_*}{\chi_*} = \frac{4U}{1 - \mu_*^2}.$$

(24)

Here \(F_\star\) is the sum of \(\nu_l\) over the optically thick lines; \(\omega_* = \nu_{\text{th}} / v_{\text{th}}\) is the growth rate; \(\chi_*^{-1} = (1 - \mu_*^2)/4 \langle D\mu Q_0 \rangle\) is the bridging length; \(U = \omega_* / \chi_*\) is a characteristic speed for radiative-acoustic waves (Abbott 1980); and \(1 - \mu_*^2 = R_*/r^2\), where \(R_*\) is the stellar radius.

IV. STABILITY-DISPERSION ANALYSIS

With the perturbed radiative acceleration tensor in hand, we are now ready to carry out a three-dimensional stability-dispersion analysis to determine the growth and propagation of linear wave modes in such a line-driven flow. Using expression (9) for \(\delta g_i\), equations (7) and (8) can be put into a \(4 \times 4\) matrix eigenvalue problem for the modes,

$$
\begin{pmatrix}
\omega - ak_1 & -ak_2 & -ak_3 & V_0 \\
-ak_1 & \omega - iT_{11} & -iT_{12} & -iT_{13} & V_1 \\
-ak_2 & -iT_{21} & \omega - iT_{22} & -iT_{23} & V_2 \\
-ak_3 & -iT_{31} & -iT_{32} & \omega - iT_{33} & V_3 \\
\end{pmatrix} = 0,
$$

(25)

where

$$V_0 \equiv \delta \rho / \rho_0 \; , \; \; \; \; V_i \equiv \delta v_i / a \; .$$

(26)

Setting the determinant of the matrix in equation (25) equal to zero gives a quartic equation for the eigenvalues \(\omega\) as functions of the wavevector \(k\) (the dispersion relation), while the corresponding solutions for \((V_0, V_i)\) define the nature of the corresponding eigenmodes.

Most of the essential physics of the flow response to perturbations can be determined from separate analyses in the two extreme limits of long and short wavelength. We shall first examine the marginally stable wave propagation that occurs in the long-wavelength limit. The reader interested primarily in the instability of short-wavelength perturbations may skip ahead to § IVb.

a) Wave Propagation in the Long-Wavelength Limit

In the limit \(k \ll Q_0\) the wavelength is long compared with the Sobolev length, and so the perturbed radiative acceleration can be evaluated using the Sobolev approximation. Abbott (1980) has previously carried out a three-dimensional propagation analysis using the Sobolev expression for the perturbed line force, but this analysis also assumed that the driving radiation is purely radial, as from a point-star source. The analysis in this section represents a generalization of this to take proper account of the finite angle of the stellar disk.

We first note that in the long-wavelength limit \(k \ll Q_0\), the perturbed scattering term \(n_i \rightarrow 0\) (see Paper II), and so our use of equation (23), which neglects this term, is well justified here. In addition, in this limit the denominator \((1 + ik_i n_i/2x_n Q_0)\) is approximately unity. Thus

$$T_{ij} = ik_i M_{ij} \; ,$$

(27)

where

$$M_{ij} = \frac{4U}{1 - \mu_*^2} \langle n_i n_j \rangle \left[ D - (1 - \epsilon) \langle DQ_0 \rangle \langle Q_0 \rangle \right].$$

(28)

We further note that the mean scattering term involving \(\langle DQ_0 \rangle / \langle Q_0 \rangle\) drops out, since the product \(n_i n_j n_i\) is odd in the vector \(n\), so that the angle average \(\langle n_i n_j n_i \rangle\) vanishes. Thus \(M_{ij}\) takes the simpler form

$$M_{ij} = \frac{4U}{1 - \mu_*^2} \langle n_i n_j n_i D \rangle.$$

(29)

It is clear that \(M_{ij}\) is symmetric under permutations of \(i, j,\) and \(l\), which greatly simplifies its evaluation. Moreover, since the quantity \(D\), defining the limb-darkening law, is a function of \(\mu\) only, the symmetry about the \(r_3\)-axis implies that many components
of $M_{ij}$ vanish. In particular, $M_{ij} = 0$ unless, among the indices $ijl$, the number of 1's is even (zero or two) and the number of 2's is even; otherwise the averaging over $\phi$ gives zero. Thus the only nonzero components are $M_{113}$, $M_{223}$, and $M_{333}$ plus all those whose indices are permutations of one of these. Symmetry about the $r_3$-axis also implies that $M_{113} = M_{223}$, so that we may take $M_{113}$ and $M_{333}$ to be the basic quantities. Therefore, using equations (9) and (27), we write

\[ \delta g_1 = M_{113}(ik_3 \delta v_1 + ik_1 \delta v_3), \]

\[ \delta g_2 = M_{113}(ik_3 \delta v_2 + ik_2 \delta v_3), \]

\[ \delta g_3 = M_{113}(ik_1 \delta v_1 + ik_2 \delta v_2) + M_{333} ik_3 \delta v_3, \]

It is easy to evaluate the quantities $M_{113}$ and $M_{333}$ for the case of no limb darkening, so that $D(\mu) = 1$ for $\mu_* < \mu < 1$, and $D(\mu) = 0$ otherwise. The following averages are required:

\[ \langle n_1 n_3 D(\mu) \rangle = \langle \cos \theta \sin^2 \theta \cos^2 \phi D(\mu) \rangle = \frac{1}{P}(1 - \mu_*^2)^2, \]

\[ \langle n_3 n_3 D(\mu) \rangle = \langle \cos^3 \theta \cos \phi D(\mu) \rangle = \frac{1}{P}(1 - \mu_*^2). \]

Thus

\[ M_{113} = \frac{1}{P}(1 - \mu_*^2)U, \quad M_{333} = \frac{1}{P}(1 + \mu_*^2)U. \]

Moving outward from the base to large radii, $\mu_*$ varies from 0 to 1, and so $M_{113}$ declines from $U/4$ to 0, while $M_{333}$ increases from $U/2$ to $U$.

The relevant values of $T_{ij}$ can be extracted from these equations by comparison with equation (9), yielding

\[ T_{ij} = \begin{pmatrix} ik_3 M_{113} & 0 & ik_1 M_{113} \\ 0 & ik_3 M_{113} & ik_2 M_{113} \\ ik_1 M_{113} & ik_2 M_{113} & ik_3 M_{333} \end{pmatrix}. \]

The dispersion relation for long-wavelength waves can be found by setting the determinant of the matrix in equation (25) to zero, using the result (33). This gives one mode with $\omega = -\xi k_3 M_{113}$, the other three values of $\omega$ being found as roots of the cubic equation

\[ \omega^3 + k_3(M_{113} + M_{333})\omega^2 + [k_3^2(M_{113} M_{333} - a^2) - (k_1^2 + k_2^2)(M_{113}^2 + a^2)]\omega - a^2 k_3 [k_3^2 M_{113} + (k_1^2 + k_2^2)(M_{333} - 2M_{113})] = 0. \]

Solving this dispersion relation is straightforward, but instead of writing out these general solutions explicitly, it is more instructive simply to consider their overall properties together with results in a few appropriate limits.

First, in the particular limit of a point star, $\mu_* = 1$, we have $M_{113} = 0$ and $M_{333} = U$, so that equations (32) reduce to

\[ \delta g_1 = 0, \quad \delta g_2 = 0, \quad \delta g_3 = ikU \delta v_3. \]

The dispersion relation (34) then reduces to that previously solved by Abbott (1980) (cf. his eq. [50]), who found that it implied wave motions that were marginally stable, neither growing nor decaying, but with phase propagation in the inward (outward) direction much faster (slower) than ordinary sound waves.

Using the full equations (33), we can show that the marginal stability of long-wavelength waves also holds for the case of a finite disk. For real wavevectors $k_i$, the $M_{ij}$ are real, and the tensor $T_{ij} = T_{ji}$ is pure imaginary. Therefore the eigenvalue problem (25) is that of a real symmetric matrix, and all modes have real $\omega$, implying marginal stability. Thus the introduction of the finite disk does not change the previous result from Abbott (1980) that radiative-acoustic waves in which the radiative force is computed from the Sobolev approximation are not unstable. As noted in Paper I (see §§ I and II there), the physical reason for this lack of instability is that, in this limit, the perturbed line force is proportional to the perturbed velocity gradient, and thus is 90° out of phase with the perturbations in the velocity and radiative acceleration. An example of this interaction can be seen in the special case of a point-star analysis of Abbott (1980).

One new effect in the case of a finite disk is that there is now an interaction between horizontal and radial components of perturbations in the velocity and radiative acceleration. An example of this interaction can be seen in the special case of a wavevector in the horizontal direction, with $k_3 = 0$. Setting the determinant of the matrix in equation (25) to zero then yields two neutral modes $\omega = 0$ plus two others satisfying

\[ \omega^2 - (a^2 + M_{113}^2)(k_1^2 + k_2^2) = 0, \]

that is, horizontal waves traveling with speed $(a^2 + M_{113}^2)^{1/2}$. Since generally $U > a$, these horizontal waves can be very supersonic near the star, where $\mu_*$ is not too close to unity, whereas far from the star they travel only at the sound speed, in agreement with the point-star analysis of Abbott (1980).

We caution that the phase speeds derived here are not related in any simple way to the speed of information propagation in the wind. As shown in Paper III, the information speed cannot be obtained from the behavior of just the long-wavelength waves but must be derived from a Green's function analysis that necessarily includes the properties of the short-scale waves as well.

To summarize, our full three-dimensional analysis shows that finite disk effects do not alter the previous results (Abbott 1980; Papers I, II, III) that long-wavelength radiative-acoustic waves are marginally stable, and that they propagate in the inward
(outward) directions at speeds that are faster (slower) than ordinary sound waves. However, finite disk effects can have substantial effects on the propagation speeds of horizontal waves.

\subsection*{b) Stability or Instability of Short-Wavelength Perturbations}

In the opposite extreme of very short wavelength, the dominant concern is not how the waves propagate but whether they are stable or unstable. As a prelude to obtaining growth or damping rates in the next subsection (§ IVb[i]), let us first obtain some simplifications for the perturbed radiative acceleration that apply in the limit of short wavelength.

\subsubsection*{i) Perturbed Radiative Acceleration in the Short-Wavelength Limit}

As described in § III, we shall for simplicity ignore the perturbed scattering terms involving $\eta_i$. In the present problem of determining the stability of short-wavelength perturbations, there are several factors that help justify this. First, for any given line, $\eta_i$ formally approaches zero in the limit $k \gg \chi_0$, since then the perturbations remain optically thin right down to line center; this implies that the perturbed scattering source function, and hence the perturbed scattering force, must then average to zero. Second, the analysis in Paper II suggests that, even when not strictly negligible, this perturbed scattering term primarily affects only the wave propagation, not the wave growth; this implies that it can quite generally be ignored when the focus is on stability, as it is here. Finally, the total perturbed radiative acceleration should be dominated by the contribution from relatively numerous lines with moderate optical thickness $\tau = \chi_0/Q_o > 1$, and for such lines the above requirement that the perturbations be optically thin reduces to the condition $k \gg Q_o$. Overall, then, we conclude that the perturbed scattering term should have little influence on the stability of perturbations with a wavelength small compared with the Sobolev length, for which $k \gg Q_o$; the neglect of this term in equation (23) is thus again well justified here.

In this short-wavelength limit $k \gg Q_o$, we find that equation (23) simplifies to

$$T_{ij} = \frac{\omega_*}{\langle D\mu Q_o \rangle} \left[ D - (1 - \epsilon) \frac{\langle DQ_o \rangle}{\langle Q_o \rangle} \right] n_i n_j Q_o. \tag{37}$$

Moreover, we note that, because of the assumed spherical symmetry of the background flow, $Q_o$ is independent of $\phi$ (see eq. [13a]), and so the terms involving the product $n_i n_j$ vanish because of the $\phi$ averaging unless $i = j$. Hence $T_{ij}$ is a diagonal tensor.

Symmetry about the $r_3$-axis further implies the equality of $T_{11}$ and $T_{22}$, or that

$$T_{11} = T_{22} = \frac{1}{2}(T_{ii} - T_{33}), \tag{38}$$

where $T_{ii}$ is the trace of the tensor $T$. Since $n_i$ is a unit vector with $n_i n_i = 1$, we see from equations (37) and (32) that the trace is simply

$$T_{ii} = \epsilon \omega_* \frac{\langle DQ_o \rangle}{\langle D\mu Q_o \rangle}. \tag{39}$$

For pure scattering ($\epsilon = 0$), which in this context is usually a very good description for most of the lines that drive such stellar winds, we thus obtain the useful result that this perturbed radiative acceleration tensor is traceless, i.e., $T_{ii} = 0$, so that $T_{11} = T_{22} = -\frac{1}{2}T_{33}$.

In the absorption or partial scattering case the nonzero components of $T_{ii}$ are thus determined by just two quantities, $T_{33}$ and $T_{ii}$, while in the pure scattering case they are determined by just the single quantity $T_{33}$. If there is no limb darkening, the result for $T_{33}$ is the same as for the quantity $g_1/v_1$ in Paper II, given in equation (II-46), that is,

$$T_{33} = \omega_* \frac{E_3 + \sigma E_3 - (1 - \epsilon) \left( \frac{1}{2} + \sigma/5 \right)(1 + \sigma E_3)}{E_2 + \sigma E_4} \tag{40}$$

where $n E_3 \equiv (1 - \mu_3^2)/(1 - \mu_3^2)$, and we have now explicitly included the term $(1 - \epsilon)$ to account for partial scattering (see eq. [II-27]). A similar calculation gives

$$T_{ii} = \epsilon \omega_* \frac{1 + \sigma E_3}{E_2 + \sigma E_4}. \tag{41}$$

The results for $T_{11}$ and $T_{22}$ are thus

$$T_{11} = T_{22} = \frac{\omega_*}{2} \frac{\epsilon(1 + \sigma E_3) - (E_3 + \sigma E_3)}{E_2 + \sigma E_4} + (1 - \epsilon) \frac{\left( \frac{1}{2} + \sigma/5 \right)(1 + \sigma E_3)}{E_2 + \sigma E_4}, \tag{42}$$

using equations (38), (40), and (41).

In Figure 1 the quantities $T_{11}$ and $T_{33}$ (in units of $\omega_*$) are plotted against radius for values of the velocity-law parameter $p = \frac{1}{3}, 1,$ and $2$ (see eqs. [II-45] and [II-46]), and for the cases of pure scattering ($\epsilon = 0$) and pure absorption ($\epsilon = 1$); since $T_{ii}$ depends linearly on $\epsilon$, results for different values of $\epsilon$ can be found by straightforward linear interpolation between the $\epsilon = 0$ and $\epsilon = 1$ curves. An important result illustrated by Figure 1 is that, for the pure scattering case ($\epsilon = 0$), $T_{11} < 0$ at all radii. In the next subsection we show that this implies that horizontally polarized velocity perturbations tend to be strongly damped in a wind driven by scattering lines.
Fig. 1.—Radial variations of the perturbed radiative acceleration tensor components $T_{11}$ and $T_{33}$, which are used to compute the growth or damping rates of horizontal and radial velocity perturbations. Results are plotted for both the pure scattering ($\epsilon = 0$) and the pure absorption ($\epsilon = 1$) cases, as well as for various values of the parameter $p$ describing the steepness of the mean velocity law $v_0(r) = v_0(\infty)\frac{1}{1 - R_w r^p}$. The ordinate is in units of the total growth rate $\omega_*$ defined in the text.

ii) Growth or Damping Rates in the Short-Wavelength Limit

Let us now obtain the growth or damping rates for wave modes in this short-wavelength limit. This requires the solution of the eigenmode problem (25) for the diagonal tensor $T_{ij}$ just derived. It is convenient to choose the coordinate system such that the wave vector lies in the $r_1$-$r_3$ plane, making $k_2 = 0$. Then equation (25) can be written

$$\begin{pmatrix}
\omega & -ak_1 & 0 & -ak_3 \\
-ak_1 & \omega - iT_{11} & 0 & 0 \\
0 & 0 & \omega - iT_{11} & 0 \\
-ak_3 & 0 & 0 & \omega - iT_{33}
\end{pmatrix}
\begin{pmatrix}
V_0 \\
V_1 \\
V_2 \\
V_3
\end{pmatrix} = 0 .
$$

The component $V_2$ is completely decoupled from the others, implying that there is one mode with eigenvector $V = (0, 0, 1, 0)$, that is, with no density amplitude and with transverse velocity polarization in the $r_2$ direction. The corresponding eigenvalue is $\omega = iT_{11}$, which describes a nonoscillating mode with growth rate $T_{11}$ (or decay rate $-T_{11}$).

The three remaining modes satisfy the simplified problem

$$\Lambda W = \omega W ,
$$

where

$$\Lambda \equiv \begin{pmatrix}
0 & ak_1 & ak_3 \\
ak_1 & iT_{11} & 0 \\
0 & 0 & iT_{33}
\end{pmatrix} ,
W \equiv \begin{pmatrix}
V_0 \\
V_1 \\
V_3
\end{pmatrix} .
$$

The dispersion relation is found by setting the determinant of this system equal to zero:

$$\det(\omega I - \Lambda) = 0 .
$$

If we let $\theta_k$ be the angle between the wavevector and the radial direction, so that $k_1 = k \sin \theta_k$ and $k_3 = k \cos \theta_k$, and define the scaled frequency and wavenumber variables

$$w \equiv -i\omega \frac{T_{33}}{T_{33}} ,
K \equiv \frac{ak}{T_{33}} ,$$

then the dispersion relation for the pure scattering case (for which $T_{11} = -T_{33}/2$) reduces to

$$w(w + \frac{1}{2}(w - 1) + K^2 \sin^2 \theta_k(w - 1) + K^2 \cos^2 \theta_k(w + \frac{1}{2}) = 0 .
$$
Fig. 2.—Growth rates (in units of the component $T_{33}$ of the radiative acceleration tensor) versus wavenumber (in units of $T_{33}/a$, where $a$ is the sound speed) for the three wave mode solutions of the dispersion relation (48), at various angles $\theta_k$ between the wavevector and the radial direction. The results are for the pure scattering case ($\epsilon = 0$) and are meant to apply to the limit, $k \gg \chi_*$, in which the wavelength is much smaller than the mean bridging length $\chi_*^{-1}$. The heavy solid curves indicate where two modes have duplicate growth rates.

This is a cubic equation with real coefficients, so it has at least one real root, while the other two roots can either be real or occur as a complex conjugate pair.

The growth and decay rates of the modes are given in Figure 2, which shows the real parts of $\omega$ plotted against $K$ for values of various values of $\theta_k$. The curves represent numerical solutions of equation (48) for arbitrary values of $K$. Recall, however, that this equation itself applies only to the limit of short wavelength. As discussed at the beginning of § IVb(ii), it is probably adequate for the present analysis of wave stability to require only that $k > Q_0$. Now, $Q_0 \sim \chi_*$, where the bridging length $\chi_*^{-1}$ is of the order of the Sobolev length, and $T_{33} \approx \omega_\epsilon$, the instability growth rate. Thus, since $\omega_\epsilon/\chi_* = U$, our assumption of short wavelength restricts our analysis to the regime $K \gg a/U$. Typically, $U \approx v_0$, and so we see that the requirement $K > a/v_0$ for the curves in Figure 2 to be valid is actually quite modest in light of the basic assumption (listed as No. 8 in Paper II) that our analysis applies only in the supersonic portions of the wind, where $a/v_0 < 1$.

iii) Analytic Results for the $K \ll 1$ and $K \gg 1$ Limits

To gain further insight into these numerical solutions of equation (48), let us now derive separate analytic results for the two asymptotic limits $K \ll 1$ and $K \gg 1$. For the limit $K \ll 1$ we may neglect the components of $\Lambda$ involving the sound speed, which leads to the simple diagonal form

$$\Lambda \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & iT_{11} & 0 \\ 0 & 0 & iT_{33} \end{pmatrix}.$$  

(49)

It follows immediately that the three eigenvalues are $0, iT_{11},$ and $iT_{33}$, which, for the case of pure scattering, give the scaled values 0, $-\frac{1}{2}$, and 1 seen on the left-hand side of Figure 2. The corresponding eigenvectors $W$ are, respectively, $(1, 0, 0), (0, 1, 0)$, and $(0, 0, 1)$. The first of these describes a neutral mode involving a density perturbation alone; the second and third describe nonoscillating velocity modes polarized in the horizontal and vertical directions with growth rates $T_{11}$ and $T_{33}$, respectively.
We thus note that, for this limit of \( K \ll 1 \), the polarizations of the velocity modes and their growth or damping rates are independent of the direction of the wavevector. In particular, the radially polarized mode is always unstable, with a growth rate equal to that of the purely radial modes described in Papers I and II. This may be understood by recognizing that the mechanism of short-wavelength growth, namely, the shifting of the line profile into a stronger continuum radiation field, works just as well for perturbation elements side by side as it does for elements arranged radially, since the perturbations are assumed optically thin at the blue edge of the line (where the radiation is being absorbed; see Paper I) and hence do not interfere with one another. Because of the possibility of a horizontal instability, it may be loosely said that the winds are subject to a "Rayleigh-Taylor"-like instability, although the physics of the instability is quite different from that of the classical Rayleigh-Taylor one.

On the other hand, from Figure 1 we note that \( T_{11} \) can take either sign, depending on \( \epsilon \), so the modes with horizontal velocity polarization can either be stable or unstable. They are stable in the case of pure scattering, but unstable for pure absorption. Furthermore, because it is always true that \( T_{11} < T_{33} \), the ratio of the horizontal to the radial velocity components should always decrease, and so any perturbation that is initially polarized in some arbitrary direction should tend to become almost radially polarized. Thus we might expect that nonlinear waves that form out of the instability will also have essentially radial velocity fields. This does not imply, however, that the wind structure is necessarily one-dimensional, since these radially polarized waves will in general still vary in the horizontal as well as the radial direction.

Now let us turn our attention to the other asymptotic limit, \( K \gg 1 \). In this case the components of \( \Lambda \) involving the sound speed dominate, and in fact lead to eigenvalues approaching infinity as \( K \to \infty \). Consequently, in order to obtain the correct imaginary parts of \( \omega \), which are of order unity, it is necessary to work to first order in perturbation theory. Let us thus write \( \Lambda \) as the sum of zeroth and first-order parts:

\[
\Lambda = \Lambda^{(0)} + \Lambda^{(1)},
\]

where

\[
\Lambda^{(0)} = ak \begin{pmatrix} 0 & \cos \theta_k & \sin \theta_k \\ \cos \theta_k & 0 & 0 \\ \sin \theta_k & 0 & 0 \end{pmatrix}, \quad \Lambda^{(1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & iT_{11} & 0 \\ 0 & 0 & iT_{33} \end{pmatrix}.
\]

Since \( K = ak/T_{33} \gg 1 \), matrix \( \Lambda^{(1)} \) can be regarded as a perturbation on the matrix \( \Lambda^{(0)} \). The three eigenvalues of \( \Lambda^{(0)} \) are easily found to be \( \omega_0^{(0)} = 0 \) and \( \omega_\pm^{(0)} = \pm ak \). The eigenvector corresponding to \( \omega_0^{(0)} = 0 \) is \( W_0^{(0)} = (0, \cos \theta_k, -\sin \theta_k) \), which describes a mode with velocity transverse to the wavevector. The two eigenvectors corresponding to \( \omega_\pm^{(0)} = \pm ak \) are \( W_\pm^{(0)} = 2^{-1/2} (\pm 1, \sin \theta_k, \cos \theta_k) \), which describe modes with velocity along the wavevector, essentially longitudinal sound waves. The perturbations of the eigenvectors are small, so the full eigenvectors are essentially the same, but now their eigenvalues are perturbed by an amount equal to the diagonal elements of the perturbation matrix \( \Lambda^{(1)} \) with respect to the zeroth-order eigenvectors. That is,

\[
\omega_0^{(1)} = (W_0^{(0)} , \Lambda^{(1)} W_0^{(0)}), \quad \omega_\pm^{(1)} = (W_\pm^{(0)} , \Lambda^{(1)} W_\pm^{(0)}).
\]

Since the zeroth-order eigenvalues are real, these perturbations completely determine the imaginary parts of \( \omega \), and thus the real parts of \( w \). After performing the matrix operations in these equations, we find the asymptotic real parts of \( w \) to be

\[
\Re \omega_0 = (T_{11} \cos^2 \theta_k + T_{33} \sin^2 \theta_k)/T_{33}, \quad \Re \omega_\pm = (T_{11} \sin^2 \theta_k + T_{33} \cos^2 \theta_k)/T_{33}.
\]

For the case of pure scattering these reduce to

\[
\Re \omega_0 = \sin^2 \theta_k - \frac{1}{2} \cos^2 \theta_k, \quad \Re \omega_\pm = \cos^2 \theta_k - \frac{1}{2} \sin^2 \theta_k,
\]

which correspond to the values found on the right-hand side of Figure 2.

Having considered the two asymptotic limits \( K \ll 1 \) and \( K \gg 1 \), we may get some insight into the intermediate cases shown in Figure 2. Starting in the regime \( K \ll 1 \), the unstable velocity modes are polarized either along or transverse to the radial direction, and independent of the wavevector. As \( K \) is increased into the regime \( K \gg 1 \), the modes adopt polarizations which are now either along or transverse to the wavevector, and independent of the radial direction. The manner in which the polarization behavior switches in the intermediate regime is complicated, as one sees from the phenomena displayed in Figure 2 for \( K \approx 1 \). In general, note that for any angle \( \theta_k \) and for any wavenumber \( k \) at least one mode is unstable; reinforcing the conclusions from Papers I and II that a radiatively driven wind is highly unstable. Moreover, the most unstable mode is that which has the most radial polarization, while modes with a predominantly horizontal polarization are either damped or only weakly unstable.

V. DISCUSSION

Let us now consider the implications of these results in conjunction with observational constraints on the three-dimensional nature of the wind structure. The strongest existing constraints are probably from UV spectral line observations in which high temporal and/or spectral resolution can partly compensate for the lack of direct spatial resolution. Superposed upon the broad P Cygni profiles of strong but unsaturated UV lines, there often appear variable "narrow absorption components" (Lamers, Gathier, and Snow 1982; Henrichs 1984; Prinja and Howarth 1988). Similar narrow components can persist between observations separated by several months, but recent high time resolution IUE observations (Prinja, Howarth, and Henrichs 1987; Prinja and Howarth 1988) indicate that components typically vary over a few hours. Often they first appear as broad absorption enhancements at a relatively low velocity (\( \approx 0.5 V_\infty \)) that then gradually narrow and shift to a higher velocity (\( \approx 0.8 V_\infty \)); after about a day, a new component often arises to replace the previous one, which gradually fades away.
These narrow absorption components imply that there exists within the wind a relatively large column depth of material with a rather narrow range of velocity. In order to produce the absorption in spatially unresolved observations, this material must cover a substantial fraction of the stellar disk and thus must extend horizontally over a substantial fraction of a stellar radius. However, the mere existence of a persistent narrow absorption component in velocity does not necessarily imply a single, distinct clump of matter extending over such a large scale. The one-dimensional, nonlinear instability simulations by OCR show, for example, that fixed narrow spectral features can form from the combined absorption by several spatially separated, dense shells, and that these features can persist for all phases of the driving wave that gave rise (via the line-driven instability) to these dense shells. This implies that such persistent narrow components could equally well arise from a three-dimensional wind model in which the dense absorbing material moves at a nearly characteristic velocity but does not necessarily have much horizontal phase coherence. These arguments do not, however, extend to the variable absorption components observed by Prinja and Howarth (1988) to show systematic shifting and narrowing, and so these probably do require a single, distinct blob of material with a velocity that does not vary much horizontally over a substantial fraction of a stellar radius.

What does the above perturbation analysis imply about the likelihood or requirements that the velocity is nearly constant over such a large horizontal scale? On one hand, it shows that horizontal coupling effects are rather weak, and so there is nothing to force the flow to have such a large horizontal length scale. On the other hand, the relative weakness of horizontal coupling also implies that there is much less tendency to amplify horizontal than radial variations; this means that if the initial excitation source were such that horizontal variation occurred only over large scales, then this property might be maintained even as radial variations are being strongly amplified.

Consider, then two types of excitation from the underlying star that could provide the source perturbations for the wind instability: sound waves and stellar pulsations. For sound waves the scale for horizontal and radial variations should generally be of the same order, and, since it is perturbations with a small radial scale that are most strongly amplified, it seems likely that excitation by sound waves would result in wind structure with a quite small horizontal as well as radial scale. On the other hand, the nonradial pulsations thought to occur in many hot stars (Vogt and Penrod 1983; Burki 1987) often have relatively low spherical harmonic quantum numbers \( l = 1-10 \); the nonlinear wind structure that arises from amplification of such pulsations (Castor 1987) thus could have velocities that are nearly constant over a significant fraction of the stellar disk. This suggests that, if the narrow absorption components that show systematic variations are to be understood in terms of the nonlinear evolution of wind instabilities, then the excitation source must be one with a large horizontal scale, such as a low-order nonradial pulsation. We caution, however, that this inference is based entirely on the above linear analysis, and could be modified if there were nonlinear effects that could suppress horizontal variation.

The above analysis does suggest that one simplification of a one-dimensional wind model, viz., radial velocity polarization, should be well justified, even though there may be horizontal variations, as implied by nonradial wavevectors. Since such winds are driven primarily by scattering lines, any horizontal velocity fluctuations will be strongly damped, at about half the very large rate for amplification of radial fluctuations. This is a consequence of the line-drag effect of the mean scattered radiation field, which was first discussed by Lucy (1984) in the context of its role in reducing or even canceling the instability of radial velocity perturbations. (See also Paper II.) The present analysis shows that, since horizontal velocity perturbations have a much weaker tendency to be amplified by the directed radiation from the stellar disk (cf. the \( T_{11} \) and \( T_{33} \) curves for the \( \epsilon = 1 \) case in Fig. 1), this line-drag effect of the diffuse radiation can lead to a net damping for the horizontal component of a velocity perturbation. We thus expect that, even if source wave excitations at the wind base have arbitrary polarization and wavevector, the nonlinear wind structures that result from the amplification of these will quickly develop velocity fluctuations that are almost totally in the radial direction.

Finally, we remark that for any star that is not rotating too rapidly, any magnetic field in the regions of flow acceleration should be primarily radial. The above analysis thus implies that short-wavelength Alfvén waves, which must propagate along the nearly radial field and which thus must have nearly horizontal velocity fluctuations, should be strongly damped if there is significant line scattering. This is of some interest because mechanisms for damping Alfvén waves are in general quite rare. It may have significance not only for the line-driven winds of hot stars but also for the winds of late-type giant stars. The latter are thought to be driven primarily by Alfvén waves, but if there is also a small component of driving by line scattering, then this could damp the Alfvén waves and thus play an important role in regulating the wind dynamics. This topic will be explored further in another paper.

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**APPENDIX**

### THE PERTURBED RADIATIVE ACCELERATION FROM AN ENSEMBLE OF LINES

In a radiatively driven stellar wind, there are in general hundreds or even thousands of lines that can contribute to the radiative acceleration. However, if we ignore line overlap effects, then the total tensor \( T_{ij} \) is simply the sum of the contributions given by equation (19) from all the lines \( L \) with various \( \kappa \), \( v \), and \( \epsilon \):

\[
T_{ij} = \sum_L \left( \frac{4 \pi \kappa_L I}{c^2} \right) \left[ D - (1 - \epsilon) \frac{\beta_0(v)}{\beta(v)} \left( n_i + (1 - \epsilon) n_i(v) \right) \frac{i_n k_i B(v)}{Q_0} n_j \right].
\]  
(A1)
In principle, equation (A1) can be used to evaluate the perturbed radiative acceleration tensor for any given list of lines with various $\kappa$, $v$, and $\epsilon$; for example, in Paper I (see § IIId there) the radial component $T_{33}$ was computed (in the point-star limit, $\mu_{*} \to 1$) for an ensemble of pure absorption ($\epsilon = 1$) lines with a frequency-integrated number distribution that was assumed to be a power law in line opacity $\kappa$.

In the present case, however, such evaluation is greatly complicated if one retains the scattering terms proportional to $(1 - \epsilon)$. The perturbed scattering term involving $\eta_{i}$ is particularly troublesome; indeed, much effort was required in Paper II to determine the role of this term for just the single-line case. Fortunately, the results of Paper II suggest that much of the essential physics of the response to perturbations can be gleaned from the two limiting cases of long- and short-wavelength perturbations, for which $\eta_{i} \to 0$. Since the analysis in this paper thus concentrates primarily on these limits, we ignore this perturbed scattering term here and set $\eta_{i} = 0$.

We still wish to retain, however, the term with $\beta_{i}/\beta$, representing the line-drag effect of the mean scattered field, since this term was shown in Paper II (see also Lucy 1984) to have significant effects on the growth rate of perturbations. Equation (15) shows that, in general, this term depends on the line opacity $\kappa$ through the optical depth $\tau_{k} = \kappa \rho_{0}/Q_{0}$, but in the present context it is a good approximation simply to use the optically thick limit result, equation (21), which is independent of $\kappa$. The reason is that we expect that the contribution of weak lines with $\tau_{k} \ll 1$ will be small, since in this case the mean line intensity becomes frequency-independent, so that the Doppler shift associated with the velocity perturbation has little effect (see Paper I).

If we use the approximation (21) for the scattering line-drag term, then obtaining the total perturbed acceleration requires only that we compute the ensemble sum of the quantity $\kappa_{B}(\kappa)$. Paper I suggests that the optically thick form for this quantity should give a good description of its wavenumber variation for an entire ensemble of pure absorption lines with a power-law number distribution in opacity $\kappa$. Assuming such a power-law number distribution, we thus use the same procedure as in § IIId of Paper I to approximate this ensemble sum with an expression quite similar to equation (22) (see Paper I, eq. [51]). Appplying this to the perturbed radiative acceleration tensor then yields

$$T_{ij} = \Omega \left[ D - (1 - \epsilon) \right] \left[ \frac{Q_{0}}{\langle DQ_{0} \rangle} \right] \left[ \frac{Q_{0}}{\langle D\mu Q_{0} \rangle} \right] \frac{in_{k}k_{i}(2\kappa_{*}Q_{0})}{1 + in_{k}k_{i}(2\kappa_{*}Q_{0})} n_{j},$$

where $\epsilon$ represents a suitable average of the destruction probability for optically thick lines, $\alpha$ is the power index introduced by Castor, Abbott, and Klein (1975), $2\kappa_{*}$ is a constant of order unity (whose numerical value is given by the factor in square brackets in eqs. [52] and [53] of Paper I), and $\Omega = 2\kappa_{*}a_{\rho_{0}}/v_{in}$ is the total growth rate. Apart from the $Q_{0}$ term and the associated difference in the definition of the constant factor, equation (A2) has the same form as equation (23), which was derived for a collection of strictly optically thick lines.

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