MODEL ATMOSPHERES OF HOT STARS: A RESCALING METHOD

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ABSTRACT A simple method is suggested that overcomes convergence difficulties encountered in calculating non-LTE model atmospheres of hot stars. The method is based on a rescaling of the radiative as well as heating/cooling rates by removing analytically the overwhelming number of scatterings and retaining only terms of the order of thermal absorptions and emissions.

1. Introduction

The method of complete linearization, introduced by Auer and Mihalas (1969), has been proven one of the most powerful methods for calculating model stellar atmospheres, taking into account departures from local thermodynamic equilibrium (LTE). However, its application is frequently plagued by convergence problems. In some cases, these problems can be overcome for instance by using the recalculation of level populations and radiative rates in certain transitions by means of the equivalent-two-level-atom approach (Hubeny 1981; 1988), or by using the collisional-radiative switching technique, developed recently by Hummer and Voels (1988). Yet, in some cases even these methods fail.

The reason is usually connected with the very nature of the non-LTE transfer of radiation. In strong resonance lines, or resonance continua of most abundant ions, both radiative as well as heating/cooling rates are numerically very large, but the corresponding net rates are by many orders of magnitude smaller. Physically, this follows from the fact that the individual rates count all the elementary processes of absorption and emission, which means that all the processes of scattering (an absorption immediately followed be a re-emission) are counted as well. However, the scatterings do not contribute to the net rates. Yet, the typical situation in stellar atmospheres is that of overwhelming dominance of the processes of scattering over the true creations/destinations of photons.

In the linearization, the upward and downward (and heating and cooling) rates are linearized separately, so that relatively small errors in the individual rates may easily yield disastrous errors in net rates. The situation is further worsened if such a strong resonance transition has a wavelength much shorter than the wavelength of the maximum of the Planck function, because the radiation intensity is extremely sensitive to very small changes in temperature, and even more so if the given transition is furthermore essential for determining the ionization balance of the corresponding atom.

The obvious cure of such a situation is to make the individual rates
numerically much smaller by subtracting analytically the contribution of scatterings. This idea is not new. For instance, Rybicki (1972) has introduced the so-called core saturation method, which is based on the same reasoning. However, this method is not well suited for an application in the complete linearization method. Another related approach was used in spherical model atmosphere calculations by Mihalas and Hummer (1974). They employed the idea of representing the radiative and heating/cooling rates by much smaller numerical values, but their approach did not employ an analytical subtraction of scatterings. I shall therefore generalize this approach and outline the idea of a quite general rescaling method.

2. Formulation

Let us consider the transition $i \rightarrow j$, which may be a line or a continuum. Let us define a frequency averaged mean intensity of radiation

$$\bar{J}_{ij} = \int_0^\infty J_\nu \varphi_{ij}(\nu) d\nu,$$

where $J_\nu$ is the mean intensity of radiation, and $\varphi_{ij}(\nu)$ is the normalized profile coefficient, defined by

$$\varphi_{ij}(\nu) = \frac{1}{\sqrt{\pi \Delta \nu D}} H\left(a, \frac{\nu - \nu_D}{\Delta \nu D}\right) \text{ for lines},$$

where $\Delta \nu D$ is the Doppler width, $H(a, x)$ the Voigt function; and

$$\varphi_{ij}(\nu) = \frac{1}{b_{ij}} \frac{4\pi}{h\nu} \sigma_{ij}(\nu) \Theta(\nu, \nu_D) \text{ for continua},$$

where

$$b_{ij} = \int_{\nu_D}^\infty \frac{4\pi}{h\nu} \sigma_{ij}(\nu) d\nu,$$

Here, $\Theta(x, x_0)$ is the Heaviside function, defined such that $\Theta = 1$ for $x \geq x_0$, and $\Theta = 0$ for $x < x_0$. $\nu_D$ is the line center frequency in the case of line, and the edge frequency in the case of continuum; $\sigma_{ij}(\nu)$ is the corresponding cross-section.

The quantity $J$ may always be written (formally) as

$$\bar{J}_{ij} = \beta_{ij} \bar{J}_{ij} + (1 - \alpha_{ij}) \bar{S}_{ij},$$

where

$$\bar{S}_{ij} = \int_0^\infty S^j_\nu \varphi_{ij}(\nu) d\nu,$$

i.e. the frequency averaged source function of the given transition. The latter is given by $S^j_\nu = \eta^j_\nu / \kappa^j_\nu$, i.e. by the ratio of the emission and absorption coefficient corresponding to the given transition. Obviously, in the case of lines, $S^j_\nu$ is independent of frequency, and therefore $\bar{S}_{ij} = S^j_\nu$.

Equation (5) is formal, i.e. for each $\beta$ there is a corresponding $\alpha$ that satisfies Eq. (5); the choice $\alpha = \beta = 1$ corresponds to the standard approach. I will consider the question of the most advantageous choice of $\alpha$ and $\beta$ in the next section.
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Radiative rates may be written generally as \((i < j)\)

\[ R_{ij} = b_{ij} \int_0^\infty \varphi_{ij}(\nu) J_\nu \, d\nu = b_{ij} J_{ij}, \tag{7} \]

and \(R_{ji} = R_{ji}^A + R_{ji}^B\), with

\[ R_{ji}^A = b_{ij} \int_0^\infty \varphi_{ij}(\nu) g_{ij}(\nu) \frac{2h\nu^3}{c^2} \, d\nu, \tag{8} \]

\[ R_{ji}^B = b_{ij} \int_0^\infty \varphi_{ij}(\nu) g_{ij}(\nu) J_\nu \, d\nu, \tag{9} \]

and where

\[ g_{ij}(\nu) = g_i/g_j \quad \text{for lines,} \tag{10} \]

\[ g_{ij}(\nu) = n_e \phi_s(T) \exp(-h\nu/kT) \quad \text{for continua,} \tag{11} \]

All quantities have their standard meaning (Mihalas 1978); the quantity \(b_{ij}\) is given by Eq. (4) for continua, and \(b_{ij} = B_{ij}\) for lines; \(B\) being the Einstein coefficient. Eqs. (8) and (9) yield for lines \(R_{ji}^A = A_{ji}\), and \(R_{ji}^B = B_{ji} J_{ji}\); \(A_{ji}\) and \(B_{ji}\) being the Einstein coefficients for spontaneous and stimulated emission, respectively.

Using Eqs. (5), (7) - (11), we find that

\[ n_i R_{ij} - n_j R_{ji} = n_i \beta_{ij} R_{ij} - n_j \left[ \alpha_{ij} R_{ji}^A + \beta_{ij} R_{ji}^B \right]. \tag{12} \]

In other words, the set of statistical equilibrium equation remains exactly the same, replacing only

\[ R_{ij} \rightarrow \beta_{ij} R_{ij}, \quad R_{ji} \rightarrow \beta_{ij} R_{ji}^B + \alpha_{ij} R_{ji}^A. \tag{13} \]

Finally, the radiative equilibrium equation is written as

\[ \int_0^\infty (\kappa_\nu J_\nu - \eta_\nu) \, d\nu = 0, \tag{14} \]

where \(\kappa_\nu\) and \(\eta_\nu\) are the total absorption and emission coefficients (not including electron scattering). Applying now the same idea as above, we find that Eq. (14) may be written as

\[ \int_0^\infty \left( \tilde{\beta}_\nu \kappa_\nu J_\nu - \tilde{\alpha}_\nu \eta_\nu \right) \, d\nu = 0, \tag{15} \]

where

\[ 1 - \tilde{\beta}_\nu = \frac{\sum \kappa_{ij}^p}{\kappa_\nu} (1 - \beta_{ij}), \tag{16} \]

\[ 1 - \tilde{\alpha}_\nu = \frac{\sum \eta_{ij}^p}{\eta_\nu} (1 - \alpha_{ij}). \tag{17} \]

In the above equations, the sum extends over all transitions \(i \rightarrow j\).

3. Rescaling parameters \(\alpha\) and \(\beta\).
Equations (13) and (15) provide the desired rescaling of radiative and heating/cooling rates. The remaining problem is an appropriate determination of the parameters $\alpha$ and $\beta$. For a two level atom, the line source function is given by $S = (1 - \epsilon)J + \epsilon B$; it is therefore natural to take $\beta = \epsilon$. Applying now the idea of the equivalent-two-level-atom formalism, one may write for any transition, line or continuum, in a multilevel atom

$$\tilde{S}_{ij} = (1 - \tilde{\epsilon}_{ij})\tilde{J}_{ij} + \delta_{ij},$$

and, therefore,

$$\beta_{ij} = \tilde{\epsilon}_{ij},$$

and, as follows from Eq. (5).

$$\alpha_{ij} = 1 - (1 - \tilde{\epsilon}_{ij})\tilde{J}_{ij}/\tilde{S}_{ij}.$$

This means that we obtain a strong cancellation of terms down to order $\epsilon$, which essentially means that only pure thermal terms are retained, while the contribution of scatterings was analytically removed - i.e. precisely what we wished to achieve.

The calculations now proceed as follows. The parameters $\alpha$ and $\beta$ for all transitions chosen to be represented by the rescaling approach are determined in the formal solution step. These parameters are then held fixed during linearization. After a completed iteration of complete linearization, they are recalculated again in order to maintain consistency with the current model parameters.

Experience shows that the above procedure is able to overcome convergence difficulties due to He II Lyman continuum, as well as due to C IV and N V resonance lines, in hot star model atmospheres essentially completely. Results of model calculations will be reported elsewhere.