ON THE INTERPRETATION OF CHROMOSPHERIC EMISSION LINES

P. G. Judge
Joint Institute for Laboratory Astrophysics, University of Colorado, Boulder
Received 1989 March 17; accepted 1989 July 1

ABSTRACT
There exist in the literature important differences in the interpretation of ultraviolet emission lines formed below \( \sim 10^4 \) K, temperatures characteristic of stellar chromospheres. This paper reexamines emission-line formation using detailed radiative transfer calculations and simpler methods based on approximate line cooling rates, including escape probabilities. Cooling integrals yield frequency-integrated line fluxes which can be estimated without a full solution of the statistical equilibrium and radiative transfer equations. In the effectively thin limit the approximations reduce to the “coronal” approximation.

Approximations for the cooling integrals are examined and are demonstrated to work well for “effectively thin” chromospheric lines. Two cases have been identified, whose behavior can be understood using Ayres’s chromospheric scaling laws relating global chromospheric structure to stellar properties:

1. For inactive stars (e.g., cool giants; the case of \( \alpha \) Tau (K5 III) is studied in detail) lines such as Mg II \( k \) thermalize in the upper photosphere, and they are effectively thin in the entire chromosphere. Chromospheric integrated line fluxes and cooling rates can be reliably calculated using complete redistribution, escape probability, and even optically thin approximations. Emission measure and similar techniques can be applied to such lines.
2. For more active stars like the Sun the lines thermalize in the chromosphere (a well-established result). Partial redistribution calculations are required to obtain reliable line-integrated fluxes and cooling rates: other approximations yield gross overestimates owing to transfer-dependent thermalization properties. Great care must be taken when applying emission measure techniques to these lines.

Analyses based on the “coronal approximation” (effectively thin lines) should be used with great care because chromospheric line fluxes are generated over regions spanning several gas pressure scale heights where \( h/2T_r \gg 1 \). Line fluxes are the result of a convolution of the atomic excitation physics and the thermal atmospheric structure, hence model atmosphere techniques are generally required to derive information on the chromospheric emitting regions. The implications of this study for past and future studies of the outer atmospheres of cool stars are discussed. Unfortunately, some authors have oversimplified the analyses of chromospheric line fluxes leading to incorrect and misleading conclusions. The arguments presented also apply in principle to emission lines formed below \( 10^4 \) K in other astrophysical situations (e.g., broad-line emitting regions of active galactic nuclei).

Subject headings: line formation — radiative transfer — stars: chromospheres — ultraviolet: spectra

I. INTRODUCTION

The steady stream of ultraviolet data from the International Ultraviolet Explorer (IUE) satellite over the last decade has revolutionized the study of the outer atmospheres of cool stars (see Jordan and Linsky 1987; Dupree and Reimers 1987). As reviewed by these authors, IUE spectra (particularly at high dispersion, \( \lambda/\Delta \lambda \approx 12 \times 10^4 \)) contain a wealth of information. Dwarf stars cooler than spectral type \( \sim F0 \) have emission-line spectra which are characteristic of regions analogous to the solar chromosphere (electron temperatures \( T_e \leq 10^4 \) K) and transition region (\( T_e \approx 10^5 \) K). Cool stars of lower gravity (giants later than \( \sim K0 \), supergiants later than \( \sim G5 \)) have quite different spectra, generally showing (with the notable exceptions of “hybrid” stars) only low-excitation chromospheric lines with evidence for substantial material outflows as deduced from the profiles of optically thick emission lines such as Mg II \( k \). Since they differ so dramatically from the solar case, such stars have been termed “nongenital” or “nonsolar.”

Ultraviolet line spectra including both permitted and inter-system (spin-forbidden) transitions, are currently the most valuable diagnostics available of the outer atmospheric structure, and they have led to major advances in our understanding of stellar chromospheres, coronae, and winds. The outer atmospheres of cool stars are of great interest to workers in the fields of chromospheric and coronal heating, mass loss, stellar evolution, and related topics. Ultraviolet emission lines play a dominant role in determining the structure of the outer atmospheres, since they provide a major energy loss channel in chromospheric gas.

Basically two different approaches have been used to examine the chromospheres of cool stars. The first consists of “semiempirical” model atmosphere techniques. Assumptions concerning the atmospheric structure are made (e.g., hydrostatic equilibrium), and a spectrum is calculated and compared with observations. The model is adjusted to obtain better agreement between synthesized and observed spectra. Such modeling techniques are reviewed by Linsky (1980, 1985). The second approach is through “empirical” techniques based on spectroscopic diagnostics (e.g., line ratios). Constraints on the atmospheric structure (e.g., gas temperatures and pressures) are derived directly from the observed emission-line diagnostics, with some simplifying assumptions. An example of an application to chromospheric lines is given by Judge (1986a).

While UV lines from the transition regions of stars with
"solar-like" spectra can be analyzed and atmospheric models deduced using the more empirical approach previously applied to the solar transition region and upper chromosphere (e.g., emission measure analysis; see Jordan and Wilson 1971; Jordan and Brown 1981), these techniques are less straightforward to apply to the "noncoronal" stars which show only chromospheric emission lines, owing to problems highlighted by Judge (1987). By solar analogy one would expect that stellar chromospheric emission lines require model atmosphere analyses such as were applied by Linsky and colleagues to a variety of cool stars (see, e.g., the reviews by Linsky 1980, 1985), and more recently by Hartmann and Avrett (1984), Drake (1985), Judge, Avrett, and Loeser (1987), and Judge (1988). Similar studies of carbon star chromospheres have also been made by Johnson and colleagues (Johnson and O'Brien 1983; Avrett and Johnson 1984; Johnson and Luttermoser 1987; Luttermoser et al. 1989).

Early work using IUE spectra of "noncoronal" stars focused on line identifications and excitation processes (Carpenter and Wing 1979; Brown and Jordan 1980; Judge 1984; Jordan and Judge 1984; Johansson and Jordan 1984). Subsequent studies adopted the more empirical spectroscopic techniques to derive initial constraints, such as emission measures, "mean" temperatures, densities, optical depths, ionization fractions, and geometric extents of the emitting regions (Stencel et al. 1981; Brown and Carpenter 1984; Carpenter, Brown, and Stencel 1983; Judge 1986a, b, c, 1987; Eaton and Johnson 1988). In some cases these analyses are apparently in strong disagreement with one another, with far-reaching implications for more detailed understanding of the outer atmospheres and winds (Judge 1987). For example, Judge (1986a) argued that there was no evidence for geometrically extended emitting regions (with thicknesses \( \sim R_\star \)) on the basis of IUE spectra of emission lines in noncoronal stars, in the sense implied by Stencel et al. (1981) and Carpenter, Brown, and Stencel (1983) in earlier analyses from identical data. Using line-ratio techniques Eaton and Johnson (1988) derived "mean" electron temperatures of M giants \( \sim 10^4 \) K and a depletion of silicon relative to the solar abundance, in apparent disagreement with Judge's (1986b) emission measure analysis of \( \beta \) Gru (M5 III), which has a typical M giant spectrum. In other cases, approximate escape probability calculations have been used to obtain estimates of chromospheric column masses (Jordan 1967; Brown, Ferraz, and Jordan 1981; Judge 1986a, b; Eaton and Johnson 1988) or to compute line fluxes (Judge 1986a, b), but the approximations used have not been checked against detailed calculations in chromospheric models. In the light of these and other problems, it is both timely and appropriate to reexamine the formation of ultraviolet chromospheric emission lines in cool stars.

The present paper has several aims:
1. To clarify the interpretation of chromospheric emission lines by analyzing approximate models for integrated fluxes \( F_\star \) which are simpler to understand than complete radiative transfer calculations (henceforth these will be referred to as "approximate cooling integral" methods).
2. To examine the use of simple line-ratio diagnostics, escape probability methods, and the more empirical analyses when applied to chromospheric lines in general.
3. To clarify the relationship between the empirical (e.g., line-ratio) and semiempirical (model atmosphere) approaches.
4. To deduce how chromospheric emission-line formation depends on stellar spectral type and activity in cool stars.
5. To test approximations for the dominant radiative cooling rates required in \textit{ab initio} hydrodynamical calculations.
6. To confront these results with others in the published literature.

Section II and the Appendix briefly review the required theory of chromospheric line formation and discuss how, by using simple approximations for the cooling rate (emissivity) of an emission line, detailed transfer calculations can be adequately replaced by simpler calculations, at least for relatively inactive stars. Section III discusses the interpretation of observed chromospheric lines in a variety of cool stars, the validity of various approximations made in the literature, and the conclusions drawn on the basis of these approximations.

II. FORMATION OF CHROMOSPHERIC EMISSION LINES

a) Previous Approximate Methods

In previous studies of chromospheric emission lines, a primary diagnostic has been the line-integrated flux, \( F_\star = \int \pi F_\nu \, dv \), where \( \pi F_\nu \) (ergs cm\(^{-2}\) s\(^{-1}\) Hz\(^{-1}\)) is the observed "astrophysical" emission-line flux profile normalized to the stellar surface. The line flux profile \( \pi F_\nu \), which clearly contains more detailed information than \( F_\star \), has also been used by workers concerned with modeling stellar chromospheres and winds where velocity fields and spherically extended geometry are important (e.g., Drake and Linsky 1983; Drake 1985). The present paper is mainly concerned with the interpretation of the line-integrated flux, \( F_\star \), as a diagnostic of chromospheric structure, and with the relation of \( F_\star \) to chromospheric energy losses (and hence atmospheric structure).

Before reviewing relevant theory, previous expressions for \( F_\star \) are discussed. A crucial consideration is whether photons created in an emitting region by, for example, electron collisions, actually escape from the atmosphere entirely. This is expressed in terms of the "effective thickness" of the emitting region: if the majority of photons, once created, eventually escape from the emitting region, then it is "effectively thin" in that particular line. Conversely, the line is "effectively thick" if a significant fraction of the created photons do not escape.

Athay (1976, § VI-4) showed how, by taking the first angle moment of the transfer equation and integrating over frequency and depth, \( F_\star \) in an effectively thin line may be reduced to the integral of the total number of collisional excitations times the energy of a photon over the height range corresponding to the emitting region \( \Delta Z \), assuming (for illustration) planeparallel geometry, i.e.,

\[
F_\star = h \nu \int_{\Delta Z} C \, dz,
\]

where \( C \) (cm\(^{-3}\) s\(^{-1}\)) represents the collisional excitation rate of the line per unit volume of emitting gas. Equation (1) represents the difference in the radiative flux between the integration limits—equating the right-hand side of this equation with \( F_\star \) implicitly assumes that \textit{all} of the photons are traveling in the outward direction. If it is assumed that half the line photons escape in the outward direction, for example, because we suspect that half are absorbed at the stellar photosphere, then this corresponds to the usual expression adopted as the starting point for emission measure analyses (Jordan and Wilson 1971; Jordan and Brown 1981). In this approximation the line flux \( F_\star \) is proportional to the usual "emission measure"
\( \int dz N_e N_i \) of the emitting region. \( N_e \) and \( N_i \) are the electron and hydrogen number densities \( [\text{cm}^{-3}] \).

Linsky and Ayres (1978) later proposed that for effectively thick lines, \( \Delta Z \) should be replaced by the height interval from above the atmosphere down to the height corresponding to the “thermalization depth” of the line, implicitly assuming that below this depth each emission is followed by an absorption and that photons created below these depths do not survive to contribute to the outward line flux. Adopting an approximate expression for the thermalization length and assuming that the line is thermalized somewhere in the chromosphere, Linsky and Ayres (1978) found that

\[
F_* \propto \exp \left( -\frac{h v}{kT^*} \right),
\]

where \( h \) is Planck's constant, \( v \) the frequency of the line, \( A \) the Einstein \( A \)-coefficient, \( \Delta v_0 \) the Doppler broadening of the line (frequency units), \( k \) Boltzmann's constant, and \( T^* \) a characteristic excitation temperature of the gas above the thermalization depth. The expression is independent of densities and hence measurement. The exponential term arises from the temperature dependence of the collisional excitation rate of the line.

The two expressions for \( F_* \) are clearly very different. As pointed out in § I, some authors have applied emission measure and line-ratio diagnostic techniques to analyze lines such as Mg ii \( k \) which, for the case of the solar chromosphere, are known to be effectively thick (e.g., Athay 1976). Calculations presented below show that care must be taken with these expressions when analyzing chromospheric spectra. We now consider these approximations using detailed radiative transfer calculations.

b) Basic Equations

To illustrate the basic line formation processes, consider the well-studied case of a two-level atom (upper level \( j \), lower level \( i \)), assuming complete frequency redistribution (CRD; complete uncorrelation of frequencies between absorbed and emitted line photons) and negligible continuum opacity (these restrictions are relaxed in calculations described later). The transfer equation, line source function (from the two-level statistical equilibrium equations), and optical depth in plane-parallel geometry are given by (e.g., Mihalas 1978: § 11-2) (here the angle variable subscript \( \mu \) on \( I \) has been suppressed for convenience):

\[
\frac{dI}{d\tau} = I - S_\nu ,
\]

\( S_\nu = S_L = (1 - \epsilon)J + \epsilon B_\nu ,
\]

\( \tau_\nu = -\int dz \kappa_\nu \phi_\nu ,
\]

where \( \mu \) is the usual angle variable, \( I \) is the mean intensity at frequency \( \nu \) in the line, \( S_\nu = S_L \) is the frequency-independent source function (all in units of ergs cm\(^{-2}\) s\(^{-1}\) Hz\(^{-1}\) sr\(^{-1}\)), \( \epsilon = C_{i//}(A_i + C_i) \) is the “thermal coupling parameter” (not corrected for stimulated emission, which is negligible at UV wavelengths), \( B_\nu \) is the Planck function, \( \phi_\nu \) is the line absorption profile (normalized so that \( \int \phi_\nu d\nu = 1 \)), \( \kappa_\nu \) is the line-center opacity \( [\text{cm}^{-1}] \), \( z \) is the height variable, and \( J = \int d\nu J_\nu \phi_\nu \) is the line mean intensity \( (J_\nu = \frac{1}{2} \int d\mu I_\mu) \). \( C_i \) and \( A_i \) are respectively the rates of collisional and radiative decay of level \( j \) \( [\text{s}^{-1}] \). The atomic level populations enter the transfer problem via \( S_\nu \) and \( \kappa_\nu \).

In traditional model atmosphere analyses, the line flux \( F_* \) is computed from a model chromosphere (e.g., Ayres and Linsky 1975) to take proper account of important radiative transfer effects such as destruction and scattering of photons as well as the variation of chromospheric properties with height. Once the radiation field and level populations have been solved self-consistently in a model calculation, the emergent line flux is obtained from the formal solution of the transfer equation for \( \pi F_* \) (e.g., Mihalas 1978):

\[
\pi F_* = 2\pi \int d\tau E_\nu (\tau_\nu)S_\nu (\tau_\nu) ,
\]

where \( E_\nu (\tau_\nu) \) is the second exponential integral. To obtain the line flux \( F_* \), \( \pi F_* \) is integrated over a suitable frequency range for comparison with observations.

Chromospheric resonance lines (e.g., Ca ii K, Mg ii \( k \)) are characterized by large line-center optical depths throughout the emitting regions across much of the line profile. In these cases, \( F_* \) at each frequency reflects (via an appropriate Eddington-Barbier relation equivalent to the \( E_\nu \) operator in eq. [6]) the source function in regions where the optical depth \( \tau_p \approx \text{unity} \) (see Ayres 1979). \( S_\nu \) is dominated by the photon scattering term \((1 - e)J \) in equation (4), even though the global scaling of equations (3) and (4) is set by the genuine source of photons \( eB_\nu \). The dominance of the scattering term causes the line formation problem to be nonlocal in the radiation field. The interpretation of the emergent line flux in terms of the original photon sources \( eB_\nu \), through the temperature-density stratification of the chromosphere, is not obvious from the formal solution of the transfer problem, in spite of the fact that \( S_\nu \propto eB_\nu \) for the case of effectively thin lines (Avrett and Hummer 1965).

The solutions to such two-level atom transfer problems are, however, well understood and documented and are easily extended to cases with continuous background opacity and an overlying continuum (e.g., Mihalas 1978, chap. 11). For the cases under study (e.g., Mg ii \( k \)) the problem is essentially linear, since the nonlinear coupling between the optical depth scale and the line radiation field is very weak. When multilevel effects are included, the problem can become highly nonlinear and to obtain a solution requires complex linearization procedures (Mihalas 1978, chap. 12).

For the case of a line which remains optically thin throughout the chromosphere (e.g., C ii \( \lambda 1335.4 \)) the emergent profile reflects directly the velocity field (“turbulence” and/or larger scale flows) in the regions where the photons are created, since the line profile (hence opacity) is dominated by these motions and, for cool stars with weak UV continua, \( S_\nu \) is dominated by the thermal source term \( eB_\nu \) alone. In this case the analysis is very much simpler than for optically thick resonance lines, since the problem is essentially independent of the radiation field in the line: the solution for \( \int d\nu \pi F_* \) (eq. [6]) in the limit \( \tau_p < 1 \) reduces to an integral of the local thermal source term over height, and is given by equation (1) (to within the factor of 2 discussed below).

The formal solution (eq. [6]) does not lend itself to simple manipulation, nor can it be used in “empirical” studies of emission lines, since both the source function and the optical depth scale must be determined. Furthermore, for optically thick lines \( S_\nu \) is dominated by the scattering term, and equation
The fraction \( f(z) \) must be estimated from detailed transfer calculations: the present study shows that the approximation \( f(z) \approx 1 \) yields correct emergent integrated fluxes for typical chromospheric lines. Assuming CRD and negligible background opacity, and integrating over frequency, the total cooling rate of the line is

\[
\Phi(z) = h \nu \{ N_j(z) [ A_{ji} + B_{ji} J(z)] - N_f(z) B_{ji} (z) \} = h \nu N_j(z) A_{ji} \rho_{ji}(z) ;
\]

therefore, we can write

\[
F_\star \approx h \nu \int dz \ N_j(z) A_{ji} \rho_{ji}(z) f(z) ,
\]

where \( N_j \) is the number density of the upper level of the transition and \( A_{ji} \) and \( B_{ji} \) are the Einstein \( A \) and \( B \) coefficients; \( \rho_{ji}(z) \), the usual “net radiative bracket” of the transition (e.g., Thomas and Athay 1961) is given by

\[
\rho(z) = 1 - \frac{J(z)}{S(z)} .
\]

Finally, from the statistical equilibrium equation for a two-level atom,

\[
N_j(z) C_{ij}(z) = N_f(z) [ A_{ji} \rho_{ji}(z) + C_{ij}(z) ] ,
\]

we obtain a generalized form of Athay’s result (eq. [1]),

\[
F_\star \approx h \nu \int dz \ N_j(z) C_{ij}(z) \left( 1 - \frac{N_j(z) C_{ij}(z)}{N_f(z) C_{ij}(z)} \right) f(z) ,
\]

where \( C_{ij} \) is the collisional rate (s\(^{-1}\)) from level \( i \) to level \( j \). In the effectively thin limit (discussed below), the bracketed term (the “net collisional bracket,” e.g., Mihalas 1978) goes to unity, and this expression is equivalent to Athay’s (1976) approximation (eq. [1]) provided that \( f(z) = 1 \).

The above equations may not at first appear to clarify the formation and interpretation of the emission lines, since in principle the transfer equation still has to be solved for \( N_j(z) \rho_{ji}(z) \). However, \( \rho_{ji}(z) \) can often be estimated much more accurately than either of the integrals of \( J \) or \( S \), over frequency individually (e.g., Mihalas 1978, p. 130), and the corresponding \( N_j \) solved using these values of \( \rho_{ji}(z) \). For instance, the above equations are closely related to the escape probability approximations for line transfer (as reviewed by Rybicki 1984), in which analytical approximations for \( \rho_{ji}(z) \) replace detailed calculations.

In the simplest “first-order” (or, equivalently, “local”) escape probability approximation, the formal solution for \( J \) in terms of \( S \) (the traditional \( \Lambda \)-operator) is replaced by a one-point quadrature (Rybicki, 1984, p. 45) such that

\[
J(z) \approx [ 1 - P^*(z) ] S(z) ,
\]

i.e. (from eq. [15]),

\[
\rho(z) \approx P^*(z) ,
\]

where \( P^*(z) \) is the local escape probability from a point at height \( z \) which can be estimated from analytic solutions to the standard transfer problem \( P^*(0) = 1 K_0(z_0) \), where \( K_2 \) is the function discussed by Avrett and Hummer 1965; see also Rybicki 1984 and the Appendix), or from scaling to numerical solutions in the case of partial redistribution (PRD) in frequency of absorbed and emitted photons (e.g., Adams 1972; Frisch 1984). Since the \( \Lambda \)-operator is local in this approx-
imination, this implies that photons, once they escape from a certain region in the atmosphere, do not interact further with the atmosphere, and they escape completely in a single flight. The importance of single-flight escape was first demonstrated for lines formed assuming that CRD is valid by Rybicki and Hummer (1969), following earlier work by Osterbrock (1962). Clearly the use of analytic expressions for \( \rho \) grossly simplifies the calculation of \( F_\lambda \).

The effectively thin and thick regimes are defined by \( P^e > \epsilon \) and \( P^e = \epsilon \), respectively, and the "thermalization depth" by \( P^e = \epsilon \). Under the assumption of effectively thin conditions, the cooling function \( \Phi(z) \) can be represented by the optically thin limit \( \tau_\nu \ll 1 \), which yields \( \bar{J}(z) = S(z) \) and \( \rho_j(z) = 1 \). Hence, in the effectively thin limit, simple line emissivities can be applied to estimate the emergent flux \( F_\lambda \) and cooling rates as if the line were optically thin. The above expressions then reduce to the usual transition-region or "coronal" limit (Jordan and Brown 1981):

\[
F_\lambda \approx f_{hv} \int dz \bar{N}(z) A_H,
\]

where the upper-level population \( \bar{N}(z) \) is determined from the statistical equilibrium equations assuming no incident (or self-created) radiation, i.e., where no radiative transfer has been treated. With this simplified expression one can proceed with, e.g., emission measure techniques or line-ratio diagnostics to derive useful information on the line-emitting regions, provided that one can physically interpret the various contributions to \( \bar{N}(z)dz \) as a function of height (see Judge 1987 and below). This approach has been successfully applied to solar and stellar transition regions for some time (see § I). Accurate transfer calculations are used in the next section to examine the usefulness of this model and to evaluate the use of approximate "escape probability" methods as applied to chromospheric lines. The restrictions of CRD and negligible background opacities are removed.

d) Exact versus Approximate Methods

Previous authors have compared escape probability methods with exact solutions; such work has been reviewed and discussed by Avrett and Loeser (1988) in the context of the total emergent line flux \( F_\lambda \) from broad-line regions of QSOs. This section makes similar comparisons in model chromospheres and discusses the relation between line cooling and the depths of formation of the emergent flux.

The exact and appropriate methods yield emergent fluxes given by equation (6) and equations (14), (17), and (20), respectively. These expressions are of a quite different nature: equation (6) expresses the summed contribution of the source of photons \( S_\nu \) at optical depth \( \tau_\nu \) to the emergent flux, whereas equations (14), (17), and (20) express the integral of the change in the radiative flux across localized regions. Global energy balance ensures that the integrals must be the same when self-consistent solutions of the transfer and statistical equilibrium equations are used in both equations.

The Appendix discusses the relationship between the formal solution for the emergent line flux (eq. [6]) and the cooling integral methods. There it is shown that the two methods yield the same emergent integrated fluxes (global energy balance). However, only in the case of the local escape probability approximation applied to the "standard problem" of line transfer (transfer of line radiation in a slab in which the line profile is independent of height with negligible background opacity) are the contributions to the line flux as a function of depth equal in both cases.

This equality has a simple physical interpretation: the number of photons escaping to the observer as evaluated by the mapping of the \( K_\nu \) function onto the source function (i.e., the frequency-integrated form of eq. [6]; Avrett and Hummer 1965) at height \( z \) corresponds to the number of photons leaving the volume element at that height, as evaluated by the cooling integral. This means that the line photons contributing to the cooling not only escape from the local region but also escape (on average) from the entire chromosphere. Such transfer properties are also approximately characterized by CRD transfer (Rybicki and Hummer 1969). However, for the more general problems encountered in stellar atmospheres where depth-dependent line profiles are present and nonlocal transfer effects may be important, there can be no such formal correspondence. This is because photons escaping from one depth can be absorbed at higher depths where the line absorption profile is physically unrelated to that where the photon was emitted. In addition, PRD transfer is characterized by diffusion of photons in space as well as in frequency (Adams 1972). In the general case detailed transfer calculations must therefore be used to identify whether the escape probability approximation is useful.

In order to compare the approximations based on the integrated line cooling with detailed transfer equations, first a suitable contribution function for the emergent line flux must be defined. Magain (1986) showed that, for the case of absorption lines, several definitions are possible, since the emergent intensity \( I_\nu \) and the observed line depression \( R_\nu = [I(\text{cont}) - I_\nu]/I(\text{cont}) \) can be formed at entirely different regions of the atmosphere \( I(\text{cont}) \) is the continuum intensity computed in the absence of the line). Magain proposed both intensity and "line depression" contribution functions \( (C_I \) and \( C_R \) which represent the contribution to the physical intensity and depression per unit of \( \Delta \log \tau(5000 \text{ Å}) \). For the purpose of interpreting the height of formation of a chromospheric emission line, the following procedure was adopted. Analogous to Magain's (1986) function \( C_I \), the formal solution for the emergent flux (eq. [6]) suggests that the integrand of equation (6) should be used for the flux contribution function \( C_I(t_r) \). Normalized to the log of the column mass, the contribution to the emergent line flux at frequency \( \nu \) between column mass \( m \) and \( m + dm \) is given by

\[
C_f, (\log m)(d \log m) = C_I(t_r) \frac{d t_r}{d (\log m)} d (\log m) = \left[ 2\pi E_2(t_r) S_\nu \ln \frac{d t_r}{d m} \right] d (\log m).
\]

However, note that no transfer equation for \( F_\lambda \) is equivalent to Magain's equations for \( C_I \) and \( C_R \), has been written to derive \( C_f, (\log m) \). \( C_f, (\log m)(d \log m) d \nu \) represents the contribution to the emergent flux between column masses \( m \) and \( m + dm \), and between frequencies \( \nu \) and \( \nu + d \nu \).

Test calculations have been made using representative lines formed in several static model chromospheres, including those of the Sun (by Vernazza, Avrett, and Loeser 1981, hereafter VAL) and of cooler stars of lower gravity (as reviewed by Linsky 1980, 1985). Here results are presented for the VAL 3C model of the Sun and the model of the K5 III star Tau of Kelch et al. (1978) using the program MULTI (Carlsson 1986, ...
The thermalization properties are very different, owing to the substantially reduced line-core escape probabilities and imprisonment of wing photons in PRD compared to the Voigt profile CRD case (Adams 1972; Milkey and Mihalas 1974). These effects, also clearly seen in the contribution functions, profiles, and cooling rates (Fig. 2a, 2b) as well as the source functions (Fig. 1) and emergent fluxes (Table 1), are emphasized by comparing the CRD limits of Voigt and Doppler core profile calculations.

2. The CRD cooling function and contribution function (Fig. 2b) correspond very closely to each other on a depth-by-depth basis, but the PRD calculations (Fig. 2a) do not. This is because the CRD transfer is dominated by single-flight escape in the regions contributing most to the line flux, but the PRD transfer is not (see § IIc and the Appendix). Photons generated near the peak of the cooling function (at column masses \(~5 \times 10^{-4}\) in the PRD case emerge from both shallower and deeper layers owing to the trapping of photons in the line core and diffusion in space (Adams 1972; Milkey and Mihalas 1974).

3. The fluxes evaluated from the PRD total cooling integral correspond very closely to the emergent flux profiles integrated between the \(k_l\) minima (Table 1). The height integration limits adopted extend from the top of the model to the "thermalization depth" in the line \(\Lambda_{th}\), identified here as the height at which the cooling function changes sign. The \(\lambda\) integration limits extend from line center to the \(k_l\) minimum. Hence, most of the photons which are emitted by Mg ii k above the thermalization depth in the chromosphere escape in the outward direction, i.e., the factor \(f(\lambda)\) is approximately unity throughout the region above \(\Lambda_{th}\), and is zero below it.

4. The cooling function is greatly overestimated when CRD, escape probability, and optically thin approximations are used. All of these approximations artificially enhance photon escape in deeper chromospheric layers, whereas in reality PRD effects inhibit diffusion of line-core photons into the transparent wings thereby trapping them near line center. For the Sun, therefore, PRD effects must be taken into account when interpreting all the properties (cooling as well as profiles) of chromospheric resonance lines. The reason for the large differences lies essentially in the different thermalization properties of the various approximations.

For the case of \(\alpha\) Tau, some of the conclusions are very different:

5. Very substantial differences exist between source functions computed in CRD and PRD, as found previously for lower gravity stars by Milkey, Ayres, and Shine (1975). Most important, the lines are not thermalized until photospheric depths are reached in this cool, low-gravity star. Unlike the solar case, the Mg ii k line is effectively thin in the entire model chromosphere, owing to the lower particle densities in the relatively inactive chromosphere.

6. Because the lines are effectively thin in the chromosphere, the cooling rates and the integrated line flux are quite accurately represented by the CRD, escape probability, and even optically thin approximations (Table 1 and Fig. 4). This is because the thermalization properties, although they differ substantially in each case, do not affect the cooling calculations in the chromosphere, since the major differences occur near photospheric regions which do not affect higher layers.

7. The PRD contribution function corresponds much more closely to the cooling function on a depth-by-depth basis for \(\alpha\) Tau than for the Sun. This initially surprising result (see, e.g.,
Fig. 2.—Flux contribution functions, cooling rates, profiles, and approximate cooling rates for Mg n k computed in the VAL 3C solar model. The contour plot shows the function $C_r(\log m)$ per unit log column mass and per unit frequency. The contribution function in the lower panel has been integrated over the frequency of the line core and shows the contribution to the integrated line flux per unit log column mass for comparison with the cooling function (also plotted in the same units). This plot was normalized to give the same area underneath both curves. (a) PRD calculation; (b) CRD (Voigt profile) calculation. Notice the approximate correspondence of the cooling function with the contribution function for the CRD case.
### TABLE 1
Representative Chromospheric Line Flux Calculations

<table>
<thead>
<tr>
<th>Line</th>
<th>Calculation</th>
<th>$\int dv \pi F_v$ (ergs cm$^{-2}$ s$^{-1}$)</th>
<th>$\Delta$ (Å)</th>
<th>$\int \pi \Phi_v$ (ergs cm$^{-2}$ s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mg $\text{ii} \ k$</td>
<td>PRD</td>
<td>3.0(5)$^b$ + 0.58</td>
<td></td>
<td>3.0(5)</td>
</tr>
<tr>
<td>Mg $\text{ii} \ k$</td>
<td>CRD</td>
<td>2.0(6) + 2.0</td>
<td>9.2(4)</td>
<td>2.4(6)</td>
</tr>
<tr>
<td>Mg $\text{ii} \ k$</td>
<td>CRD (Doppler)</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Mg $\text{ii} \ k$</td>
<td>Escape probability</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Mg $\text{ii} \ k$</td>
<td>Optically thin</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$\alpha$ Tau</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mg $\text{ii} \ k$</td>
<td>PRD</td>
<td>4.1(4) + 1.5</td>
<td></td>
<td>4.2(4)</td>
</tr>
<tr>
<td>Mg $\text{ii} \ k$</td>
<td>CRD</td>
<td>4.6(4) + 5.0</td>
<td></td>
<td>4.6(4)</td>
</tr>
<tr>
<td>Mg $\text{ii} \ k$</td>
<td>CRD (Doppler)</td>
<td>1.9(4) + 0.4</td>
<td></td>
<td>4.2(4)</td>
</tr>
<tr>
<td>Mg $\text{ii} \ k$</td>
<td>Escape probability</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Mg $\text{ii} \ k$</td>
<td>Optically thin</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>C $\text{i} \lambda3235.4$</td>
<td>CRD</td>
<td>2.1(3) + 0.25</td>
<td></td>
<td>3.3(3)</td>
</tr>
<tr>
<td>C $\text{i} \lambda1657$</td>
<td>CRD</td>
<td>5.8(1) + 1.0</td>
<td></td>
<td>2.6(2)</td>
</tr>
<tr>
<td>C $\text{i} \lambda1994$</td>
<td>CRD</td>
<td>3.3(1) + 0.2</td>
<td></td>
<td>7.7(1)</td>
</tr>
</tbody>
</table>

*$^a\Delta$ is the bandwidth over which the integration over wavelength of the emergent profile was made. For Mg $\text{ii} \ k$ this was chosen to extend from line center to the computed $k_1$ minimum feature.

*$^b3.0(5) = 3.0 \times 10^5$.

*$^c$ Pure absorption line owing to the artificially high intensities in the line wings.

Milkey, Ayres, and Shine 1975, who showed the greater importance of PRD effects on source functions and profiles in low-gravity stars) can be understood in terms of the overriding importance of the thermalization properties (point 6).

In addition, the following more general statements concerning chromospheric emission lines can be made:

8. Points 3 and 6 confirm the validity of the approximation proposed by Linsky and Ayres (1978) for the emergent line flux, provided that the thermalization properties can be adequately estimated for stars like the Sun.

9. Although the frequency-integrated contribution functions peak in a very narrow region near the top of the atmosphere (log $m \sim -5.1$ for the Sun; see point 12 below), the largest contributions to the integrated line flux in Mg $\text{ii}$ come from much deeper regions (log $m \sim -2.7$) with high line-center optical depths ($\tau_0 \approx 10^3$). This is reflected in the line profiles by a significant broadening of the emission components beyond the line-profile width because photons escape at frequencies nearer the line wings at line-center optical depths greater than unity.

10. The contribution and cooling functions are significant over large regions of the chromospheric models which extend over a factor of 100 in gas pressure. This has very important consequences concerning the interpretation of $F_\nu$ (§ III).

11. In general, the cooling functions $\Phi_v(\tau)$ do not correspond as a function of depth to the contribution functions summed over frequency ranges covering the emission-line cores. Therefore, although the net integrated flux of radiation is adequately reproduced by the cooling-integral methods, a depth-by-depth correspondence does not exist between the cooling and flux integrands. This demonstrates graphically that single-flight escape is not the dominant process determining the transfer of chromospheric radiation in resonance lines such as Mg $\text{ii} \ k$. This point deserves further attention following the early study of Adams (1972).

12. The regions influencing the profile near line center (the usual "$k_3$" component) are physically distinct from the deeper regions where the $k_2$ peaks are formed and the contribution to the emergent flux is highest. The emergent profile near line center is determined by the optical depth scale and source function in material at much lower column masses than the regions from which most of the photons contributing to the line emission originate. (The peak of the contribution functions at the top of the chromospheric models results from a combination of the weak coupling of the source function to the sharply increasing Planck function in the models’ transition regions and the peak of $E_2(\tau)$ for $\tau < 1$. Thermal emission in the optically thin background continua (dominated by the

![Fig. 3: Source functions and Planck functions for the Mg $\text{ii} \ k$ line in the Kelch et al. (1978) model chromosphere of a Tau, marked as in Fig. 1. Notice that both the PRD and CRD (Voigt profile) calculations thermalize ($S = B$) not in the chromosphere but in the photosphere: the emission line is effectively thin in the chromosphere, in both the PRD and CRD calculations. Hence the emergent integrated flux is independent of the details of the line transfer (see text).](image-url)
Fig. 4.—Flux contribution functions, cooling rates, profiles, and approximate cooling rates for Mg II k computed in the Kelch et al. (1978) model chromosphere of a Tau, marked as in Fig. 2. (a) PRD calculation; (b) CRD (Voigt profile) calculation.
Balmer continuum) contributes a small amount to the increase of the source function. There is no contribution to the emergent flux from material at smaller column masses, since the sharp increase in $T_e$ in the models forces the chromospheric lines to form deeper owing to ionization.

Calculations have also been made for the C II λ2325 multiplet (and similar intersystem multiplets) as an example of an optically thin chromospheric emission line. The conclusions concerning $F_r(\text{Mg II k})$ are very similar for the C II multiplet, the major difference being that the contribution and cooling functions correspond almost exactly on a depth-by-depth basis, as expected for an optically thin emission line. Calculations for a ζ Tau are given in Table 1. (Results are not given for the Sun, since photospheric continuum emission dominates the source function and hence the emergent flux.)

To summarize, the total cooling integral for resonance and intersystem chromospheric lines corresponds approximately to the emergent flux integral in these representative calculations. Hence, if the cooling integral (including the thermalization limit) can be estimated without solving for the source functions explicitly, then meaningful information can be derived from the emission-line flux without the need for detailed transfer solutions in model chromospheres, thereby permitting the application of simpler "empirical" techniques (e.g., emission measure analysis) to chromospheric integrated line fluxes.

e) Effective Thin or Effectively Thick Chromospheric Lines?

The above calculations reveal that a critical quantity determining the interpretation of $F_r$ is the effective thickness of the emission line, which depends on the thermalization length $\Lambda_{th}$. Although $\Lambda_{th}$ for CRD-type transfer is well defined ($S_i \rightarrow B_i$) and understood (Rybicki and Hummer 1969), this is not the case for lines when PRD-type transfer dominates, because $\Lambda_{th}$ depends strongly on the frequency in the line profile. As a first approximation, the present paper adopts $\Lambda_{th}$ as the point where the cooling function of the line, $\Phi(t)$, changes sign at large $\tau_0$, i.e., radiative cooling changes to heating. This criterion was chosen because (1) it involves quantities related to photon escape (see the Appendix) integrated over frequency (and 2) it represents a local approximation to the global problem of photon escape. Alternatively, analytical estimates for average escape probabilities could have been adopted [Adams 1972 found $P(\tau_0) \sim 1/\tau_0$ for lines where PRD effects dominate]. These points deserve further study.

Essentially two types of behavior were found:

1. In the solar chromospheric model the Mg II k line is effectively thick, hence Linsky and Ayres's (1978) approximation (eq. [2]) can be used. Effectively thin approximations substantially overestimate the emergent flux in Mg II k; errors made in using, e.g., escape probabilities or CRD are a factor of 10 or more. For the Sun and stars of similar chromospheric activity levels, the line flux depends critically on the thermalization properties. Hence an emission measure derived from a line flux has a restricted meaning: it represents the sum of the thermal sources above the thermalization depth, not the total sum of the sources in the chromosphere.

2. In the ζ Tau model the Mg II k line is effectively thin, hence the simple line emissivities can be used to estimate the emergent line flux. In stars as inactive as ζ Tau the Mg II k line is effectively thin throughout the chromospheric emitting regions. Therefore, $F_r$ is essentially independent of $\Lambda_{th}$ and a meaningful "chromospheric emission measure" can be derived from the line flux.

Questions then arise concerning the behavior of a given line (e.g., Mg II k) with stellar spectral type and chromospheric activity levels, and the behavior of different lines (e.g., resonance vs. intersystem lines) in a given photospheric model. In order to address these important questions, the chromospheric scaling laws of Ayres (1979) are adopted, together with approximations for thermalization lengths based on CRD and PRD escape probabilities. (Judge 1986b, 1987 has presented empirical evidence for the validity of these scaling laws in addition to the arguments presented by Ayres himself.)

In units of line-center optical depth $\tau_0$, thermalization lengths $\Lambda_{th}$ are approximately given by equating $P(\tau_0)$ with $\varepsilon$ (e.g., Avrett and Hummer 1965). With the usual approximations for escape probabilities (e.g., Rybicki 1984; Frisch 1984),

$$
\Lambda_{th} \sim \frac{1}{\varepsilon} \quad (\text{CRD}; \: \alpha_0 \ll 1),
$$

$$
\Lambda_{th} \sim \frac{\alpha}{\varepsilon^2} \quad (\text{CRD}; \: \alpha_0 \ll 1),
$$

$$
\Lambda_{th} \sim \frac{1}{\varepsilon} \quad (\text{PRD}; \: (\alpha_0)^{1/3} > 1),
$$

where $\varepsilon$ was defined earlier and $\alpha$ is the usual Voigt parameter. Converting to a column mass scale $m$ given by

$$
m \propto \frac{N_E N_{\text{ion}} N_i}{N_H N_E N_{\text{ion}} A_{ji}},
$$

where $N_E$/$N_{\text{ion}}$ is the element abundance, $N_{\text{ion}}/N_E$ the ionization fraction of the ion under study, and $N_i/N_{\text{ion}}$ the fraction of the ion in level $i$, the lower level of the transition, the column mass for thermalization $m_{th}$ for CRD (small $\tau_0$ limit) and PRD (the most common cases) reduces to

$$
m_{th} \propto \left( \frac{R_{ji} N_E N_{\text{ion}} N_i}{N_H N_E N_{\text{ion}}} \right)^{-1},
$$

where $R_{ji}$ is the smaller of $C_{ji}$ and $A_{ji}$. Typically, for UV lines the level $i$ is the ground level of a dominant stage of ionization [hence $(N_{\text{ion}}/N_E)(N_i/N_{\text{ion}}) \sim 1$], so that $m_{th}$ scales as

$$
m_{th} \propto \left( \frac{R_{ji} N_E}{N_{\text{ion}}} \right)^{-1}.
$$

This expression yields several results, depending on the value of $m_{th}$ relative to $m_*$ (the column mass of the temperature minimum region) and on the relative values of $C_{ji}$ and $A_{ji}$. For resonance lines $C_{ji}(\text{res}) \ll A_{ji}(\text{res})$, but for intersystem lines the ratio $C_{ji}(\text{int})/A_{ji}(\text{int})$ can take values both greater and less than unity. Typically $C_{ji}(\text{res}) \sim 10C_{ji}(\text{int})$ (e.g., Allen 1973).

Ayres (1979) has proposed that the location of the temperature minimum in column mass and the mean chromospheric electron densities should scale as

$$
m_* \propto \tilde{A}_F \tilde{F}_e^{1/2} g^{1/2} T_e^{1/2} \Sigma^{1/2},
$$

$$
\langle N_e \rangle \propto \tilde{A}_F \tilde{F}_e^{1/2} g^{1/2} T_e^{1/2} \Sigma^{1/2},
$$

where $\tilde{A}_F$, $\tilde{F}_e$, $g$, and $T_e$ are the stellar metal abundance, chromospheric heating parameter ($\sim 1$ for the quiet Sun; $\sim 10$ for active regions), gravity, and effective temperature, respectively. Since $C_{ji}$ with $\langle N_e \rangle$ weak temperature dependence ($\propto T_e^{-1/2}$), for a given line the thermalization depth (eq. [26]) relative to the depth of the temperature minimum region (eq. [27]) scales as

$$
\frac{m_{th}}{m_*} \propto (\tilde{F}_e^{1/2} \tilde{F}_e^{1/2} \Sigma^{1/2})^{-1} \propto F_r^{-1}.
$$
where \( F_{\text{tot}} \) (ergs cm\(^{-2}\) s\(^{-1}\)) is the total chromospheric heating rate above the temperature minimum region.

Hence, for more active regions on stars (i.e., higher chromospheric heating rates per unit area), thermalization occurs farther out in the chromosphere relative to the temperature minimum. This is precisely the result found in the detailed calculations above. It follows that effectively thick approximations must be applied to stars with chromospheric surface fluxes equal to and in excess of those of the Sun, and that effectively thin analyses (e.g., emission measures) can be sensibly applied only to stars of substantially lower fluxes. The critical surface flux at which the change from effectively thin to effectively thick lines occurs, measured in \( F_{\text{tot}}(\text{Mg } \hbar + k) \), lies near \( 2 \times 10^5 \) ergs cm\(^{-2}\) s\(^{-1}\) on the basis of calculations in a range of stellar model chromospheres (from Kelch et al. 1978 and earlier references). The star \( \beta \) Gem (KO III) is effectively thick in the Mg \( \perp \) resonance lines \( [F_{\text{tot}}(\text{Mg } \hbar + k) \sim 2.4 \times 10^4 \) ergs cm\(^{-2}\) s\(^{-1}\) but \( \alpha \) Boo (K1 III) is effectively thin \( [F_{\text{tot}}(\text{Mg } \hbar + k) \sim 1.7 \times 10^4 \) ergs cm\(^{-2}\) s\(^{-1}\)\]. This result has important consequences (see § IV).

f) Effects of Line Formation over Several Pressure Scale Heights

The results of § IIId emphasize graphically that chromospheric emission lines are quite unlike transition-region or coronal emission lines, because their fluxes have substantial contributions from regions of widely differing gas pressures (analogous to the equivalent width of a strong photospheric absorption line). A clearly observable consequence of this is the opacity broadening of optically thick lines leading to a general explanation of the Wilson-Bappu effect (Ayres 1979). There are additional important consequences.

For an effectively thin emission line with \( f = 1 \) we can write equation (17) as

\[
F_{\text{tot}} \approx \hbar \int_{\Delta z} dz N_i(z) C_i(T_i(z)),
\]

where

\[
C_i(T_i(z)) = 8.63 \times 10^{-6} \frac{\Omega_{ij}}{g_i} e^{-\hbar / kT_i(z)} T_i^{1/2}(z) N_i(z)
\]

and \( \Omega_{ij} \) is the usual “collision strength.” The point emphasized here is that \( \Delta z \) covers regions where line cooling is important, i.e., extending over several chromospheric pressure scale heights. This expression involves a convolution of atomic excitation rates [through \( C_i(T_i(z)) \)] and atmospheric structure \( [N_i(z)N_e(z)dz] \), where one cannot (without care or further information) remove any of the atomic or atmospheric components from the integral. In the case of stellar chromospheres \( N_i(z)N_e(z) \) will change by typically 2 orders of magnitude over the integration limits, and any small variation in \( T_i(z) \) over this region will produce large changes in \( C_i(T_i(z)) \). I conclude that, in the general case, one cannot apply standard plasma diagnostic techniques (such as temperature line-ratio diagnostics) to chromospheric emission lines without great care. Attempting to assign a single physical quantity to the ratio of two line fluxes without other constraints will therefore result in numbers which will generally be misleading. In general cases (exceptions are discussed below), model atmosphere techniques are required to deduce reliable information from chromospheric integrated line fluxes. The requirement of such techniques has been recognized for some decades as applied to the solar chromosphere (e.g., Thomas and Athay 1961).

Lines formed in transition regions (e.g., C iv \( \lambda 1550 \)) or coronae are fundamentally different because it is generally possible to identify the temperature regimes in which the lines are formed, since the \( N_e \) values are sharply peaked as a function of \( \log T_e \) (e.g., Judge 1987 and references therein), and meaningful separations of the atmospheric and atomic contributions in the above integral can therefore be made.

9) Nonplanar Geometries

The above calculations are based on one-component model atmospheres with the assumption of hydrostatic equilibrium in plane-parallel geometry. There is increasing evidence (e.g., from radio measurements [Drake and Linsky 1986], eclipsing binaries [e.g., Reimers 1987], and other arguments [see the review by Linsky 1985]) for “geometrically extended” chromospheres and spherical outflows and/or circulation patterns in the outer atmospheres of cool, low-gravity stars. Therefore, one should also examine the validity of the above conclusions in these models.

The best studied star for which a spherical outflowing chromospheric model is available is \( \alpha \) Ori (M2 Iab). This star has a relatively high mass-loss rate \( (dM_{\text{ej}} / dt \sim 10^{-6} M_\odot \text{yr}^{-1}) \) (e.g., Hartmann and Avrett 1984). It shows strong UV line asymmetries indicative of large departures from hydrostatic equilibrium at the levels of the atmosphere where the most opaque parts of the line profiles (e.g., Mg \( \perp \) k) are formed. Examination of Hartmann and Avrett’s (1984) model reveals several important, perhaps unexpected, conclusions:

Despite a large radial extent \( (\geq 10 R_\odot) \) of material at chromospheric temperatures \( (T \geq T_\odot) \), the integrated line fluxes of Ca ii and Mg ii, as computed from Hartmann and Avrett’s (1984) Figure 2 (showing the cooling as a function of height), are generated mostly in the region where hydrostatic equilibrium dominates the momentum balance, i.e., \( R / R_\odot \leq 2 \). The structure of the atmosphere in these regions is determined by the balance of gravity and “turbulence” in their model, and not by the velocity field gradient term \( \langle V (dV / dr) \rangle \) in the momentum equation. Hartmann and Avrett adopted a gravity which was a factor of 7 smaller than that in the plane-parallel, hydrostatic equilibrium model computed by Basiri, Linsky, and Eriksson (1981), which accounts for the difference in the geometrical extent of the two models in the hydrostatic regions. The bulk of the line cooling is confined to regions close to the stellar surface because the emission measure \( \langle J_\nu dh N_e N_i \rangle \) is dominated by the higher density regions, and not by an extended geometry (i.e., large \( \Delta H \)). For this reason, the integrated fluxes of UV emission lines may not be used to examine the relatively lower density regions associated with the extended geometry of the wind. Such regions must be examined using diagnostics sensitive to “optical depth” \( \langle \gamma \rangle dh N_e N_i \rangle \), such as profiles, rather than the diagnostics based upon integrated fluxes.

Scaling from these results, it is inferred that for the chromospheres of evolved stars with mass-loss rates comparable to or less than that of \( \alpha \) Ori, the extended geometry and velocity fields are critically important in determining the shape of the emergent profile—but not its integral over frequency. For \( \alpha \) Ori, the extended, outflowing gas serves mainly to scatter photons and redistribute them in frequency, rather than to create or destroy them.

Furthermore, an important selection effect exists which enhances the line optical depth without enhancing the line-integrated flux as the surface gravity of a star becomes lower:
Judge (1986b) showed empirically that Ayres's (1979) scaling laws adequately described the variation of chromospheric densities and column masses for representative giant stars between spectral types K2 III and M5 III. According to Ayres, the optical depths increase with lower gravity and the electron densities decrease (eqs. [27] and [28]). The integrated fluxes scale as the product of these parameters, which essentially cancel each other to first order. Therefore, it becomes progressively easier to see effects on the line profiles (via the increased optical depth), such as asymmetries caused by outflowing gas, rather than on the line-integrated fluxes, in lower gravity stars.

h) Chromospheric Inhomogeneities

Atmospheric inhomogeneities have been observed directly in the chromosphere of the Sun and inferred spectroscopically in chromospheres of cool stars (e.g., Heasley et al. 1978). The arguments made above in the context of one-component models will hold if the size of the individual emitting structures exceeds the representative line thermalization length. For smaller scale structures the escape of photons is enhanced, and the integrated line fluxes will only be substantially affected according to whether contributions to the cooling integral come from regions from which, in a homogeneous model, no substantial cooling could occur. This sensitivity of the line fluxes to inhomogeneous structure potentially provides a diagnostic of the scales of atmospheric inhomogeneities (e.g., Mihalas, Auer, and Mihalas 1978), and it deserves further attention.

III. APPLICATIONS AND CONSEQUENCES FOR PREVIOUS WORK

a) Chromospheric Electron Temperatures

Several authors have derived "mean" chromospheric electron temperatures $T_e$ using line-ratio techniques, essentially by examining the integrated flux ratios of lines whose collisional excitation rates $C_{ij}$ have different sensitivities to $T_e$. For example, Brown and Carpenter (1984) used the ratio $F_{\lambda 12325}/F_{\lambda 13355}$; Eaton and Johnson (1988) used ratios within certain Fe II multiplets. Judge (1986a, b) used a variety of lines of different elements, ions, and excitation energies to constrain temperatures using emission measure analysis.

The results of § II imply that, to apply this line-ratio technique, a meaningful "mean" temperature can be derived only if the contribution functions [equivalently $\Phi(\lambda)$ for optically thin lines] overlap; otherwise the lines are formed in physically separate parts of the atmosphere. However, the practical use of this type of analysis in chromospheres is dubious, and the meaning of the derived temperatures is unclear because (1) chromospheric contribution functions cover a wide range of temperatures (a factor of 2 is typical), and (2) if the contribution functions do overlap, then the line ratios necessarily also have a relatively weak dependence on temperature. Finally, to assess points (1) and (2), one needs to have a working model chromosphere, exactly what one is attempting to avoid in this type of initial study.

When many lines are available whose contribution functions cover a wide range of excitation temperatures, the above problems still remain. However, by using emission measure analysis for a sample of effectively thin lines (Judge 1986a, b) the temperature regimes are better defined than for the two-line ratio case, and one has greater confidence that the lines have contribution functions which do actually overlap (e.g., Judge 1986a,

Figs. 2 and 5). In support of this argument, Judge's (1986a, b) emission measure calculations for α Boo and α Tau also show that the derived temperatures correspond to those of the peaks of the cooling functions in model computations of the hydrostatic chromospheres of Ayres and Linsky (1975) and Kelch et al. (1978), confirming that the emission measure techniques yield, for these stars, correct chromospheric temperature estimates, at least in the context of these models.

b) Chromospheric Particle Densities

The comments concerning electron temperatures also apply, in principle, to the determination of particle densities. Several studies have been made using line ratios to estimate "mean" electron densities, for example, using lines within the C II j $\lambda 12325$ multiplet (Brown, Ferraz, and Jordan 1981; Stencel et al. 1981; Carpenter, Brown, and Stencel 1985; Judge 1986a, b; Lennon et al. 1985; Byrne et al. 1988; Eaton and Johnson 1988). These determinations rely on the fact that the optically thin cooling (or emissivity) decreases with increasing electron density when $C_{ji} \geq A_{ji}$, hence the emergent fluxes depend on the electron density (eqs. [14] and [16], with $p_{ii} = 1.0$).

Since chromospheric line contribution functions extend over several pressure scale heights, electron densities might be expected to change by several orders of magnitude ($N_e \sim N_{fe} \sim \rho_{fe}^{-1}$) (e.g., Bohm-Vitense 1986), and therefore the attempts to derive a "mean" are essentially meaningless. However, although total particle pressures (mainly neutral hydrogen) vary according to this law in hydrostatic equilibrium, the electron pressure is a minor contributor to the total pressure, and the ionization equilibrium of hydrogen (which is the primary source of the electrons) yields electron densities which are almost constant with height throughout the chromosphere (e.g., Ayres 1979). A similar situation also occurs in the spherical outflowing chromospheric model of α Ori of Hartmann and Avrett (1984).

The relatively constant value of electron density with height in available chromospheric models results from the derived thermal structures, which reflect the average heating as a function of height. As argued by Ayres (1979), the electron density ultimately controls the chromospheric energy balance—a constant value of $N_e$ implies that the energy input per gram of material (equivalently, per particle) is approximately constant with height. This is true for the VAL models of the Sun from the chromosphere through to the corona.

In conclusion, there are good physical reasons to believe that "mean" chromospheric electron densities can be derived from line ratios of intersystem lines such as C II j $\lambda 12325$, and their ratios with permitted transitions (Judge 1986a, b). However, "mean" densities of other species (e.g., neutral hydrogen and any atomic or molecular species) cannot be derived using simple line-ratio techniques, since these particle densities change by several orders of magnitude over the line flux generation regions. This argument also shows that attempts to derive "mean ionization fractions" (Eaton and Johnson 1988) will lead to misleading results. Estimating particle densities other than for electrons really requires model chromosphere calculations.

c) Geometric Sizes and Velocity Fields of Chromospheric Emitting Regions

Recently controversy has existed concerning the geometric extent of the chromospheric emitting material around cool stars, as derived from integrated line fluxes and electron tem-
temperature and density estimates based on line ratios (see the discussion by Judge 1987). Since these analyses are based on integrated line fluxes, we can apply the theory discussed in § II, e.g., to the C \( n \) \( \lambda 2325 \) intersystem multiplet. We can write (assuming, for illustration only, that the lines are effectively thin and are excited by electron collisions from the ground term): \( F_\star(\text{C } n \lambda \lambda 2325) \approx \frac{1}{2} \nu \int dz N_i C_{ij} \), (32)

where \( f = \frac{1}{2} \) was assumed. To interpret this further, we derive an emission measure locus which represents the amount of emitting material as if the line were formed at one temperature \( T_e \):

\[
EM(T_e) = \int dz N_i N_H = \frac{F_\star}{(8.63 \times 10^{-6} \nu \Omega_i / g_i) g(T_e) N_i / N_H},
\]

where

\[
g(T_e) = \frac{N_i}{N_{\text{ion}}} N_E e^{-\nu/K T_e T^{-1/2}}.
\]

Judge (1988) has emphasized fundamental problems in the interpretation of these emission measures, revealing shortcomings in the analysis of C \( n \) \( \lambda \lambda 2325 \) fluxes of Carpenter, Brown, and Stencel (1985), who argued that chromospheres of giants later than \( \sim K0 \) are extended (i.e., \( \Delta z \sim R_\star \)). Since UV emission lines from chromospheres always have \( \nu / K T_e \gg 1 \), emission measures rise steeply with decreasing electron temperature, and line fluxes are generated over regions where the ground-level populations \( N_i \) increase very rapidly (exponentially) inward, leading to a very large contribution to the integral for \( F_\star \) at relatively low temperatures (\( \sim 5000 \) K). This fundamental property means that \( EM(T_e) \) cannot simply be divided by the product \( N_i N_H \) to yield a geometric extent \( \Delta z \), since \( N_H \) is varying rapidly over the emitting volume. Other authors (e.g., Eaton and Johnson 1988) have interpreted Judge's (1986a, b) conclusions of smaller geometric extents (\( \Delta z \sim \lambda \)) for line profiles. Since UV emission lines of practical use at present (e.g., lines of C \( i \), O \( i \), and Fe \( ii \)) the excitation mechanisms are not known or are poorly determined (e.g., Judge 1987). As mentioned in § IIIa, a crucial requirement is that the contribution functions do overlap, i.e., that the fluxes are generated in the same regions of the atmosphere.

For reasons given above, it is difficult to assess the meaning of such “mean” quantities, since the flux contribution functions are so broad and, although less important, for many of the lines of practical use at present (e.g., lines of C \( i \), O \( i \), and Fe \( ii \)) the excitation mechanisms are not known or are poorly determined (e.g., Judge 1987). As mentioned in § IIIa, a crucial requirement is that the contribution functions do overlap, i.e., that the fluxes are generated in the same regions of the atmosphere.

Figure 5 shows the flux contribution functions computed for (1) the 1657 Å and (2) the 1936 Å lines of C \( i \) in the model chromosphere of \( \alpha \) Boo, assuming CRD for the radiative transfer calculations. (In this model the line excitation is driven by electron collisions, with an artificially enhanced collision strength to produce a reasonably strong emission line above the continuum. The excitation of this line is not presently known, but it is not important for the present study.)
Fig. 5.—Flux contribution functions, cooling rates, and profiles for the opacity-sensitive C i lines as an example of lines which share a common upper level (1657.4, 1993.6 Å), marked as in Fig. 2. CRD and Voigt profiles were used in the calculations. Note that the line fluxes are formed over substantially different regions of the model chromosphere. Also marked is the column mass which would be derived from the escape probability method using the computed line flux ratio. (a) 1657.4 Å; (b) 1993.6 Å.
marked on the plot is the "mean" column mass, which is derived from the computed line ratio (Table I) using the escape probability formalism. From the figure it is clear that (a) the two line fluxes are formed over substantially different regions of the chromospheric model (the weaker 1933.6 Å line is formed much deeper), but that (b) the "escape probability" method using line ratios does indeed yield sensible mean column masses which are characteristic of the regions where much of the emergent fluxes are generated.

The success of the simple escape probability formalism can be accounted for by the importance of single-flight photon escape in the transfer of resonance line radiation assuming CRD. PRD effects will change the absolute values of the derived column masses, since the mean escape probability is determined not by single-flight escape but by diffusion of photons in space and frequency. Compared with CRD calculations, the approximate PRD escape probabilities are higher than for the pure Doppler profiles or Voigt (CRD) profiles (eqs. [22]–[24]). Hence, a given observed line ratio (e.g., C i λ19396/λ196574) will require higher column mass values than predicted using the CRD/Doppler profile escape probabilities.

IV. DISCUSSION AND CONCLUSIONS

This paper has been concerned with the formation of chromospheric emission lines, mainly in the ultraviolet region of the spectrum, in cool stars from dwarfs like the Sun to cool giant stars such as α Tau (K5 III) and supergiants such as α Ori (M2 Iab). Specific problems addressed include the interpretation of emergent line fluxes using both model atmosphere and more empirical techniques, the radiative losses of such lines, and the behavior of these with stellar spectral type and level of activity. By adopting "exact" and approximate solutions to the transfer problem, a correspondence between flux contribution functions and the cooling integral has been found which provides a link between the formal transfer methods and those based on simple line emissivities (e.g., effectively thin emission measure techniques) or escape probability approximations.

A crucial factor in determining the emergent line flux is the "effective thickness" of the chromosphere in a given line. Adopting the chromospheric scaling laws of Ayres (1979) together with estimates for thermalization lengths reveals that for relatively active stars like the Sun, the important resonance lines of Mg ii (h and k) are effectively thick, whereas for inactive giant stars such as α Tau the lines are effectively thin. These estimates are confirmed by detailed calculations.

In the case of the Sun, a full partial redistribution calculation is required because the thermalization properties are very sensitive to the nature of the redistribution and trapping of photons in the line core. Simple effectively thin and even complete redistribution approximations substantially overestimate the emitted flux and line cooling rates, for optically thick lines. Therefore, emission measure and similar empirical techniques should not be applied to optically thick chromospheric lines in such stars with great care. Full model calculations are required to interpret the chromospheric line flux and compute radiative loss rates for comparison with theory.

For stars less active than, e.g., β Gem (KO III), characterized by \( F_\text{L}(\text{Mg} \, \text{II} \, h + k) \lesssim 2 \times 10^8 \text{ ergs cm}^{-2} \text{ s}^{-1} \), thermalization occurs so deep in the chromosphere that the integrated line flux and cooling rates are insensitive to the details of the transfer. Hence, simple effectively thin calculations can be used to estimate both the emergent line fluxes and the cooling rates. This simplification will grossly simplify model hydrodynamical calculations requiring reliable estimates for radiative losses in relatively inactive stars (e.g., Ulmschneider and Stein 1982; Cuntz 1987). It also confirms the validity of applying emission measure techniques to noncoronal stars (Judge 1986a, b).

These results will have important ramifications for the interpretation of the so-called flux-flux relations (e.g., Ayres, Marstad, and Linsky 1981; Oranje 1986), in which the integrated fluxes of chromospheric lines (e.g., Mg ii h + k) scale empirically with the flux of transition-region lines, to the power of \( \sim \lambda \). The different thermalization properties of the Mg ii h and k lines will result in a redistribution of radiative loss channels (for example, the cooling effects of Ca ii H and K and Fe ii lines may be enhanced; see Anderson and Athay 1989), for certain levels of chromospheric heating. This will be the subject of a future paper. In addition, the simplest model proposed by Cram and Giampapa (1987) for the interpretation of Ca ii H and K profiles in dwarf stars should be reexamined.

Several problems are encountered when applying standard plasma diagnostic techniques (e.g., line-ratio temperature and density diagnostics) to stellar chromospheres. These problems essentially result from the nature of stellar chromospheres themselves rather than in specific cases. Chromospheres are quite unlike transition regions because of the resiliency of the electron density \( N_e \) owing to the partial ionization of hydrogen, and the effect of \( N_e \) on strong radiative coolants such as Mg ii k. This results in a chromospheric "temperature plateau" (and line-forming layer) covering several gas pressure scale heights (e.g., Athay 1981). Line-ratio techniques, even in the case of inactive stars with effectively thin chromospheres, should be used with great care, since emergent fluxes are complicated convolutions of atomic excitation physics and atmospheric structure (and radiative transfer effects, for the more active stars). Unfortunately, some authors have applied such techniques to obtain "first estimates" of the temperatures and densities of stellar chromospheres without taking into account the important line formation physics, leading to incorrect and misleading results. However, there are good physical reasons to believe that meaningful chromospheric electron densities can be derived using suitable diagnostics (e.g., ratios within the Ca ii H-and K multiplet; Stencel et al. 1981; Lennon et al. 1985).

It is perhaps surprising that the interpretation of chromospheric emission-line integrated fluxes in relatively inactive stars has been found to be simpler than in more active stars like the Sun. This results entirely from the scaling of thermalization lengths versus the scaling of the thickness of the chromosphere, as measured by the column mass at the temperature minimum. In contrast, interpretation of the emergent line profile becomes very complex: Basri (1980) found that, owing to the extreme dominance of scattering in the (frequency-dependent) PRD source function and the very weak coupling to the thermal source of photons, the interpretation of emission-line profiles such as Mg ii k becomes very difficult and nonunique in stars with very low gravities, particularly the cool supergiants. In principle, the methods described here should also apply to such stars, but in practice it may not be possible to estimate the thermalization properties sufficiently reliably to apply the approximate cooling integral methods and thereby derive emission measures. This problem could usefully be addressed observationally by comparing the fluxes of optically thick and thin chromospheric emission lines (e.g., Mg ii k and Al ii) 22670), which, for inactive stars like α Ori, should be effectively thin and hence in the ratio of their respective emissivities.
Finally, there exists an additional important selection effect: cool stars of low gravity exhibit stronger spectral features resulting from enhanced opacity, such as line asymmetries and evidence for extended geometry and winds, than do stars of higher gravity. This effect will certainly contribute to the overall behavior of spectral features observed as a function of position in the H-R diagram (e.g., Dupree and Reimers 1987).

I wish to thank M. Carlsson, E. Avrett, and D. Luttermoser for very useful discussions, and J. Linsky, R. Stencel, D. Luttermoser, and especially T. Ayres and I. Hubeny for very valuable comments on the original manuscript. This work has been supported by NASA grants NAG5-985 and NGL 06-003-057 to the University of Colorado.

APPENDIX
DEPTHS OF FORMATION AND COOLING OF EMISSION LINES

Here the line formation is considered assuming CRD in a phase-parallel atmosphere with a depth-independent line profile, paying attention to the cooling integral and the formal solution for the emergent flux.

With the usual definitions (see Avrett and Hummer 1965 and the review by Rybicki 1984), the $K_1$ and $K_2$ operators are given by

\[ K_1(t) = \frac{1}{2} \int_{-\infty}^{\infty} dx E_1(\tau \phi(x)) \phi^2(x), \]  
\[ K_2(t) = \int_{-\infty}^{\infty} dx E_2(\tau \phi(x)) \phi(x), \]  

where $E_1$ and $E_2$ are the usual first and second exponential integrals and $\phi(x)$ is the absorption profile at frequency $x$ (x in Doppler units). The optical depth scale is defined as the mean optical depth $\tau = \tau_0/\phi(0)$. The zeroth and first moments of the transfer equation are given by the traditional $A$- and $\Phi$-operators (e.g., Kourganoff 1963), yielding expressions for the mean intensity $\bar{J}(\tau)$ and frequency-integrated Eddington flux $H(\tau)$:

\[ \bar{J}(\tau) = \int_0^T dt K_1(|t - \tau|) S(t), \]  
\[ H(\tau) = \frac{1}{2} \int_0^T dt K_2(|t - \tau|) \text{sign}(t - \tau) S(t), \]  

where $\text{sign}(t - \tau)$ is +1 for $t > \tau$, 1 for $t < \tau$.

In addition, since $K_1(t)dt$ represents the probability of a photon being absorbed between depths $\tau$ and $\tau + dt$, the total escape probability is given by

\[ P(\tau) = 1 - \int_0^T dt K_1(|t - \tau|), \]  
\[ = \frac{1}{2} \left[ K_2(\tau) + K_2(T - \tau) \right], \]  

where the latter equality is from Hummer (1964). The terms $\frac{1}{2}K_2(\tau)$ represents the mean probability of photon escape in the line in the outward direction.

The flux emerging from the top of the atmosphere computed from the cooling integral is $-\Delta H$, where

\[ \Delta H = H(\tau = T) - H(\tau = 0) = \int_0^T dt [\bar{J}(t) - S(t)] \]  
\[ = \int_0^T dt \left\{ \left[ \int_0^T dt' K_1(|t - t'|) S(t') \right] - S(t) \right\}. \]  

Since $t$ and $t'$ are independent dummy variables to be integrated over the atmosphere, the order of integration can be changed to give

\[ \Delta H = \int_0^T dt' \left\{ \int_0^T dt K_1(|t - t'|) S(t') \right\} - \int_0^T dt S(t) \]  
\[ = \int_0^T dt' \left[ S(t') \int_0^T dt K_1(|t - t'|) \right] - \int_0^T dt S(t). \]  

From the equality of Hummer (1964) (eqs. [A5] and [A6] above), this yields

\[ \Delta H = \int_0^T dt' S(t') [1 - \frac{1}{2} \left( K_2(\tau) + K_2(T - \tau) \right)] \]  
\[ = -\frac{1}{2} \int_0^T dt' S(t') [K_2(\tau) + K_2(T - \tau)]. \]  

© American Astronomical Society • Provided by the NASA Astrophysics Data System
The formal solution (eq. [A4]) yields
\[
\Delta H = \frac{1}{2} \left\{ \int_0^T dt S(t)[ -K_2(t - T') - K_2(t)] \right\}
\]
\[
= -\frac{1}{2} \int_0^T dt S(t)(K_2(t) + K_2(T - t)).
\]  
(A10)

Hence the formal solution (A10) and cooling integral (A9) methods yield the same net change in the radiative flux, as required by energy conservation.

The contribution functions to the radiative flux from the cooling integral and formal solution are, in general, quite different, and are given by the integrands of equations (A7) and (A10), respectively (i.e., dh/dt'). The order of integration of equation (A7) cannot, of course, be changed analogous to equation (A8), since this would yield dh/dt'. When depth-dependent profiles and PRD effects are taken into account, then these equations and the interpretation of depths of formation and line cooling become more complex.

Useful insight can be gained by investigating the first-order escape probability approximation in equation (A7), where S(t') is assumed to be varying slowly with respect to the kernel K_2(t - t'), so that
\[
\int_0^T dt K_2(t - t')S(t') - S(t) \approx \left( \int_0^T dt K_3(t' - t) \right)[S(t) - S(t')],
\]  
(A11)

and then the cooling integral (A7) reduces to
\[
\Delta H = -\frac{1}{2} \int_0^T dt S(t)(K_2(t) + K_2(T - t)).
\]  
(A12)

with the same integrand and hence contribution function as the formal solution (eq. [A10]). Hence in the local escape probability approximation, the contribution function for the emergent flux is identically equal to the cooling function. The contribution function is the product of the single-flight escape probability (eq. [A6]) and the source function. This is consistent with physical intuition, since the adopted mathematical approximation implies that strict single-flight escape dominates the line transfer. Hence photons escaping from a local region (evaluated by the cooling function) escape to infinity and contribute to the total emergent flux. Since CRD transfer is predominantly determined by single-flight escape (Rybicki and Hummer 1969), then the contribution function should also correspond to the cooling function, as found in the text.

REFERENCES


© American Astronomical Society • Provided by the NASA Astrophysics Data System


P. G. Judge: Joint Institute for Laboratory Astrophysics, P.O. Box 0440, University of Colorado, Boulder, CO 80309