NUMERICAL SIMULATION OF CORONAL HEATING BY RESONANT ABSORPTION OF ALFVÉN WAVES

STEFAAN POEDTS, MARCEL GOOSENS
Astronomisch Instituut, Katholieke Universiteit Leuven, B-3030 Heverlee, Belgium

and

WOLFGANG KERNER
Max-Planck-Institut für Plasmaphysik, Euratom Association, D-8046 Garching bei München, F.R.G.

(Received in revised form 9 March, 1989)

Abstract. The heating of coronal loops by resonant absorption of Alfvén waves is studied in compressible, resistive magnetohydrodynamics. The loops are approximated by straight cylindrical, axisymmetric plasma columns and the incident waves which excite the coronal loops are modelled by a periodic external driver. The stationary state of this system is determined with a numerical code based on the finite element method. Since the power spectrum of the incident waves is not well known, the intrinsic dissipation is computed. The intrinsic dissipation spectrum is independent of the external driver and reflects the intrinsic ability of the coronal loops to extract energy from incident waves by the mechanism of resonant absorption.

The numerical results show that resonant absorption is very efficient for typical parameter values occurring in the loops of the solar corona. A considerable part of the energy supplied by the external driver, is actually dissipated Ohmically and converted into heat. The heating of the plasma is localized in a narrow resonant layer with a width proportional to $n^{1/3}$. The energy dissipation rate is almost independent of the resistivity for the relevant values of this parameter. The efficiency of the heating mechanism and the localization of the heating strongly depend on the frequency of the external driver. Resonant absorption is extremely efficient when the plasma is excited with a frequency near the frequency of a so-called 'collective mode'.

1. Introduction

The heating of the solar corona is a major problem in solar and stellar physics since X-ray observations showed that many classes of stars have a corona like the Sun (see, e.g., Mullan and Stencel, 1982; Rosner, Golub, and Vaiana, 1985; Jordan and Linsky, 1987). The existence of stellar coronae (hot plasma overlying cooler plasma) requires a non-radiative heating mechanism. The temperature can only rise further away from the surface of the Sun (and other stars) when there is a flow of energy from the Sun towards the corona and this requires that work is done. This leads in a natural way to the picture that convective upwelling motions in the photosphere do work and convert mechanical energy in the form of waves. These waves propagate upwards through the atmosphere and the dissipation of the wave energy in the corona causes the heating of the corona. This picture contains three aspects to be verified: the generation of the waves, the propagation of the waves in the atmospheric plasma, and the dissipation of the wave energy in the corona. In this paper the focus is entirely on the dissipation of waves in the solar corona. The propagation and dissipation of acoustic waves in the solar atmosphere has already been studied profoundly. Short periodic sound waves


© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System
(10–50 s) may heat the lower chromosphere but the acoustic flux in the higher chromosphere and corona is by far too small to deliver the energy necessary for the heating of these layers. This is one of the reasons why it is now generally assumed that the magnetic field plays a fundamental role in the heating of the higher atmospheric levels.

The effect of magnetic fields on the generation, the propagation and the dissipation of waves in the solar atmosphere has been investigated (see, e.g., Osterbrock, 1961; Zwiebel, 1980; Habbal and Rosner, 1979). A general photospheric magnetic field disturbance produces waves of several types such as fast and slow magnetosonic waves and Alfvén waves. The magnetosonic waves steepen into shocks and dissipate in a similar manner to pure acoustic waves. The strong inhomogeneity of the solar corona magnetic field and its strong concentration in intense kilogauss fields, leads to the suggestion that Alfvén waves play a dominant role in the heating of the solar corona (Hollweg, 1979, 1981). Due to the strong magnetic fields in the solar atmosphere, Alfvén waves can propagate very easily without becoming evanescent or being internally reflected. In fact, the problem with Alfvén waves is not how they can propagate from the photosphere through the chromosphere to the corona, but rather how they are dissipated.

Observations of the solar corona in soft X-rays from satellites such as Skylab have revealed the dominant influence of the magnetic field on the structure of the corona. The corona consists of open and closed magnetic structures. The open regions are known as coronal holes and give the most important contributions to the solar wind. The regions in which the magnetic field is mainly closed consist of myriads of loops. These loops are outlined by the magnetic field and show up as bright structures (in X-ray observations), hotter than their surroundings. By consequence, the loop heating mechanism is of primordial importance.

In an inhomogeneous plasma magnetic waves and Alfvén waves in particular can be dissipated very efficiently by means of resonant absorption. A qualitative description of this heating mechanism can be given in ideal (dissipationless) magnetohydrodynamics. The spectrum of oscillation frequencies of an inhomogeneous plasma contains two continuous parts in linear ideal MHD: an Alfvén continuum and a slow magnetoacoustic continuum. The solutions of the linearized ideal MHD operator that correspond to the continuum frequencies, are non-square integrable. This characteristic feature of the continuum solutions lies at the basis of the process of resonant absorption of energy.

Consider a diffuse cylindrical axisymmetric plasma with equilibrium quantities only varying in the radial direction. The local Alfvén frequency, \( \omega_\Lambda \), and the local cusp frequency, \( \omega_c \), are then (in general) functions of the radial coordinate. When the plasma is excited periodically by an external source with a frequency, say \( \omega_0 \), within the range of the Alfvén continuum, a resonance occurs on the magnetic surface \( r_s \) where \( \omega_0 = \omega_\Lambda (r_s) \). The radial velocity component has, in general, a logarithmic singularity and the tangential components of the velocity field possess non-square integrable singularities in \( r = r_s \) (see, e.g., Goedbloed, 1983). The plasma energy will, therefore,
accumulate unbounded (in ideal MHD) in an ever diminishing plasma layer around the resonance point. However, due to dissipative effects, the system attains a stationary state after a finite time. In this stationary state all physical quantities oscillate in time with the frequency of the driver, \( \omega_0 \), and the power of the external driver exactly balances the energy dissipation rate in the resonant layer. The absorbed energy is thermalized by the dissipation.

Resonant absorption of Alfvén waves was first studied as a heating mechanism for laboratory plasmas in the context of controlled thermonuclear fusion research. The objective of the thermonuclear fusion research programme is to confine a sufficiently hot and dense plasma for a long enough time so that the power produced by the fusion reactions exceeds the power necessary to heat the plasma (Gill, 1981; Chen, 1984). The ‘tokamak’ is, of all experiments where the plasma is confined by means of magnetic fields, the most promising candidate for a future fusion reactor. A tokamak is an axisymmetric toroidal configuration characterized by a strong toroidal magnetic field, mainly generated by external coils, and a weaker poloidal magnetic field which is a superposition of the field created by the toroidal plasma current and an external contribution. The plasma current not only generates a poloidal magnetic field that confines the plasma, but also provides a primary heating of the plasma by Ohmic dissipation. However, due to the fast decrease of the resistivity with the increasing temperature, supplementary heating is necessary to bring the plasma in the ignition regime. Resonant absorption of Alfvén waves is one of the methods investigated for this purpose (see, e.g., Tataronis, 1975; Tataronis and Grossmann, 1973, 1976; Grossmann and Tataronis, 1973; Kappraff and Tataronis, 1977; Chen and Hasegawa, 1974; Hasegawa and Chen, 1974, 1975; Appert et al., 1980, 1981).

Resonant absorption of Alfvén waves was first proposed as a heating mechanism for coronal loops by Ionson (1978). Since then this heating mechanism has been investigated by many authors in this context (see, e.g., Ionson, 1982, 1984; Rae and Roberts, 1981, 1982; Heyvaerts and Priest, 1983; Nocera, Leroy, and Priest, 1983; Sakurai, 1985; Sakurai and Granik, 1984; Hollweg, 1984a, b; Hollweg and Sterling, 1984; Mok and Einaudi, 1985; Lee and Roberts, 1986; Steinolfson, 1984; 1985; Pritchett and Dawson, 1978; Grossmann and Smith, 1988; Hollweg, 1987a, b; Hollweg and Yang, 1988).

Most of the plasma physics literature and the solar physics literature deals with ideal MHD. Of course, ideal MHD is a conservative theory. The energy absorbed by the plasma is not dissipated in this model but accumulates in an ever diminishing layer around the resonant surface. It is then assumed that this energy is converted into heat by dissipative effects but this is, of course, outside the framework of ideal MHD. However, damping has been taken into account by some authors in analytical and numerical studies, but only for very simple one-dimensional models for coronal loops with slab geometry. Grossmann and Smith (1988) recently reported numerical results for a simple model with a force free magnetic field and cylindrical geometry, in the ideal MHD context. To our knowledge the energy absorption rate has so far not been determined in a consistent manner in dissipative MHD for realistic one-dimensional models of coronal loops. This is, therefore, the objective of this paper.
Resonant absorption of Alfvén waves is investigated in a resistive compressible plasma. The resistive MHD equations are solved numerically by means of a very accurate numerical code based on the finite element method. The numerical tackling of the problem makes it possible to study the heating process in more realistic equilibrium models of coronal loops and also to describe the coupling of the external driver to the plasma accurately.

This paper is arranged as follows. In Section 2 we present the basic equations of linear resistive MHD that govern the stationary state of a cylindrically-symmetric plasma in the presence of an external periodic driver. Section 2 also includes the calculation of the boundary conditions. The numerical method for determining the stationary state of the plasma response to the external driver is discussed in Section 3. In particular, we explain how the external periodic driver enters in the finite element discretization of the equations. Section 4 deals with the energetics in resistive MHD. We derive an energy equation which enables us to obtain a quantitative picture of the energy deposition in the plasma. In addition, this energy equation provides us with a stringent test for the accuracy of the numerical code and for the validity of the physical assumptions. In Section 5 we give results concerning the dependence of the efficiency and the localization of the energy deposition on the plasma resistivity and on the wave numbers and the frequency of the external driver. Finally, our conclusions are drawn in Section 6.

2. Physical Model

2.1. Resistive MHD Equations

We use the linearized equations of resistive MHD which allow compressible displacements for describing resonant absorption of Alfvén waves and reduce the study of resonant absorption to the study of linear displacements about an ideal static equilibrium state that are excited by an external source. Strictly speaking magnetostatic equilibria do not exist in resistive MHD because the resistive diffusion generates flow. However, linearization around an ideal magnetostatic equilibrium yields a good approximation for the description of phenomena with a characteristic time-scale τ that is long compared with the time Alfvén waves need to rearrange the magnetic field lines (the dynamic time-scale, \( \tau_{\text{dyn}} \)) and short compared to the time-scale of resistive diffusion, \( \tau_{\text{diff}} \) (Roberts, 1967). So the conditions for linear theory and magnetostatic equilibrium are

\[
\tau_{\text{dyn}} = \frac{l_0}{v_A} \ll \tau \ll \tau_{\text{diff}} = \frac{l_0^2}{\eta},
\]

with \( l_0 \) a characteristic length scale and \( v_A \) the Alfvén velocity. Since these conditions are satisfied for coronal plasmas, it is meaningful to consider magnetostatic equilibria and to apply the theory of linear motions around a magnetostatic equilibrium for the study of waves in these plasmas. Resonant absorption can be treated within this theory because the characteristic time-scale related with this phenomenon is proportional to \( \eta^{-1/3} \) and for small values of \( \eta \) much smaller than \( \tau_{\text{diff}} (\sim \eta^{-1}) \) but much larger than the dynamic time-scale.
We consider a one-dimensional cylindrically-symmetric plasma column with equilibrium quantities varying only in the radial direction. The resistive MHD equations that govern linear displacements about this ideal static equilibrium are:

\[
\rho \frac{\partial \mathbf{v}}{\partial t} = -\mathbf{\nabla} p + (\mathbf{\nabla} \times \mathbf{B}) \times \mathbf{b} + (\mathbf{\nabla} \times \mathbf{b}) \times \mathbf{B},
\]

(2.1)

\[
\frac{\partial p}{\partial t} = -\mathbf{v} \cdot \mathbf{\nabla} p - \gamma p \mathbf{\nabla} \cdot \mathbf{v},
\]

(2.2)

\[
\frac{\partial \mathbf{b}}{\partial t} = \mathbf{\nabla} \times (\mathbf{v} \times \mathbf{B}) - \mathbf{\nabla} \times (\eta \mathbf{\nabla} \times \mathbf{b}),
\]

(2.3)

with \(P, \rho,\) and \(B,\) respectively, the equilibrium plasma pressure, density, and magnetic field (magnetic induction) and \(v, p,\) and \(b,\) the Eulerian variation of, respectively, the velocity, the plasma pressure, and the magnetic field. \(\eta\) is the electric resistivity and \(\gamma\) the adiabatic index. Equations (2.1)–(2.3) are expressed in dimensionless units. The distance is normalized to the plasma radius \(r_p\) and the time to the Alfvén-transit-time \(t_\Lambda = r_p/V_\Lambda,\) where \(V_\Lambda\) is the Alfvén speed \(V_\Lambda = B_z(0)/\sqrt{\mu \rho(0)}\) given by the longitudinal magnetic field and the density on the axis \((r = 0)\). The following transformation,

\[
\frac{t}{t_\Lambda} \rightarrow t, \quad \frac{r}{r_p} \rightarrow r, \quad \frac{r_p \mathbf{\nabla}}{V_\Lambda} \rightarrow \mathbf{\nabla}, \quad \frac{\mathbf{v}}{V_\Lambda} \rightarrow \mathbf{v},
\]

\[
\frac{\mathbf{B}}{B_z(0)} \rightarrow \mathbf{B}, \quad \frac{\mu r_p}{B_z(0)} \mathbf{J} \rightarrow \mathbf{J}, \quad \frac{\rho}{\rho(0)} \rightarrow \rho,
\]

(2.4)

\[
\frac{\mu p}{B_z^2(0)} \rightarrow p, \quad \text{and} \quad \frac{\eta}{\mu r_p V_\Lambda} \rightarrow \eta,
\]

is used to make the equations dimensionless. \(\mu\) is the magnetic permeability and is for coronal plasmas well approximated by its value in vacuum, \(\mu_0.\)

Equation (2.1) is the momentum equation for a non-viscous plasma. Equation (2.3) is the induction equation which includes the Ohmic term (due to the finite electric conductivity of the plasma). Equation (2.2) is the equation for the variation of the internal energy not including non-ideal effects. In particular the term due to finite electrical conductivity is ignored in this equation (the adiabatic law). The inclusion of finite electric conductivity in the generalized form of Ohm’s law but not in the energy equation, is an approximation often made in magnetohydrodynamics and is based on the fact that finite electrical conductivity has its most important effect in Equation (2.3). The adiabatic law (2.2) makes it possible to describe the behaviour of a compressible plasma.

Other authors (e.g., Kappraff and Tataronis, 1977) consider an incompressible plasma \((\mathbf{\nabla} \cdot \mathbf{v} = 0)\). The assumption of incompressibility is only justified when the
acoustic speed \( v_s = \sqrt{\gamma p/\rho} \) is much larger than the other typical plasma speeds. Since the plasma velocities are often of the order of the Alfvén speed \( v_A = B/\sqrt{\mu \rho} \), the condition for incompressibility becomes \( v_s \gg v_A \). In coronal loop plasmas, however, the opposite is true, \( v_s \ll v_A \), and, therefore, incompressibility is not a good approximation. Moreover, it is impossible to describe the coupling of the driver to the plasma in an incompressible model. In the process of resonant absorption the energy of the driver is transported by means of fast magnetosonic waves which only appear in a compressible plasma. The coupling of the driver to the plasma is an important aspect of the heating mechanism since this coupling determines the fraction of the supplied energy that is actually converted into heat, in other words the efficiency of the process of resonant absorption.

The equilibrium state is taken to be a cylindrical magnetostatic plasma with equilibrium quantities varying only in the radial direction. In cylindrical coordinates \((r, \theta, z)\) the equilibrium quantities depend only on \( r \) and the equilibrium force balance takes the following form:

\[
\frac{\mathrm{d}P}{\mathrm{d}r} + B_z \frac{\mathrm{d}B_z}{\mathrm{d}r} + \frac{B_\theta}{r} \frac{\mathrm{d}(rB_\theta)}{\mathrm{d}r} = 0. \tag{2.5}
\]

Two of the three profiles \( P(r), B_\theta(r), \) and \( B_z(r) \) may be chosen arbitrarily. The remaining profile is determined by Equation (2.5). The mass density \( \rho(r) \) does not appear in the equilibrium equation and can be chosen arbitrarily.

Since the equilibrium quantities do not depend on \( \theta \) and \( z \), the perturbed quantities can be Fourier-analyzed in these coordinates and each Fourier term can be studied separately. The following separation ansatz is made for the perturbed quantities \( f \):

\[
f(r, \theta, z; t) = f(r; t) e^{i(m\theta + n kz)}. \tag{2.6}
\]

The number \( k = \pi/L \) defines a quantization factor to allow an integral number of half-wavelengths on the column (the ends of the loop are tied in the dense photosphere) with \( L \) the length of the loop in the \( z \)-direction \((L = \pi \varepsilon, \) with \( \varepsilon \) the aspect ratio of the loop\). The cylindrical geometry closely approximates that of a large aspect ratio loop. The results presented in this paper were obtained with the aspect ratio \( \varepsilon = 20 \). \( n \) is the longitudinal mode number and \( m \) is the mode number in the \( \theta \)-direction.

We assume that linear displacements about the static equilibrium are excited by an external periodic source with frequency \( \omega_p \). After a time proportional to \( \eta^{-1/3} \) (Kappraff and Tataronis, 1977) the dissipative system reaches a stationary state. In this stationary state all physical quantities vary periodically in time with the frequency \( \omega_p \) of the external driver. With this in mind, it is much more efficient to compute immediately the stationary state than to do the time integration where hundreds (or even thousands, depending on the value of \( \eta \)) of time steps are needed. The stationary state can be computed directly by substituting the temporal dependence

\[
f(r; t) = f(r) e^{i\omega_p t}. \tag{2.7}
\]

in Equations (2.1)–(2.3).
With the Fourier decomposition in the ignorable directions (2.6) and substitution of the time dependence (2.7) the linearized equations of resistive MHD (2.1)–(2.3) are reduced to a set of ordinary differential equations, which has to be complemented with the appropriate boundary conditions.

2.2. Physical model

The power spectrum of the waves that are incident on the magnetic loops in the solar corona is only poorly known. This is in contrast with laboratorium configurations in controlled thermonuclear fusion research where the driving spectrum is very well known. Let us, therefore, first explain how heating by resonant absorption can be studied in tokamak configurations. Subsequently we explain how this approach has to be modified to be applicable to solar coronal loops. We consider a cylindrical plasma with radius \( r_p \) (region I) surrounded by a vacuum (regions II and III) and by a perfectly conducting wall with radius \( r_w \). We assume that the linear displacements of the plasma column are excited by an idealized external antenna situated in the vacuum region, with radius \( r_a \) (\( r_p \leq r_a \leq r_w \)). A schematic representation of the configuration is given in Figure 1. We have normalized the distance with the choice \( r_p = 1 \). We choose in accordance with Appert et al. (1984) a helical antenna specified by

\[
\begin{align*}
\mathbf{j}_{\text{ant}}(r) &= 0, \\
\mathbf{j}_{\text{ant}}(\theta) &= \frac{n k I}{2} \delta(r - r_a) e^{i(m \theta + nkz + \omega_p t)}, \\
\mathbf{j}_{\text{ant}}(z) &= -\frac{m I}{2r} \delta(r - r_a) e^{i(m \theta + nkz + \omega_p t)},
\end{align*}
\]

with \( I \) the current in a 'wire'. This choice satisfies \( \nabla \cdot \mathbf{j}_{\text{ant}} = 0 \). We treat this antenna as a discontinuity of the perturbed magnetic field in the vacuum region at \( r = r_a \). Note that in the tokamak case the number \( k = 2\pi / L \) defines a periodicity length in the \( z \)-direction (\( L \)) in order to simulate a large aspect ratio tokamak (\( L = 2\pi R / r_p \), with \( R \) the major radius and \( r_p \) the minor radius of the tokamak). Of course, coronal loops are not excited by a current in external coils neither are they surrounded by a vacuum region and a perfectly conducting wall. However, the intrinsic energy dissipation rate in coronal loops can be computed by placing (in the numerical code) the driver at the plasma-vacuum interface (\( r = r_p = r_a \)) and by taking the wall away from the plasma column (\( r_w \to \infty \)). In this set-up the periodic driver simulates a wave with particular wave numbers and frequency that is incident on the loop. The solution is normalized by taking a unit current in the antenna (\( I = 1 \)). This normalization is not essential since the main concern is to determine the intrinsic energy dissipation rate which is independent of the amplitude of the incident wave.

2.3. Boundary conditions

The appropriate boundary conditions to the ordinary differential equations of linear resistive MHD that govern the displacements of the one-dimensional plasma column
excited by the external source, are obtained by integrating the resistive MHD equations across the surfaces separating the different regions of the configuration. The resulting boundary conditions then have to be linearized and expressed in terms of the perturbed quantities and the perturbed surfaces between the different regions. We consider a class of equilibria with plasma profiles $P(r)$, $B_z(r)$, and $B_\rho(r)$ joining smoothly onto the corresponding vacuum profiles so that no surface currents occur in the equilibrium configuration. We obtain the following boundary conditions:

At $r = r_p$ (the plasma-vacuum-interface) the tangential components of the perturbed electric field, $\mathbf{e}$, and of the perturbed magnetic field have to be continuous:

$$[\mathbf{n} \times \mathbf{e}]_I^{II} = 0,$$

and

$$[\mathbf{n} \times \mathbf{b}]_I^{II} = 0,$$

where $[Q]_I^{II}$ denotes the jump a quantity $Q$ makes at the boundary of regions I and II. From Equation (2.9) and Faraday’s law it follows that

$$[\mathbf{n} \cdot \mathbf{b}]_I^{II} = 0.$$  

Moreover, the radial component of the perturbed electric current density, $\mathbf{j}$, has to be continuous at $r = r_p$ as a consequence of Equation (2.10):

$$[\mathbf{n}, \mathbf{j}]_I^{II} = 0.$$  

However, as $\mathbf{e}$ and $\mathbf{j}$ are eliminated in the set (2.1)–(2.3) the appropriate boundary conditions are (2.10) and (2.11).

In the limit $\eta \to 0$ a magnetohydrodynamic boundary layer, $L_h$, develops, in which the resistive terms are of crucial importance. The components of $\mathbf{n} \times \mathbf{b}$ change very rapidly throughout this boundary layer and there is an important electric current in $L_h$. The volume integral of this current over $L_h$ converges to a finite limit for $\eta \to 0$,

$$[\mathbf{n} \times \mathbf{b}]_I^{II} = J_{\text{surf}},$$

the surface current. In ideal magnetohydrodynamics boundary condition (2.10) is replaced by (2.13). Equation (2.13) is not a restriction on the solution but only the definition of the surface current. Ideal MHD can not give any information about the structure of the boundary layer $L_h$ since the dissipative terms, which play a crucial role in it, are ignored. In resistive magnetohydrodynamics it can be shown that the width of $L_h$ is proportional to $\eta^{1/2}$ (see, e.g., Roberts, 1967). The surface current, $j_{\text{surf}}$, in ideal MHD is related to a discontinuity in the pressure profile by the boundary condition

$$[p + \mathbf{B} \cdot \mathbf{b}]_I^{II} = 0.$$  

According to boundary condition (2.14) the total pressure (plasma pressure plus magnetic pressure) has to be continuous at the plasma-vacuum-interface. In ideal MHD boundary conditions (2.11) and (2.14) form a complete set. For our choice of equilibria $P(r_p) = P^*(r_p) = 0$ so that $p(r_p) = 0$ and boundary condition (2.14) follows automatically from (2.10).
At \( r = 0 \), finally, the equations have a singular point. For physical reasons the solutions have to be regular at the axis. It can be shown that the regularity conditions on \( b_r \) and \( v_r \) form a complete set. In terms of \( rv_r \) and \( rb_r \) we have

\[
rv_r(0) = 0, \tag{2.15}
\]

\[
rb_r(0) = 0. \tag{2.16}
\]

The six boundary conditions, namely (2.15) and (2.16) at the axis and (2.10), (2.11), and (2.14) at the plasma-vacuum interface, determine the plasma solution completely when the solution is known in the external region.

3. Numerical Method

In this section we present the numerical method that is used to solve the set of ordinary differential equations governing the linear displacements that are excited in the plasma column by the external driver. Particular attention is given to the treatment of the driving term resulting from the external source.

3.1. External Solution

The external region is approximated by a vacuum. The vacuum solution can be derived analytically. For completeness, we first derive the vacuum solution for the tokamak case (as coded) and then stress the modifications made for the coronal loop case. The perturbed magnetic field in the vacuum region, \( b_v \), satisfies the equations

\[
\nabla \times b_v = 0 \quad \text{and} \quad \nabla \cdot b_v = 0 \tag{3.1}
\]

and the boundary conditions

\[
b_r(r = r_w) = 0 \tag{3.2}
\]

at the perfectly conducting wall, and

\[
[n \cdot b]_{II}^{III} = 0 \tag{3.3}
\]

and

\[
[n \times b]_{II}^{III} = j_{ant} \tag{3.4}
\]

at the external antenna (\( r = r_a \)). \( j_{ant} \) denotes the antenna current defined by Equation (2.8). Equations (3.1) can be rewritten as a second order differential equation for \( b_{vr} \),

\[
2r^2 \frac{d^2 b_{vr}}{dr^2} + r \frac{db_{vr}}{dr} - [(nk)^2 + m^2] b_{vr} = 0, \tag{3.5}
\]

and expressions for \( b_{vr} \) and \( b_{v\theta} \) in terms of \( b_{vr} \) and its first-order derivative:

\[
b_{vr} = -i \frac{db_{vr}}{nk \, dr}, \tag{3.6}
\]
and

\[ b_{\vartheta \theta} = \frac{m}{nkr} b_{\vartheta z}, \]  

(3.7)

Equation (3.5) is the equation for the modified Bessel functions and the solutions of this equation in regions \( \text{III} \) \( (r_a \leq r \leq r_w) \) and \( \text{II} \) \( (r_p \leq r \leq r_a) \) are:

\[ b_{\vartheta z} = A_{\text{III}} I_m(nkr) + B_{\text{III}} K_m(nkr), \]  

(3.8)

and

\[ b_{\vartheta z} = A_1 I_m(nkr) + B_1 K_m(nkr), \]  

(3.9)

with \( I_m \) and \( K_m \) the modified Bessel functions of order \( m \) and \( A_{\text{III}}, B_{\text{III}}, A_1, \) and \( B_1 \) four constants. Three of these constants can be determined upon imposing the boundary conditions (3.2)–(3.4). This yields the following solution for \( b_{\vartheta z} \) in region II:

\[ b_{\vartheta z} = f_d(nkr) + B_{\text{III}} f_i(nkr), \]  

(3.10)

with

\[ f_d(nkr) = j_{\text{ant}} \frac{[I'_m(nkr_a)K_m(nkr) - K'_m(nkr_a)I_m(nkr)]}{[I'_m(nkr_a)K_m(nkr_a) - K'_m(nkr_a)I_m(nkr_a)]}, \]  

(3.11)

the magnetic field driven by the external source (‘driven field’) and

\[ B_{\text{III}} f_i(nkr) = B_{\text{III}} \left[ K_m(nkr) - \frac{K'_m(nkr_w)}{I'_m(nkr_w)} I_m(nkr) \right], \]  

(3.12)

the induced field. The driven field is proportional to the current in the antenna. The solution for the induced field contains a constant \( B_{\text{III}} \) determined by the response of the plasma to the excitation by the external source. We recall that for the coronal loop case the \( (I = 1) \) source is placed at the plasma boundary \( (r_a = r_p = 1) \) while \( r_w \to \infty \) which is numerically equivalent to making \( r_w \) very large but finite. The results given in the present paper are obtained with \( r_w = 100 \).

### 3.2. Plasma Response

With the Fourier decomposition (2.6) and the substitution of the time dependence (2.7) the equations governing the linear oscillations, (2.1)–(2.3), can be written in the following form:

\[ i \omega_p \frac{\rho}{r} v_1 = -\left[ \frac{\bar{p}}{r} + \frac{1}{m} B_\vartheta b_i + \left( B_z - \frac{nkr}{m} B_\vartheta \right) \frac{b_3}{r} \right]^{'} + \]

\[ + \frac{1}{r} \left( \frac{m}{r} B_\vartheta + nkB_z \right) b_1 - \frac{2}{rm} B_\vartheta b_1 + \frac{2nk}{rm} B_\vartheta b_3, \]
$i\omega_p \rho v_2 = \frac{m}{r} \bar{p} + \left( \frac{1}{r} B_\theta + B_\theta' \right) b_1 - \frac{nkr}{m} B_z b_1' + \left( \frac{n^2 k^2 r + m}{m} \right) B_z b_3',$

$\frac{i\omega_p}{r} v_3 = \frac{nk}{r} \bar{p} - \left( \frac{m}{r^2} + \frac{n^2 k^2}{m} \right) B_\theta b_3 + \frac{nk}{m} B_\theta b_1' + B_z' \frac{1}{r} b_1,$

$\frac{i\omega_p}{r} \bar{p} = - \frac{1}{r} P' v_1 - \gamma P \frac{1}{r} v_1' - \gamma P \frac{m}{r} v_2 - \gamma P \frac{nk}{r} v_3,$

$\frac{i\omega_p}{r} b_1 = - \left( \frac{m}{r} B_\theta + nk B_z \right) v_1 + \eta \left[ b_1'' + \frac{1}{r} b_1' - \left( \frac{m^2}{r^2} + n^2 k^2 \right) b_1 - \frac{2nk}{r} b_3 \right],$

$\frac{i\omega_p}{r} b_3 = -B_z v_1' - m B_z v_2 + \frac{m}{r} B_\theta v_3 - B_z' v_1 + \eta \left[ \left( b_3' - \frac{1}{r} b_3 \right)' - \left( \frac{m^2}{r^2} + n^2 k^2 \right) b_3 \right] + \eta \left[ b_3' - \frac{1}{r} b_3 - nk b_1 \right].$

Here, and in the following, the prime denotes the derivative with respect to $r$. The following transformation,

$v_1 = r v_r, \quad v_2 = i v_\theta, \quad v_3 = r v_z,$

$\bar{p} = r \bar{p}, \quad b_1 = r b_r, \quad b_3 = r b_z,$

yields only real quantities. The perturbed form of the divergence equation for the magnetic field,

$\nabla \cdot \mathbf{b} = -\frac{i}{r} (b_1' - m b_2 - nk b_3) = 0,$

has been used to eliminate $b_2$ ( = $b_\theta$) (we only consider cases with $m \neq 0$). The $\theta$-component of the Maxwell–Ohm equation (2.3) is then automatically satisfied.

The solutions that correspond to the continuum frequencies are non-square integrable in ideal MHD. With finite electrical conductivity included in the equations, the solutions become regular. However, since $\eta$ is very small, the resistive solutions exhibit a nearly singular behaviour: they are localized in the neighbourhood of the radius where the singularity occurs in ideal MHD. In addition, the small factor $\eta$ appears in the coefficient of the second-order derivative of $b_1$ and the order of the system of differential equations (3.13) increases from two (in ideal MHD) to six when $\eta \neq 0$. As a consequence, boundary layers may appear, in which the solutions are rapidly changing. These features complicate the numerical treatment of the system (3.13) and demand a special tackling of the problem.

The system of differential equations (3.13) is solved in its weak form. We introduce a state vector $\mathbf{u}$ which contains the perturbed velocity, pressure, and magnetic field:

$\mathbf{u}' = (v_1, v_2, v_3, \bar{p}, b_1', b_3).$
The system of differential equations (3.13) can then be written in the form $L \cdot u = 0$, where $L$ denotes the linear matrix operator. The vector $u(r)$ is a weak solution if, for any function $v(r)$ of the admissible Sobolev space satisfying the boundary conditions, the product $\langle L \cdot u, v \rangle$ vanishes (Strang and Fix, 1973). The components of $u$ are approximated by a finite linear combination of local expansion functions:

$$u^k(r) \approx \tilde{u}^k(r) = \sum_{j=1}^{N} a_j^k h_j^k(r), \quad k = 1, \ldots, 6,$$

where the $a_j^k$ are coefficients to be determined and the $h_j^k(r)$ are the chosen expansion functions. We apply the Galerkin method in which the basis functions $h_j^k(r)$ are used in the weak form. This yields the following system of linear equations:

$$\langle L \cdot \tilde{u}, h_j \rangle = 0, \quad j = 1, \ldots, N.$$

The error $E(r)$ introduced in the differential equations through the approximation $\tilde{u}(r)$ for $u(r)$, $L \cdot \tilde{u} = E(r)$, is orthogonal to every basis function.

The operator $L$ is represented by matrices $\mathcal{R}$ and $\mathcal{S}$,

$$\mathcal{R} \cdot \tilde{u} = i\omega_p \mathcal{S} \cdot \tilde{u},$$

with $\mathcal{S}$ a diagonal matrix and $\mathcal{R}$ a matrix containing differential operators and equilibrium quantities. The Galerkin procedure leads to the problem

$$\mathcal{A} \cdot a = i\omega_p \mathcal{B} \cdot a$$

with $a$ the vector of the $6N$ expansion coefficients. The matrix $\mathcal{B}$ is symmetric and positive definite but $\mathcal{A}$ is a non-Hermitian matrix. The matrices $\mathcal{A}$ and $\mathcal{B}$ consist of $N \times N$ subblocks with identical structure and dimension $6 \times 6$. All of the non-diagonal elements of a subblock are zero for the matrix $\mathcal{B}$. Since we use finite elements as expansion functions, only the subblocks that form the main diagonal and their nearest neighbours contain non-zero elements. The matrices $\mathcal{A}$ and $\mathcal{B}$ thus possess a tridiagonal block structure. The dimension of the matrices is $6N \times 6N$, with $N$ the number of finite elements used in the expansion.

The development of the spectral codes for ideal MHD has indicated that every component of the state vector, $u(r)$, has to be represented by appropriate finite elements. If the same elements are used for all components, poor discretization is achieved because the condition that the transverse divergence of the velocity vanishes exactly in every interval (Appert et al., 1975), i.e.,

$$\nabla \cdot \mathbf{v} = \frac{1}{r} (v_1' + mv_2) = 0,$$

cannot be satisfied. If for $v_1$ and $b_1$ finite elements of order $n + 1$ are chosen and elements of order $n$ for the remaining components, the divergence condition (3.21) can be satisfied and the spectrum is well approximated numerically without pollution (Rappaz, 1975). Linear elements for $v_1$ and $b_1$ and piecewise constant elements for $v_2$, $v_3$, $\bar{v}$, and $b_2$ – as used in ideal spectral codes – yield a good numerical representation.
for zero resistivity. For finite $\eta$, however, the component $b_3$ has to be differentiated, which also calls for the use of finite elements which are (at least) linear. We, therefore, introduced higher-order elements. Cubic Hermite spline functions are used for $b_1$ and $v_1$ and quadratic finite elements for $v_2$, $v_3$, $\bar{p}$, and $b_3$. In each case two orthogonal functions define a complete set (Kerner et al., 1985; Strang and Fix, 1973).

### 3.3. THE DRIVING TERM

The fact that the linear oscillations of the plasma column are excited by an external source, is taken into account by imposing the boundary conditions at the plasma-vacuum interface (described in Section 2.3) and by connecting in this way the solutions in the plasma column to the solutions in the vacuum region. In the finite element method the regularity conditions on the axis, (2.15) and (2.16), are called the essential boundary conditions because they have to be imposed on the basis functions themselves. In other words, the Sobolev-space of the basis functions has to be limited to the space of functions satisfying the regularity conditions (2.15) and (2.16). The boundary conditions at the plasma-vacuum interface are called natural boundary conditions as they are satisfied automatically after a slight modification of the weak form of Equation (3.13). The so-called driving term or force term resulting from this operation, is derived below.

The continuity of $b_r$ at $r = r_p = 1$ (boundary condition (2.11)) yields

$$b_1(r_p) = \frac{r_p}{nk} \left[ f_d'(nkr_p) + B_{\text{III}} f_i'(nkr_p) \right], \quad (3.22)$$

where we have made use of Equations (3.6) and (3.10). With the use of Equation (3.22) the scale factor of the induced magnetic field in the vacuum region can be expressed in terms of the value of $b_1$ on the plasma boundary. The solution in the plasma region is continued in the vacuum region by means of Equation (3.22) so that boundary condition (2.11) is always satisfied.

The elimination of the constant $B_{\text{III}}$ from Equation (3.22) and the continuity of the parallel component of the perturbed magnetic field, Equation (2.10), gives an expression for the value of the total pressure, $\Pi = p + B \cdot \mathbf{b}$, at the plasma boundary ($r = r_p = 1$):

$$\Pi(1) = A_1 b_1(1) + A_2, \quad (3.23)$$

with

$$A_1 = \left[ \frac{mB_0}{r + nkB_z} \frac{f_i(nkr)}{f_i'(nkr)} \right]_{r=1}$$

and

$$A_2 = \left[ \frac{mB_0}{nk} \left( f_d(nkr) - \frac{f_d'(nkr)}{f_i'(nkr)} f_i(nkr) \right) \right]_{r=1},$$

where we have used Equations (3.7) and (3.10).
From the conditions (2.10) and Equation (3.7) it follows that

\[
b'_1(1) = \left( \frac{m^2}{nk} + nk \right) b_3(1). \tag{3.24}\]

The two boundary conditions (3.23) and (3.24) that remain after elimination of the scale factor \(B_{111}\) in the vacuum solution, are called natural boundary conditions. The solution automatically satisfies these boundary conditions when the weak form of Equation (3.13) is slightly modified. Partial integration in the weak form of the radial component of the equation of motion and in the components of the induction equation yields three surface terms, namely:

\[
[ - v_1 \Pi ]_{r = 1}, \tag{3.25}
\]

\[
[ b_1 \eta b'_1 ]_{r = 1}, \tag{3.26}
\]

and

\[
[ b_3 \eta j_\theta ]_{r = 1}. \tag{3.27}
\]

After substitution of Equation (3.23) in the surface term (3.25) we get:

\[
[ - v_1 \Pi ]_{r = 1} = - [ v_1 A_1 b_1 ]_{r = 1} - [ v_1 A_2 ]_{r = 1}. \tag{3.28}
\]

The first term in the right-hand side of Equation (3.28) gives a contribution to the coefficient matrix of the discretized form of the equations. The second term in the right-hand side gives a contribution to the driving term. The substitution of the expression for \(b'_1(r_p)\), Equation (3.24), in the surface term (3.26) yields

\[
[ b_1 \eta b'_1 ]_{r = 1} = \left[ b_1 \eta \left( \frac{m^2}{nk} + nk \right) b_3 \right]_{r = 1}. \tag{3.29}
\]

The right-hand side term in this equation again gives a contribution to the coefficient matrix of the discretized problem. The third surface term, (3.27), is not used to impose a boundary condition. The tangential components of the electric current density of the perturbation do not have to be continuous at \(r = r_p\).

In an externally driven system the application of the Galerkin procedure therefore leads to a system of linear equations of the following form:

\[
(\mathcal{A} - i\omega_p \mathcal{B})\mathbf{a} = \mathbf{f}, \tag{3.30}
\]

with \(\mathbf{a}\) the vector of the \(6N\) expansion functions and \(\mathbf{f}\) the 'driving term', i.e., the vector that contains the terms resulting from the modification of the weak form to impose the boundary condition (3.23). Due to the local nature of the finite elements only one component of the vector \(\mathbf{f}\) is actually different from zero,

\[
f(6(N - 1) + 1) = h^1_N(1)A_2, \tag{3.31}
\]

and the modification of the coefficient matrix \((\mathcal{A} - i\omega_p \mathcal{B})\) is very small. The only finite elements that are non-zero in \(r = 1\) and, therefore, related to the boundary conditions
(3.23) and (3.24), are the finite elements defined on the very last interval so that the modification only involves the last subblock of the coefficient matrix.

Due to the choice of two orthogonal finite elements per interval and per component of \( \mathbf{u} \) the dimension of the subblocks in the matrices \( \mathcal{A} \) and \( \mathcal{B} \) is increased to \( 12 \times 12 \). The dimension of the matrices \( \mathcal{A} \) and \( \mathcal{B} \) is then \( 12N \times 12N \), with \( N \) the number of gridpoints. Even for a relatively small number of gridpoints the matrices \( \mathcal{A} \) and \( \mathcal{B} \) therefore, have considerable dimensions. In order to save memory the matrix \( (\mathcal{A} - i\omega_p \mathcal{B}) \) is stored in ‘band storage mode’. The coefficient matrix is \( LU \)-factorized:

\[
(\mathcal{A} - i\omega_p \mathcal{B}) = LU,
\]

and further inverted by means of LINPACK routines.

Most of the results presented in this paper where obtained with 1201 gridpoints in the interval \([0, 1] \) so that \( 12N = 14412 \). For very small values of the resistivity \( (\eta \leq 10^{-11}) \), however, 2501 gridpoints where needed so that \( 12N = 30012 \). The numerical code is very accurate (see below) and very efficient.

4. Energetics in Resistive MHD

4.1. Resistive Energy Equation

We are now concerned with the determination of the energy dissipation rate. Moreover, a complete picture of energy conservation in the system provides us with physical insight and is an excellent test for the accuracy of the numerical code and for the validity of the physical assumptions made.

The equation for the change of electromagnetic energy,

\[
- \nabla \cdot (\mathbf{e}^{\ast} \cdot \mathbf{b}) = \eta j^2 + \mathbf{v}^{\ast} \cdot \mathbf{j} \times \mathbf{B} + \mathbf{b} \cdot \frac{\partial \mathbf{b}^{\ast}}{\partial t} \tag{4.1}
\]

and the mechanical energy condition

\[
\rho \mathbf{v}^{\ast} \cdot \frac{\partial \mathbf{v}^{\ast}}{\partial t} = -\mathbf{v}^{\ast} \cdot \nabla p + \mathbf{v}^{\ast} \cdot \mathbf{j} \times \mathbf{B} + \mathbf{v}^{\ast} \cdot \mathbf{J} \times \mathbf{b} \tag{4.2}
\]

are combined and yield, after integration over the volume \( V_p \) of the plasma, the following equation:

\[
- \int_{V_p} \nabla \cdot (\mathbf{e}^{\ast} \times \mathbf{b}) \, dV =
\]

\[
= \int_{V_p} \rho \mathbf{v}^{\ast} \cdot \frac{\partial \mathbf{v}^{\ast}}{\partial t} \, dV + \int_{V_p} \left\{ \mathbf{v}^{\ast} \cdot \nabla p - \mathbf{v}^{\ast} \cdot \mathbf{J} \times \mathbf{b} + \mathbf{b} \cdot \frac{\partial \mathbf{b}^{\ast}}{\partial t} \right\} \, dV + \int_{V_p} \eta j^2 \, dV. \tag{4.3}
\]

\[\equiv K\]

\[\equiv W_p\]

\[\equiv OD\]
and \( \mathbf{j} \) are the Eulerian perturbations of respectively the electric field and the electric current density. The * denotes the complex conjugate. According to Equation (4.3) an inflow of electromagnetic energy produces a rise of the kinetic energy of the plasma \( (K) \), a change of the potential energy of the plasma \( (W_p) \) and heat by Ohmic dissipation \( (\text{OD}) \). The change of the potential energy of the plasma comprises the work done by \(- \nabla p\) and \( \mathbf{J} \times \mathbf{b} \) and the rise of the magnetic energy of the plasma.

For a tokamak configuration the left-hand side term of Equation (4.3) is related to power emitted by the antenna since it can be rewritten as

\[
- \int_{V_p} \nabla \cdot (\mathbf{e}^* \times \mathbf{b}) \, dV = - W_v + P, \tag{4.4}
\]

where

\[
W_v = \int_{V_v} b_v \frac{\partial b^*_v}{\partial t} \, dV \tag{4.5}
\]

is the change of the magnetic energy in the vacuum region and

\[
P = - \int_{V_v} \mathbf{j}_{\text{ant}} \cdot \mathbf{e}_v^* \, dV \tag{4.6}
\]

is the amount of energy emitted by the antenna per unit of time. \( V_v \) is the volume of the vacuum region and \( \mathbf{j}_{\text{ant}} \) is the electric current density in the antenna defined by Equation (2.8). Substitution of Equation (4.4) in Equation (4.3) then gives

\[
P = K + W_p + W_v + \text{OD}. \tag{4.7}
\]

The physical interpretation of Equation (4.7) is that in a laboratorium configuration where the plasma is excited by a current in external coils the power emitted by the antenna produces a rise of the kinetic energy of the plasma \( (K) \), a change of the potential energy of the plasma \( (W_p) \), a rise of the magnetic (potential) energy of the vacuum \( (W_v) \), and heat by Ohmic dissipation \( (\text{OD}) \).

4.2. Resistive Energy Balance

In an ideal system that is excited periodically by an external driver with a frequency within the range of the ideal Alfvén continuum, the energy transferred to the plasma accumulates without dissipation about the singular magnetic surface where \( \omega_p \) equals the local Alfvén frequency (see, e.g., Tataronis, 1975). The physical quantities in these surfaces grow in time without bound as a result of the energy accumulation. However, dissipation prevents the system of evolving so far. Finite electrical conductivity (or any other physical dissipation mechanism), however small, causes the system to attain a stationary state where the energy dissipation in the neighbourhood of the resonant surface just balances the energy inflow from the external driver. The analysis of Kappraff and Tataronis (1977) reveals that two time-scales characterize the energy dissipation in a resistive plasma. Initially the energy accumulates in time about the singular layer in
the same manner as in ideal MHD up to a critical time \( t_h \) proportional to \( \eta^{-1/3} \). For \( t = t_h \) the growing physical quantities in the resistive layer reach a saturation amplitude. From that time on the solution is purely oscillatory with a time behaviour \( \sim e^{i\omega_p t} \), where \( \omega_p \) is the frequency of the external driver. This oscillatory time behaviour was substituted in the equations describing the energy absorption process, so that our numerical code computes the stationary state for a given value of \( \omega_p \). Tackling the problem in this way saves a lot of CPU.

The resistive energy balance for \( t > t_h \) is obtained by substituting the time behaviour \( e^{i\omega_p t} \) in the resistive energy equation (4.7). The terms appearing in Equation (4.7) are all the product of two linear perturbations. The real and the imaginary parts of these terms (in the complex notation) are related to terms \( \sim \cos^2(\omega_p t) \) or \( \sim \sin^2(\omega_p t) \) (real parts) and \( \sim \cos(\omega_p t) \sin(\omega_p t) \) (imaginary parts) in real notation. Integration in time over a period \( (P_p) \) of the external driver gives \( P_p/2 \) when the integrand is \( \sim \cos^2(\omega_p t) \) or \( \sim \sin^2(\omega_p t) \) and zero when the integrand is \( \sim \cos(\omega_p t) \sin(\omega_p t) \). The real parts of the terms in Equation (4.7) correspond to cumulative or persistent effects, while the imaginary parts correspond to non-persistent effects having a vanishing mean over a period of the external driver.

In the stationary state all physical quantities oscillate with the frequency of the driver and the resistive energy balance reads

\[
\text{Re}(P) = \text{OD}.
\]

This means that for \( t > t_h \) the supply of energy from the antenna exactly balances the rate at which energy is converted into heat by Ohmic dissipation.

The resistive energy equation (4.7) and the resistive energy balance (4.8) provide excellent tests for the numerical accuracy of the code and for the validity of the physical approximations made (ideal static equilibrium, adiabatic law). These approximations are good as long as Equations (4.7) and (4.8) are satisfied within reasonable bounds.

4.3. Efficiency of the Process

In a system that is excited by an external source the power absorbed by the system can be derived from the source impedance. The source impedance is proportional to the power, \( P \), delivered by the driver and given by Equation (4.6) (see, e.g., Appert et al., 1984; Tataronis and Grossmann, 1976). With the definition (4.6) the real part of \( P \), \( \text{Re}(P) \), denotes the power resonantly absorbed by the plasma and converted into heat by Ohmic dissipation (see Equation (4.8)), and the imaginary part, \( \text{Im}(P) \), describes the energy circulating in the system according to Equation (4.7). The ratio of these two quantities, \( Q = \text{Im}(P)/\text{Re}(P) \), is called the coupling factor and is a measure for the efficiency of the process of resonant absorption. A large coupling factor implies little energy absorption and bad coupling between the external antenna and the plasma. A small value of \( Q \) means good coupling (for \( Q = 0 \) the coupling would be perfect and all the energy emitted by the antenna would be converted into heat). In the context of coronal loop heating it is more convenient to express the efficiency of the heating process in terms of the fractional absorption which is defined as \( f_a = \text{OD}/|P| \), i.e., the ratio of
the dissipated energy to the total energy delivered by the external source. The closer \( f_a \) is to one, the more efficient the heating takes place. The fractional absorption is independent of the amplitude of the external driver and is a measure for the ability of the plasma column to be heated by resonant absorption of waves.

Inserting the expression for \( j_{ant} \), (2.8) in Equation (4.6) yields

\[
P = \pi LI(nk^2 - me^2)_{r=r_a},
\]

with \( L \) the length of the cylinder and \( I \) the current in the antenna which is put equal to 1 in the coronal loop case for reasons of normalization. With the use of Faraday’s law and Equations (3.6) and (3.10) we find after elimination of \( B_{III} \) with Equation (3.22)

\[
P = -\pi LI \frac{\omega_p}{nk} \left[ f'_{d}(nk_{r_a}) - \frac{(nkb_1(r_p) - f'(nk_{r_p}))}{f'(nk_{r_p})} f'(nk_{r_a}) \right].
\]

The value of \( P \) can be determined from the value of the radial component of \( b \) at the plasma boundary.

### 4.4. Ideal Limit

Before we present numerical results on resistive Alfvén wave heating in the next section we first briefly recall how energy absorption rates have been computed in the context of ideal magnetohydrodynamics. In ideal MHD the energy absorption rate in a driven system may be calculated due to the singular nature of the ideal MHD equations. The radial components of \( b \) and \( v \) possess a logarithmic singularity at the resonant surface, \( r = r_s \), so that near the resonant surface we can write (see, e.g., Chen and Hasegawa, 1974)

\[
v_r = \begin{cases} 
C \ln(r - r_s) & r > r_s, \\
C \left[ \ln| r - r_s | + i\pi \right] & r < r_s.
\end{cases}
\]

The imaginary part in Equation (4.11) appears because of the analytic continuation of the solution across the singularity and represents the resonant absorption of the external perturbation by the plasma. The absorption rate can be shown to be proportional to \( C^2 \) (Chen and Hasegawa, 1974; for a linear density profile) so that in absence of the singularity, i.e., \( C = 0 \), there is no absorption. Indeed, if there is no singularity, \( b_1 \) is real and \( P \) is purely imaginary (for real \( I \)). This implies no absorption and the total power in the system is circulating.

### 5. Results

The heating of coronal loops by resonant absorption of Alfvén waves is now analyzed quantitatively. We consider a class of equilibria given by the profiles

\[
B_z = 1,
\]
\[
J_z = j_0 (1 - r^2)^\nu,
\]
\[
\rho = 1 - (1 - d)r^2,
\]

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System
in dimensionless units. The distance is normalized with the radius of the plasma column \( r_p \). The constants \( d, v, \) and \( j_0 \) are free parameters with \( j_0 = 2k/q_0 \) and \( q_0 \) the safety factor on the axis \( q(r) = r k B_z/B_0 \). \( d \) denotes the density at the plasma boundary, \( d = \rho(1) \). The pressure can be derived from the equilibrium equation (2.5). In this paper we discuss results for two specific equilibria corresponding to particular choices of the free parameters. We consider a constant finite electrical conductivity \( (\eta = \text{constant}) \).

We have used transformation (2.4) to obtain dimensionless quantities. The computed dimensionless frequencies are then related to physical frequencies via the transformation formula

\[
\sigma(\text{Hz}) = 0.1417 \omega \left( \frac{r_p}{10^6 \text{ m}} \right) \left( \frac{B_z(0)}{10 \text{ G}} \right) \left( \frac{\rho(0)}{10^{15} \text{ m}^{-3}} \right)^{-1/2}.
\]  

(5.2)

Here we assumed a mean atomic weight of 0.6 (for a fully-ionized \( H \) plasma \( \bar{\mu} = 0.5 \); in the solar atmosphere the presence of extra elements makes \( \bar{\mu} \approx 0.6 \) (Priest, 1984)). The dimensionless shear Alfvén frequency \( \omega \) is typically 0.1 so that \( \sigma \) is typically 0.01 Hz. The range of frequencies at which the coronal loops can be resonantly excited agrees very well with the observed motions in the solar atmosphere which have periods of typically 100 s.

5.1. Model I

The first equilibrium has a parabolic current profile, \( v = 1 \), so that the ratio \( q(r = 1)/q_0 = v + 1 = 2 \). We take a constant density profile \( (d = 1) \), an aspect ratio

![Schematic representation of a plasma-vacuum-antenna-wall system.](image_url)
\( \varepsilon = 20 \) and the wave numbers \( m = 2, n = 1 \). We choose \( q_0 = 0.5 \) so that the current density on the axis, \( j_0 \), equals 0.2. The plasma beta (ratio of plasma pressure over magnetic pressure) is a function of \( r \) and \( \beta_{\text{max}} = 0.0139 \) (= \( \beta(0) \)). For this choice of the equilibrium parameters and wave numbers the profile of the local Alfvén frequency is monotonic and parabolic, with \( 0.15 \leq \omega_{\alpha}(r) \leq 0.25 \), and is displayed in Figure 2.

5.1.1. Typical Result

Let us now consider the forced oscillation of this equilibrium that corresponds to the pump frequency \( \omega_p = 0.195 \) with a resonant surface at \( r = 0.7416 \) (= \( r_s \)) as illustrated in Figure 2. We choose the magnetic Reynolds number, \( R_m \), normalized with the toroidal Alfvén velocity on the magnetic axis, as \( 10^8 (\eta = 10^{-8}) \).

![Figure 2. Local Alfvén frequency versus radial coordinate \( r \) for \( d = 1, \nu = 1, q_0 = 0.5, m = 2 \), and \( n = 1 \).](image)

The real and imaginary parts of the components of the perturbed magnetic field in the plasma column and the real and imaginary parts of the total perturbed pressure \( \pi \) and the divergence of the velocity field are shown in Figures 3, 4, and 5, respectively. The solutions are characterized by sharp variations and nearly-singular behaviour in a narrow layer about the resonant surface \( r = r_s \). This typical behaviour of the solutions can be understood in terms of the results from ideal MHD. In ideal MHD, a solution that corresponds to a continuum frequency is characterized by a logarithmic singularity and a jump contribution to the normal \( (r - r) \) component of the perturbed magnetic field and by a \( 1/(r - r_s) \) singularity and a \( \delta(r - r_s) \)-contribution to the tangential components of the perturbed magnetic field (Goedbloed, 1975, 1983). In addition, for a frequency in the Alfvén continuum the tangential component of the perturbed magnetic field perpendicular to the magnetic field lines, dominates. This characteristic behaviour of the ideal continuum modes is still clearly recognizable in the nearly-singular behaviour of the components of the plasma response in resistive magnetohydrodynamics (see Figure 3). The forced oscillation resembles an ideal Alfvén continuum mode since it is
Fig. 3. The components of the perturbed magnetic field for \( \eta = 10^{-8} \) and \( \omega_p = 0.195 \) (\( d = 1, \nu = 1, q_0 = 0.5, m = 2, \) and \( n = 1 \)).

characterized by the dominant behaviour of the perpendicular component of the perturbed magnetic field in the vicinity of the resonant surface \( r = r_s \). The \( b_\perp \)-component shows strong traces of the ideal \( 1/(r - r_s) \) singularity and the ideal \( \delta(r - r_s) \)-contribution, which are spread over a narrow layer around \( r = r_s = 0.7416 \). The logarithmic singularity of the ideal continuum mode and the jump contribution are clearly recognizable in respectively the imaginary part and the real part of the normal component of the perturbed magnetic field. The jump of \( \text{Re}(irb_\perp) \) in \( r = r_s \) is proportional to the singular part in the ideal solution, in other words the factor \( C \) in Equation (4.11). The \( b_\parallel \) component has a small amplitude and the same nearly-singular behaviour as the normal component, as expected from ideal MHD.
Fig. 4. The components of the total perturbed pressure for $\eta = 10^{-8}$ and $\omega_p = 0.195$ ($d = 1$, $v = 1$, $q_0 = 0.5$, $m = 2$, and $n = 1$).

Fig. 5. The divergence of the velocity field for $\eta = 10^{-8}$ and $\omega_p = 0.195$ ($d = 1$, $v = 1$, $q_0 = 0.8$, $m = 2$, and $n = 1$).

Fig. 6. The Ohmic dissipation rate for $\eta = 10^{-8}$ and $\omega_p = 0.195$ ($d = 1$, $v = 1$, $q_0 = 0.5$, $m = 2$, and $n = 1$) (dimensionless units).

The coupling of the external source to plasma can be studied from the variation of the normal and parallel components of the perturbed magnetic field (see Figure 3), $\pi$ (Figure 4), and $\text{div}(v)$ (Figure 5). The relatively large values of $b_r$, $b_\theta$, $\pi$, and $\text{div}(v)$ at the plasma-vacuum interface ($r = 1$) show that the incoming perturbation is a compres-
sible magnetosonic wave while the 'jumps' of these quantities near \( r = r_s \) indicate that a significant part of the energy supplied by this magnetosonic wave is absorbed via the excitation of the Alfvén mode at \( r = r_s \). The \( Q \)-factor is 1.4253 corresponding to a fractional absorption \( f_a = 57\% \).

In resistive MHD the absorbed energy is converted into heat. The heating of the plasma by Ohmic dissipation is shown in Figure 6. The Ohmic dissipation is concentrated in the neighbourhood of the resonant surface \( r = 0.7416 \) as expected from ideal MHD but also in a narrow boundary layer. However, the energy emitted by the antenna is mainly transferred to the resonant layer and especially the plasma in a narrow layer around the resonant surface is heated by Ohmic dissipation.

The resistive energy equation (4.8) provides an excellent test for both the numerical accuracy of this result and the validity of the physical assumptions we made (see Section 2). The left-hand side of this equation is calculated by means of the expression (4.10) for \( P \), whereas the right-hand side is determined by means of a numerical integration using Gauss's rule (with four points in every subinterval). The difference between these two quantities is – for a sufficient number of gridpoints – very small \( (> 10^{-6}) \) so that the numerical approximation of the solution (with 1201 gridpoints and mesh accumulation at the nearly singular layer and at the plasma-vacuum interface) is extremely good. The difference between OD and \( \text{Re}(P) \) is proportional to the value of \( \eta \) and is for \( \eta = 10^{-8} \) smaller than \( 1 \times 10^{-5} \). The relative error made by taking an ideal magnetostatic equilibrium and the adiabatic law, is smaller than \( 0.0001\% \). These physical assumptions yield, therefore, a very good approximation.

5.1.2. \( \eta \)-Dependence

For a less resistive plasma (with a smaller value of \( \eta \)) the nearly-singular behaviour of the solution (stationary state) is more pronounced and the resemblance with the singular continuum mode is stronger. This is illustrated in Figure 7 where we show the three components of \( b \) for \( R_m = 10^{10} \) \((\eta = 10^{-10})\) and the same driving frequency \((\omega_p = 0.195)\). The plasma response shows the same characteristics as for \( R_m = 10^8 \) (Figure 3) but the extremum of the solution in \( r = r_s \) is now more pronounced than it was for \( R_m = 10^8 \). Because the width of the resonant layer is smaller, the Ohmic dissipation is almost the same as for \( R_m = 10^8 \). From Figures 3 and 7 it is clear that the jump contribution in the \( \text{Re}(b_1) \)-profile too is the same for \( R_m = 10^8 \) and for \( R_m = 10^{10} \).

The \( \eta \)-dependence of the Ohmic dissipation rate (OD) is illustrated in Figure 8 for \( \omega_p = 0.195 \). Figure 8 shows how strikingly insensitive the Ohmic dissipation rate is to the electric conductivity of the plasma: enlarging \( R_m \) by a factor of \( 10^6 \) only changes the value of OD (in dimensionless units) by \( 1.4\% \). Moreover, the dissipative absorption reaches an asymptotic value for \( R_m > 10^8 \). The relevant values of \( R_m \) for coronal loop plasmas are in the range \( 10^8 \) to \( 10^{12} \). Since the nearly-singular behaviour of the solution on the resonant layer is very pronounced for these values of \( R_m \), the heating too is localized in the immediate proximity of the resonant layer. In the ideal MHD limit \((\eta = 0)\) the tangential components of the perturbed magnetic field are discontinuous at
the plasma-vacuum interface. When $\eta \neq 0$ this discontinuity is smoothened and the transition is spread over a narrow boundary layer $\Delta_{b_\eta}$. The width of this boundary layer is proportional to $\eta^{1/2}$. Kappraff and Tataronis (1977) have shown analytically that the width of the resonant layer, $\Delta_{\text{res}}$, is proportional to $\eta^{1/3}$ in an incompressible slab. From our numerical results it follows that this result obtained by Kappraff and Tataronis for a very simple model has a more general validity as it holds for monotonic profiles of the local Alfvén frequency, $\omega_A(r)$, in a compressible cylindrical plasma. The Ohmic dissipation at the plasma boundary (in the boundary layer $\Delta_{b_\eta}$) can be ignored for the relevant (small) values of the resistivity and almost all the energy of the driver that couples to the plasma, is absorbed and converted into heat in the resonant layer.
5.1.3. Frequency Scan of Ideal Continuum

Let us now investigate how the energy dissipation rate varies with the frequency of the driver in order to get a clear and general picture of the fractional absorption resulting for the chosen mode $(m, n) = (2, 1)$. The fractional absorption as a function of the frequency of the external driver is shown in Figure 9 for $R_m = 10^8$. From Figure 9 it is immediately clear that not all driving frequencies couple to the plasma with the same efficiency. For the $\omega_p = 0.195$ (discussed above) $f_a = 0.57$ as indicated on Figure 9. However, the process can be much more efficient as the profile of the fractional absorption is characterized by a maximum at $\omega_p = 0.1911$. For this optimal frequency $f_a = 1$ which means that the coupling is perfect and all the energy supplied by the external driver is converted into heat! In order to understand this surprising maximum
in the fractional absorption profile we have displayed in Figure 10 $\text{Re}(rv_r)$ and $\text{Re}(iv_\perp)$ of the solutions corresponding to six frequencies in a range around $\omega_p = 0.1911$ (the corresponding values of $f_a$ are indicated on Figure 9). The radial component of $v(rv_r)$ consists for $\omega_p = 0.1700$ of a logarithmic part and a jump contribution. This corresponds to an $(r - r_s)^{-1}$-singularity and a $\delta(r - r_s)$-contribution in the tangential component $v_\perp$. The influence of the logarithmic term is negligible for $\omega_p = 0.1844$. The profiles of $v_r$ and $v_\perp$ are characterized, respectively, by a jump and a $\delta$-function spread out over a narrow plasma layer ($\eta \neq 0$). For $\omega_p = 0.1911$ the solution again consists of a logarithm and a Heaviside function ($rv_r$) but the logarithmic contribution now has the

---

**Fig. 10a–c.**
opposite sign. For $\omega_p = 0.1936$ the $rv_r$-singularity is almost purely logarithmic and the Heaviside contribution is ignorable (the peaks in the $v$ profile have about the same amplitude). For $\omega_p = 0.2044$ both singular contributions in the solution have changed sign compared to the first frequency in this series, $\omega_p = 0.1700$.

The maximum in the profile of the energy dissipation rate versus the driving frequency is due to a global motion of the plasma which has received many different names in the literature such as ‘collective mode’ (Balet, Appert, and Vaclavik, 1982; Sedlacek, 1971), ‘external kink’ (Chance et al., 1977), ‘surface mode’ (Appert et al., 1984), etc. According to Chance et al. this global solution is related to the external kink mode. In absence of a fixed wall, the external kink mode is a global perturbation of the plasma surrounded by a vacuum region. A rigid wall has a stabilizing effect on this external kink mode and this stabilizing effect increases as the wall is shifted towards the plasma surface (for coronal loops line-tying has the same stabilizing effect on the external kink mode). From a certain position of the wall on, the external kink is stable and for particular values of the equilibrium parameters its frequency is situated in the range of the ideal continuum. The frequency of this mode is then complex, even in ideal MHD. By consequence, this oscillation is not an eigenmode of the ideal system. However, it is clear that this ‘mode’ of the plasma is of physical interest. Although the real part of the frequency of the global
'mode' is located inside the continuous spectrum, the external kink does not exhibit the singular behaviour that is characteristic for the continuous spectrum. This is so because the factor $C$ in the ideal solution (4.11) depends on the equilibrium parameters and can be zero for certain frequencies and for certain choices of these parameters. In ideal MHD it can be shown that the constant $C$ of nearby continuum modes approaches zero (Chance et al., 1977). In the dissipative model ($\eta \neq 0$) we can locate the external kink in the same way. Indeed, we can identify the global mode in the continuous spectrum as the mode corresponding to the locus of the point in the continuum where the imaginary part of the absorbed power $P$ (and, hence, $Q$) changes sign.

The heating of the plasma by resonant absorption is extremely efficient when the plasma is excited with a frequency in the neighbourhood of the real part of the frequency of the global mode. Indeed, as the imaginary part of $P$ changes sign at the locus of the external kink in the continuum so does the $Q$-factor. This is illustrated in Table I showing the fractional absorption and the $Q$-factor for the 5 driving frequencies the plasma responses of which are shown in Figure 10 and discussed just above. $Q$ is almost zero ($-0.0053$) for $\omega_p = 0.1911$ and changes sign between $\omega_p = 0.1911$ and 0.1936. $Q = 0$ stands for perfect coupling, i.e., all the energy supplied by the external antenna is dissipated and there is no energy circulating in the system.

The result that optimal heating is obtained with a global mode which has no logarithmic contribution, is – at first glance – in contradiction with the ideal MHD result (power absorption rate $\sim C^2$). However, the cited ideal MHD result was obtained in a simple model (infinite slab). For a finite configuration the coefficients of the logarithmic term and of the Heaviside function in the radial component of the velocity field, may differ and so it is indeed possible that the logarithmic term vanishes while the Heaviside contribution (responsible for the power absorption) becomes very pronounced.

The bandwidth of frequencies around the real part of the frequency of the global 'mode' which are very efficiently absorbed is rather small (see Figure 9). This is, however, not always the case. The efficiency of resonant absorption of Alfvén waves depends strongly on the profiles of the equilibrium quantities and the wave numbers of the external driver. Moreover, the damping of the global modes in the continuous
spectrum also depends on the variation of the equilibrium quantities. In order to illustrate these statements we now consider an equilibrium with a (more realistic) density variation.

5.2. Model II

The second equilibrium model is characterized by the profiles (5.1) with $v = 2$, $q_0 = 1$, and $d = 0.25$. The current density is more peaked than in model I and the density profile is parabolic. This is a more realistic density profile for coronal loops in view of Foukal's observations (1975, 1976, 1978). The aspect ratio is still chosen as 20 and we now consider wave numbers $m = 1$ and $n = 1$. The local Alfvén frequency then is 0.1 on the magnetic axis, 0.13333 at the plasma boundary and reaches a minimal value in $r_{\text{min}} = 0.515$ where $\omega_A(r = r_{\text{min}}) = \omega_{A,\text{min}} = 0.09822$. The profile of the local Alfvén frequency differs from that in equilibrium model I in two ways. The profile of $\omega_A(r)$ is non-monotonic now and reaches its maximal value at the plasma boundary. The variation of $\omega_A(r)$ is illustrated in Figure 11.

![Graph showing the variation of $\omega_A(r)$ with $r$.](image)

Fig. 11. Local Alfvén frequency versus radial coordinate $r$ for $d = 0.25$, $v = 2$, $q_0 = 1.0$, $m = 1$, and $n = 1$.

5.2.1. Frequency Scan of Ideal Continuum

We obtain a global picture of the energy dissipation rate for the $(m, n) = (1, 1)$ mode by means of a parametric scan of the Alfvén continuum with respect to the pump frequency $\omega_p$. The Ohmic dissipation versus frequency $\omega_p$ is displayed in Figure 12 for $0.09943 \leq \omega_p \leq 0.13212$ and for $R_m = 10^8$. The maximum in the profile of the energy dissipation rate is less sharp in this case. The fractional absorption is over 90% for all the (ideal) continuum frequencies and reaches a maximum ($\approx 1$) for the driving frequency $\omega_p = 0.11275$. Since the resonant layer for this optimal pump frequency is close to the plasma boundary, $r_s \approx 0.905$ (see Figure 11), the perturbation with frequency $\omega_p = 0.11275$ couples to a continuum mode that is localized around $r = 0.905$ and the
energy is dissipated in a narrow layer close to the boundary of the plasma column. However, for this equilibrium loop model the bandwidth of frequencies which are very efficiently absorbed by the plasma (and, therefore, yield very efficient heating of the plasma column) covers the whole ideal Alfvén continuum.

6. Conclusions

The heating of coronal loop plasmas by means of resonant absorption of Alfvén waves has been analyzed quantitatively for one-dimensional, cylindrically-symmetric equilibrium models within the framework of linear resistive MHD. The compressibility of the plasma has been taken into account which made it possible to describe the energy transfer from the external driver to the plasma. The energy is transferred by fast magnetoacoustic waves which do not appear in an incompressible plasma. Finite electrical conductivity of the plasma was included in the generalized Ohm’s law and the heating of the plasma by resonant absorption was calculated consistently in a dissipative model.

We have developed a numerical code which calculates the heating that results when a one-dimensional, cylindrically-symmetric plasma is excited by a periodic external driver at a given frequency. The numerical solutions are determined by means of the finite element method. This method is extremely suitable for the numerical approximation of the solutions of the linearized resistive MHD equations which are strongly localized in a resistive boundary layer and a narrow resonant layer for the relevant (very small) values of the resistivity (for $\eta = 0$ the equations are singular).

Resonant absorption of Alfvén waves turns out to be very efficient for typical coronal loop parameter values. The energy dissipation rate due to Ohmic heating is independent of the resistivity of the plasma in an asymptotic sense ($\eta \to 0$). Since the resistivity of coronal loop plasmas is very small ($10^{-8} \leq \eta \leq 10^{-12}$), the Ohmic dissipation is quasi
independent of \( \eta \) for the relevant values of this parameter. The heating of the plasma is localized in a resonant layer with a width proportional to \( \eta^{1/3} \) and in a boundary layer with a width \( \sim \eta^{1/2} \). The Ohmic dissipation in this boundary layer is ignorable for the relevant values of \( \eta \). We remark that Hollweg (1987b) has pointed out that, similarly, for a viscous plasma the thickness of the resonant layer is proportional to \( \nu^{1/3} \) (where \( \nu \) is the viscosity) when the viscous stress tensor takes the usual form for classical fluids.

The efficiency of the process of resonant absorption of Alfvén waves and the localization of the heating are both strongly affected by the profiles of the equilibrium quantities and by the wave numbers and frequency of the external driver. Resonant absorption is extremely efficient when the system is driven at a frequency close to the real part of the frequency of a so-called ‘collective mode’. In particular, we have investigated the heating resulting from a pump frequency in the neighbourhood of the real part of the frequency of the external kink mode. For smooth equilibrium profiles the external kink mode is only damped weakly and the variation of the fractional absorption as a function of the pump frequency \( \omega_p \) has a sharp and pronounced maximal value at the real part of the frequency of the external kink mode. For this optimal driving frequency the efficiency of the heating mechanism is extreme: all the energy supplied by the driver couples to the plasma and is converted into heat by the Ohmic dissipation. For driving frequencies in the neighbourhood of the optimal frequency the heating mechanism is also very efficient. However, for smooth equilibrium profiles this result only applies for a narrow bandwidth around the frequency of the global mode. For more realistic profiles of the equilibrium quantities the peak in the profile of the fractional absorption is broader and the bandwidth of pump frequencies for which the heating mechanism is very efficient, comprises the complete range of the ideal continuous spectrum.

We recall that our numerical simulations of coronal loop heating by resonant absorption of Alfvén waves are based on earlier work by Poedts, Kerner, and Goossens (1988) in which the plasma is surrounded by a vacuum and a perfectly conducting wall and driven by a current in external coils situated in the vacuum region between the plasma and the wall (tokamak case). For the coronal loop case we have recast our calculations in terms of a unit driver situated at the plasma boundary. In this way the external antenna and the vacuum region are removed from the problem but the calculation is reduced to the determination of the intrinsic dissipation. The dissipation rates and fractional absorptions presented in this paper, therefore, only depend on the characteristics of the equilibrium models and on the assumed unit amplitude of the driver. The actual heating of the coronal loops can only be determined when the power function of loop driving mechanism is known. Grossmann and Smith (1988) have suggested a two-step method to determine the actual heating of coronal loops: (1) for a given equilibrium model, determine the intrinsic absorption spectrum (depending only on the wave numbers and the driving frequencies but not on the driving spectrum); (2) relate a given input spectrum of driving oscillations to a spectral weighting function, \( S(m, n, \omega) \), and make the convolution of \( S(m, n, \omega) \) with the intrinsic absorption spectrum. We think this is indeed the procedure to be followed to determine the actual
heating of coronal loops. In order to be successful, however, we think that it is not sufficient to consider only \( m = 1 \) modes in the calculations, as suggested by Grossmann and Smith. Indeed, as is clear from the results presented above for the first model, \( m \neq 1 \) modes can be very efficiently absorbed too. The efficiency of the resonant absorption process strongly depends on the equilibrium parameters and on the characteristics of the driver (wave numbers, frequency, \ldots). Therefore, a parametric study is currently carried out, the results of which will be published in a following paper.

The heating mechanism in principle also applies for driving frequencies within the range of the ideal slow magnetosonic continuum. We, therefore, also simulated coronal loop heating with these low frequencies. For the two cases discussed in this paper, however, heating with frequencies within the ideal slow continuum range turns out to be very inefficient: the fraction absorption is less than 0.01 which means that less than 1\% of the energy supplied by the driver is coupled to the plasma and converted into heat by Ohmic dissipation. It is currently investigated whether this is always so in the parameter domain characteristic for coronal loops.

Acknowledgements

This work was carried out partially while S. Poedts was Research Assistant of the Belgian National Fund for Scientific Research and partially in his current capacity of Research Assistant of the Research Council of the K.U. Leuven. The support by both the B.N.F.S.R. and the K.U.L. Research Council are greatly acknowledged. We thank J. P. Goedbloed, J. V. Hollweg, and K. Appert for stimulating discussions and suggestions. The K.U. Leuven–MPIPP collaboration is supported by the European Economic Community Scientific Cooperation Contract No. ST2J–0275–C.

References