USE OF Al\textsc{XII} AND Mg\textsc{XI} LINES AS SOLAR PLASMA DIAGNOSTICS

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Abstract. We present three sets of observations of \( n = 1 \) to \( n = 2 \) lines due to helium-like aluminium (Al\textsc{XII}), made during two solar flares (25 August, 1980 and 19 October, 1986), using the X-Ray Polychromator on the SMM satellite. The observed temperature-sensitive line ratio \( G \) is shown to be consistent with the close-coupling calculations of Keenan and McCann (1987), although the ratio \( R \), which is both temperature and density-sensitive for lower-\( Z \) elements, is not sufficiently well determined from these data to say more than that the observed values of \( R \) are not inconsistent with the theoretical calculations. This region of the spectrum also includes the helium-like magnesium (Mg\textsc{XI}) \( 1^1S - 3^1P \) line, and it is shown that the ratio of this line to the Al\textsc{XII} resonance \( (1^1S - 2^1P) \) line is a more sensitive indicator of electron temperature than are the Al\textsc{XII} \( G \) and \( R \) ratios. We demonstrate that the three ratios may be used together in order to derive values of emission measure, electron temperature and electron density during these flares.

1. Introduction

Lines due to helium-like ions in the soft X-ray spectrum of solar active regions and flares have been studied extensively over the past twenty years with the aim of using the diagnostic capability of their intensity ratios. The principal \( n = 1 \) to \( n = 2 \) lines are referred to as the resonance (\( r \)) \( 1s^2 1^1S - 1s2p^1P \) transition, the intercombination (\( i \)) \( 1s^2 1^1S - 1s2p^3P \) transition, and the forbidden (\( f \)) \( 1s^2 2^1S - 1s2s^3S \) transition. Gabriel and Jordan (1969) showed that, for lower-\( Z \) elements, such as oxygen and neon, the intensity ratio \( R = f/i \) is a strong function of electron density \( (N_e) \); there is also an electron temperature \( (T_e) \) dependence, which in principle may be found from the flux ratio \( G = (f + i)/r \), a weak function of \( T_e \). For heavier elements in solar plasmas, the \( R \) ratio is observed to be nearly equal to a low density limit. A number of theoretical calculations of these ratios as functions of \( T_e \) and \( N_e \) have been made since Gabriel and Jordan’s paper (see, for example, Keenan et al., 1987, and references therein). The most accurate calculations are currently those using the close-coupling method (Burke and Robb, 1975), and these include the effect of auto-ionising resonances in the collisional excitation cross-sections which, in particular, make large contributions to the \( i \) and \( f \) line intensities. Recently, Keenan and McCann (1987) presented the results of their calculations of the \( G \) and \( R \) ratios for Al\textsc{XII}, comparing them to observations of two solar flares (Phillips et al., 1982; McKenzie et al., 1985). The measured \( G \) ratios were seen to be consistent with the theoretical value at the temperature of maximum Al\textsc{XII}
emissivity of $7.9 \times 10^6$ K. However, Keenan and McCann point out in their paper that the value of $R$ ($\simeq 2.0$) obtained by Phillips et al. would imply that $\log N_e \approx 12.7$ (in c.g.s. units) yet Phillips et al. deduce from the absence of certain Fe XXI and Fe XXII lines that $\log N_e$ must be $\leq 12$. In this paper we look again at the flare of 25 August, 1980; the region of the spectrum between 7.6 and 8.0 Å is examined closely and compared with simulations (see Figure 4 and Section 4) generated assuming the theoretical $G$ and $R$ ratios presented by Keenan and McCann together with line emissivities published by Mewe, Gronenschild, and van den Oord (1985).

A line due to the transition $1s^2 1S_0 \rightarrow 1s3p^1 P_1$ in helium-like magnesium (Mg XI) is also present in this region and it became apparent in the course of producing these simulations that the intensity ratio of the resonance line of Al XII and this Mg XI line was sensitive to $T_e$ (see Section 3). In order to study the simulation more objectively, a program was written to fit Voigt profiles to the lines present using an automatic least-squares curve-fitting algorithm (see Section 4). As expected, it was found that the Mg XI/Al XII ratio could be derived with greater precision than the Al XII $G$ and $R$ ratios and yielded more precise values of $T_e$. The next step was to employ the Mg XI/Al XII ratio in conjunction with the $G$ and $R$ ratios in a program which fitted for emission measure, $T_e$ and $N_e$ simultaneously, combining the information provided by all three ratios to give well-determined electron temperatures and emission measures, together with a (poor) estimate of electron density (this might be better determined in cases where $R$ is below the low density limit, corresponding to densities above about $10^{12}$ cm$^{-3}$).

2. The Observations

The three spectra examined here were recorded by the Flat Crystal Spectrometer (FCS), part of the X-Ray Polychromator instrument, on board the Solar Maximum Mission (SMM) satellite. Instrument details are given by Acton et al. (1980). The region under discussion was covered on a channel which uses an ADP crystal ($2d = 10.64$ Å) as dispersing agent, and covers the range 7.3–10.1 Å. The first spectrum was recorded during the decay of an M1 flare which occurred on 25 August, 1980, with the FCS scanning and accumulating data for 0.512 s at every point, with a step size of $\sim 0.0017$ Å. The total time taken to scan the region 7.6–8.0 Å was $\sim 2$ min.

The other two spectra were recorded in the early stages of the decay of a long-lived M5 flare which occurred on 19 October, 1986. In these two cases the spectrum was sampled in wavelength steps of $\sim 0.0010$ Å, accumulating counts for 0.256 s. The total time taken to scan 7.6–8.0 Å was approximately 100 s. The time which elapsed between recording these two spectra was approximately 14 min.

The region of the spectrum containing the lines which were measured spanned about 0.15 Å. This took about 40 s to record, during which time the flare intensity declined only slowly; any change in the plasma parameters which would have occurred between recording the first and last lines in the 7.6–8.0 Å region was considered small enough to be neglected in the data analysis.
3. Atomic Data

3.1. Mg\textsc{xi}/Al\textsc{xii} Line Ratios

Mewe et al. (1985) have published tables of line emissivities for He-like ions. Figure 1 shows these data for Mg\textsc{xi} and Al\textsc{xii}, plotted as log (emissivity) against log $T_e$. The two curves are very similar, which suggests that the emission observed in these two lines may be taken to come from the same volume of plasma. However, although the two curves are very similar, they cross near the peak-emissivity values, rendering the line ratio near the peak very sensitive to $T_e$. The variation of line ratio with log $T_e$ is also presented in Figure 1, clearly showing this enhanced sensitivity for values of log $T_e$ between about 6.6 and 7.1, corresponding to a range of temperature approximately $4 \times 10^6$–$12 \times 10^6$ K. Above this range the ratio is insensitive to temperature but at lower temperatures the ratio can still be used as long as there is sufficient emission to measure the line fluxes. This dependence on temperature can be compared with the $G$ ratio curve.

![Figure 1](image_url)  

Fig. 1. Curves of log (emissivity) for the Al\textsc{xii} resonance line and the Mg\textsc{xi} line, together with a curve of the flux ratio of these two lines, all as functions of log $T_e$. 

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for \( \text{Al XIII} \) calculated by Keenan and McCann (1987) and reproduced here in Figure 2. Considering the \( \log T_e \) range 6.4–7.0, \( G \) varies correspondingly from about 1.0–0.74, whereas the \( \text{Mg XI/Al XIII} \) ratio varies between 2.5 and 0.8, an increase in sensitivity of about a factor of 6. Moreover, the \( G \) ratio involves the weaker \( f \) and \( i \) lines, making the errors of measurement greater. So we can expect about an order-of-magnitude improvement in the precision of \( \log T_e \) using the \( \text{Mg XI/Al XIII} \) ratio as a temperature diagnostic.

3.2. Data Used in the Fitting Programs

The wavelengths shown in Table I were taken from the relativistic calculations of Ermolaev and Jones (1974). Wavelengths of the \( \text{Al XIII} \) lines calculated by Vainshtein and Safronova (1978) are about 0.0008 Å less than these values, whilst those of Safronova (1981) are up to 0.0008 Å larger. Note that the intercombination line \( i \) is, in fact, made up of both the \( 1^1S_0 - 2^3P_1 \) line (\( i_1 \) in Table I) and the magnetic-quadrupole line \( 1^1S_0 - 2^3P_2 \) (\( i_2 \)), and that for helium-like ions, \( i_2 \) has an intensity relative to line \( i_1 \) which increases with increasing \( Z \). From the line ratio calculations of Keenan and McCann (1987) for \( \text{Al XIII} \), \( i_2/i_1 \) was determined to be 0.19.

<table>
<thead>
<tr>
<th>Line</th>
<th>Transition</th>
<th>Wavelength (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Al XIII} ) ( \tau )</td>
<td>( 1s^2 1S_0 - 1s2p^1P_1 )</td>
<td>7.7571</td>
</tr>
<tr>
<td>( \text{Al XIII} ) ( i_2 )</td>
<td>( 1s^2 1S_0 - 1s2p^3P_2 )</td>
<td>7.8036</td>
</tr>
<tr>
<td>( \text{Al XIII} ) ( i_1 )</td>
<td>( 1s^2 1S_0 - 1s2p^1P_1 )</td>
<td>7.8067</td>
</tr>
<tr>
<td>( \text{Mg XI} )</td>
<td>( 1s^2 1S_0 - 1s3p^1P_1 )</td>
<td>7.8503</td>
</tr>
<tr>
<td>( \text{Al XIII} ) ( f )</td>
<td>( 1s^2 1S_0 - 1s2s^2S_1 )</td>
<td>7.8719</td>
</tr>
</tbody>
</table>

There also exist a number of dielectronic satellite lines of \( \text{Al XI} \) within this wavelength region. The intensities, \( I_{\text{sat}} \), relative to the resonance line of \( \text{Al XIII} \), \( I_r \), were calculated using the expression

\[
I_{\text{sat}} = \frac{16000}{T_e} e^{5.34 \times 10^6/T_e} F_{\text{sat}}.
\]

The values of \( F_{\text{sat}} \) and the wavelengths, \( \lambda_{\text{sat}} \), of the satellites (taken from Vainshtein and Safronova, 1978) are shown in Table II. The satellite notation is that of Gabriel (1972).

4. Data Analysis

4.1. The Simulation Program

On entry to the program, the user is asked to supply values for the electron temperature \( (T_e) \), electron density \( (N_e) \) and emission measure \( (\text{e.m.}) \), together with the required
TABLE II
AI XI satellites

<table>
<thead>
<tr>
<th>Satellite</th>
<th>$\lambda_{sat}$ (Å)</th>
<th>$F_{sat}$ (s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>j</td>
<td>7.8759</td>
<td>7.30</td>
</tr>
<tr>
<td>k</td>
<td>7.8716</td>
<td>4.77</td>
</tr>
<tr>
<td>q</td>
<td>7.8465</td>
<td>4.0$^a$</td>
</tr>
<tr>
<td>m</td>
<td>7.7994</td>
<td>1.29</td>
</tr>
<tr>
<td>n</td>
<td>7.7958</td>
<td>0.566</td>
</tr>
<tr>
<td>n = 3</td>
<td>7.765</td>
<td>5.0$^a$</td>
</tr>
<tr>
<td>n $\geq$ 4</td>
<td>7.775</td>
<td>5.0$^a$</td>
</tr>
</tbody>
</table>

$^a$ These values estimated, not taken from Vainshtein and Safronova (1978).

spectral resolution. The spectrum was simulated over the range 7.6–8.0 Å, generating five lines in all. The wavelengths and transitions for these lines are shown in Table I (Section 3.2). The flux (photons cm$^{-2}$ s$^{-1}$) at the Earth of each line is determined from the product of an emissivity (photons s$^{-1}$ per unit emission measure) and the emission measure (e.m.), divided by 4$\pi$ (astronomical unit)$^2$, where

$$e.m. = \int N_e^2 \, dV,$$

Fig. 2. The line ratio $G((f + i)/r)$ vs log $T_e$, from the atomic calculations of Keenan and McCann (1987), for AlXII.
and the integral is performed over the emitting volume \( V \). The emissivities of the Al\(\text{XII} \) resonance (\( r \)) line and the Mg\(\text{XI} \) 1\(s^2\) \(1S_0 \) – 1\(s3p^1P_1 \) line are straightforward to estimate because they do not have a density dependence. These emissivities for temperature \( T_e \) were obtained by linear interpolation between tabular values taken from Mewe et al. (1985); multiplication by e.m. yielded the fluxes. From these, the intercombination (\( i \)) and forbidden (\( f \)) transition line fluxes, which do have a density dependence, were derived using the ratios

\[
G = (f + i)/r \quad \text{and} \quad R = f/i,
\]

with \( i = (i_1 + i_2) \) and \( i_2/i_1 = 0.19 \) (see Section 3.2).

The function \( G(T_e) \) was approximated by:

\[
G = 0.53625 (\log T_e)^2 - 7.68575 (\log T_e) + 28.25261.
\]

This was obtained by fitting a quadratic to the values calculated by Keenan and McCann in the region \( T_e = 6.6 \)–7.2. The function \( R(T_e, N_e) \) was also taken from calculations performed by Keenan and McCann. These values were approximated by the following function:

\[
R = \frac{1.944 + 5.7 \times 10^{-8} T_e}{2.59 \times 10^{13}} \left(4.995 - 0.577 \log T_e\right) + 1
\]

![Log Ne vs Ne](image)

Fig. 3. For Al\(\text{XII} \), the line ratio \( R(f/i) \) vs \( \log N_e \), for a series of values of \( \log T_e \). The curves represent the function \( R(N_e, T_e) \) used in the computer simulation and fitting; the triangles correspond to the points calculated by Keenan and McCann (1987).
where 

\[
\text{COR}(T_e) = \begin{cases} 
0 & \text{for } \log T_e \leq 7.1, \\
0.3(\log T_e - 7.1) & \text{for } \log T_e > 7.1.
\end{cases}
\]

These functions, together with the calculated values of Keenan and McCann are shown in Figures 2 and 3.

The line shapes were taken to be Voigt profiles, with the instrumental component a Lorentzian, to which the rocking curve of the FCS ADP crystal approximates, and with the Gaussian component resulting from the thermal Doppler profile for temperature \( T_e \). The Lorentzian half-half-width was calculated as a function of wavelength using a linear approximation (over the range 7.3–8.5 Å) to the rocking-curve obtained from pre-launch measurements on the ADP crystal, while the Gaussian half-1/e-width \( g_{\text{wid}} \) was determined, for each line, from

\[
g_{\text{wid}} = \lambda \sqrt{\frac{2kT_e}{mc^2}},
\]

where \( m \) is the atomic weight of the ion; \( \lambda \), central wavelength of the line; \( k \), Boltzmann’s constant; and \( c \) the speed of light.

A linear background was added to the spectrum before displaying the results. Figure 4 shows two examples of simulated spectra, superimposed on the 25 August data.

4.2. The line-fitting programs

Two fitting programs were developed; the first used a Voigt function to fit the line shapes and will optimise \( T_e \) and the line fluxes to give a best fit in the least-squares sense; in the second program the fitting function is formulated in terms of \( N_e \), \( T_e \), and e.m., but again using a Voigt line shape. Both programs derive from the same root, a nonlinear optimisation program written by Fraser and Suzuki (1973) to fit infra-red spectra. The optimisation method used is based on an algorithm by Marquardt (1963); it is a modified Gauss-Newton nonlinear least-squares optimisation technique. This method is described in more detail by Bromage (1982).

The program has been modified to weight the data points individually. Initially the weighting function is obtained from the raw data counts, then after the first cycle it is set proportional to the fitted function at each point. A facility is provided to specify ranges of data which will be zero-weighted, so that spurious effects may be ignored.

Both programs can also fit a linear background, using the background values at the two extremes of the wavelength range as parameters to be optimised. The only differences between the two programs are the form and parameters of the function fitted to the data.

In either program any of the parameters may be fixed at their initial values and may be allowed to vary in a second or subsequent fit. The fitting function may optionally include a number of satellite lines whose fluxes will be calculated as functions of \( T_e \).

As well as a routine to evaluate the fitted function, the programs require a routine which will provide derivatives of the function with respect to each of the parameters.
A routine was written to calculate these derivatives using a finite difference method, incorporating a test for the validity of the result.

The progress of the fit and the final result may be displayed graphically, if required.

Fig. 4. Two simulations of the flare spectrum of 25 August, 1980: (a) using $T_e = 7.9 \times 10^6$; (b) using $T_e = 6.0 \times 10^6$. The simulated spectrum is shown as a dashed line superimposed on the observed data.
5. Results and Discussion

Figure 4 shows two simulations of the 25 August, 1980 data: the first uses values of $T_e$ ($7.9 \times 10^6$ K) and $N_e$ ($5.0 \times 10^{12}$ cm$^{-3}$) deduced by Keenan and McCann (1987); the second shows the effect of reducing $T_e$ to $6.0 \times 10^6$ K and $N_e$ to below $10^{12}$ cm$^{-3}$. Figure 4 reveals that the flare temperature of $7.9 \times 10^6$ K is probably an overestimate. From Figure 3 it can be seen that if the temperature is reduced then the corresponding curve of $R$ vs log$N_e$ is shifted down in value. The value of $R$ expected for a density consistent with the observations of the Fe xxii and Fe xxiii line ratio (which suggest that log$N_e$ is $< 12$) then lies within the error bars of the observed value. This is borne out by the improvement obtained in Figure 4(b) of the simulation using $T_e = 6.0 \times 10^6$ K. The agreement between observation and the calculations of Keenan and McCann is well demonstrated by the results presented in this figure.

It is clear from these simulations that the determination of $T_e$ is heavily dependent on the ratio of the two resonance lines, partly because these two lines are stronger and, therefore, may be better determined than the remaining Al xii lines, and partly because, as explained in Section 3 and Figure 1, the ratio itself is more sensitive to $T_e$ than the Al xii G-ratio is.

However, comparison of the simulation and the observations is a subjective process, with the eye tending to focus on the peak values of the lines. To obtain a more objective comparison, and to quantify the errors in the deduced temperature, emission measure and density, two programs were developed to fit the lines with Voigt profiles (see Section 4.2). In the first program, the fluxes of the lines were considered to be independent of each other (except $i_2/i_1$, which was taken to be 0.19, see Section 3.2). However, the width of the lines was dependent on $T_e$, which was assumed to be the same for each line. The wavelengths of the lines were fixed at the values shown in Table I and the spectra were fitted, optimising $T_e$, the line fluxes and the two background parameters (for linear background).

Originally the program contained the capability to fit a quadratic background. However, it often appeared that this produced an artificially curved background in an attempt to fit the noise in the data. So the function was restricted to a linear shape. The value of $T_e$ appeared sensitive to variations in the background parameters so the dependence of $T_e$ on errors in the background was investigated. A fixed constant background was assumed for the first spectrum from the 19 October data set while fitting the rest of the parameters, including $T_e$. The background level was varied in a range from 13 to 19 counts and the fit repeated. Figure 5 shows how the fitted value of $T_e$ falls from about $8.5 \times 10^6$ K to about $6.5 \times 10^6$ K as the background level is increased over this range. The standard deviation of the observations from the fitted curve is also shown, indicating that the optimum background level is about 16 counts in this case. It can be seen from Figure 5 that it is important to get a good estimate of the background or there will be a systematic error in the fitted $T_e$ value. The third curve in Figure 5 shows the dependence on background error of the value of $T_e$ inferred from the line ratio Mg xi/Al xii, obtained from the fitted fluxes. It shows that $T_e$ determined in this way was not so sensitive to background error.
Fig. 5. Curves showing the variation of $T_e$ with background level, for the value of $T_e$ obtained from the fit (×) and that obtained from the ratio of the Mg ii line and the resonance line of Al ii (■). Also shown in the figure is the variation of the resulting unbiased standard deviation of the fitted points from the observed data (○).

Obviously, the background parameters must be allowed to vary in the fit. In order to improve the background determination, the wavelength range included in the fit was extended on either side of the lines in question to give as many background data points as possible, while not invalidating the assumption of linearity. However, care must be taken to ensure that the spectral region included in the fit does not contain lines which

<table>
<thead>
<tr>
<th>Spectrum</th>
<th>$T_e$ ($\times 10^6$ K)</th>
<th>$G$</th>
<th>$R$</th>
<th>$b_0$</th>
<th>$b_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 Aug., 1980</td>
<td>6.3 ± 0.7</td>
<td>0.81 ± 0.09</td>
<td>1.80 ± 0.34</td>
<td>62 ± 1</td>
<td>43 ± 1</td>
</tr>
<tr>
<td>19 Oct., 1986 (1st)</td>
<td>6.9 ± 1.2</td>
<td>0.57 ± 0.11</td>
<td>1.37 ± 0.46</td>
<td>16.8 ± 0.4</td>
<td>15.7 ± 0.4</td>
</tr>
<tr>
<td>19 Oct., 1986 (2nd)</td>
<td>7.4 ± 1.4</td>
<td>1.10 ± 0.19</td>
<td>2.06 ± 0.61</td>
<td>11.0 ± 0.3</td>
<td>9.9 ± 0.3</td>
</tr>
</tbody>
</table>
have not been accounted for in the fitting function, otherwise they will cause an overestimate of the background level. For this reason, a number of minor satellite lines of Al xi were included in the fitting function (see Table II, Section 3.2).

Also, if it was suspected that a region might contain unassigned emission lines this was zero-weighted so that it did not influence the background determination. Two such regions were treated in this way: 7.8138–7.8304 Å and 7.8535–7.8678 Å.

Fits were obtained for all three spectra and the results are shown in Table III. The Mg xi/Al xii line ratio was also calculated from the fitted line fluxes and $T_e$ was inferred from this for each spectrum. These results (which can be seen in Table IV) were consistent with the temperatures obtained from the line-fitting and had smaller error bars. These temperatures imply the $G$-ratios shown in Table IV. It can be seen that the observed $G$-ratio agrees well with the expected value for the 25 August data but the other

<table>
<thead>
<tr>
<th>Spectrum</th>
<th>$T_e$ ($\times 10^6$ K)</th>
<th>$G$ (implied)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 Aug., 1980</td>
<td>5.8 ± 0.4</td>
<td>0.80 ± 0.01</td>
</tr>
<tr>
<td>19 Oct., 1986 (1st)</td>
<td>7.1 ± 0.6</td>
<td>0.77 ± 0.01</td>
</tr>
<tr>
<td>19 Oct., 1986 (2nd)</td>
<td>5.6 ± 0.7</td>
<td>0.81 ± 0.03</td>
</tr>
</tbody>
</table>

two do not. The 19 October data are, however, noisier with error bars on the observed $G$-ratios of about 20%. It can be seen from the errors estimated for the observed $R$-ratios that they cannot be expected to yield meaningful densities.

In Table III $b_0$ and $b_1$ are the values of the background at the initial and final wavelength positions. They have been optimised with the other parameters, so the effects of uncertainties in the background are reflected in the estimated uncertainties in the other parameters. Care has been taken (see above) to avoid systematic error in the background determination. The quality of the fit may be observed in the plots of Figure 6.

In Table IV it can be seen that the values of $T_e$ inferred from the Mg xi/Al xii ratio are obtained with greater precision than those obtained by the line-profile fitting. Looking at the data from 25 August, the observed $G$-ratio of 0.79 implies a temperature of about $6 \times 10^6$ K which is consistent with $T_e$ obtained by the other two means. However, the uncertainties in the measurement of $G$, which are relatively high because of the dependence on the weak $f$ and $i$ lines, mean that temperatures deduced from this line ratio would be very unreliable (see the $G$ vs log $T_e$ plot, Figure 2). The temperature obtained from the line-fitting itself is, in fact, more reliable. Better still is the temperature obtained from the Mg xi/Al xii ratio. If we take these temperatures and then deduce the corresponding values of $G$ which would be expected from the Keenan and McCann calculations, all of the observations of $G$ are consistent, within the error bars, to within 8% of the calculations, with the cleaner data of 25 August showing complete consistency.
with the calculations. The observed values of $R$ are also consistent, within the error bars, with a density $\simeq 10^{12}$ cm$^{-3}$, equal to the low-density limit calculated by Keenan and McCann (1987). This was also the limit deduced by Phillips et al. (1982) from Fe XXI and Fe XXII lines. However, the uncertainties in $R$ are very large, being $\sim 20\%$, even for the 25 August data.
Fig. 6. The three spectra with the final fitted curves superimposed (from the fit for e.m., \( N_e \) and \( T_e \)). In each case, the fitted function is represented by a dashed line.

If we now assume the theoretical calculations of Keenan and McCann, adding this information to that of the Mg\textsc{xii}/Al\textsc{xii} line ratio dependence, and the profile dependence on \( T_e \), we can produce a fit to optimise e.m., \( N_e \), and \( T_e \). The results of this, second fitting program are shown in Table V and Figure 6. The plots show good fits to the data and the precision of the results obtained for e.m. and \( T_e \) are good, and \( T_e \) is consistent with values obtained from the line-fitting above. In particular, increased precision is achieved in fitting the noisier spectra. The densities, as expected, are unreliable, because \( R \) is insensitive to density near the low-density limit (see Figure 3).

### Table V

<table>
<thead>
<tr>
<th>Spectrum</th>
<th>e.m. ( (\times 10^{48} \text{ cm}^{-3}) )</th>
<th>( N_e ) ( (\times 10^{12} \text{ cm}^{-3}) )</th>
<th>( T_e ) ( (\times 10^6 \text{ K}) )</th>
<th>( b_0 )</th>
<th>( b_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 Aug., 1980</td>
<td>9.2 ± 1.0</td>
<td>7 ± 6</td>
<td>5.9 ± 0.3</td>
<td>61.7 ± 1.0</td>
<td>42.9 ± 0.9</td>
</tr>
<tr>
<td>19 Oct., 1986 (1st)</td>
<td>2.2 ± 0.1</td>
<td>15 ± 10</td>
<td>6.8 ± 0.5</td>
<td>16.8 ± 0.4</td>
<td>15.6 ± 0.4</td>
</tr>
<tr>
<td>19 Oct., 1986 (2nd)</td>
<td>1.6 ± 0.1</td>
<td>3 ± 9</td>
<td>6.3 ± 0.5</td>
<td>11.0 ± 0.3</td>
<td>9.9 ± 0.3</td>
</tr>
</tbody>
</table>

### 6. Conclusions

These observations are consistent with the \( G \) and \( R \) ratios calculated for Al\textsc{xii} by Keenan and McCann (1987). However, it has been found that more precise estimates
of $T_e$ may be inferred from the Mg XI (1$^1S - 3^1P$)/Al XII (1$^1S - 2^1P$) line ratio. Combining the information from all three ratios, together with the assumption that the emission lines may be fitted with a Voigt profile which has a Gaussian component due to thermal motions, provides a good method of determining e.m. and $T_e$ for flares which have temperatures such that Al XII and Mg XI are observable ($T_e$ about $5 \times 10^6 - 2 \times 10^7$ K). Under some circumstances an idea of the electron density may also be obtained with this method, although in general this quantity cannot be determined with sufficient precision to yield more than a suggestion of whether or not the low density limit pertains.

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References