AVERAGE PROPERTIES OF BIPOLAR MAGNETIC REGIONS
DURING SUNSPOT CYCLE 21

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Abstract. We examine the statistical properties of some 2700 bipolar magnetic regions (BMRs) with magnetic fluxes \( \geq 3 \times 10^{20} \) Mx which erupted during 1976–1986. Empirical rules were used to estimate the fluxes visually from daily magnetograms obtained at the National Solar Observatory/Kitt Peak. Our analysis shows the following: (i) the average flux per BMR declined between 1977 and 1985; (ii) the average tilts of BMRs relative to the east–west line increase toward higher latitudes; (iii) weaker BMRs had larger root-mean-square tilt angles than stronger BMRs at all latitudes; (iv) over the interval 1976–1986, BMRs with their leading poles equatorward of their trailing poles contributed a total of 4 times as much flux as BMRs with ‘inverted’ tilts, but the relative amount of flux contributed by BMRs with inverted or zero tilts increased as the sunspot cycle progressed; (v) only 4% of BMRs had ‘reversed’ east–west polarity orientations; (vi) although the northern hemisphere produced far more flux during the rising phase of the sunspot cycle, the southern hemisphere largely compensated for this imbalance during the declining phase; (vii) southern-hemisphere BMRs erupted at systematically higher latitudes than northern-hemisphere ones through most of sunspot cycle 21.

1. Introduction

A knowledge of the magnetic properties of active regions is essential for understanding the mechanism of the solar cycle and the evolution of the Sun’s large-scale field. The early statistical studies of sunspot groups by Hale et al. (1919) and Brunner (1930) provided an empirical basis for solar dynamo models such as those of Babcock (1961) and Leighton (1969). More recent statistical studies using magnetograph data have concentrated on the properties of bipolar magnetic regions (BMRs), including both small ephemeral regions (Harvey and Martin, 1973) and longer-lived active regions (Tang, Howard, and Adkins, 1984; Tang, 1982; Howard, 1989).

During the course of sunspot cycle 21, one of us (N.R.S.) compiled a list containing the coordinates and estimated fluxes of BMRs that appeared on daily magnetograms obtained at the National Solar Observatory/Kitt Peak. These data were used as input for numerical modelling of the Sun’s large-scale magnetic field (see Wang, Nash, and Sheeley, 1989, and references therein), but until now the properties of the listed BMRs (or ‘sources’) have not been examined in detail. (A brief analysis of the magnetic fluxes and moments of the sources may be found in Sheeley, DeVore, and Boris, 1985.)

In the present study, we employ a slightly modified version of our source list (see Section 2) to determine the statistical properties of longer-lived BMRs that erupted during the interval 1976–1986. One of our motives was to find out what fraction of the BMRs had orientations consistent with the Babcock model for polar-field reversal. However, we have extended our analysis to include other properties such as average flux.


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per BMR, latitude distribution, and north–south asymmetries. Our study thus supplements the statistical surveys of Tang, Howard, and Adkins (1984) and Howard (1989), which were based on digitized magnetograph data from Mount Wilson Observatory extending back to 1967.

2. The Source List

Our 'source list' contains the approximate times of eruption, the centroidal coordinates for each polarity, and the estimated leader fluxes of some 2700 BMRs that appeared on the Sun between 12 August, 1976 and 5 April, 1986. Although the method for compiling this list was described in Sheeley, DeVore, and Boris (1985), for completeness we summarize the procedure in this section.

The BMRs were identified and measured from photographic prints of daily full-disk magnetograms taken at the National Solar Observatory/Kitt Peak. The individual magnetograms were organized into 27-day rows and examined on both the preceding and following rotations, so as to keep track of regions that erupted on the back side of the Sun and to facilitate the identification of new flux eruptions within active-region complexes (Gaizauskas et al., 1983). Usually, it was possible to distinguish a substantial new injection of flux from an older one that occurred during a previous rotation, by the greater intensity and more compact spatial structure of the later eruption. However, a significant amount of flux could have been overlooked in the years around sunspot maximum, because of the number and complexity of the regions during that time. The failure to include every source would not seriously affect most of the average statistical properties considered in this study.

Each new BMR was idealized as a pair of equal and opposite fluxes concentrated at the centers-of-mass of the respective polarities. The heliographic latitude and longitude coordinates of each 'pole' were estimated to the nearest degree, using Stonyhurst overlay grids which correct for the Sun's axial tilt. (It was sometimes necessary to lift the overlays slightly to compensate for small day-to-day variations in the apparent size and shape of the solar disk. This may have resulted in random errors of a degree or two in the absolute coordinates assigned to the BMRs, but the relative locations of the poles would have been less affected.)

The amount of flux in each BMR was estimated from the photographic images using a combination of empirical rules. One such rule relates the sum of the (unsigned) leader and follower fluxes of the BMR, \( \Phi_l \), to the total area \( A_{ss} \) covered by all of its sunspots: thus \( \Phi_l = 2.3A_{ss} \), where \( \Phi_l \) is expressed in \( 10^{21} \) Mx and \( A_{ss} \) is in units of \( 10^{18} \) cm\(^2\) (see Sheeley, 1966; Mosher, 1977). An alternative rule that was also applied states that the total flux in a sunspot is equal to one-fourth of its central field strength \( B_m \) times its (umbral plus penumbral) area (Mattig, 1953). The quantity \( \Phi_l \) can then be estimated by summing the fluxes in the dominant sunspots (which have \( B_m \sim 3000 \) G), and allowing for an equal amount of flux outside sunspots (Sheeley, 1981). (In both of these methods, the leader and follower fluxes are assumed to be in balance, when both sunspot and non-sunspot contributions are included.) If sunspots were not visible, the flux estimates
were based on comparisons between the areal sizes of the BMRs, in which the average field strength within a compact new BMR was taken to be \( \sim 100 \) G.

In many individual cases, we have compared the flux estimates obtained by the above methods to the corresponding digitized magnetograph data, and found agreement to within a factor of 2. We note that the digitized data may themselves be subject to similarly large errors, not only because of the usual uncertainties involving seeing and line calibration, but also because of their tendency to underestimate the flux in sunspots.

Whenever possible, measurements were performed when the BMR flux reached its peak. Unlike Tang, Howard, and Adkins (1984) and Howard (1989), we counted each BMR only once. For those sources that erupted or reached their maximum on the back side of the Sun, the measurements were made 3–4 days after the region appeared at the east limb. (Of course, shorter-lived BMRs that erupted and faded on the back side, or very close to the limb, were missed.)

Only BMRs with leader fluxes estimated to be greater than or equal to \( 0.3 \times 10^{21} \) Mx were included in the present survey; this lower threshold excludes most of the ephemeral regions whose properties have been described by Harvey and Martin (1973). In the original source list used for numerical modelling of the Sun’s large-scale field (Sheeley, DeVore, and Boris, 1985), many weak BMRs with leader fluxes less than \( 1 \times 10^{21} \) Mx were intentionally omitted prior to 1982, because of their negligible contribution to the large-scale field. For the purpose of this statistical survey, we have recounted the weaker sources that appeared during 1976–1981 and added the omitted ones to our list.

Hereafter, the ‘flux’ or ‘strength’ of a BMR (‘source’) should be understood to refer to the amount of leader flux \( \Phi \) (unsigned and assumed to be equal in magnitude to the amount of follower flux). Similarly, by the ‘total flux’ of all BMRs or sources we mean the combined amount of leader flux.

Figure 1(a) shows how the 2710 sources measured between August 1976 and April 1986 were distributed with respect to their strengths. The number distribution peaks near \( \Phi = 4 \times 10^{21} \) Mx, and fully 71% of the sources have strengths in the range \( 1 \times 10^{21} \text{ Mx} \leq \Phi < 10 \times 10^{21} \text{ Mx} \). Only 11% of the sources have strengths \( \Phi \geq 10 \times 10^{21} \) Mx, with the largest strengths recorded being \( \Phi = 50 \times 10^{21} \) Mx. We note that the number of sources with strengths \( 0.3 \times 10^{21} \text{ Mx} \leq \Phi < 1 \times 10^{21} \text{ Mx} \) has probably been underestimated by more than a factor of 2, because such weak, short-lived sources would have been missed if they erupted on the back side of the Sun, near the limb, or within a much stronger region.

Figure 1(b) shows how the total flux that erupted during 1976–1986 was distributed with respect to source strength \( \Phi \). It can be seen that BMRs with \( \Phi \sim 10 \times 10^{21} \) Mx contributed the most flux (even though there were considerably more sources with \( \Phi \sim 4 \times 10^{21} \) Mx). On the other hand, a negligible amount of flux originated from sources with \( \Phi < 1 \times 10^{21} \) Mx.

In Figure 2, the strengths of the 2710 BMRs are plotted on a log-log scale against their pole separations in degrees. We define the pole separation of a BMR located at a mean latitude \( \lambda \) to be the quantity \( s = [(A\phi)^2 (\cos \lambda)^2 + (A\lambda)^2]^{1/2} \), where \( A\phi \) denotes
Fig. 1.  (a) Histogram showing how the 2710 BMRs that erupted during 1976–1986 were distributed with respect to their (leader) fluxes $\Phi$. The source numbers are binned in 0.2-wide intervals of $\log_{10} \Phi$ (in Mx). Our survey includes only sources with $\Phi \geq 3 \times 10^{20}$ Mx. (b) Histogram showing how the total (leader) flux that erupted during 1976–1986 was distributed with respect to the source strengths $\Phi$. The total flux (in units of $10^{21}$ Mx) is again binned in 0.2-wide intervals of $\log_{10} \Phi$.

Fig. 2. The strengths $\Phi$ (in Mx) of all BMRs that erupted during 1976–1986 are plotted in log-log format against their pole separations $s$ (in degrees). Individual sources are denoted by plus symbols. The dashed line shows a linear least-squares fit to the data points, which may be represented by the function $\Phi(s) = 4 \times 10^{20} \text{Mx} \ s^{1.3}$.

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the longitudinal separation and $\Delta \lambda$ the latitudinal separation of the poles in degrees. The $\cos \lambda$ factor has been included so that $s$ scales as the actual distance between the poles. The plot shows that stronger sources tend to have larger pole separations than weaker sources, although there is a wide scatter in the fluxes corresponding to any given pole separation; the correlation coefficient is 0.75. A least-squares fit to the data (dashed line in Figure 2) yields an average relationship of the form $\Phi = 4 \times 10^{20} \text{ Mx} \cdot s^{1.3}$, where $s$ is expressed in degrees.

3. Average Properties of Sources Having Differing Strengths

In this section, we look for systematic differences between the properties of BMRs having different strengths. For convenience, we arbitrarily define ‘strong’ sources as those with (leader) fluxes $\Phi \geq 5.0 \times 10^{21} \text{ Mx}$, ‘medium’ sources as those with fluxes $2.0 \times 10^{20} \text{ Mx} \leq \Phi < 5.0 \times 10^{21} \text{ Mx}$, and ‘weak’ sources as those with fluxes $\Phi < 2.0 \times 10^{20} \text{ Mx}$. Over the entire interval August 1976 to April 1986, we counted 710 strong sources with an average flux of $9.8 \times 10^{21} \text{ Mx}$, 1071 medium sources with an average flux of $2.9 \times 10^{21} \text{ Mx}$, and 929 weak sources with an average flux of $0.86 \times 10^{21} \text{ Mx}$.

3.1. Numbers and Total Fluxes

Figure 3(a) shows the yearly numbers of ‘strong’, ‘medium’, and ‘weak’ sources during the interval 1977–1985, as well as the yearly numbers of all sources combined. The plot suggests that the distributions were phase-shifted relative to one another, such that the stronger sources reached their maximum earlier than the weaker sources. Indeed, from Figure 3(b), which displays yearly ratios of the number of sources in each category to the total number of sources, we see that the relative number of strong sources declined through the sunspot cycle, whereas the relative number of weak sources rose steadily after 1979.

In Figure 3(c), we plot the fraction of the total source flux contributed by each of the three source categories as a function of time. It can be seen that the strong sources provided most of the flux during every year, whereas the weak sources provided at most $\sim \frac{1}{10}$ of the total flux. However, the relative amount of flux in strong sources declined over the sunspot cycle, while the relative amount of flux in the weaker sources steadily increased.

Figure 3(d) shows the total amounts of (leader) flux contributed by strong, medium, weak, and all sources as a function of sine latitude. Here, the fluxes erupting in both hemispheres have been combined and summed over the entire interval 1976–1986. The (equal-area) sine-latitude bins have a width of 0.1, except for the last bin, which has been ‘stretched’ to include sources at all latitudes poleward of $\sin^{-1} 0.6 = 37^\circ$. In every latitude zone, strong sources contribute the majority of the flux, and weak sources contribute the least flux. The noticeable ‘phase shift’ between the distributions of weak and strong sources can be attributed to the rise (decline) in the number of weak (strong) sources after sunspot maximum, together with the equatorward migration of source
Fig. 3a–d. Numbers and total fluxes of sources in different strength categories. Here, plain solid lines are used to denote ‘strong’ sources with \( \Phi \geq 5.0 \times 10^{21} \text{ Mx} \), long dashed lines denote ‘medium’ sources with \( 2.0 \times 10^{21} \text{ Mx} \leq \Phi < 5.0 \times 10^{21} \text{ Mx} \), dotted lines denote ‘weak’ sources with \( \Phi < 2.0 \times 10^{21} \text{ Mx} \), and connected asterisks denote all categories combined. (a) Yearly numbers of sources in each category. (b) Yearly ratios of the numbers of strong, medium, and weak sources to the total number of sources. (c) Yearly ratios of the total fluxes of strong, medium, and weak sources to the combined flux of all sources. (d) Cumulative distributions of source flux with respect to sine latitude. The total amount of (leader) flux erupting in each latitude zone during 1976–1986 is shown for each source category. Each sine-latitude bin combines data from both hemispheres and has a width of 0.1, except for the last bin, which includes all sources poleward of \( \sin^{-1} 0.6 = 37^\circ \).

eruptions over the sunspot cycle (Section 3.3). The combined source-flux distribution peaks near a latitude of \( 15^\circ \).

3.2. AVERAGE FLUX PER SOURCE

As noted above, the temporal distributions of ‘strong’, ‘medium’, and ‘weak’ sources were displaced relative to each other in such a way that strong sources were relatively numerous earlier in the cycle and weak sources were relatively numerous later in the cycle. This behavior suggests that the flux per source, averaged over all sources, declined as the sunspot cycle progressed. In fact, Figure 4(a) shows that the average flux per source decreased by almost a factor of 2 between 1977 and 1985.
Because of the correlation between source strength and pole separation indicated by Figure 2, one might expect the average pole separation per BMR to display a similar evolution. However, Figure 4(b) shows that the (unweighted) average pole separation attained a maximum around 1980 and thereafter declined, rather than decreasing steadily from 1977 onwards like the average flux. The result shown in Figure 4(b) is consistent with the variation of the average areal size of active regions, which peaks near sunspot maximum (Tang, Howard, and Adkins, 1984).

The most likely explanation for the somewhat differing evolutions of the average flux and the average pole separation is that the correlation between the fluxes and pole separations of BMRs is nonlinear: $\Phi \sim s^{1.3}$ according to the least-squares fit shown in Figure 2. Thus the very strong sources that were relatively common during the rising phase of the cycle would have contributed more to the average value of the flux than to the average value of the pole separation. However, we cannot rule out the possibility that we have systematically overestimated the fluxes of such strong sources during 1977–1978 or underestimated their numbers and fluxes around sunspot maximum.

3.3. AVERAGE SOURCE LATITUDES

We found no systematic differences between the average latitudes of eruption of strong, medium, and weak sources, which all showed the usual equatorward drift during 1977–1985. However, weak sources tended to be more widely spread in latitude than strong sources (cf. Tang, Howard, and Adkins, 1984), particularly after 1983, when they erupted both very close to the equator and at high latitudes. This difference may be seen by comparing Figures 5(a) and 5(b), where we have plotted separate butterfly diagrams for the strong and weak sources, respectively (the data for the two hemispheres have been combined). As discussed in Section 4, the latitudinal spread in the weak sources after 1983 is partly attributable to BMRs with ‘reversed’ east–west polarity, of which those at high latitudes heralded the start of sunspot cycle 22.
Fig. 5a–b. Butterfly diagrams of strong and weak BMRs. (a) The asterisks mark the latitudinal positions of the 710 'strong' sources (with $\Phi \geq 5.0 \times 10^{21}$ Mx) that erupted during 1976–1986. (b) The plusses mark the latitudinal positions of the 929 'weak' sources (with $\Phi < 2.0 \times 10^{21}$ Mx) that erupted during 1976–1986. In these plots, the data from the two hemispheres have been combined.

3.4. Latitude dependence of tilt angles

We now consider the axial inclinations of BMRs having different strengths. We define the tilt angle $\alpha$ of a BMR to be the angle between the bipole axis and the heliographic east–west line, such that $\tan \alpha = \Delta \lambda / [(\Delta \phi) \cos \lambda]$. The tilt angle ranges from $-90^\circ$ to $+90^\circ$, and is taken to be positive if the leading (westward) pole is located equatorward of the trailing pole ('normal' orientation), negative if the trailing pole lies equatorward of the leading pole ('inverted' orientation). In most cases, the tilt angle is determined when the BMR flux reached its peak (see Section 2).
Fig. 6a–b. Latitude dependence of average and r.m.s. tilt angles (cumulative, 1976–1986). The tilt angle of a BMR is defined as positive if its leading pole is located equatorward of its trailing pole. The latitude bins are 5° wide, except for the polemost bin which includes all sources poleward of 30°. (a) The arithmetically-averaged tilt angles of 'strong', 'medium', and 'weak' BMRs are indicated by the thin solid line, dashed line, and dotted line, respectively. The thick line shows the latitude distribution of the flux-weighted average tilt angles based on all sources. (b) Root-mean-square tilt angles of 'strong', 'medium', and 'weak' BMRs (symbols as in Figure 6(a)).

Figure 6(a) shows the average tilt angles of strong, medium, and weak BMRs as a function of latitude. Here the latitude bins are 5° wide, except for the polemost bin which includes all latitudes above 30°. The value of the tilt angle corresponding to each latitude zone and source category represents an arithmetic average (for that latitude zone and source category) taken over the entire interval 1976–1986. Also plotted in Figure 6(a) are the flux-weighted average tilt angles (thick line) based on all sources that erupted within each of the latitude zones. The average tilt angles all display a progressive increase toward higher latitudes. For all three strength categories, the average tilts lie between 1° and 5° in the 0°–5° latitude zone, and between 11° and 14° in the 25°–30° latitude zone; the tilt angles thus increase by an average of ~4° for every 10° in latitude.

We emphasize that the tilt angles of individual sources will have a very wide scatter about the average values shown in Figure 6(a): the standard deviations are comparable in magnitude to the mean values themselves. In order to make this clear, we plot in Figure 6(b) the root-mean-square (r.m.s.) tilt angles of strong, medium, and weak BMRs as a function of latitude. It can be seen that the r.m.s. tilt angles show less of a systematic increase toward higher latitudes than do the arithmetic averages of Figure 6(a). Also, irrespective of latitude, weak sources have larger r.m.s. tilts than medium sources, which in turn have larger r.m.s. tilts than strong sources (whereas no such systematic differences are apparent in the average tilt angles). We infer from this that the tilt angles of weaker sources tend to be larger (in absolute value) and to have a wider scatter about their average values than the tilt angles of stronger sources (cf. Howard, 1989).
The tendency for the average tilt angles to increase with latitude was found to be present during individual years of the sunspot cycle. The latitudinal variation did not show any clear systematic time evolution over the interval 1977–1985.

Figure 7 shows the year-to-year evolution of the tilt angles, after averaging over all latitudes. The tendency for the tilt angles to decrease as the sunspot cycle progresses reflects the equatorward migration of the average source latitudes and the latitudinal distribution of the average tilt angles (Figure 6(a)).

Hale et al. (1919) were the first to note the increase of the average axial tilts of sunspot groups with latitude. This result was confirmed by Brunner (1930), who also found that larger, well-developed sunspot groups tend to have smaller tilts than smaller, less-developed ones. More recently, however, Tang, Howard, and Adkins (1984) were unable to detect any systematic latitudinal variation in the orientations of BMRs in Mount Wilson observations during 1967–1981. The reason for the absence of a latitude dependence in their data is unclear to us. Although the discrepancy between the results of Tang, Howard, and Adkins and those of the early investigators might appear to reflect differences in the orientation properties of BMRs and sunspot groups, even our relatively weak BMRs (which do not always contain sunspots) displayed a strong latitudinal variation in their average tilt angles (see Figure 6(a)).
4. Polarity Orientations

For the purposes of this Section, we define the following two main categories of sources, based on their east–west polarity orientations:

(I) ‘Hale’: BMRs having positive-polarity leading flux in the northern hemisphere or negative-polarity leading flux in the southern hemisphere (this was the ‘expected’ orientation for cycle 21 sources, according to the polarity laws of Hale et al., 1919).

(II) ‘Anti-Hale’: BMRs having negative-polarity leading flux in the northern hemisphere or positive-polarity leading flux in the southern hemisphere.

We further divide the ‘Hale’ sources into the following sub-groups, based on their north–south polarity orientations:

(Ia) ‘Hale/normal tilt’: Hale sources having their leading pole located equatorward of their trailing pole (positive tilt angle).

(Ib) ‘Hale/untilted’: Hale sources aligned with the east–west direction to within 0.5°.

(Ic) ‘Hale/inverted tilt’: Hale sources having their trailing pole located equatorward of their leading pole (negative tilt angle).

Figure 8(a) shows the yearly numbers of sources in categories I (solid line), Ia (connected plusses), Ib (connected diamonds), Ic (long dashes), and II (dotted line), while Figure 8(b) (using the same symbols) shows yearly ratios of the total flux in each category to the total flux from all sources. Throughout the period 1977–1985, ‘Hale’ sources vastly outnumbered ‘anti-Hale’ sources, and provided practically all of the flux. However, the relative number of anti-Hale sources, and the proportion of the total flux that they contributed, began to increase in 1984.

Among the Hale sources themselves, those with ‘normal’ tilts were more numerous and contributed more flux than the remaining Hale sources throughout the interval 1976–1986. On the other hand, the fraction of ‘untilted’ and ‘inverted’ Hale sources tended to increase as the sunspot cycle progressed. Thus, in 1977, Hale sources with inverted tilts contributed only 20% as much flux as those with normal tilts, whereas in 1985 they contributed 65% as much flux. Taken over the whole sunspot cycle, however, Hale sources with normal tilts contributed over 4 times as much flux as Hale sources with inverted tilts.

Figure 8(c) shows the average flux per source for Hale, normal Hale, inverted Hale, and anti-Hale sources. Each source category displays the progressive decline in the average flux noted earlier. The average flux per normal Hale source was systematically larger than the average fluxes of the other source categories. Over the entire interval 1976–1986, Hale, normal Hale, inverted Hale, and anti-Hale sources had average fluxes of $4.0 \times 10^{21}$ Mx, $4.5 \times 10^{21}$ Mx, $3.6 \times 10^{21}$ Mx, and $3.3 \times 10^{21}$ Mx, respectively.

Figure 8(d) shows the annual, flux-weighted average tilt angles of Hale, normal Hale, inverted Hale, and anti-Hale sources during 1977–1985 (the data from all latitudes have been combined in the averages). It is interesting to note that, even though the average tilts of normal Hale sources increased after 1980, the average tilts of all Hale sources combined decreased, because of the steady rise in the relative numbers of inverted and untilted Hale sources (see Figure 8(b)). The increase in the average tilts of normal Hale
sources is related to the increase in the relative amount of flux contributed by weaker sources after sunspot maximum (Figure 3(c)); according to Figure 6(b), such sources have comparatively large r.m.s. tilt angles. The erratic behavior displayed by the anti-Hale sources in Figure 8(d) is due both to their small numbers and to the wide scatter in their tilt angles. Taken over the entire sunspot cycle, anti-Hale sources had an r.m.s. tilt angle of 34°, as compared to 20° for Hale sources.

In Figure 9, we plot a histogram of the total flux (in units of $10^{21}$ Mx) contributed by Hale sources during 1976–1986 as a function of their tilt angles. As expected, the flux distribution is displaced toward positive tilt angles (for which the leading pole is located equatorward of the trailing pole), with the peak occurring in the +5° to
Fig. 9. Histograms showing how the total (leader) flux that erupted during 1976–1986 was distributed with respect to the tilt angles of the erupting sources. The solid line histogram represents the distribution of total flux (in units of $10^{21}$ Mx) for 'Hale' sources, while the dotted line histogram represents the distribution of total flux (in units of $10^{20}$ Mx) for 'anti-Hale' sources. (In both cases, positive values of the tilt angle indicate that the leading pole is located equatorward of the trailing pole.) The bins of tilt angle are 10° wide, with the central bin extending from $-5°$ to $+5°$. In order to emphasize the asymmetry of the distributions, the vertical dashed line marks the position of zero tilt.

+ $15°$ bin. Figure 9 also shows a histogram of the total flux contributed by anti-Hale sources as a function of their tilt angles. For visibility, the flux in this case (dotted line) is plotted in units of $10^{20}$ Mx, not $10^{21}$ Mx. Again, the flux distribution is displaced toward positive tilt angles, but is less concentrated toward small values of the tilt angle than is the distribution of Hale sources. The peak in the distribution of anti-Hale flux occurs in the $+25°$ to $+35°$ bin. Similar results were found by Howard (1989) from his analysis of Mount Wilson data since 1967.

Figure 10(a) shows the yearly-averaged latitudes for normal Hale, untilted Hale, inverted Hale, and anti-Hale sources (here the averages have not been flux-weighted). The low average latitude of the anti-Hale sources during 1977 suggests that some of these sources were remnants of the previous cycle. It can also be seen that Hale sources with normal tilts tended to erupt at higher latitudes than those with no tilt or with inverted tilts. This behavior is related to the general increase in the average tilt angles toward higher latitudes (Figure 6(a)).

In Figure 10(b), we have plotted the latitudinal positions of all the anti-Hale sources on top of a butterfly diagram of the remaining (Hale) sources. Here we have combined
the data from both hemispheres; the asterisks and plusses represent anti-Hale sources with fluxes $\Phi \geq 2 \times 10^{21}$ Mx and $\Phi < 2 \times 10^{21}$ Mx, respectively. The overall impression from Figure 10(b) is that the anti-Hale sources do not show the systematic progression from higher to lower latitudes characterizing the butterfly diagram of the Hale sources, which are indicated by the background of small dots. However, the scatter is mainly a result of the low-latitude BMRs that erupted during 1976–1977 and may be regarded as remnant 'Hale' sources from the previous sunspot cycle, and the weak, high-latitude BMRs that appeared from 1983 onwards and were forerunners of sunspot cycle 22. It is also interesting to note the occurrence of anti-Hale sources very close to the equator, as if these were 'Hale' sources displaced from the other hemisphere.
TABLE 1

BMRs classified by polarity orientation (cumulative statistics, 1976–1986)

<table>
<thead>
<tr>
<th>Source category</th>
<th>Number</th>
<th>Total flux a (10^{21} Mx)</th>
<th>Average flux a (10^{21} Mx)</th>
<th>Tilt angle b average/r.m.s.</th>
<th>Average pole separation b</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I) Hale</td>
<td>2593</td>
<td>10488</td>
<td>4.0</td>
<td>10.0°/20.1°</td>
<td>6.9°</td>
</tr>
<tr>
<td>(Ia) Hale/normal</td>
<td>1671</td>
<td>7464</td>
<td>4.5</td>
<td>18.1°/21.5°</td>
<td>7.2°</td>
</tr>
<tr>
<td>(Ib) Hale/untitled</td>
<td>430</td>
<td>1248</td>
<td>2.9 c</td>
<td>0.0°/0.0°</td>
<td>6.1°</td>
</tr>
<tr>
<td>(Ic) Hale/inverted</td>
<td>492</td>
<td>1776</td>
<td>3.6</td>
<td>-16.7°/21.3°</td>
<td>6.2°</td>
</tr>
<tr>
<td>(II) anti-Hale</td>
<td>113</td>
<td>374</td>
<td>3.3</td>
<td>5.5°/34.5°</td>
<td>5.5°</td>
</tr>
</tbody>
</table>

a As always, fluxes refer to one polarity only.
b Flux-weighted average/r.m.s. values.
c A factor contributing to the low average flux of Hale/untitled sources is the difficulty in estimating the tilt angles of very small BMRs, whose latitudinal pole separations are often close to 0°.

Table I summarizes the overall statistical properties of the BMRs belonging to each of the five orientation categories. The cumulative (1976–1986) data for each category include the total number of sources, the total and average (leader) fluxes, the flux-weighted average and r.m.s. tilt angles, and the flux-weighted average pole separation.

5. North–South Asymmetries

We next consider asymmetries between the northern and southern hemispheres. In Figures 11(a) and 11(b), we display the total number of sources and the total (leader) flux erupting in each hemisphere as a function of year. During the rising phase of the cycle, the northern hemisphere was consistently more active than the southern hemisphere: between 1977 and 1979, 23% more sources and 35% more flux erupted in the north than in the south. On the other hand, during 1983–1985, 1.8 times as many sources and 2.0 times as much flux erupted in the south as in the north. When integrated over the entire interval 1976–1986, the two hemispheres contributed approximately equal amounts of flux (to within 3.6%), but their yearly flux distributions were phase-shifted relative to each other.

Figure 11(c) shows how the average flux per source in each hemisphere evolved with time. Comparison between Figures 11(b) and 11(c) suggests a tendency for the asymmetry in the total flux to occur in phase with the asymmetry in the average flux, so that if the total flux was greater in one hemisphere than the other, then so was the average flux. However, it is clear that the asymmetry in the total flux was also strongly correlated with an imbalance in the number of sources (Figure 11(a)), so that the dominant hemisphere had more BMRs as well as stronger ones. Over the entire sunspot cycle, the average flux per source was $4.2 \times 10^{21}$ Mx for the northern hemisphere and $3.8 \times 10^{21}$ Mx for the southern hemisphere.

Finally, Figure 11(d) shows the yearly, flux-weighted average latitudes for the northern- and southern-hemisphere sources. Except in 1981, the southern-hemisphere
sources erupted at systematically higher latitudes than the northern-hemisphere sources. The maximum differences of $3^\circ-4^\circ$ occurred in 1979 and 1985.

6. Summary

We now summarize our main findings concerning the statistical properties of BMRs measured from August 1976 to April 1986 (see also Table I):

1) Of the total of $1.1 \times 10^{25}$ Mx of (leader) flux that erupted during 1976–1986, 64% was contributed by `strong' sources having leader fluxes $\Phi \geq 5 \times 10^{21}$ Mx, 29% was contributed by `medium' sources having $2 \times 10^{21}$ Mx $\leq \Phi < 5 \times 10^{21}$ Mx, and only 7% was contributed by `weak' sources having $0.3 \times 10^{21}$ Mx $\leq \Phi < 2 \times 10^{21}$ Mx. The relative amount of flux contributed by weak sources was small during every year of the cycle,
even allowing for the possibility that their number was underestimated by more than a factor of 2.

(2) The peak in the distribution of source strengths was located at $\Phi \sim 4 \times 10^{21}$ Mx. This may be compared with the corresponding value of $\sim 2 \times 10^{21}$ Mx obtained by Howard (1989) from Mount Wilson data during 1967–1988 (here we have multiplied his measured result by 1.8 to correct for the saturation of the FeI 5250 Å line profile). One factor that would tend to make our source strengths larger is that the digitized magnetograph data used by Howard may substantially underestimate the flux in sunspots.

(3) The average flux per source declined as the sunspot cycle progressed. Stronger sources were relatively numerous early in the cycle, and weaker sources were relatively numerous late in the cycle. The average pole separation per BMR peaked in 1980 and declined thereafter.

(4) For BMRs in all strength categories, the average tilt angles relative to the east–west line showed a progressive increase toward higher latitudes. When arithmetically averaged over all latitudes and over the entire period 1976–1986, the tilt angles of strong, medium, and weak sources all had values close to $+9^\circ$ (the plus sign indicates that the leading pole lies equatorward of the trailing pole).

(5) The root-mean-square tilt angles of BMRs were inversely correlated with their strengths, irrespective of latitude (cf. Brunner, 1930; Howard, 1989). Calculated over the entire interval 1976–1986 and over all latitudes, the r.m.s. tilt angles of strong, medium, and weak BMRs were 19.2°, 21.9°, and 27.7°, respectively.

(6) ‘Hale’ sources obeying Hale’s polarity law for sunspot cycle 21 contributed a total of 28 times as much flux as ‘anti-Hale’ sources having reversed east–west polarity orientations. The average flux per ‘Hale’ source was $4.0 \times 10^{21}$ Mx, while the average flux per ‘anti-Hale’ source was $3.3 \times 10^{21}$ Mx. The relative fraction of ‘anti-Hale’ sources increased noticeably in 1985. This increase can be attributed both to the eruption of high-latitude sources marking the onset of cycle 22, and to the appearance of low-latitude sources that almost straddled the equator.

(7) Among the ‘Hale’ sources themselves, those with ‘normal’ or positive tilts (leading pole equatorward of trailing pole) contributed a total of 4 times as much flux as those with ‘inverted’ tilts, and 6 times as much flux as those with no discernible tilts. The average flux per ‘normal Hale’ source was $4.5 \times 10^{21}$ Mx, as compared with an average flux of $3.6 \times 10^{21}$ Mx per ‘inverted Hale’ source. The relative amount of flux contributed by normal Hale sources declined steadily after sunspot maximum.

(8) Substantially more sources and more flux erupted in the northern hemisphere than in the southern hemisphere during 1977–1979, whereas the reverse occurred during 1983–1985. The more active hemisphere also tended to have stronger sources. Over the whole sunspot cycle, a total of 4% more flux erupted in the northern hemisphere, but the southern hemisphere had 7% more sources. Southern-hemisphere sources erupted at systematically higher latitudes than northern-hemisphere sources through most of the cycle.
7. Discussion

One purpose of the present study was to determine the extent to which the polarity orientations of BMRs during sunspot cycle 21 fulfilled the conditions for Babcock-type models of polar-field reversal (Babcock and Babcock, 1955; Babcock, 1961; Leighton, 1964, 1969). Such models require the vertical dipole moments of the individual BMRs to be anti-parallel to the Sun’s large-scale dipole field at the start of the sunspot cycle. (By the ‘vertical’ dipole moment of a BMR, we mean the product of its flux and its pole separation projected along the solar rotation axis.) According to our statistical survey, BMRs with the ‘correct’ orientations contributed more than 4 times as much flux as BMRs with ‘incorrect’ orientations during sunspot cycle 21. Indeed, the net vertical dipole moment of all sources that erupted during 1976–1986 was almost 3 times as large as the (oppositely-directed) dipole moment of the Sun’s large-scale field in 1976. The conditions for polar-field reversal were thus amply satisfied, as we have confirmed elsewhere using a flux-transport model that includes the effects of source eruptions, supergranular diffusion, and a poleward bulk flow (Wang, Nash, and Sheeley, 1989). On the other hand, it is perhaps surprising that BMRs with ‘incorrect’ orientations or with no discernible tilts were so common: together, they comprised 37% of the sources that erupted during sunspot cycle 21.

Brunner (1930) was the first to show that smaller, less-developed sunspot groups tend to have greater inclinations relative to the east–west line than larger, well-developed groups. Weart (1970), from a study of Hα pictures, found that emerging active regions initially have largely random orientations, but that the regions that survive the first few days soon take up the ‘correct’ tilt, in which the leading polarity lies slightly closer to the equator. Consistent with these results, Howard (1989) concluded that smaller active regions are more likely to show deviations from the ‘usual’ polarity orientation. Our findings concerning the tilt angles of BMRs during sunspot cycle 21, summarized in paragraphs (5)–(7) above, are in agreement with those of the previous studies.

Brunner (1930) noted that the observed ‘proper motion’ of leader spots in longitude would act to reduce the axial tilts of sunspot groups as they developed in time, thus accounting for the tendency for well-developed groups to have relatively small tilt angles. The reason for the rapid westward motion of leader spots during the early development of active regions is still not understood. This relative motion may initially occur at a rate of $\sim 1^\circ \text{ day}^{-1}$, but ceases once the flux has stopped emerging (see Zirin, 1988). By comparison, photospheric differential rotation would separate the poles of a BMR, located at latitudes of 14° and 16°, at a rate of only 0.05° day$^{-1}$. This rate would be too slow to account for the observed decrease in the tilt angles of active regions during their first few days of development.

Howard (1989) attributed the abnormal orientations of weaker BMRs, as well as their relatively complex polarity configuration, to random twistings of the rising flux tubes by the supergranular velocity field. According to Zirin (1988), the arch filament systems marking newly-emerging flux regions in Hα are invariably straight and untwisted. Complex structures result when successive bipoles come up at angles to each other. It
may be that, even though the weaker flux tubes are strongly twisted as they rise through the convection zone, the field lines comprising each flux tube straighten out along their rotated axis once they break through the surface, where the lower densities require them to maintain an approximately current-free state.

The kinematic dynamo model of Babcock (1961) provides a possible explanation for the observed latitude dependence of the average tilt angles of BMRs. Babcock suggested that the sub-surface poloidal field would be weaker near the equator (where its flux fills a greater volume) than at mid-latitudes. The low-latitude field lines must then be wound up more tightly before attaining the threshold field strength at which the flux ropes become buoyant. Low-latitude BMRs would thus emerge with smaller average tilt angles, as found here and in earlier studies of sunspot groups (Hale et al., 1919; Brunner, 1930).

The tendency for the relative numbers of BMRs with zero or negative tilt angles to increase as the sunspot cycle progressed may simply reflect the equatorward migration of source eruptions and the latitudinal dependence of the average tilt angle. Another possibility is that the increased occurrence of such abnormal tilts is related to a change in the north–south pitch of the sub-surface field after the Sun’s dipole moment reverses, leading to the ‘unwinding’ of the toroidal field (see Leighton, 1964). In that case, the average tilt angles might be expected to decrease with time in each latitude zone during the latter part of the cycle, but we were unable to detect such a systematic effect.

In Babcock’s (1961) model, magnetic buoyancy sets in and amplification ceases near a critical field strength which is independent of time. If the flux ropes rising to the surface have a fixed distribution of strengths, the average flux per BMR would remain unchanged throughout the sunspot cycle. However, we have found that the average flux per BMR underwent a progressive decline during cycle 21. In order to account for this behavior, we might suppose that there is no single ‘critical’ field strength: instead, as the amount of sub-surface toroidal flux decreases at a given latitude, so would the average strength of the buoyant flux tubes emerging at that latitude. Weaker flux tubes would in any case require more time to reach the surface than stronger ones, perhaps becoming twisted and further fragmentated in the convection zone. A time-varying average flux may also result if the toroidal field continues to be amplified beyond the assumed threshold strength for eruption (cf. Leighton, 1969).

North–south asymmetries in sunspot activity have been well-documented in the past (e.g., Kiepenheuer, 1953; Tang, Howard, and Adkins, 1984). Our conclusion that southern-hemisphere sources tended to erupt at somewhat higher latitudes than northern-hemisphere sources during sunspot cycle 21 is consistent with the rotation of the coronal magnetic field during that period (Wang et al., 1988). We found that the coronal rotation profile, obtained by means of a potential-field source-surface extrapolation of the observed photospheric field, was markedly asymmetric during 1977–1979, with the southern hemisphere rotating more slowly than the northern hemisphere. Because the coronal rotation rate is determined by the latitudinal distribution of the non-axisymmetric photospheric flux, we attributed the rotational asymmetry to a latitudinal asymmetry in the source distribution. We also noted that the effect of such
latitudinal asymmetries would be magnified at high latitudes, where the gradients in the photospheric rotation profile are largest. This explains why the observed north–south asymmetry in the average source latitude had its greatest effect on the coronal rotation profiles during the rising phase of the sunspot cycle.

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