POLARIZATION PROPERTIES OF A ‘ZEISS-TYPE’ COELOSTAT: THE CASE OF THE SOLAR TOWER IN ARCETRI

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Abstract. A theoretical model of the polarization properties of a ‘Zeiss-type’ coelostat is presented and discussed in detail. The Muller matrix describing the modification of the Stokes vector of the incident radiation as a result of the multiple reflections on the coelostat mirrors is derived as a function of the solar coordinates, the geometrical configuration of the coelostat, and the parameters defining the optical properties of the mirrors. These parameters, or more particularly, the index of refraction $n$ and the extinction coefficient $k$, have been evaluated by means of laboratory measurements performed on a series of specimens having characteristics similar to those of the coelostat mirrors. The geometry of the coelostat configuration is described in full detail. The theoretical model has been then particularized to the case of the Donati Solar Tower in Arcetri, and some experimental measurements have been performed to check the correctness of the model. These measurements show the basic adequacy of the mathematical model, although some offset terms are found in the Stokes parameters $U$ and $V$.

1. Introduction

Modern research in solar physics has pointed out the extreme importance of magnetic fields in determining the shape, evolution, and, in many cases, the existence itself of the typical structures that are observed on the active Sun. As a consequence, if a suitable diagnostic has to be given for the solar plasma, also the magnetic field vector has to be determined together with the more conventional parameters such as temperature, pressure and electronic pressure. On the other hand, the magnetic field vector can in practice be measured only through the polarimetric signal that it is able of introducing in suitable spectral lines; for this reason spectropolarimetry has become in recent years one of the most powerful tools of solar physics and many efforts have been devoted to the attempt of obtaining suitable instruments capable of measuring the Stokes parameters profiles in solar spectral lines.

Two different philosophies can in principle be followed to obtain careful measurements of Stokes parameters profiles. The first is the one of building dedicated instruments, especially devised for spectropolarimetric observations. In these instruments the optical design is generally conceived to avoid oblique mirror reflections before the polarimetric analysis of the incident radiation has taken place. The prototype of such instruments can be considered the HAO Stokes polarimeter which has been operating at the Sacramento Peak Observatory in the late 1970s and early 1980s (Baur, House, and Hull, 1980; Baur et al., 1981).

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A totally different approach consists in adapting a pre-existing telescope, initially conceived for the observation of ordinary, unpolarized radiation, into a polarimeter. Particularly relevant is the case of the existing solar towers that are generally provided with sophisticated pieces of instrumentation capable of attaining extremely high values of spectral resolution. If the spectroscopic analysis of the radiation could be preceded by a careful polarimetric analysis of the radiation itself, a solar tower would be turned in an extremely powerful Stokes polarimeter – a unique piece of instrumentation for the analysis of solar phenomena.

Unfortunately, in usual solar towers the imaging system involves oblique reflections on one or more mirrors (the coelostat) that significantly distort the polarization of the solar radiation. These distortion effects, that are ultimately due to the differential reflectivity of the mirrors to electric fields directed either orthogonally or along the plane of incidence, can be accounted for in two different ways. A first approach consists in introducing along the ray-path one or more optical devices capable of compensating for the instrumental polarization. This approach has been followed by Stenflo, Twrenbold, and Harvey (1983) to obtain linear polarization recordings of the solar spectrum at the Kitt Peak McMath telescope with a modification of the Fourier transform spectrometer conceived by Brault (1978).

A different approach consists in developing a mathematical model of the instrumental polarization introduced by the oblique reflections on the coelostat mirrors as a function of the time of observation. The mathematical model translates the operation of the solar tower into a Muller matrix $M$ relating the Stokes vector $S$ of the incident radiation to the Stokes vector $S'$ found in the point where the polarimetric analysis is performed. Such an approach has been followed by Balasubramaniam, Venkatakrishnan, and Bhattacharyya (1985) to derive the theoretical Muller matrix of the Kodaikanal Solar Tower.

As a preliminary step toward adapting the Donati Solar Tower in Arcetri into a polarimeter, we have developed a mathematical model for its Muller matrix and we have checked its soundness by means of a series of experimental tests aimed at a direct measurement of the instrumental polarization. The aim of the present paper is to give a detailed description of this work that can be considered a prototype study of the polarization effects introduced by a ‘Zeiss-type’ coelostat. A strictly similar approach could be used to evaluate the polarization effects introduced by coelostats of more conventional types.

## 2. The Imaging System of the Arcetri Solar Tower

The imaging system of the Arcetri Solar Tower (Abetti, 1926) consists of a two-mirrors coelostat (40 cm in diameter) and a zenithal refractor (30 cm in diameter, 1792 cm in focal length). The coelostat mirrors are mounted in the so-called ‘Zeiss-type’ assembly on top of the tower (see Figure 1).

The first mirror reflects the sunlight towards the second mirror which, in turn, illuminates the vertical telescope. The first mirror revolves around a diameter set parallel
to the Earth's axis and, usually, it has the center lying in the meridian plane passing through the optical axis of the telescope. The surface of the second mirror faces the first mirror and the objective of the vertical telescope. This mirror revolves around two axes:

Fig. 1. The coelostat of the Donati Solar Tower in Arcetri.
the first one is parallel to the vertical E–W plane passing through the telescope optical axis, while the second one is horizontal and lies in the meridian plane.

In our mounting the position of the center of the second mirror is fixed. Therefore, the centre of the first mirror (the one facing the sky) must be displaced while the solar declination changes, in order to make it lie on the line passing through the center of the second mirror and parallel to the direction of the reflected rays. For this purpose, the mounting of the first mirror can be moved on its polar axis.

In the case of small or negative solar declinations, the mounting of the second mirror casts a shadow on the first mirror when the Sun is around noon. This disadvantage, typical of this kind of mounting, may be avoided by displacing laterally the coelostat mirror from the meridian. In this case, however, image rotation is introduced and much larger angles are involved in the reflections.

A further description of the geometry of the coelostat, particularly relevant from the point of view of its polarization properties, is given in Section 3.3.

3. Polarization Properties of a ‘Zeiss-type’ Coelostat

3.1. Muller matrices for rotations and mirror reflections

In order to characterize the polarization properties of a light beam, two mutually orthogonal directions have to be specified in the plane perpendicular to the direction of propagation. In the following we assume a right-handed, orthogonal reference system \((e_1, e_2, e_3)\) to be associated with each light beam, with the unit vector \(e_3\) directed along the beam itself. The polarization properties of the beam can then be characterized by the Stokes vector \(S^t \equiv (I, Q, U, V)\), which we assume here to be defined according to Shurliff (1962) (we denote by \(S\) the Stokes column vector and by \(S^t\) its transposed vector); we recall that \(e_1\) specifies the positive \(Q\) direction.

When the light beam is reflected by a flat metallic mirror, the Stokes vector is changed according to \(S' = TS\), where \(S\) refers to the incident light and \(S'\) to the reflected light; Figure 2(a) shows the reference systems \((e_1, e_2, e_3)\) and \((e'_1, e'_2, e'_3)\) for \(S\) and \(S'\), respectively. The Muller matrix \(T\) is given (Jager and Oetken, 1963a, b) by:

\[
T = \frac{r_\perp^2}{2} \begin{pmatrix}
X^2 + 1 & X^2 - 1 & 0 & 0 \\
X^2 - 1 & X^2 + 1 & 0 & 0 \\
0 & 0 & 2X \cos \tau & 2X \sin \tau \\
0 & 0 & -2X \sin \tau & 2X \cos \tau
\end{pmatrix}
\]

where \(X^2 = r_\parallel^2/r_\perp^2\) is the ratio between the Fresnel amplitude reflection coefficients, for the electric vector components parallel and perpendicular to the incidence plane, and \(\tau\) is the difference between their phase changes on reflection.

From the theory of metallic reflection (cf. Berning and Berning, 1960; Abélès, 1963) the parameters \(X^2\) and \(\tau\) may be expressed as functions of the index of refraction \(n\), of
the extinction coefficient $k$, and of the incidence angle $i$ on the mirror surface. The resulting expressions are:

$$X^2 = \frac{f^2 + g^2 - 2f \sin i \tan i + \sin^2 i \tan^2 i}{f^2 + g^2 + 2f \sin i \tan i + \sin^2 i \tan^2 i}, \quad (2a)$$

$$\tan \tau = \frac{2g \sin i \tan i}{\sin^2 i \tan^2 i - (f^2 + g^2)}, \quad (2b)$$

where

$$f^2 = \frac{1}{2}[n^2 - k^2 - \sin^2 i + \sqrt{(n^2 - k^2 - \sin^2 i)^2 + 4n^2 k^2}],$$

$$g^2 = \frac{1}{2}[k^2 - n^2 + \sin^2 i + \sqrt{(n^2 - k^2 - \sin^2 i)^2 + 4n^2 k^2}].$$

Note that the quantity $\tau$ in Equation (2b) is defined in the interval $[0, \pi]$, the values 0 and $\pi$ corresponding, respectively, to $i = \frac{1}{2}\pi$ and $i = 0$.

As described in Section 2, the sunlight is reflected twice in the coelostat system (on the first and second mirror, respectively), so that two distinct incidence planes are involved. As a consequence, we need to know how the Stokes vector is changed when the reference system $(e_1, e_2, e_3)$ is suitably rotated through an angle $\theta$ around $e_3$, in order to bring $e_1$ in the incidence plane as specified in Figure 2(a). The effect of such a rotation is given (see, e.g., Shurcliff, 1962) by $S' = R(\theta)S$, where $S$ and $S'$ refer, respectively, to the system $(e_1, e_2, e_3)$ and $(e'_1, e'_2, e'_3)$ shown in Figure 2(b). The rotation matrix $R(\theta)$ has the expression:

$$R(\theta) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos 2\theta & \sin 2\theta & 0 \\
0 & -\sin 2\theta & \cos 2\theta & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}, \quad (3)$$

where the angle $\theta$ is measured from $e_1$ in the counterclockwise direction, as seen by an observer looking towards the light source. It appears from Equation (3) that the Stokes vector is unaffected by a rotation $\theta = \pi$.

3.2. Evaluation of the optical parameters of the mirrors

In order to evaluate the optical parameters of the coelostat mirrors, we have measured in the laboratory the polarization properties of a beam reflected, at different incidence angles, by a series of aluminum coating specimens characterized by different coating thicknesses (0.55 $\mu$m (No. 1), 0.108 $\mu$m (No. 2), 0.110 $\mu$m (No. 3), 0.166 $\mu$m (No. 4)). These specimens were prepared at the same time as the reflecting surfaces of the coelostat mirrors, and, in particular, the same aluminum thickness as for the coelostat mirrors was used for the second specimen (0.108 $\mu$m). The analysis of the polarization of the reflected beam has been carried out at the wavelength of 6500 Å by using suitably
oriented retarders and linear polarizers and by measuring the transmitted beam intensity. For each specimen we have obtained the plots of $X^2$ and $\tau$ as functions of the incidence angle (Figure 3(a)). The errors in our measurements are due to the photon noise in the detector and may be estimated not to exceed about 0.7%. In the measurements, great care has been taken to keep the spurious modulation due to the rotating optical elements as low as 0.5%. We note that specimens 1, 2, and 3 give results which are very similar, while specimen 4 shows a different behavior.

Fig. 2. Polarization reference systems for reflection (a) and for rotation (b). The unit vectors $\mathbf{e}_1, \mathbf{e}_1'$ in (a) lie in the incidence plane.
Fig. 3. (a) The quantities $X^2$ and $\tau$ as determined from our measurements are plotted versus the incidence angle. The various symbols refer to different specimens ($\Delta$ for specimen No. 1, $\Box$ for No. 2, $\bigcirc$ for No. 3, $\lozenge$ for No. 4). The full lines refer to the analytical expressions (Equation 2) calculated from the values for $n$ and $k$ given by Schultz (1954) and Schultz and Tangherlini (1954) ($n = 1.13$, $k = 6.39$). (b) Experimental values for $X^2$ and $\tau$ relative to specimen No. 2 (square symbols $\Box$). The full line represents the best fit obtained for the values $n$ and $k$ given in the text.

From our measurements we have derived, by means of a nonlinear least-squares fit, the values for $n$ and $k$ at 6500 Å relative to specimen No. 2 (Figure 3(b)). The derived values were $n = 1.036 \pm 0.004$, $k = 5.89 \pm 0.01$, which appreciably differ from the values given by Schultz (1954) and Schultz and Tangherlini (1954) for the same wavelength. Such values have been employed in our model to describe the polarization properties of the coelostat mirrors.

3.3. GEOMETRY OF THE ZEISS COELOSTAT SYSTEM

In Figure 4 the celestial sphere centered on the first mirror $C_1$ of the coelostat is shown; $S$ is the position of the Sun, $C_2$ is the point where the direction joining the first with the second mirror meets the celestial sphere, $N_1$ is the direction perpendicular to the first mirror, and $\phi$ is the latitude of the solar tower. Since the first mirror revolves around the polar axis, $N_1$ lies on the celestial equator.
Fig. 4. The celestial sphere centered on the first mirror of the coelostat (see text for details). The incidence angles $i_1, i_2$ and the rotation angles $\theta_1, \theta_2, \theta_3$ entering the Muller matrices of the telescope are indicated.

In the following we assume the south point on the horizon to be specified by a null value of the hour angle and azimuth, and that both coordinates increase westward.

As mentioned in Section 2, the first mirror of the Arcetri Solar Tower can be displaced laterally from the meridian to avoid the shadow due to the second mirror when the Sun is around noon; the geometry of Figure 4 refers to a configuration where the first mirror is displaced eastward with respect to the second mirror. However, the formulae that will be deduced in the present section still hold for the 'opposite' configuration (the first mirror displaced westward).

Let the Stokes vector $\mathbf{S}$ of the light coming from the Sun be defined in the right-handed reference system $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$, where $\mathbf{e}_3$ points toward $C_1$ and $\mathbf{e}_1$ is tangent to the celestial meridian through the center of the Sun and points toward the north celestial pole $P$. In other words this means that we are measuring the positive $Q$ direction of the solar radiation along the celestial meridian.

In order to evaluate how the polarization changes in the reflection on the first mirror, a preliminary rotation of the reference system $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ has to be performed, to bring $\mathbf{e}_1$ in the incidence plane $SC_1C_2$ of the first reflection; this is done through a rotation around the unit vector $\mathbf{e}_3$ of the angle $\theta_1$ shown in Figure 4.

The Stokes vector $\mathbf{S}'$ of the radiation reflected by the first mirror is then given by $\mathbf{S}' = \mathbf{T}(i_1)\mathbf{R}(\theta_1)\mathbf{S}$, where $\mathbf{T}$ and $\mathbf{R}$ are the reflection and rotation matrices defined in
Equations (1) and (3), respectively, and $i_1$ is the incidence angle $SC_1N_1$ on the first mirror.

Analogously, the Stokes vector $S''$ of the radiation reflected by the second mirror $C_2$ is obtained by the Stokes vector $S'$ through the relation $S'' = T(i_2)R(\theta_2)S'$, where $i_2$ is the incidence angle $\frac{1}{2}C_1C_2V$ on the second mirror and $\theta_2$ is the angle shown in Figure 4.

The Stokes vector $S''$ characterizes the radiation falling on the horizontal plane of the telescope. For practical purposes it is found convenient to specify the polarization with respect to a reference system having the $e_1$ unit vector pointing toward the North point on the horizon. To this aim we must perform a further rotation characterized by the angle $\theta_3$ shown in Figure 4.

We finally obtain that the overall effect due to the reflections on the mirrors of the coelostat is to modify the Stokes vector $S$ of the solar radiation according to:

$$S' = MS = R(\theta_3)T(i_2)R(\theta_2)T(i_1)R(\theta_1)S,$$

where $S'$ is the Stokes vector of the radiation propagating vertically along the tower, and is referred to the system specified above.

It is now necessary to express the rotation angles $\theta_1$, $\theta_2$, $\theta_3$ and the incidence angles $i_1$, $i_2$ as functions of the position of the Sun ($\delta_\odot$, $H_\odot$) and of the geometrical parameters of the coelostat.

From the laws of reflection, we immediately have $C_2C_1N_1 = \frac{1}{2}SC_1C_2$ (see Figure 4), so that the following relations hold:

$$\delta_{C_2} = -\delta_\odot,$$

$$H_{N_1} = \frac{1}{2}(H_{C_2} + H_\odot),$$

where $\delta$ and $H$ denote declination and hour angle.

From the spherical triangle $PSN_1$ we have:

$$\sin \theta_1 \sin i_1 = -\sin H,$$

$$\cos \theta_1 \sin i_1 = -\sin \delta_\odot \cos H,$$

$$\cos i_1 = \cos \delta_\odot \cos H,$$

where

$$H = H_{N_1} - H_\odot = \frac{1}{2}(H_{C_2} - H_\odot).$$

From Equations (6a and 6b) we obtain, for the elements appearing in the rotation matrix $R(\theta_1)$:

$$\sin 2\theta_1 = \frac{2 \sin \delta_\odot \sin H \cos H}{\sin^2 \delta_\odot \cos^2 H + \sin^2 H},$$

$$\cos 2\theta_1 = \frac{\sin^2 \delta_\odot \cos^2 H - \sin^2 H}{\sin^2 \delta_\odot \cos^2 H + \sin^2 H},$$
while Equation (6c) allows the trigonometrical functions contained in the elements of the reflection matrix $T(i_1)$ to be evaluated.

The incidence angle $i_2$ is easily obtained from the triangle $C_1 C_2 V$:
\[ i_2 = \frac{1}{2} (90^\circ - h_{C_2}) . \]  
(8)

From the spherical triangle $S C_2 N$ we obtain:
\[ \sin \theta_2 = - \frac{\cos h_\odot \sin A}{\sin 2i_1} , \]  
(9a)
\[ \cos \theta_2 = \frac{\sin h_{C_2} \cos 2i_1 - \sin h_\odot}{\cos h_{C_2} \sin 2i_1} , \]  
(9b)

where $h$ denotes altitude and $A = A_{C_2} - A_\odot$ is the difference between the azimuth of $C_2$ and the azimuth of the Sun. From Equations (9) the trigonometrical functions $\sin 2\theta_2$ and $\cos 2\theta_2$, entering the rotation matrix $R(\theta_2)$, are directly obtained.

As far as the $\theta_3$ angle is concerned, we immediately have from Figure 4:
\[ \theta_3 = 360^\circ - A_{C_2} , \]  
(10)
which allows the elements of $R(\theta_3)$ to be evaluated.

The hour angle $H_{C_2}$ appearing in the right-hand side of Equation (6d) is obtained from the spherical triangle $P Z C_2$:
\[ \sin H_{C_2} = \frac{\cos h_{C_2} \sin A_{C_2}}{\cos \delta_\odot} , \]  
(11a)
\[ \cos H_{C_2} = \frac{\sin h_{C_2} + \sin \phi \sin \delta_\odot}{\cos \phi \cos \delta_\odot} , \]  
(11b)

where Equations (5) have been taken into account.

The altitude $h_\odot$ and the azimuth $A_\odot$ of the Sun are related to the equatorial coordinates $\delta_\odot$, $H_\odot$ by the relations:
\[ \cos h_\odot \sin A_\odot = \sin H_\odot \cos \delta_\odot , \]  
(12a)
\[ \cos h_\odot \cos A_\odot = \cos \delta_\odot \cos H_\odot \sin \phi - \sin \delta_\odot \cos \phi , \]  
(12b)
\[ \sin h_\odot = \cos \delta_\odot \cos H_\odot \cos \phi + \sin \delta_\odot \sin \phi . \]  
(12c)

Finally, the quantities $A_{C_2}$ and $h_{C_2}$ can be expressed through the geometrical parameters of the coelostat, schematized in Figure 5.

The center $C_2$ of the second mirror is a fixed point lying on the vertical axis $(C_2 V)$ of the telescope. The first mirror revolves around a slide set parallel to the polar axis ($\phi$ is the latitude). As mentioned in Section 2, this mirror can be shifted along the slide: $C_1$ is its lowest position, and $d$ is the distance $C_1 C_1$; $\pi$ is the horizontal plane passing through $C_1$. Moreover, the slide itself can be moved along the east-west direction, so
that $C_1$ is shifted along the line $r$. The point $O$ is the intersection between $r$ and the meridian plane containing the telescope axis ($C_2V$); the position of $C_1$ along the line $r$ is specified by the quantity $a$, which is measured positively from $O$ in the eastward direction. Finally, $c$ is the distance between $C_2$ and the plane $\pi$, and $b$ the distance between the telescope axis and the line $r$.

It appears from Figure 5 that the altitude $h_{C_2}$ and the azimuth $A_{C_2}$ are given by:

$$\cos h_{C_2} = \frac{\sqrt{a^2 + (b + d \cos \phi)^2}}{\sqrt{a^2 + (b + d \cos \phi)^2 + (c - d \sin \phi)^2}},$$  \hspace{1cm} (13a)$$

$$\sin h_{C_2} = \frac{c - d \sin \phi}{\sqrt{a^2 + (b + d \cos \phi)^2 + (c - d \sin \phi)^2}},$$  \hspace{1cm} (13b)$$

$$\cos A_{C_2} = \frac{b + d \cos \phi}{\sqrt{a^2 + (b + d \cos \phi)^2}},$$  \hspace{1cm} (13c)$$

$$\sin A_{C_2} = \frac{a}{\sqrt{a^2 + (b + d \cos \phi)^2}}.$$  \hspace{1cm} (13d)
In practice, the quantities \( b \) and \( c \) are held fixed to the values \( b = 49.5 \text{ cm} \), \( c = 137.5 \text{ cm} \), while \( a \) and \( d \) are varied during the year to avoid the shadow cast by the second mirror (or its mounting) on the first mirror. It is not necessary here to describe in detail the procedure by which the values \( a \) and \( d \) are chosen on a day by day basis; the interested reader is referred to Cavallini et al. (1985) for details. What is important to notice is that, for a four months period centered around the summer solstice, \( a \) is set equal to zero, while, for the remaining part of the year, \( a \) is set to a positive value \( a_0 \) before noon, and to \( -a_0 \) after noon, \( a_0 \) being a function of the day.

Taking into account this sign inversion of \( a \), a direct inspection of the telescope Muller matrix \( \mathbf{M} \) shows that two symmetric positions of the Sun with respect to the meridian give rise to the symmetry properties schematized below:

\[
\begin{pmatrix}
  + & + & - & - \\
  + & + & - & - \\
  - & - & + & + \\
  - & - & + & + \\
\end{pmatrix},
\]

(14)

where a positive sign means that the matrix element is unchanged under the transformation:

\[
H_{\odot} \rightarrow -H_{\odot}, \quad a \rightarrow -a,
\]

while a negative sign means that the matrix element changes its sign under the same transformation.

Note that these last matrix elements may exhibit a discontinuity at \( H_{\odot} = 0 \) when \( a \neq 0 \).

3.4. EVALUATION OF THE ELEMENTS OF THE MULLER MATRIX

The instrumental effect on the Stokes parameters is twofold: an attenuation described by the diagonal elements of the Muller matrix, and a 'crosstalk', which is accounted by the off-diagonal elements; both effects depend on the position of the Sun in the sky.

Let us first consider the diagonal elements. In Figure 6 we plot these terms for some values of the Sun's declination versus the hour angle. It appears that close to the summer solstice the attenuation effects are generally less remarkable, and, moreover, they increase with increasing distance fro the meridian.

The off-diagonal elements describing the crosstalk among \( Q, U, \) and \( V \), are shown in Figure 7. Again the best observing conditions are close to the summer solstice, since the first and second mirrors are on the meridian. At the equinoxes and at the winter solstice, due to the off-meridian position of the coelostat, the crosstalk strongly increases not only for large hour angles, but also for the meridian position of the Sun. This is particularly relevant for crosstalk between \( U \) and \( Q \). One may note the partial symmetry existing between transposed elements of the Muller matrix.

The other Muller matrix elements, namely those which connect the intensity with the Stokes parameters \( Q, U, \) and \( V \), are plotted in Figure 8. Their absolute value is not as
Fig. 6. The diagonal elements of the telescope's Muller matrix are plotted versus the hour angle. Only positive values for the hour angle are considered because of the symmetry properties described by Equations (14). The full line refers to the equinox ($\delta_0 = 0$) and is obtained for the values $a = -69.0$ cm, $d = 59.3$ cm that define the proper position of the first mirror at this epoch. The dotted line corresponds to the winter solstice ($\delta_0 = -23^\circ$, $a = -41.2$ cm, $d = 0$); the arrowed line corresponds to the summer solstice ($\delta_0 = 23^\circ$, $a = 0$, $d = 116.0$ cm).

large as that of the elements which describe the crosstalk between the other parameters; however, since the intensity of the unpolarized light is predominant in solar measurements, they cannot be ignored, especially when unpolarized stray light adds to the image of the region under observation.

We have computed the same matrix elements for the latitude of the Canary Islands ($\phi = 28^\circ$). The general result is that both the crosstalk and the attenuation effects are less important with respect to those corresponding to the Arcetri Solar Tower. This is obviously due to the fact that the incidence angles on the first and second mirrors are smaller.

4. Preliminary Measurements of the Telescope Instrumental Polarization

As an attempt to test the mathematical model described in the former sections, we have performed some preliminary measurements of the instrumental polarization of the
Arcetri Solar Tower coelostat system in the particular case of unpolarized incident light (continuum from the center of the solar disk). For the polarization measurements we have used the spectro-interferometer installed at the focal plane of the telescope (Cavallini et al., 1985, 1987), mounting a polarimeter package in the objective focal plane before the spectro-interferometer entrance slit. This package consisted of a pair of quarter waveplates and a linear polarizer. Each waveplate was connected with a step motor to change the orientation of the fast axis in order to modulate the polarization coming out from the telescope. The linear polarizer, which was the last element of the
package, was kept fixed with respect to the spectro-interferometer. This precaution was necessary since we do not know the effect of this instrument on the incident polarization.

The intensity of the light that falls on the photomultiplier for each configuration of the polarimeter is given by

\[ K(I' \pm S') \]

where \( K \) is a constant factor and \( S' \) is, alternatively, \( Q' \), \( U' \), or \( V' \), depending on the configuration selected for the polarimeter. Taking sums and differences of the intensities
measured by the photomultiplier, we find a set of four quantities proportional to the Stokes parameters, from which we can derive the normalized parameters \( S'/I' \).

Some polarization measurements have been performed on July 1987 for a solar declination of about 20° and hour angles ranging between −80° and 80°. The measurements refer to a wavelength window of 150 mÅ in the continuum, centered at 120 mÅ to the red from the center of the 6306.6 Å line of telluric O₂. This wavelength approximately corresponds to the one used for calibrating the reflecting properties of the mirrors.

In Figure 9 the experimental ratios \( Q'/I' \), \( U'/I' \), and \( V'/I' \), as observed on July 22, 1987 are shown. A comparison with the results deduced from our mathematical model shows a good agreement with the experimental results for \( Q'/I' \) (Figure 9(a)). On the other hand, we found that the experimental values of \( U'/I' \) are systematically higher than the theoretical ones (dashed line in Figure 9(b)). The full line in the same figure shows the theoretical curve with an offset of 0.68% arbitrarily introduced in the mathematical model. Analogously, for \( V'/I' \) we also find that the experimental results are, with a few exceptions, systematically lower than the theoretical curve (dashed line in Figure 9(c)). The full line in the same figure is the theoretical curve with a negative offset of 0.67%. The two offsets in Figures 9(b) and 9(c) have been deduced by means of a least-squares method.

![Graph](image)

Fig. 9a. The measured values of \( Q'/I' \) are plotted as a function of the hour angle. The error bars are deduced from the photon noise statistics. The solid line is the theoretical curve deduced from the mathematical model described in the text. The measurements refer to July 22, 1987.

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Fig. 9b. Same as Figure 9(a) for $U''/I'$. The dashed line results from the theoretical model, while the full line is obtained from the previous one by adding an offset of +0.68%.

Fig. 9c. Same as Figure 9(a) for $V''/I'$. The dashed line results from the theoretical model, while the full line is obtained from the previous one by adding an offset of −0.67%.
It is difficult to single out the origin of the additional polarization which introduces the observed offsets. However, it is important to note that the offsets do not depend on the hour angle of the Sun and, therefore, they are most likely introduced by the optics mounted below the coelostat system.

5. Conclusions

We have developed an analytical model for computing the instrumental polarization effects introduced by a ‘Zeiss-type’ coelostat mounting. The previsions of the model have been largely confirmed by the experimental results. This work suggests that it is possible, when using optical mountings with oblique reflections at variable angles, to correct the data for instrumental polarization. This requires that we measure the complex index of refraction of the aluminum coating of the mirrors and know the instantaneous geometry of the optical system.

However, the complete test of the analytical model requires us to retrieve all the elements of the Muller matrix. To this purpose the Stokes parameters should be measured also when illuminating the telescope with polarized light. However, the scheduling of the observing time at the solar tower and some technical difficulties in placing a polaroid sheet in front of the coelostat, prevented us from performing these measurements. In fact the mechanical structure needed to hold the polarizing film should follow the Sun independently of the motion of the coelostat, and should provide also the possibility of rotating the polarizer around its optical axis, what implies non-trivial mechanics. These measurements are scheduled for the next observing seasons.

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