THE STEADY-STATE SOLUTIONS OF RADIATIVELY 
DRIVEN STELLAR WINDS FOR A NONSOBOLEV, PURE- 
ABSORPTION MODEL

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ABSTRACT We summarize the reasons for the principle 
conclusion of the Poe et al. paper: To the extent that a pure-
absorption model is applicable, radiatively driven stellar winds have 
no well defined steady state. Using the nonSobolev, pure-
absorption radiation force from Owocki et al., we find that the 
solution topology at the sonic point is a node, not a saddle or "x" as 
in the solar case. The number of transonic solutions increases from 
one unique solution for the "x" type to a range of solutions for the 
ode type topology. Thus, in the pure-absorption approximation, 
line driven winds can have a range of possible mass-loss rates and 
terminal velocities.

INTRODUCTION

In this paper, we examine the existence of a unique, well defined, steady state 
for a radiatively driven wind. In problems of outflow from stars, a unique 
mas-loss rate and velocity structure is determined by the condition that the flow 
is smooth from the subsonic flow near the photosphere to the supersonic flow 
at large radii. Finding the steady solution is reduced to an analysis near a 
critical point where the solutions from the photosphere are matched onto the 
solutions from large radii.

Previous models based on the Sobolev approximation (Castor, et al. 1975; 
CAK) and based on a more complete Co-Moving Frame (CMF; Pauldrach, et 

al. 1986) calculation found solutions that shows generally good agreement. In 
the CAK model, the solution is unique, requiring the solution to satisfy a 
regularity condition at a critical point. The CMF description of the radiation 
force is so complex, however, that a critical point analysis is not possible and 
we cannot test for the uniqueness of the solution.
Owocki et al. (1988; OCR) developed a simple description of the radiation force that does not make the Sobolev approximation. They assumed that the radiation force is given by the pure absorption approximation and, similar to CAK, the lines are distributed as a power law in opacity. We will use this force to analyze the solutions near the sonic point and then look for the smooth, transonic solutions. For more details, see Poe et al. (1989; POC).

THE EQUATIONS

The equation of motion for an isothermal, spherically-symmetric, steady wind is given by:

$$\frac{d v}{d r} = \frac{\frac{2a^2}{r} - \frac{GM_{\text{eff}}}{r^2} + g_{\text{rad}}}{v - \frac{a^2}{v}} = \frac{N}{D} \quad (1)$$

The forces included are gas pressure, gravity, and line radiation. The radiation force is given by (see OCR),

$$g_{\text{rad}} = \frac{K L_*}{r^2} \left( \frac{v_{\text{th}}}{c} \right) \alpha \int_{-\infty}^{\infty} d x \phi \left( x - \frac{v}{v_{\text{th}}} \right) \eta^{-\alpha} \quad (2)$$

where $\eta$ is the profile-weighted mass column depth at each frequency given through the differential equation,

$$\frac{d \eta}{d r} = \rho \phi \left( x - \frac{v}{v_{\text{th}}} \right) \quad (3)$$

$\phi$ is the (gaussian) profile function.

The ratio of the thermal speed of the ion to the sound speed, $v_{\text{th}}/a$, depends only on the atomic weight of the ion driving the wind and is independent of the gas temperature. In the Sobolev limit, $v_{\text{th}}/a \to 0$ while OCR assumed $v_{\text{th}}/a = 0.5$. Since C, N, and O primarily drive the winds of O stars, $v_{\text{th}}/a$ should be approximately 0.3.
CRITICAL POINT ANALYSIS

The critical point occurs in the flow when $D = 0$ (or $v = a$) in equation 1. The transonic solution must pass through this point with $N = 0$ for a smooth transition. Figure 1 shows the solutions, or topology, of equation 1 in a small region around the sonic point. Figure 1a shows the situation for the solar wind when $\text{grad} = 0$. Only one unique solution passes through the sonic point smoothly.

If, instead, $\text{grad}$ is given by equation 2, the topology near the sonic point are nodal as shown in figure 1b. Now there appears to be a range of solutions that pass through the sonic point. At the sonic point, the solution can have either one of two slopes. The steeper slope is well defined and unique. However, the smaller slope at the sonic point has a degeneracy of solutions in both the subsonic and the supersonic regions.

We identify the steady-state solution found by OCR as the steeper slope solution in the nodal topology. In the Sobolev/CAK limit ($v_{th}/a \to 0$), the steeper slope becomes infinite at the sonic point and the smaller slope solution becomes the CAK solution. For the realistic value of $v_{th}/a = 0.3$, the slope at the sonic point is always the smaller, CAK-type slope.

Figure 1 -- The solution topologies near the sonic point for a) the saddle or "x" type topology of the solar wind ($\text{grad} = 0$), and b) the nodal type topology of the pure absorption radiation force in equation 2. The "x" topology has one unique transonic solution while the nodal topology has a continuous range of possible transonic solutions. The ordinate is proportional to $v-a$ and the abscissa is proportional to $r - r_c$. 

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The radiation force in equation 2 assumes that the star is a point source to the wind. For this steady-state model calculation, we can improve the radiation force by including the finite disk of the star. The topology at the sonic point is still nodal and for realistic wind parameters, the smaller slope (degenerate) solution is still the correct solution.

CONSEQUENCES OF THE DEGENERATE SOLUTION

The subsonic region of the flow specifies the mass-loss rate. Because of solution degeneracy, the wind can have a continuous range of mass-loss rates. Similarly, for a given mass-loss rate, there is a range of solutions that extends into the supersonic part of the flow, each with a different terminal velocity. Consequently, we have lost our ability to determine a unique mass-loss rate and terminal velocity to the radiation-driven wind problem.

Figure 2 shows the range of mass-loss rates and terminal velocities allowed with the pure absorption line force. For a given mass-loss rate, the upper limit on the terminal velocity is the terminal velocity of the steeper slope solution. The larger the mass-loss rate, the smaller the velocity that the radiation can push the wind. If the mass-loss rate is too high, the wind material does not reach infinity. All values of the mass-loss rates and terminal velocities to the upper right (hatched region) are not allowed.

In addition to the problem of finding a unique steady-state solution, we have to question the stability of the steady state. As shown in Owocki et al. (1989; this volume), the degenerate steady-state solutions appear to be unstable in a time dependent model.

REFERENCES

Figure 2 -- The range of allowed mass-loss rates and terminal velocities (solid curve) for \(v_{\text{th}}/a = 0.3\). The range of values in the hatched region are not allowed because the wind does not reach infinity. The "X" denotes the CAK value for the same parameters. The dashed curve is the Sobolev/CAK model with zero sound speed.