THE EVOLUTION OF LOOP STRUCTURES IN FLUX RINGS
WITHIN THE SOLAR CONVECTION ZONE

ARNAB RAI CHAUDHURI

Department of Physics, Indian Institute of Science, Bangalore 560012, India*

and

High Altitude Observatory, National Center for Atmospheric Research, Boulder, CO 80307, U.S.A.**

(Received 10 March, 1989; in revised form 5 May, 1989)

Abstract. Chaudhuri and Gilman (1987) considered certain implications of the hypothesis that the magnetic flux within the Sun is generated at the bottom of the convection zone and then rises through it. Taking flux rings symmetric around the rotation axis and using reasonable values of different parameters, they found that the Coriolis force deflects these flux rings into trajectories parallel to the rotation axis so that they emerge at rather high latitudes. This paper looks into the question of whether the action of the Coriolis force is subdued when the initial configuration of the flux ring has non-axisymmetries in the form of loop structures. The results depend dramatically on whether the flux ring with the loops lies completely within the convection zone or whether the lower parts of it are embedded in the stable layers underneath the convection zone. In the first case, the Coriolis force supresses the non-axisymmetric perturbations so that the flux ring tends to remain symmetric and the trajectories are very similar to those of Chaudhuri and Gilman (1987). In the second case, however, the lower parts of the flux ring may remain anchored underneath the bottom of the convection zone, but the upper parts of the loops still tend to move parallel to the rotation axis and emerge at high latitudes. Thus the problem of the magnetic flux not being able to come out at the sunspot latitudes still persists after the non-axisymmetries in the flux rings are taken into account.

1. Introduction

The conventional wisdom about the solar magnetic fields is that they are produced by the dynamo action. Though the solar dynamo theory is at present encountering many serious difficulties, no satisfactory alternative theory has emerged so far (Cowling, 1981; Gilman, 1986). This makes it imperative to look at different aspects of the solar dynamo theory as carefully as possible before passing a verdict on it.

Since the photospheric surface constitutes an opaque screen for the observer, we can observe magnetic activities only at and above the photosphere. As the magnetic fields below the photosphere cannot be observed directly, their properties have to be inferred from other considerations. First, the magnetic fields in the photosphere seem to be of a fibril and intermittent nature. Whether subsurface fields are also intermittent or continuous is a question which is by no means easy to settle. However, theoretical investigations on magnetoconvection (Proctor and Weiss, 1982) seem to suggest that magnetic fields probably exist in an intermittent fashion within the convection zone also. We take this point of view in the present paper and model the magnetic field within the convection zone in the form of flux tubes. Another inference about subsurface fields can be made from Hale’s polarity law of bipolar regions. Though it is quite unclear how the

* Present address.
** National Center for Atmospheric Research is sponsored by the National Science Foundation.

field lines of sunspots connect to the global fields underneath the surface (Parker, 1979; Choudhuri, 1986), the polarity law indicates that the global field in the convection zone must be predominantly azimuthal. So it is probably not unreasonable to assume that the magnetic flux within the convection zone basically exists in the form of flux rings going around the rotation axis and forming loop-like structures in places that may give rise to bipolar regions. Lastly, the existence of the 22-year solar cycle compels us to look for a mechanism capable of producing an oscillatory behaviour in the global field.

If the solar cycle is caused by the dynamo action, we have to figure out where this dynamo action takes place. Since the operation of the hydromagnetic dynamo requires convective motions, it used to be tacitly assumed that the convection zone is the site of the dynamo action. However, it has become increasingly clear in recent years that there are several serious difficulties in building a satisfactory model of a convection zone dynamo, one of the main difficulties being the destabilizing effect of magnetic buoyancy (Parker, 1975). This has led several theorists to suggest that the dynamo action takes place in the overshoot region at the bottom of the convection zone. A discussion of this topic with references to the appropriate literature can be found in Section I of Choudhuri and Gilman (1987; hereafter Paper I).

If we assume the dynamo to operate in the overshoot region, we can get around the difficulties confronting a convection zone dynamo model, but we seem to encounter new difficulties which are hardly any less serious. Since the overshoot region is believed to have a thickness of only a few thousand kilometers (van Ballegooijen, 1982), one has to consider whether it is thick enough for the storage of the appropriate amount of magnetic flux. A straightforward arithmetical estimate by Parker (1987) gives the answer that in order to pack the right amount of flux in the overshoot region, it is necessary for the magnetic field there to be at least one order of magnitude larger than the equipartition value. A more serious difficulty is pointed out in Paper I, which studies the influence of the Coriolis force on the dynamo-generated flux as it rises through the convection zone.

When the dynamo was believed to operate within the convection zone, the magnetic flux on the photospheric surface could be regarded as a direct signature of the dynamo action. If the magnetic flux were generated in the convection zone just below the photospheric surface and then emerged to the surface due to magnetic buoyancy, then the latitudes at which the sunspots were appearing could be regarded as indicating the regions where the flux was actually being generated by the dynamo. However, if the dynamo action takes place in the overshoot region, then the magnetic flux at the photospheric surface can no longer be regarded as a direct signature of the dynamo. The whole of the convection zone separates the overshoot region where the magnetic flux is generated and the photospheric surface where the flux ultimately emerges. In order to understand the connection between the dynamo-generated magnetic field and the magnetic fields on the solar surface, one then has to study the intermediate physical processes which cause the dynamo-generated flux to emerge eventually on the photospheric surface after making a transit through the whole of the convection zone.

As far as we know, Paper I was one of the first attempts to look into this problem
of how the dynamo-generated flux traverses the connection zone to reach the photosphere. It was treated as an initial-value problem. If the dynamo operates in the overshoot region, then it is expected that the magnetic flux will first be deposited at the bottom of the convection zone from where it will rise. Accordingly, Paper I took flux tubes at the bottom of the convection zone as the initial state and studied their time-evolution. Axisymmetry was assumed to simplify the problem so that the flux tubes were taken to be rings symmetric around the rotation axis. It was found out that the Coriolis force played a very dominant role in the problem. Unless the magnetic field in the flux ring at the bottom of the convection zone was of the order of $10^5$ G or more, it was found that the flux rings moved parallel to the rotation axis and eventually emerged on the solar surface significantly poleward of the sunspot zone. If the magnetic field at the bottom of the convection zone is assumed to have the equipartition value of about $10^4$ G, then we are at a loss to understand how magnetic flux appears at all in the sunspot latitudes.

The present paper extends these calculations to include non-axisymmetric effects. This is not a mere mathematical generalization, but involves a considerable amount of new physics also. Instead of flux tubes which just go around the rotation axis symmetrically, we now allow wave-like undulations to be present on a ring-like flux tube (Figure 1). If the undulated ring lies entirely within the convection zone, then the Coriolis force can suppress further growth of the non-axisymmetric undulations so that the results turn out to be very similar to those in Paper I. However, the results are more dramatic when the undulated ring is located such that its lower parts are embedded within the overshoot region, whereas its upper parts are within the convection zone (Figure 2(a)). This is not just a hypothetical situation. If the dynamo operates in the overshoot region and parts of the dynamo-generated flux enter the convection zone either due to statistical fluctuations or due to some instability, then we indeed expect configurations like the one in Figure 2(a) to result. We show that the upper part of the flux tube will be subject to the destabilizing effect of magnetic buoyancy and will rise through the convection zone in the form of a loop, whereas the lower part within the overshoot region may remain anchored there, provided certain conditions are satisfied. Figure 2(b) gives a schematic sketch of how the configuration in Figure 2(a) would look.

![Fig. 1. A ring-like configuration with non-axisymmetric undulations.](image)
Fig. 2a. A flux tube with its upper part sticking into the convection zone and the lower parts remaining in the overshoot region.

Fig. 2b. The appearance of the flux tube in Figure 2(a) after some time.

after some time. It is possible that the bipolar regions are caused by such an anchoring mechanism which allows only the loop-like upper parts of the flux tubes to rise and eventually 'pierce' through the photospheric surface.

In this paper, however, we are more concerned with the question whether the upper parts of the loops can appear within the sunspot latitudes. As in Paper I, we assume constant angular velocity inside the Sun and treat the convection zone as a passive region through which the flux tubes rise due to magnetic buoyancy. The possible effects of convective motions on the dynamics of flux tubes will be discussed in another paper (D'Silva and Choudhuri, 1989). We also use the thin flux tube approximation which should be good enough until the flux tubes reach the uppermost layers of the convection zone. When the calculations are carried out, we find that the Coriolis force is as overpowering in the present situation as it was in the axisymmetric case. If the initial field strength is less than $10^5$ G, the upper parts of the loops move parallel to the rotation axis and travel far beyond the sunspot latitudes. We are thus unable to resolve the difficulty pointed out in Paper I by incorporating non-axisymmetric effects in the present calculations. Though some authors have previously done two-dimensional calculations of the dynamics of flux tubes (Moreno-Insertis, 1986; Chou and Fisher, 1989), as far as we know, this paper presents the first full three-dimensional calculations incorporating the Coriolis force. As in these other simulations, we also have not included any twist in the magnetic field of the flux tube. The presence of twist imparts a cohesiveness to the flux tube and may influence its interactions with the surrounding convection zone (Tsinganos, 1980). How much twist a flux tube at the bottom of the convection zone may have is a question to which no dependable answer is available. However, if there
is enough twist for the flux tube to maintain its identity and cohesiveness, then the
dynamical evolution of the flux tube is not expected to depend much on the value of
the twist (provided it is modest and does not lead to any instabilities). So one would
hope that the present calculations without including twist would be valid for a
moderately twisted flux tube.

The basic equations of our problem are introduced in the next section. Then Section 3
presents a discussion of magnetic buoyancy, including an analysis of the concept of
anchoring. The next three sections present the results of the numerical simulation. First
we discuss the effect of non-axisymmetry without introducing anchoring in Section 4.
Then Section 5 is devoted to a study of the dynamics of anchored flux tubes in an inertial
frame of reference (i.e., without Coriolis force). Finally Section 6 describes how the
Coriolis force influences the motions of upper parts of the anchored flux tubes. Section 7
summarizes the significance of the numerical results.

2. Formulation of the Problem

2.1. The equation of motion

A systematic study of the equations governing the motions of thin flux tubes was first
presented by Spruit (1981). However, he did not consider the Coriolis force term in his
equations. In the present problem, as was done in Paper I, we assume the Sun to rotate
as a rigid body and set up our equations in the frame of reference of the Sun. So we
replace \( \text{d}v/\text{d}t \) by \( \text{d}v/\text{d}t + 2\Omega \times v \) in Equation (14) of Spruit (1981) to obtain

\[
\frac{\text{d}^2 \mathbf{r}}{\text{d}t^2} + 2\mathbf{\Omega} \times \frac{\text{d} \mathbf{r}}{\text{d}t} = -\frac{1}{\rho} \partial_t p \hat{\mathbf{i}} + g_t \hat{\mathbf{i}} + \frac{\rho}{\rho + \rho_e} \frac{B^2}{4\pi \rho} \mathbf{k} + \\
+ \frac{\rho - \rho_e}{\rho + \rho_e} (\hat{\mathbf{i}} \times \mathbf{g}) \times \hat{\mathbf{i}}.
\]  \( (1) \)

Here \( \mathbf{r} \) is the position vector of a point of the thin flux tube where the magnetic field
is \( B \), internal pressure \( p \), and internal density \( \rho \), whereas the density of the surrounding
external medium is \( \rho_e \). The unit vector \( \hat{\mathbf{i}} \) is tangent to the flux tube at that point, and
\( \mathbf{k} \) is the curvature vector. Denoting the space derivative along the flux tube (i.e., in the
direction of \( \hat{\mathbf{i}} \)) by \( \partial_t \), we have \( \mathbf{k} = \partial_t \hat{\mathbf{i}} \). The acceleration due to gravity at \( \mathbf{r} \) is \( \mathbf{g} \), and
\( g_t \) is its component along \( \hat{\mathbf{i}} \). We do not include a drag term in any of the calculations
in this paper. It was shown in Paper I that the inclusion of drag does not change the
qualitative behaviour of the system. Since the flux tube is in pressure equilibrium with
the surroundings, we have

\[
p_e = p + \frac{B^2}{8\pi}.
\]  \( (2) \)
Using the fact $\frac{\partial}{\partial t} p_e = \rho_e g_t$, it follows that

$$-\frac{1}{\rho} \frac{\partial}{\partial t} p + g_t = \frac{\rho - \rho_e}{\rho} g_t + \frac{1}{\rho} \frac{\partial}{\partial t} \left( \frac{B^2}{8\pi} \right).$$

Writing $\rho + \rho_e \approx 2\rho$, and using the vector identity $(\hat{\mathbf{i}} \times \mathbf{g}) \times \hat{\mathbf{i}} = \mathbf{g} - g_t \hat{\mathbf{i}}$, we obtain from (1):

$$\frac{d^2 \mathbf{r}}{dt^2} + 2\Omega \times \frac{d\mathbf{r}}{dt} = \frac{\rho - \rho_e}{2\rho} \mathbf{g} + \frac{B^2}{8\pi \rho} \mathbf{k} +$$

$$+ \left[ \frac{\rho - \rho_e}{2\rho} g_t + \frac{1}{\rho} \frac{\partial}{\partial t} \left( \frac{B^2}{8\pi} \right) \right] \hat{\mathbf{i}}. \quad (3)$$

Equation (1) is not suitable for numerical simulation, because the two large terms $(1/\rho) \frac{\partial}{\partial t} p$ and $g_t$ nearly balance out to give a small difference, and it is difficult to make accurate calculations of such small differences between large terms in a numerical scheme. However, (3) is free from this difficulty and can be used for numerical work.

The obvious choice for the origin of the coordinates is the centre of the Sun. As in Paper I, we introduce dimensionless coordinates $\xi = r/R_\odot$ and $\tau = 10^{-3} t (g_s/R_\odot)^{1/2}$, where $g_s$ is the surface gravity. In these new units, 1 unit of time corresponds to 18 days. We denote the radial distance by $\xi$, whereas $\xi$ is the position vector of the point $(\xi, \theta, \phi)$.

Equation (3) now becomes

$$\frac{d^2 \xi}{d\tau^2} + 2\Omega \times \frac{d\xi}{d\tau} = -M \frac{g}{g_s} + \frac{\rho_*}{\rho_e} QFk' +$$

$$+ \left[ -M \frac{g_t}{g_s} + \frac{\rho_*}{\rho_e} Q \frac{\partial F}{\partial l'} \right] \hat{\mathbf{i}}. \quad (4)$$

We use the notation $\Omega = 10^3 \Omega (R_\odot/g_s)^{1/2}$, $M = 10^6 (\rho_e - \rho)/2\rho$, $F = B^2/B_{\ast}^2$, and $Q = 10^6 B_{\ast}^2/(8\pi \rho_e g_s R_\odot)$, where $B_{\ast}$ and $\rho_{\ast}$ are suitably chosen as discussed later. Taking $l'$ to be the distance along the flux tube measured units of $R_\odot$, the curvature vector in the same units is $k' = \frac{\partial}{\partial l'} = R_\odot k$.

Once $B_{\ast}$ and $\rho_{\ast}$ are specified, $Q$ becomes a constant. As in Paper I, $\omega = 4.4$, and we take $g = -g_s/\xi^2$. The convection zone around the flux tube is modeled the same way as in Paper I (see Equations (10)–(12) thereof with the values of the parameters given in Section III of that paper) so as to give $\rho_e$ readily. We then need to know how to calculate $F$ and $M$ in order to be able to solve (4) and study the time evolution of the flux tube. We now discuss how to specify $\rho_{\ast}$ and $B_{\ast}$ suitably to describe the initial state. Details about the calculation of magnetic buoyancy $M$ and the associated phenomenon of ‘anchoring’ will be found in Section 3. The Appendix gives a discussion about how to estimate $F$ and the curvature vector $k'$, and then solve (4). The Cray–XMP at the National Center for Atmospheric Research was used to carry out the calculations.
2.2. Specification of the Initial State

If one starts with an axisymmetric initial state, then obvious choices for $B_*$ and $\rho_*$ would be to take them equal to the initial values of the magnetic field and the density. However, in the present non-axisymmetric calculations, we usually start with initial configurations of flux tubes lying in the $\theta$-cone with a sinusoidal variation in $\phi$, i.e.,

$$\xi_0 = \tilde{\xi}_* + \xi_1^* \sin m\phi.$$  \hspace{1cm} (5)

Then $B_*$ and $\rho_*$ are taken as values of magnetic field and density at the point $\xi_*$. We found it convenient to start with initial states for which the initial magnetic buoyancy $M_0$ is constant along the flux tube which is initially in thermal equilibrium. Of course, this is one of the many possibilities. One could start with other choices also, though it is found that the qualitative behaviour of the system does not depend much on such choices. If $M_0$ is constant and the tube is in thermal equilibrium, then we have

$$2 \times 10^{-6} M_0 = \frac{B_0^2}{8\pi p_{e,0}} = \frac{B_*^2}{8\pi p_{e,*}},$$  \hspace{1cm} (6)

where $B_0$ and $p_{e,0}$ are the values of $B$ and $p_e$ at different points along the flux tube in its initial configuration, and $p_{e,*}$ is the value of $p_e$ at $\xi_*$. Thus, it is seen that $B_0$ is not constant along the flux tube but is given by

$$\frac{B_0}{B_*} = \left(\frac{p_{e,0}}{p_{e,*}}\right)^{1/2}.$$  \hspace{1cm} (7)

In most of our calculations, we start with flux tubes with given values of initial magnetic buoyancy $M_0$ (corresponding to given values of $B_*$) and having initial configurations given by (5) with suitable choices of $\xi_*$, $\xi_1^*$, and $m$. Then the time evolution of the flux tube is followed. It is thus seen that the non-axisymmetric case allows many more possible choices of the initial state compared to the axisymmetric case.

3. On Magnetic Buoyancy and Anchoring

Paper I considered motions of flux rings within the convection zone under different thermal conditions (see Figure 2 in Paper I) and found that the results were qualitatively similar. This was due to the nearly adiabatic stratification of the convection zone with only a small superadiabatic gradient. However, thermal conditions may greatly influence the outcome if the flux tubes have parts of them embedded in the subadiabatic layers underneath the convection zone. The motion of the plumes in the overshoot region does not allow the temperature gradient there to depart too much from the adiabatic value (van Ballegooijen, 1982). So we expect the magnitude of the subadiabatic gradient in the overshoot region to be sufficiently low, though probably not as low as the magnitude of the superadiabatic gradient at the bottom of the convection zone where fluid motions are more vigorous and, consequently, the stratification is more nearly adiabatic. Since
the mean free path of photons is rather small at the bottom of the convection zone and heat leaks in or out of the flux tubes rather slowly, we assume the flux tubes to move adiabatically. We would see that this assumption makes it possible for the flux tubes to be anchored in the subadiabatic layers which would not be the case if the flux tubes were always in temperature equilibrium with the surroundings.

So we consider non-axisymmetric flux tubes initially in thermal equilibrium with constant magnetic buoyancy $M_0$ (as discussed in Section 2) and then allow them to move adiabatically. As pointed out in Paper I (see also Moreno-Insertis, 1983), a part of the flux tube which has moved from $\xi_0$ to $\xi$ will have magnetic buoyancy

$$M = M_0 \left[ \left(1 - \frac{1}{\gamma} \right) + \frac{1}{\gamma} \frac{B^2}{B_0^2} \frac{p_e,0}{p_e} \right] + S(\xi_0, \xi).$$  (8)

The first term always lies within the range $M_0(1 - 1/\gamma)$ to $M_0$ (i.e., it is of the order of $M_0$), and the second term $S(\xi_0, \xi)$ is caused by departures from the adiabatic stratification. The Appendix describes how to calculate the factor $B^2/B_0^2$. It can be shown that

$$S(\xi_0, \xi) = \frac{10^6}{2} \int_{\xi_0}^{\xi} \frac{\nabla \Delta T}{T_e} R_\od R \, d\xi,$$  (9)

where

$$\nabla \Delta T = \left| \frac{dT}{dr} \right| - \left| \frac{dT}{dr} \right|_{\text{adiabatic}}$$

is the superadiabatic gradient. If the gradient is subadiabatic, then the contribution to the integral in (9) is negative, and if this negative contribution is sufficiently large (of order $M_0$), we see from (8) that it is possible for $M$ to be negative. A negative $M$ implies a downward force instead of an upward directed magnetic buoyancy, and this downward force gives rise to the possibility of the flux tube being anchored in subadiabatic layers. If the flux tube were in thermal equilibrium with the surroundings, then the magnetic buoyancy would always be positive and anchoring would not have been possible.

To make an estimate of $\nabla \Delta T$ within the overshoot region or at the base of the convection zone and then to calculate $S(\xi_0, \xi)$ is a rather difficult problem, and the answers depend very much on the assumptions in one's model (van Ballegooijen, 1982). To get an insight into the problem, let us evaluate $S(\xi_0, \xi)$ in the following simple-minded fashion. Let $\xi'$ be the bottom of the convection zone. If a part of a flux tube is moved from a point $\xi_0$ within the convection zone to a higher point $\xi$, then certainly $S(\xi_0, \xi)$ is positive and increases monotonically as $(\xi - \xi_0)$. We take $S(\xi_0, \xi)$ to be linear in $(\xi - \xi_0)$:

$$S(\xi_0 > \xi_0, \xi > \xi_0) = (\xi - \xi_0) \Gamma.$$  (10a)

Similarly, if a part of the flux tube is moved from a point $\xi_0$ within the overshoot region
to another higher point $\xi$ still within the overshoot region, then also we take $S(\xi_0, \xi)$ to be linear in $(\xi - \xi_0)$ though with a minus sign:

$$S(\xi_0 < \xi_s, \xi < \xi_s) = -(\xi - \xi_0)\alpha \Gamma.$$

(10b)

Finally, for a displacement from $\xi_0$ within the overshoot region to $\xi$ within the convection zone, we have the composite relation:

$$S(\xi_0 < \xi_s, \xi > \xi_s) = -(\xi_s - \xi_0)\alpha \Gamma + (\xi - \xi_s)\Gamma.$$

(10c)

The relation $L_\odot = 4\pi r^2 \rho_e c_p \kappa \nabla \Delta T$ can be used to find the superadiabatic gradient $\nabla \Delta T$. Taking $\kappa = 10^{13}$ cm$^2$ s$^{-1}$ and substituting values of different quantities appropriate for the bottom of the convection zone in (9), the constant of proportionality $\Gamma$ in the expression (10a) is found to have the value

$$\Gamma = 2.3.$$

(11)

Taking $\alpha = 1$ would imply a subadiabatic gradient in the overshoot region as small in magnitude as the superadiabatic gradient in the convection zone. If plumes are not very vigorous, a value $\alpha = 10$ may not be unreasonable. A look at Figure 1 of Paper I shows that the superadiabatic gradient term increases fairly rapidly at the top of the convection zone. So the simple expression (10a) taken with (11) would tend to underestimate the superadiabatic gradient term in the top layers in which the results of our calculations cannot in any way be trusted too much, because of the breakdown of the thin flux tube approximation. For motions of the flux tubes within the main bulk of the convection zone where various quantities change slowly, however, using (10) to estimate $S(\xi_0, \xi)$ should be quite satisfactory.

Let us take the overshoot region to have a thickness $\delta \xi = 0.015$ ($\approx 10^4$ km). If a segment of a flux tube starts from the bottom of the overshoot region and reaches the top, then the contribution to $S(\xi_0, \xi)$ will be $-\delta \xi \alpha \Gamma = -0.035 \alpha$. In order for anchoring to be possible, the initial magnetic buoyancy $M_0$ should not be larger than this, i.e.,

$$M_0 < 0.035 \alpha.$$

(12)

The magnetic field $B_0$ corresponding to this $M_0$ has to satisfy the condition

$$B_0 < 9.9 \times 10^3 \alpha^{1/2} \text{ G}.$$

(13)

Thus, for reasonable values of $\alpha$, flux tubes with fields of the order of $10^4$ G may be anchored. It should be noted that (12) and (13) are only necessary conditions and may not be sufficient conditions. Our numerical simulations will provide specific examples of anchoring.

4. Results without Anchoring

Before considering flux tubes which have parts anchored in the overshoot region, let us study flux rings which lie entirely in the convection zone but possess non-axisymmetries in the initial state. In order to isolate the effects of Coriolis force, we first consider the
motions of such flux rings in a non-rotating frame (i.e., without the Coriolis force term in the equation of motion), and then, with identical initial conditions, we switch on the Coriolis force to look at the differences in the subsequent time-evolutions. It turns out that the non-axisymmetries can be suppressed in a rotating frame of reference, provided the flux ring is entirely within the convection zone and there is no anchoring. In other words, even if non-axisymmetries are present in the initial state, they tend to be smeared out by the influence of the Coriolis force. Throughout this section and the next two sections, we shall be considering flux rings located at a latitude $5^\circ$ initially. In those cases in which the Coriolis force is 'switched off', however, all motions are in the radial direction (apart from negligible deviations due to magnetic tension) and the particular value of latitude does not have much significance, we still have used initial locations of $5^\circ$ latitude throughout the calculation in order to compare the results of the cases with and without Coriolis force.

Let us present the results for a flux ring with initial magnetic buoyancy $M_0 = 1$, corresponding to $B_* = 5.4 \times 10^4$ G at the bottom of the convection zone. This case was studied in Paper I (see the middle row of either Figure 2 or Figure 5 of Paper I), and it was seen that the Coriolis force is quite dominant in this case. Right now, however, instead of starting from an axisymmetric initial state as in Paper I, we consider initial states which have some non-axisymmetric features. The non-axisymmetric features can be of different kinds. One possibility is to consider wave-like variations in the initial configurations as given by (5). For flux tubes which are not anchored, however, the consequences are more dramatic if we introduce non-axisymmetries through an asymmetric initial velocity distribution. So let us first describe this case. We consider a flux tube which has the shape of a symmetric ring at the bottom of the convection zone at time $\tau = 0$. All flux rings were supposed to start with zero velocity in Paper I. Here we take the initial velocity to have non-zero $v_r$-component given by

$$v_{r,0} = 0.1(1 + \sin m\phi),$$

where the velocity is expressed in the dimensionless units following the convention of Paper I (a velocity of 1 corresponds to 0.43 km s$^{-1}$). Figure 3(a) shows a plot of $\xi$ vs $\phi$ for the flux ring at intervals of $S = 0.1$ ( = 1.8 days) as it evolves in time. We had taken $m = 4$ in Equation (14) for this run, and the Coriolis force term was dropped from the equations. The marks on the curves indicate Lagrangian positions of fluid particles which had $\phi$-values differing by $15^\circ$ at $\tau = 0$. Though the flux ring starts from a symmetric configuration at the bottom of the convection zone, it is clearly seen that the velocity perturbations give rise to ever-increasing undulations as the flux ring rises through the convection zone. The reader, however, should be warned that Figure 3(a) gives an exaggerated notion of the sizes of the undulations. In order to see clearly how large the undulations are, it is necessary to make a polar plot of $\xi$ vs $\phi$. Accordingly, Figure 3(b) displays the same result as Figure 3(a) in a polar plot. We made several runs for different values of $m$. Since the undulations are rather gentle for small values of $m$ and magnetic tension does not play any significant role, it was found that the perturbations grew at comparable rates up to a value of $m = 8$. For larger values of $m$, growth
Fig. 3a. The evolution of a flux ring without anchoring and without Coriolis force shown in $\zeta$ vs $\phi$ plot. The flux ring had initial magnetic buoyancy $M_0 = 1.0$ (corresponding to $B_\ast = 5.4 \times 10^4$ G) and initial velocity given by (14). The successive configurations differ by the interval $S = 0.1$ ($= 1.8$ days).

Fig. 3b. A polar plot of the same configurations as Figure 3(a).

rates of the undulations are somewhat slowed down due to the action of magnetic tension.

Similar runs were made for initial states with position perturbations of the type indicated in Equation (5), but with zero initial velocities. Care was taken that no part of the initial flux ring was within the overshoot region so that there was no anchoring.
In this case, the undulated flux rings rose through the convection zone without the undulations growing much as in the case of initial velocity perturbations.

Finally the Coriolis force term was put into the equations in order to see explicitly what differences it makes. Figure 4(a) presents the results for a run in which everything else was exactly the same as in the case of Figure 3(a), the only difference being that the Coriolis force was 'switched on' and the time interval between the successive positions is taken be \( S = 0.3 \) (\( = 5.4 \) days) instead of \( S = 0.1 \) (\( = 1.8 \) days) as in Figure 3(a). The numbers appearing with the curves in Figure 4(a) indicate the succession of configurations as the flux ring evolves in time. The dramatically different appearance of Figure 4(a) from Figure 3(a) clearly demonstrates the importance of the Coriolis force. In order to interpret what is happening, our programme calculated the average values of \( \xi \) and \( \theta \) for the flux ring at each time-step of integration. These average values of \( \xi \) and \( \theta \) trace out a trajectory which can be plotted in exactly the same way that we plotted the trajectories for axisymmetric flux rings in Paper I. Figure 4(b) shows the trajectory corresponding to the case of Figure 4(a). This trajectory appears exactly similar to the trajectories in Paper I for the case of \( M_0 = 1.0 \) starting at a latitude of \( 5^\circ \). Looking at both Figures 4(a) and 4(b), one realizes what is happening. The Coriolis force is suppressing the non-axisymmetries so that the ring rises very much like a ring symmetric around the rotation axis. As a result of the oscillations of the flux ring trajectories discussed in detail in Paper I (see Appendix A of Paper I), \( \xi \) for a fluid particle on the ring does not increase monotonically with time, resulting in the complicat-

![Figure 4a](image-url)  

Fig. 4a. The evolution of a flux ring identical to the flux ring in Figure 3(a) except that the Coriolis force is 'switched on'. The successive configurations indicated by the numbers differ by the interval \( S = 0.3 \) (\( = 5.4 \) days).
ed behavior shown in Figure 4(a). We thus conclude that when there is no anchoring, it makes little difference whether we consider a strictly symmetric initial state or an initial state with some amount of non-axisymmetry. The Coriolis force smears out the non-axisymmetries and makes the results come out almost the same in both cases.

Comparing Figure 4(b) with the trajectories in Figure 2 of Paper I, one finds more resemblance with the trajectories in 2(d) or 2(e) rather than those in 2(f), which included the superadiabatic gradient and trajectories became radial at the top of the convection zone. Though the superadiabatic gradient is included in the present calculations also, the difference is caused by the fact that its inclusion through the formulae (10) makes an underestimation of its value in the uppermost layers as pointed out already and, consequently, the trajectory does not turn radial at the top in the present calculations, though the qualitative behaviour within the bulk of the convection zone is the same for all the thermal conditions.

5. Anchoring without Coriolis Force

In order to understand the physics of how the flux tube can get anchored at the bottom of the convection zone, we first look at this problem of anchoring in the absence of the Coriolis force. As we have pointed out in Section 3, it is probably not unrealistic to take the magnitude of the subadiabatic gradient within the overshoot region to be about ten times the magnitude of the superadiabatic gradient at the bottom of the convection zone.
So we take $\alpha = 10$ in (10). It then follows from (12) that a flux tube with $M_0 = 0.1$ (corresponding to $B_\star = 1.7 \times 10^4 \text{ G}$) is expected to be anchored if it starts from the bottom of the overshoot region.

Here we present the results for a flux ring with $M_0 = 0.1$ having an initial configuration of the type (5) given by

$$\xi_0 = 0.7 + 0.015 \sin 4\phi$$

(15)

at $5^\circ$ latitude and with zero initial velocity. Since the bottom of the convection zone is taken at $\xi_s = 0.7$, (15) represents a sinusoidal perturbation at the bottom of the convection zone such that the upper half of the flux ring is in the convection zone and the lower half is within the overshoot region. Since the thickness of the overshoot region is estimated to be about 0.015 in our dimensionless units, the lowestmost points of the initial configuration (15) lies at the bottom of the overshoot region just before the radiative core is reached. Figure 5(a) shows the time-evolution of this flux ring plotted in a polar diagram at intervals of $S = 0.4$ (= 7.2 days).

It is clearly seen in Figure 5(a) that the lower parts of the flux tube remain anchored within the overshoot region, whereas the upper parts rise in the form of loops. As these upper parts rise through the convection zone, they tend to get accelerated. This is due to the superadiabatic gradient term in the expression of magnetic buoyancy as given by (8). There is a downflow of plasma from the upper parts of the loops to the lower parts. This sort of flow is an essential aspect of the type of instability usually called Parker instability (Parker, 1966), which is related to the present problem. However, if we look at the marks which indicate the Lagrangian positions of particular fluid
elements, we find that the downflow is not very noticeable. It turns out that the superadiabatic gradient is sufficiently strong to make the upper parts move rather fast and the marks representing the Lagrangian fluid particles do not have enough time to slide down the trough during the relatively rapid time of rise.

In reality, however, we expect the upper parts of the loops to be much less buoyant, since heat exchange becomes more efficient in the upper layers of the convection zone and the actual magnetic buoyancy should be much less than the value obtained by assuming adiabatic conditions with a superadiabatic gradient. The other extreme limit would be to assume the upper parts of the loops to be in thermal equilibrium with the surroundings. In Paper I, however, we also considered another condition intermediate between these two limits: the adiabatic condition without including the contribution due to the superadiabatic gradient (see Section III of Paper I). Since the real situation lies between the two extreme limits, this intermediate condition may actually be closer to reality than the two extreme conditions. In any case, Figure 2 of Paper I shows that this intermediate condition and thermal equilibrium with surroundings give very similar results for motions within the convection zone. We now present some calculations done by using this intermediate condition within the convection zone, i.e., by neglecting the contribution to magnetic buoyancy made by the superadiabatic gradient within the convection zone. If we retain the contribution due to the subadiabatic gradient in the overshoot region where heat exchange is very slow, then anchoring is still possible. Mathematically, this means that while calculating magnetic buoyancy from (8), we replace (10) by the following expressions:

\[ S(\xi_0 > \xi_s, \xi > \xi_s) = 0, \]
\[ S(\xi_0 < \xi_s, \xi < \xi_s) = - (\xi - \xi_0) \alpha \Gamma, \]
\[ S(\xi_0 < \xi_s, \xi > \xi_s) = - (\xi_s - \xi_0) \alpha \Gamma. \]  

Figure 5(b) presents the results for this case where everything else is the same as in Figure 5(a), only (10) is replaced by (16) while calculating magnetic buoyancy. A comparison between Figures 5(a) and 5(b) shows that the upper parts of the loops do not get accelerated as much in Figure 5(b) as in Figure 5(a). The slower motion allows the downflow to continue for a longer time and certainly it is quite visible in Figure 5(b).

The results obtained in this section are quite similar to the results obtained by other authors who have carried out two-dimensional simulations of flux tube evolution in Cartesian geometry (Moreno-Insertis, 1986; Chou and Fisher, 1989). Moreno-Insertis (1986) started with a flux tube originally in mechanical equilibrium with the surroundings rather than in thermal equilibrium as we have done in this paper. He showed that even if a flux tube with a wave-like perturbation is initially entirely within the convection zone, still its lower parts could go underneath the bottom of the convection zone and get anchored there. Results were presented for a flux tube with a rather high magnetic field of $10^{5.5}$ G which showed this type of anchoring. It was possible for a flux tube with such high field to sink below the convection zone and get anchored only because of the rather artificial initial condition of mechanical equilibrium. This meant that the flux tube was
colder than the surroundings, which produced a stabilizing influence that persisted under the adiabatic conditions assumed by Moreno-Insertis (1986). When we begin with a flux tube in thermal equilibrium with the surroundings, the necessary initial conditions for anchoring are that the lower parts of the flux tubes are within the overshoot region and the inequality (13) is satisfied. Chou and Fisher (1989) did not consider any physical mechanism for anchoring. They produced anchoring in their calculations merely with the mathematical specification of the boundary condition that the end-points of the flux tube did not move. In spite of these differences, the evolution of the upper parts of the loop structures turns out to be qualitatively similar in all the three simulations.

6. Anchoring with Coriolis Force

At last we are ready to look at the final problem of how an anchored flux tube evolves when the Coriolis force is taken into account. In order to isolate the effect of the Coriolis force, we repeat the same calculations presented in the last section with the Coriolis force term ‘switched on’ now. We consider a flux ring with $M_0 = 0.1$ (corresponding to $B_\star = 1.7 \times 10^4$ G) with an initial configuration given by (15) at 5° latitude. Instead of the thermal condition (10), we use (16), which is probably closer to reality and has the added advantage of slowing down the motions of loop structures so that the dynamics of rise can be studied more easily.

A polar plot of $\xi$ vs $\phi$ showing the successive configurations of the flux ring at intervals of $S = 0.8$ ( = 14.4 days) is presented in Figure 8(a). This figure is comparable to Figure 5(b) in all other respects (including the thermal condition) except that the Coriolis force is present now and the interval between the successive configurations is changed from $S = 0.4$ ( = 7.2 days) to $S = 0.8$ ( = 14.4 days). Looking at the Lagrangian markers,
Fig. 6a. A polar plot of the evolution of a flux ring under identical conditions (including the thermal condition) as the flux ring shown in Figure 5(b) except that the Coriolis force is 'switched on' \( (B_\phi = 1.7 \times 10^4 \text{ G}) \). The successive configurations differ by the interval \( S = 0.8 \) (= 14.4 days).

we see that there is clear evidence of a flow in the \( \phi \)-direction as we expect from considerations of angular momentum in a rotating frame of reference (see Paper I for a detailed discussion of the \( \phi \)-flow). Also the rising loops appear somewhat asymmetric now. Figure 6(b) shows the same \( \zeta \) vs \( \phi \) plot as Figure 6(a) in a Cartesian representation and makes some of the details clearer. However, the real significance of the Coriolis force becomes apparent only when we plot the trajectories of the highest and lowest
points of the flux ring (i.e., points with the maximum and minimum value of $\xi$ at any instant of time) in the $\xi - \theta$ plane. This is done in Figure 6(c), which shows that the highest points of the loops move more or less parallel to the rotation axis and have trajectories very similar to those of axisymmetric flux rings of Paper I. The markers on the trajectories in Figure 6(c) show the positions of the highest and lowest points at those moments for which the detailed configurations are presented in Figures 6(a) and 6(b).

It is seen in Figure 6(c) that the lowest points also slide to somewhat higher latitudes. This is due to the fact that the upper parts of the loops moving to high latitudes exert a force of magnetic tension on the lower parts and make them move. We carried out some simulations in which we imposed the boundary condition that the lowest point could not move in the $\theta$-direction, thereby pinning down the lowest point at a particular latitude. In those simulations also, the upper parts of the loops moved parallel to the rotation axis. We thus could not avoid the inescapable conclusion that the upper parts of the loops can emerge only at high latitudes, well beyond where the sunspots are observed.

7. Conclusion

Recent observations of emerging active regions seem to suggest that they are caused by magnetic loops rising through the solar surface (Zwaan, 1985). The fact that such rising loops would be influenced by the Coriolis force is obvious. In fact, the initial orientations of the emerging active regions have been thought to be due to the action of the Coriolis
force which may make the flux loops rotate as they rise (Garcia de la Rosa, 1986). However, we find that the effect of the Coriolis force is much stronger than anybody suspected before, and unless we figure out some mechanism by which the rising loops can get out of the grips of the Coriolis force, we fail to understand how the rising loops could, to begin with, appear at the latitudes where they seem to appear.

In this paper, we have discussed an anchoring mechanism in which parts of flux tubes are within the overshoot region and thereby get anchored there. If the dynamo operates within the overshoot region, as suggested by many authors (see Section I of Paper I), then this might be a natural anchoring mechanism. However, making the dynamo operate within the overshoot region is not the only way of suppressing magnetic buoyancy. Other suggestions also have been made to show that there may be a stable layer at the bottom of the convection zone where magnetic buoyancy is suppressed and dynamo action may take place. Parker (1987) pointed out that thermal shadows at the bottom of the convection zone may produce the desired stabilizing effect. van Ballegooijen and Choudhuri (1988) showed that if there is an equatorward meridional flow at the bottom of the convection zone satisfying certain conditions, then this flow may be able to suppress the magnetic buoyancy. The basic common feature of all these scenarios is that they all predict a stable layer at the bottom of the convection zone where magnetic flux can be stored and when parts of flux tubes come out of this stable layer, those parts may rise in the form of loop structures. Though our calculations in this paper were done by specifically assuming one particular anchoring mechanism, i.e., the subadiabatic temperature gradient of the overshoot region, our basic conclusions should be equally applicable to all the proposed scenarios. No matter how the anchoring is accomplished, the main fact is that the rising parts of the loops tend to move parallel to the rotation axis and, hence, appear at high latitudes.

How could one hope to resolve this difficulty? Right now it is a totally open question. One possibility is whether the convective motions could entangle the magnetic flux and carry it to the solar surface. It should be remembered that the Coriolis force will still act on the rising magnetic flux while it is being dragged by convective motions. This is a very complicated problem to tackle. However, some calculations are currently under way and we hope to be able to present the results in a forthcoming paper (D'Silva and Choudhuri, 1989). It was concluded in Paper I that the differential rotation could not be of much help in making axisymmetric flux rings come out at low latitudes. However, when non-axisymmetric loops are formed, the differential rotation may have a more pronounced effect. We plan to look at this problem in the future. As the new science of helioseismology comes of age, we may know more about differential rotation and magnetic fields within the interior of the Sun, giving us an additional handle on the dynamo problem. Another possibility is that the magnetic fields at the bottom of the convection zone may get compressed by some means to values more than one order of magnitude higher than the equipartition value. If the field can be compressed to $10^5$ G, then magnetic buoyancy would overpower the Coriolis force to make the flux come out radially. At first sight, this may not look like a totally impossible proposition, since the magnetic fields in sunspots also are known to have values considerably above equipar-
tribution values in the photosphere. However, a look at the condition (13) will show that such a high field would make anchoring impossible, making it very difficult to explain the origin of bipolar active regions. Furthermore, such a high field cannot be stored even in the stable layers at the bottom of the convection zone for a sufficiently long time for dynamo amplification. Thus, with respect to the value of the magnetic field strength at the bottom of the convection zone, we seem to be between Scylla and Charybdis. If the value is too low, then the Coriolis force drives the flux to high latitudes, and if the value is too high, then anchoring or storage becomes difficult.

It is perhaps fair to say that a major uncertainty remains at the present time regarding the question of where exactly the dynamo process takes place. If we assume the dynamo to operate within the main body of the convection zone, then the magnetic buoyancy poses a serious problem. On the other hand, until we understand how the magnetic flux can get out of the clutches of the Coriolis force, the hypothesis that the dynamo operates at the bottom of the convection zone remains at best a far-fetched speculation. One may even take an extreme point of view and raise the question whether the dynamo theory itself is the correct theory for the generation of solar magnetic fields. There have been some recent claims that the solar cycle may involve oscillations penetrating to the core of the Sun (Gough, 1988). If these claims turn out to be true, then it will be necessary to understand their implications for the dynamo theory. More theoretical and observational work will certainly be needed before the fate of the dynamo theory can be decided.

Acknowledgements

This work began when I was a Visiting Scientist at High Altitude Observatory and was finished after I joined the faculty of Indian Institute of Science. First and foremost, I wish to thank Peter Gilman. This work started as a joint venture between the two of us, and Peter contributed many important inputs at the early stages of the work. Afterwards, however, Peter became too preoccupied with his duties as the Director of High Altitude Observatory, and at his own suggestion, he is not listed as a co-author. I am also very grateful to Jack Miller who learnt how to use my code developed for the Cray-XMP at NCAR. While I was away from NCAR, Jack managed to run my code and sent me some results that I needed. I thank Sydney D'Silva for his comments on the manuscript.

Appendix. A Note on the Numerical Code

The dynamics of a thin flux tube is essentially a problem which involves the evolution of a 1-dimensional string in 3-dimensional space. We represented the flux ring with 120 Lagrangian points such that the configuration of the flux ring at a particular instant could be obtained by connecting these 120 points. The basic Equation (4) was used in the numerical code to find out how the fluid particles at these 120 Lagrangian points moved in space. The Lagrangian points themselves were chosen in such a way that the initial values of longitude $\phi$ for successive points differed by $3^\circ$ at $\tau = 0$. The initial $\zeta$-coordi-
nates of the points could, for example, be given in accordance with Equation (5), whereas the latitude \((\frac{1}{2} \pi - \theta)\) was chosen to be \(5^\circ\) for all the calculations in this paper. Since (4) is second-order in time, in addition to the initial position coordinates for the Lagrangian points, we have to specify the initial velocities of those points also. Calculations in Sections 5 and 6 took the initial velocity to be zero.

In order to solve (4), it is necessary to evaluate the curvature vectors at these Lagrangian points. This is done as follows. The tangent vector at a point intermediate between the Lagrangian points \(\xi_{n-1}\) and \(\xi_n\) can be approximated by

\[
\hat{\mathbf{t}}_{n\pm1/2} = \frac{\xi_{n \mp 1} - \xi_{n-1}}{|\xi_{n \mp 1} - \xi_{n-1}|}, \tag{A1}
\]

Similarly the tangent vector at a point intermediate between \(\xi_n\) and \(\xi_{n+1}\) is

\[
\hat{\mathbf{t}}_{n+1/2} = \frac{\xi_{n+1} - \xi_n}{|\xi_{n+1} - \xi_n|}, \tag{A2}
\]

Since the curvature vector is defined as \(\mathbf{k}' = \partial \hat{\mathbf{l}} / \partial l'\), the curvature vector at the \(n\)th Lagrangian point can be taken as

\[
\mathbf{k}'_n = \frac{\hat{\mathbf{t}}_{n+1/2} - \hat{\mathbf{t}}_{n-1/2}}{|\xi_{n+1} - \xi_{n-1}|/2}, \tag{A3}
\]

where \(\hat{\mathbf{t}}_{n-1/2}\) and \(\hat{\mathbf{t}}_{n+1/2}\) are given by (A1) and (A2). To calculate this curvature vector numerically, we need to find out the components of quantities like \((\xi_{n+1} - \xi_n)\). If we erect local Cartesian axes at the point \(\xi_n\), then it can be easily shown that the local 'Cartesian' components of \(\Delta \xi_{n+1,n} = \xi_{n+1} - \xi_n\) in the \((\xi, \theta, \phi)\) directions are given by

\[
(\Delta \xi_{n+1,n})_\xi = \xi_{n+1} \sin \theta_{n+1} \sin \theta_n \cos (\phi_{n+1} - \phi_n) + \\
\xi_{n+1} \cos \theta_{n+1} \cos \theta_n - \xi_n, \\
(\Delta \xi_{n+1,n})_\theta = \xi_{n+1} \sin \theta_{n+1} \cos \theta_n \cos (\phi_{n+1} - \phi_n) - \\
- \xi_{n+1} \cos \theta_{n+1} \sin \theta_n, \\
(\Delta \xi_{n+1,n})_\phi = \xi_{n+1} \sin \theta_{n+1} \sin (\phi_{n+1} - \phi_n). \tag{A4}
\]

Using these expressions, one can easily find out the components of the tangent vector or the curvature vector from (A1)–(A3).

To solve (4), we also have to evaluate \(F = B^2 / B_*^2\) at the \(n\)th point. Let \(B_n\) be the magnitude of the magnetic field at the \(n\)th point and \((B_n)_0\) its initial value. From the conservation of flux, it follows

\[
\frac{B_n}{(B_n)_0} = \frac{\rho_n}{(\rho_n)_0} \frac{|\xi_{n+1} - \xi_{n-1}|}{|\xi_{n+1} - \xi_{n-1}|_0}, \tag{A5}
\]

where \((\rho_n)_0\) and \(|\xi_{n+1} - \xi_{n-1}|_0\) are the initial values of these quantities. Once we know
\( B_n/(B_n)_0 \), the magnetic buoyancy as given by (8) can be evaluated. The quantity \( F \) at the \( n \)th point is now given by

\[
F_n = \frac{B_n^2}{B_0^2} = \frac{B_n^2}{(B_n)_0^2}.
\]

(A6)

Since (A5) gives \( B_n/(B_n)_0 \) and (7) gives \((B_n)_0/B_0\), we can readily calculate \( F_n \).

To find out how the coordinates \((\xi_n, \theta_n, \phi_n)\) of the \( n \)th Lagrangian point change in time, we take the three components of (4) written in the following fashion:

\[
\ddot{\xi}_n \dot{\theta} - \dot{\xi}_n \dot{\phi}_n^2 \sin^2 \theta_n - 2\omega \ddot{\xi}_n \dot{\phi}_n \sin^2 \theta_n =
\]

\[
= M_n \frac{\xi_{n+1} - \xi_{n-1}}{\xi_{n+1} - \xi_{n-1}} + c_n(k'_n)_{\xi},
\]

(A7)

\[
\xi_n \dddot{\theta}_n + 2 \dddot{\xi}_n \dot{\theta}_n - \dddot{\xi}_n \dot{\phi}_n^2 \sin \theta_n \cos \theta_n - 2\omega \ddot{\xi}_n \dot{\phi}_n \sin \theta_n \cos \theta_n =
\]

\[
= L_n \frac{\xi_{n+1} - \xi_{n-1}}{\xi_{n+1} - \xi_{n-1}} + c_n(k'_n)_{\theta},
\]

(A8)

\[
\xi_n \dddot{\phi}_n \sin \theta_n + 2 \dddot{\xi}_n \dot{\phi}_n \sin \theta_n + 2 \dddot{\xi}_n \dot{\theta}_n \sin \theta_n + 2\omega (\dddot{\xi}_n \dot{\theta}_n \cos \theta_n + \dddot{\xi}_n \sin \theta_n) = 
\]

\[
= L_n \frac{\xi_{n+1} - \xi_{n-1}}{\xi_{n+1} - \xi_{n-1}} + c_n(k'_n)_{\phi}.
\]

(A9)

Here a dot represents differentiation with respect to \( \tau \), \( M_n \) is magnetic buoyancy at the \( n \)th point as given by (8), \( c_n = \{\rho_*/\rho_e(\xi_n)\}QF_n, ((k'_n)_{\xi}, (k'_n)_{\theta}, (k'_n)_{\phi}) \) are the components of the curvature vector \( k'_n \) and \( L_n \) is given by

\[
L_n = \frac{M_n}{\xi_n^2} \frac{\xi_{n+1} - \xi_{n-1}}{\xi_{n+1} - \xi_{n-1}} + \frac{\rho_*/Q}{\rho_e(\xi_n)} \frac{F_{n+1} - F_{n-1}}{\xi_{n+1} - \xi_{n-1}}.
\]

The integration in time was carried on with the help of the programme ODEN for solving ordinary differential equations. The Cray-XMP at NCAR was used for the numerical simulations.

References


© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System