THE UPPER BOUNDARY OF THE SOLAR CONVECTION ZONE: HYDRODYNAMICAL ASPECTS

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ABSTRACT Using spectrograms of high spatial resolution we measured the horizontal rms velocity of the granulation at different depths in the photosphere. We find a steep vertical gradient of the horizontal velocity indicating strong dissipation in the first 100 km above \( \tau = 1 \). Using the boundary layer concept we estimate the dissipation to be 10% of the total energy. Beyond 200 km, granulation triggers gravity waves. The turbulent viscosity is estimated to be \( 10^{11} \text{ cm}^{-2} \text{ sec}^{-1} \).

The granulation is a characteristic observable phenomenon which defines the upper boundary of the solar convection zone. Related to the granulation are convective motions which occupy this region and control, thus, their hydrodynamics. Granular velocities and their height gradients determine, furthermore, the nonradiative energy and momentum flux in these layers. The dynamics of the deep photospheric layers is strongly affected by the height gradients, \( \frac{\partial V_z}{\partial z} \) and \( \frac{\partial V_x}{\partial z} \), of the horizontal and vertical granular velocity, respectively. Their significance for the transport of momentum and energy in these layers is described by the Navier-Stokes equation and the energy equation.

In the present investigation we are especially interested in the height gradient of the horizontal velocity \( \frac{\partial V_x}{\partial z} \), which determines the friction force and controls the associated dissipation losses in the photospheric layers. The friction force is given by \( \tau_{xz} = \eta \left( \frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x} \right) \), where \( \eta \) represents the viscosity. For the present purpose we can assume safely that the vertical velocity \( V_z \) remains practically constant over the mean horizontal size of a granulum (\( \approx 1000 \text{ km} \)), so that \( \tau_{xz} = \eta \frac{\partial V_x}{\partial z} \). Associated with this friction force is an energy dissipation per unit time and area given by the product \(-V_z \tau_{xz}\) of the horizontal velocity \( V_x \) and the friction force \( \tau_{xz} \). Thus, due to the viscosity, part of the energy associated with horizontal granular velocities dissipates in the upper boundary layers of the solar convection zone, thus affecting the thermodynamic as well as the hydrodynamic state of these layers.

Our study is based on granular velocities and their variations with height in the photosphere as measured by Nesis and Mattig (1989). Fig. 1 shows the gradient of the horizontal velocity \( \frac{\partial V_x}{\partial z} \) as a function of height \( z \) in the solar photosphere. The three curves correspond to the following determination of the horizontal velocity: the theoretical calculation of Nelson and Musman (1977), the semiempirical calculation of Keil and Canfield (1978) and the empirical estimation by Nesis and Mattig (1989).

These three gradients differ most markedly at the height of 200 km above \( \tau_{5000} = 1 \): here, the theoretical gradient (NM) shows a maximum, whereas the empirical gradient (NeMa) decreases rapidly with height, in contrast to the slowly varying semiempirical
gradient (KC).

Recalling that the gradients reflect the friction forces $\tau_{xz}$ and the dissipation losses $-V_x\tau_{xz}$ in the photospheric layers, the different behaviour of the gradients signifies different locations of the dissipation losses in these layers: the dissipation losses related to the semiempirical gradient (KC) are distributed over the entire photosphere up to 400 km above $\tau_{5000} = 1$. By contrast, the losses associated with the theoretical and empirical gradients are localized at about 200 km (NM) and within the first 100 km (NeMa) above $\tau_{5000} = 1$, respectively.

We now concentrate on the empirical gradient (NeMa) in fig. 1. Here, the overshoot of the horizontal convective motions implies large horizontal shear stresses $\tau_{xz}$ and dissipation losses $-V_x\tau_{xz}$, both are concentrated within a small region ($\leq 100$ km) above $\tau_{5000} = 1$. In the region between 100 — 200 km above $\tau_{5000} = 1$ the motions of granular origin disappear and gravity waves are initiated (Staiger, 1987).

![Graph of vertical gradient of horizontal velocity](image)

**Fig. 1** (left) Vertical gradient of the horizontal velocity $V_x$ as a function of height $z$, above $\tau_{5000} = 1$, according to three different authors. KC: Keil, S.L., and Canfield, R.C. (*semiempirical*); NM: Nelson, G.D., and Musman, S. (*theoretical*); NeMa: Nesis, A., Mattig, W. (*empirical*). Spacial velocity gradients correspond to characteristic times, which are given on the rhs of the figure.

**Fig. 2** (right) Kinematic viscosity $\nu$ as a function of height in the photosphere.

The velocity gradient $\frac{\partial V_x}{\partial z}$ is a local quantity with a dimension of $sec^{-1}$. It assigns to every height $z$ in the photosphere a time rate ($sec^{-1}$), which characterizes the transport processes at this height. The characteristic time $t_{\text{char}}$ associated with this time rate is shown on the right hand side of fig. 1. Small time scales (related to dissipation) characterize the layers up to 100 km above $\tau_{5000} = 1$ (cf. fig. 1). Here, the granular intensity-velocity correlation disappears also (Nesis et al., 1988) and the temperature fluctuations $\Delta T$ associated with the granulation diminish drastically (Kneer et al., 1980). On the basis of very different arguments, Atroschenko et al. (1989) arrived at a similar subdivision of the deep photospheric layers.

For an estimation of the dissipation losses in these layers we used the *boundary layer* concept. The concentration of a large horizontal shear stress in a small region supports this choice. The *boundary layer* concept is commonly used to describe the behaviour of real fluids near rigid boundaries, where the horizontal velocity changes rapidly in vertical
direction (Tritton, 1977). In the solar atmosphere the convectively stable photospheric layers form a "rigid" boundary for the convectively unstable layer underneath, as far as hydrodynamical stability is concerned. Consequently, the horizontal velocity of the convective flow has to decrease with height in the photosphere and to "vanish" at the convectively stable layers. This is the case in the overshoot layers as can be seen in fig. 1 (NeMa).

In a boundary layer the inertial and viscous forces are of the same order (Tritton, 1977). Therefore, the convective and viscous terms in the energy equation also have to be comparable. The convective term of the energy equation associated with the vertical velocity $V_z$ is

$$\frac{\partial}{\partial z} \left[ \rho V_z (c_p \Delta T + \frac{1}{2} V^2) \right].$$

With $V_z = 1.0 \text{ km sec}^{-1}$ (Nesis and Mattig, 1989) or $1.4 \text{ km sec}^{-1}$ (Komm, 1989) and temperature fluctuations $\Delta T = 600 K$ (Kneer et al. 1980) we estimate the convective term $\left[ \rho V_z (c_p \Delta T + \frac{1}{2} V^2) \right]$ in the $\tau_{5000} = 1$ layers. We find a convective energy flux associated with the vertical motion $V_z$ of 8—10% of the total energy. In the case of an energy "equipartition" between the horizontal and vertical motions (somewhere) in these layers, the horizontal convective term of the energy equation $\frac{\partial}{\partial z} \left[ \rho V_z (c_p \Delta T + \frac{1}{2} V^2) \right]$ has to be of the same order.

Since in a boundary layer the convective and viscous terms in the energy equation are comparable we conclude that the horizontal viscous flux $\frac{\partial}{\partial z} (V_z \tau_{xx})$ is again of the order of 8 — 10% of the total energy flux. Thus, in the overshoot region there is not only a convective energy flux associated with the vertical motions, but also a significant convective energy flux (presumably again 8—10%) of the total) associated with the horizontal velocities. Part of this energy is radiated in the stable photospheric layers. However, due to the viscosity, another part of the energy associated with the horizontal motions is dissipated. This affects the thermodynamical state of these layers and the process of line formation (Komm et al., 1989).

The kinematic viscosity $\nu = \frac{a}{\tau}$ of the overshoot layers, which is associated with the dissipation, can be estimated using the comparability of the inertial force $V_z \frac{\partial V_z}{\partial z}$ and the viscous force $\nu \frac{\partial^2 V_z}{\partial z^2}$. Fig. 2 shows the variation of $\nu$ with height $z$ in the photosphere. The viscosity exceeds $10^{11} \text{ cm}^2 \text{ sec}^{-1}$ up to 100 km above $\tau_{5000} = 1$. As a consistency check the turbulent kinematic viscosity can be estimated by $\nu_{turb} = u \cdot l$. For a characteristic velocity $u = V_z \approx 0.8 \text{ km sec}^{-1}$ and a characteristic length $l = 100 \text{ km}$ we find $\nu_{turb} = u \cdot l \approx 10^{11} \text{ cm}^2 \text{ sec}^{-1}$ — the same value was already derived by Schwarzschild (1959) for the photosphere.

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