ROTATIONAL ENHANCEMENT OF DOPPLER MEASUREMENTS OF SOLAR AND STELLAR HEXADECAPOLE OSCILLATIONS

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Abstract. Rotational enhancement of the sensitivity of whole-disk Doppler observations of solar oscillations may permit the measurement of five-minute modes with \( l = 4 \). We estimate from superposed power spectra of artificial solar data that there might be identifiable power lying above the noise in the data acquired by Pallé et al. (1986), which could provide confirmation of the rotational splitting measured by Duvall and Harvey (1984).

1. Introduction

Low-degree solar oscillations were first detected in whole-disk Doppler measurements, which use light integrated over the entire disk of the solar image (Claverie et al., 1979). This method of observation is of great importance since it is one of the means of observing other stars. Because there is severe cancellation of the signal for all modes except those of lowest degree, one obtains sufficiently few peaks in the power spectrum of the data to permit the degrees \( l \) of the modes to be identified. Initial estimates predicted that only modes with \( l \leq 3 \) are detectable. It is the purpose of this paper to suggest why it might be possible also to detect modes with \( l = 4 \).

On the Sun only the five-minute modes have high amplitudes. These are high-order acoustic oscillations, which satisfy the asymptotic dispersion relation for modes of order \( n \), degree \( l \), and azimuthal order \( m \) (Tassoul, 1980):

\[
v \sim (n + \frac{1}{2}l + \delta) v_0 - \varepsilon(n, l, m),
\]

as \( n/(l + \frac{1}{2}) \to \infty \), where \( \delta \) and \( v_0 \) are constants that depend on the structure of the star, and \( |\varepsilon| \ll v_0 \). Accordingly the power spectrum of the data is essentially a sequence of uniformly spaced groups \( G_x \) of peaks, where \( x = n + l/2 \), separated in frequency by \( \frac{1}{2} v_0 \) and containing power from modes of alternately odd and even degree; this is confirmed by numerically computed eigenfrequencies of theoretical solar models (e.g., Iben and Mahaffy, 1976; Christensen-Dalsgaard, Gough, and Morgan, 1979). According to Christensen-Dalsgaard and Gough (1980a) the modes with odd \( l \) should provide the same power on average as those with even \( l \). The earliest observations (Claverie et al.,


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1979) showed this overall structure, but were of too short a duration to resolve frequency differences of order \( \varepsilon \) and, hence, to separate the modes within a group.

Grec, Fossat, and Pomerantz (1980) went to the South Pole to obtain a longer data set, which was sufficient to resolve part of the contribution to \( \varepsilon \), namely the separation due to different \( l \). If the angular velocity \( \Omega \) in the interior of the Sun were to have been approximately the same as that at the surface, rotational splitting of otherwise degenerate modes with like \( n \) and \( l \) could not have been fully resolved. In order to reduce the effect of noise, and of mode beating between unresolved components of the spectrum, Grec, Fossat, and Pomerantz averaged the peaks from modes with like \( l \). It is evident from the approximately uniform spacing of the groups \( G_x \) that the integrated effect of modes of like \( l \) can be seen by dividing the power spectrum into segments of length \( \Delta v \approx v_0 \) and superposing the spectra. The result was Figure 2 of Grec et al., which is superficially very similar to Figure 4 of this paper.

It was from this superposed frequency spectrum that the degrees \( l \) of the modes could be determined. There are two properties of the spectrum to consider: the frequency spacing and the power \( P_1 \) in the peaks due to modes of degree \( l \). By comparison with an artificial superposed spectrum computed from theoretical eigenfrequencies and instrumental response functions it was demonstrated that both the frequency spacing and the power in the peaks of the superposed spectrum of the real data are in good agreement with theory (Christensen-Dalsgaard, 1980; Christensen-Dalsgaard and Gough, 1980a). There is evidence that the energies of the low-degree modes depend only on frequency \( v \), and not on \( l \) and \( m \) at fixed \( v \) (Christensen-Dalsgaard and Gough, 1982; Libbrecht et al., 1986); this is to be expected if the dominant excitation and damping takes place in the outer layers of the convection zone. In that case the velocity amplitudes in the photosphere would also depend only on \( v \), and consequently the power in the observations would depend on \( l \) only through the instrumental response function from the averaging of the spectral line over the solar image (Christensen-Dalsgaard and Gough, 1980b, 1982). According to this the power \( P_1 \) in dipole modes is the greatest, being about twice that in the monopole and in the quadrupole modes, and about ten times the power \( P_2 \) in the octupole modes. The power \( P_4 \) in the modes with \( l = 4 \) was anticipated to be about \( 2 \times 10^{-3} P_1 \), which should have been too small to have been distinguished from the noise, and the power in any group of modes of higher degree should have been even smaller. Nevertheless, Grec, Fossat, and Pomerantz (1980) found a peak in the superposed power spectrum of their data at just the frequency expected of modes with \( l = 4 \).

2. Response Function for a Rotating Star

Brookes, Isaak, and van der Raay (1978) have pointed out that when the surface of a star is rotating the sensitivity of the modulation of an observed spectrum line to a line-of-sight velocity component on the star varies with position on the stellar surface. In particular, if one were to measure line shift from the intensities in two small wavelength bands placed at the points of steepest slope either side of the centre of the
time-averaged spectrum line, greatest sensitivity to line-of-sight velocity would be achieved along the central meridian. The effect of this is essentially to reduce the effective area over which the Doppler signal is averaged, which tends to enhance the sensitivity of the instrument to some of the higher-degree modes (Christensen-Dalsgaard, 1988). The original theoretical estimates (Christensen-Dalsgaard and Gough, 1980b, 1982) of the power one would expect from modes with \( l = 4 \) were made ignoring rotation. Here we make estimates when rotation is taken into account, and discuss potential observational implications.

It is convenient to introduce a spatial response function \( S_{lm}^{(e)} \), defined such that the instrumental response, in velocity units, to a mode whose vertical velocity component has r.m.s. amplitude \( V_{nlm} \) when averaged over the visible surface of the star is \( V = S_{lm}^{(e)} V_{nlm} \). For high-order \( p \) modes, the contribution from the horizontal component of velocity is small and can be neglected. The response function depends not only on the geometry of the mode, the angular velocity of the star, and the presence of other velocity fields producing effects such as the convective blueshift, but also on the way the instrument samples the spectrum line. In the case of resonance spectroscopy, where the stellar line is sampled at wavelengths located symmetrically either side of the centre of the line produced in the cell, the response depends on differences, caused by the gravitational redshift and the motion of the star relative to the observer, in the

<table>
<thead>
<tr>
<th>( l )</th>
<th>( m )</th>
<th>( S_{lm}^{(e)} ) [NR]</th>
<th>( S_{lm}^{(e)} )</th>
<th>Ratio</th>
<th>( S_{lm}^{(e)} ) [NR]</th>
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Values of the response function \( S_{lm}^{(e)} \) are for the Sun, which is assumed to be rotating rigidly with an angular velocity \( \Omega = 2.86 \times 10^{-6} \text{ s}^{-1} \) about an axis perpendicular to the line-of-sight. The limb-darkening function in the line was taken to be \( 1 - u(1 - \mu) - v(1 - \mu^2) \), where \( \mu \) is the cosine of the angle from the disk centre; \((u, v) = (0.89, -0.23)\) for Na D\(_1\) and \((u, v) = (0.75, -0.22)\) for K D\(_1\). In addition are listed the corresponding values \( S_{lm}^{(e)} \) [NR] ignoring rotation, which are given also by Christensen-Dalsgaard and Gough (1982). (Notice that in Equation (A18) of that paper, \( n \) should be replaced by \( \sqrt{2l + 1} \).)
wavelengths as measured in the frames of the star and the resonance cell. The computation of $S'_{lm}^{(c)}$ in this case, under the assumption that the sole effect on the line is a Doppler shift without change of shape, is discussed by Christensen-Dalsgaard (1988).

Some examples of response functions are listed in Table I. They have been computed for the solar Na D$_1$ line with zero line-of-sight motion of the Sun relative to the Earth, which corresponds to the conditions of the observations by Grec, Fossat, and Pomerantz (1980), and for the K D$_1$ line. For comparison, corresponding response functions computed by Christensen-Dalsgaard and Gough (1982) ignoring the influence of rotation are also listed. In general the response is enhanced by rotation. Among the modes listed, the enhancement is greatest for modes with $l = 4$. In particular, for Na D$_1$, the response to those modes is increased by up to almost a factor 3; the anticipated power, summed over $m$, is increased by a factor 5, bringing it up to a level at which it might be detectable.

3. Simulated Superposed Power Spectra

To estimate what might be detectable in the South Pole data we have simulated observations under ideal conditions. We used computed frequencies of the zonal modes for Model 1 of Christensen-Dalsgaard (1982), and obtained the frequencies of the nonaxisymmetric modes using the rotational splittings reported by Duvall and Harvey (1984). The amplitudes were assumed to vary with frequency in accordance with Equation (D12) of Christensen-Dalsgaard and Gough (1982), which is simply a Gaussian function centred at 3.3 mHz with a standard deviation of 0.3 mHz. No random fluctuations were added. The values of $S'_{lm}^{(c)}$ were those for the Na D$_1$ line with rotation listed in Table I. The axis of rotation of the Sun was assumed to be perpendicular to the line-of-sight. The artificial power spectrum was simply a sum of appropriately scaled $\text{sinc}^2$ functions centred at the frequencies included in the set and corresponding to the continuous observing interval of 6 days; thus beating between unresolved modes was ignored, and it was assumed that damping and excitation could be neglected so that the modes have constant amplitudes and are coherent over the observing interval. Therefore, our simulation represents the expectation of the South Pole data, in the absence of noise and mode beating and with perfect phase coherence. Subsequently the spectrum was divided into segments, in the various ways described below, and the segments were superposed.

The frequencies of a sequence of modes of like $l$ and $m$ are not strictly in a linear relation with $n$. This is true of both observed and theoretical frequencies, and is evident from the existence of the small term $\epsilon$ in Equation (1), whose dominant behaviour is proportional to $(n + \frac{1}{2}l + \delta)^{-1}$. Consequently, chopping the spectrum into equal segments is not the optimal procedure. Grec, Fossat, and Pomerantz (1983) recognized this, and used instead variable frequency segments. We have adopted a similar procedure, taking segments of magnitude

$$\Delta \nu = \beta \nu + 2\gamma(x - x_0),$$

(2)
where \( x = n + \frac{1}{2} l \), \( x_0 \) corresponds to the central group in the sample of modes selected, and \( \beta_l \) and \( \gamma_l \) are independent of \( x \); it results from approximating Equation (1) as a quadratic function of \( x - x_0 \), namely

\[
v \simeq \alpha_l + \beta_l (x - x_0) + \gamma_l (x - x_0)^2. \tag{3}\]

As indicated, the parameters \( \beta_l \) and \( \gamma_l \) depend on \( l \); indeed, according to Tassoul's (1980) asymptotic formula, \( \alpha_l, \beta_l, \) and \( \gamma_l \) should be linear functions of \( l(l + 1) \) (cf. Scherrer et al., 1983), and we have confirmed that this is a property both of the solar data and of numerically computed eigenfrequencies of several stellar models. Consequently, the dissections of the frequency interval optimally suited for superposing the spectra of modes of different degrees should be different. We have accordingly determined \( \alpha_l, \beta_l, \) and \( \gamma_l \) separately for each value of \( l \), by fitting the formula (3) by least squares to the frequencies over the same frequency range as that used to construct the superposed spectrum. All frequencies were weighted equally, rather than being weighted according to power, so the resulting \( dV/l \) is not necessarily that which gives the minimum width to the chosen peak in the superposed power spectrum. The results are listed in Table II. For comparison, the table also includes the corresponding coefficients determined from the compilation of observed frequencies in Duvall et al. (1988).

### Table II

<table>
<thead>
<tr>
<th>Model 1</th>
<th>Duvall et al.</th>
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<td>( l )</td>
<td>( \beta_l )</td>
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<tr>
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<td>137.61</td>
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</table>

Expansion parameters \( \beta_l \) and \( \gamma_l \) (cf. Equation (3)) obtained from computed adiabatic frequencies of Model 1 of Christensen-Dalsgaard (1982), and from the observed frequencies of Duvall et al. (1988), including all modes between 2500 and 4100 \( \mu \)Hz. The reference value \( x_0 \) of \( x = n + l/2 \) was 23. The values in the row marked ‘a’ were obtained from the values for \( l = 0 - 3 \), by linear regression in \( l(l + 1) \).

An example of such a synthetic spectrum is illustrated in Figure 1, with parameters chosen to optimize the coincidence of the radial modes. In this, as in all other illustrations in this paper, the frequency range is restricted to 2.5–4.1 mHz, outside of which the spectrum of the South Pole data appears to be quite noisy. Notice the small contribution from the modes having \( l = 4 \). The appropriate portion of the spectrum is
Fig. 1. Superposed artificial power spectrum of solar oscillations, based on frequencies of Model 1 of Christensen-Dalsgaard (1982) and the response function with rotation in Table I for the Na D₁ line. The frequencies of the nonaxisymmetric modes were computed as \( \nu_{nlm} = \nu_{nl0} + m \Delta \nu_\text{rot} \), with rotational splittings \( \Delta \nu_\text{rot} \) of 0.66, 0.50, 0.46, and 0.44 \( \mu \text{Hz} \) for \( l = 1, 2, 3, \) and 4, respectively (Duvall and Harvey, 1984). A 6-day time series was assumed. The abscissa is the reduced frequency \( \tilde{\nu} \), determined by the dissection of the spectrum, which used the values of \( \beta_0 \) and \( \gamma_0 \) for \( l = 0 \) (see Table II). Before superposition, the spectrum was normalized to have unit maximum power. The degrees of the modes contributing to each peak are given in the figure.

shown on an expanded scale as a dashed line in Figure 2. There is a slight indication of two peaks in the power, due mainly to the contributions from the rotationally split sectoral modes. Amongst modes of like \( l \), it is the sectoral modes which have the greatest response function \( S_{nlm}^{(c)} \) (see Table I), and when \( l = 4 \) the frequency splitting between these modes is \( 8 \times 0.44 \mu \text{Hz} = 3.5 \mu \text{Hz} \), which in principle is great enough to be detected with the 1.9 \( \mu \text{Hz} \) temporal resolving power of the South Pole data sets.
Fig. 2. The dashed curve is an enlargement of the central part of Figure 1. The continuous curve was obtained from a similarly superposed spectrum, but using values of $\beta_I$ and $\gamma_I$ corresponding to $l = 4$. The origin of the dissection for the continuous curve was chosen to produce the best simultaneous alignment of the four peaks corresponding to $l = 0, 1, 2, 3$ with those of the dashed spectrum.

One might expect that the $l = 4$ modes are not shown to greatest advantage in Figure 1, because $\beta_I$ and $\gamma_I$ were chosen to optimize the superposition of the radial modes. When values of $\beta_I$ and $\gamma_I$ corresponding to another value of $l$ are chosen, there is some tendency for the power in the corresponding identifiable peak of the superposed power spectrum to be increased. If parameters suitable for $l = 4$ are selected, for example, the $l = 4$ modes should be optimally enhanced. In Figure 2 the resulting spectrum is shown as a continuous line; to simulate analysis of whole-disk observations, where a priori only identification of modes with $l = 0–3$ would be expected, the parameters for $l = 4$ were determined by extrapolation in $l(l + 1)$ using a linear regres-
sion with the values for \( l = 0 - 3 \). There is a clear increase in the height and concentration of the superposed peak, and the rotational splitting is now evident. Greater enhancement might be possible if \( \beta_r \) and \( \gamma_r \) were obtained by minimizing power-weighted frequency departures from the fit; however, in the case of the more noisy data that one would expect from distant stars, the resulting effective reduction in the frequency range makes it more difficult to obtain reliable estimates of \( \beta_r \) and \( \gamma_r \).

In no case was it possible to resolve rotational splitting in modes with \( l < 4 \). This is as one would expect, given the fact that the frequency splitting of the sectoral modes is close to the limit of resolution. The imperfect alignment of power resulting from inaccuracy of the fitting formula (3) broadens the superposed peaks; for \( l > 1 \), additional power is provided by the modes with \( m = l \), which have a higher response, relative to the sectoral modes, for \( l < 4 \) than they do for \( l = 4 \). Even for \( l = 3 \) this property ensures that in the absence of mode beating only a single peak is produced. It is interesting to record, however, that there are slight indications of splitting in the \( l = 0 \) and \( l = 1 \) peaks in the superposed spectrum designed to align modes with \( l = 4 \). This arises from poorly aligned peaks, and is quite unrelated to rotation.

We can estimate the effect of the \( l \)-dependent dissection of the spectrum. Neglecting quadratic and higher-order terms in Equation (3), the total spread of the peaks of degree \( l \), in a spectrum superposed according to the parameters corresponding to degree \( l' \), is of order

\[
(v_2 - v_1) \left| \frac{\beta_l - \beta_{l'}}{\beta_l} \right|,
\]

where \([ v_1, v_2 ]\) is the range of frequencies included in the superposition. For the values relevant for Figure 2 this predicts a spread of order 10 \( \mu \)Hz for the superposed hexadecapolar modes in the spectrum optimized for the radial modes, in agreement with what is actually obtained.

Unless the spread in Equation (4) is substantially larger than the width of the individual peaks in the spectrum caused by the finite duration of the observations or the finite life-time of the modes, little gain can be expected by choosing values of \( \beta_l \) and \( \gamma_r \) appropriate to a certain degree. For the 6-day time string assumed in Figures 1 and 2, this condition is only marginally satisfied, except perhaps for \( l = 4 \). If the duration of the time string is increased to 12 days, the effect is considerably enhanced, as illustrated in Figure 3, which is analogous to Figure 2. Now the rotational splitting of the \( l = 4 \) sectoral modes is almost completely resolved, when the superposition is optimized for these modes; the spread in the superposition corresponding to \( l = 0 \) is similar to that obtained for the shorter time string. In this case there are also indications that the rotational splitting of the \( l = 2 \) and \( l = 3 \) modes can be resolved, in the appropriate superpositions. For a 24-day time string this effect is further enhanced, and even the dipolar modes show rotational splitting. It should be pointed out, however, that such a high resolution is unrealistic over the frequency range considered here, due to the finite lifetime of the modes. At 3.3 mHz the observed full width at half maximum
is about 2 $\mu$Hz (Libbrecht, 1988; Christensen-Dalsgaard, Gough and Libbrecht, 1988), which is similar to the width of the sinc$^2$ function corresponding to a 10-day time series. Thus it is unlikely that resolution higher than in the 12-day case can be achieved for solar modes in the vicinity of 3.3 mHz.

4. Superposed Power Spectra of the South Pole Data

The same procedure for superposing peaks in the synthetic spectrum was also carried out on the spectrum of the South Pole data. The superposition used the parameters $\beta_i$ and $\gamma_i$ obtained from the analysis of the observed frequencies in Duvall et al. (1988). The results are illustrated in Figures 4 and 5, which correspond to the synthetic Figures 1
Fig. 4. Superposed spectrum, based on the observations of Grec et al. (1980). The dissection used $\beta_i = 135.5 \, \mu\text{Hz}$ and $\gamma_i = 0.117 \, \mu\text{Hz}$, computed to optimize the alignment of the $l = 0$ modes. Unlike Figure 2 of Grec, Fossat, and Pomerantz (1980), for which the dissection was uniform, and was obtained as a compromise for all significant peaks, no peak is visible where the $l = 4$ modes are expected.

and 2. The striking resemblance between Figures 1 and 4 leaves no room to doubt the identification of the values of $l$. It is evident that the superposed spectrum designed for aligning modes with $l = 0$ shows only minimal evidence of modes with $l = 4$.

The superposed spectrum in the vicinity of the $l = 4$ modes with $\beta_i$ and $\gamma_j$ chosen to optimize $l = 4$ alignment is plotted as a continuous line in Figure 5. The power is not significantly different from that for the $l = 0$ alignment, though there is, perhaps, some slight evidence for a qualitative change in the character of the spectrum; the more jagged appearance of the continuous line provides a hint of the existence of poorly resolved discrete frequencies within the noise. To compare with the properties of the solar
hexadecapole modes, as determined from spatially resolved observations, we constructed a synthetic spectrum analogous to that illustrated in Figures 1 and 2, but based on the frequencies in Duvall et al. The result is similar to Figure 2, but with a less pronounced effect of selecting the appropriate $\beta_l$ and $\gamma_l$. Figure 6 shows the superposed spectrum optimizing the $l = 4$ alignment; for comparison the corresponding spectrum based on the South Pole data, after the subtraction of an estimated constant background power of 0.15, is also shown. It is striking, though hardly significant, that the two peaks in the South Pole spectrum in the middle of the figure are positioned at just the location of the modes with $l = 4$ and $m = \pm 4$, as obtained from the synthetic spectrum.
5. Discussion

We have argued that rotational enhancement of the response function for whole-disk solar and stellar Doppler observations raises hope for detecting $p$ modes with $l = 4$. This increases the range of penetration depths of potentially detectable modes, and so adds substantially to the diagnostic power of the data to measure conditions in stellar cores. Even for the Sun the case for analyzing whole-disk data is strong, for although hexadecapole modes are in principle more readily detected by observing with limited spatial resolution, of the data that are in hand at present those with greatest temporal resolution are from whole-disk observations (Isaak, 1986; Pallé et al., 1986). It is interesting to note from Table I that rotational enhancement of the $l = 4$ response functions is greater for the K line used by the Birmingham group than it is for the broader Na line used by Grec, Fossat, and Pomerantz (1980, 1983).

If the signal-to-noise ratio is too low to obtain reliable observations of isolated modes, useful information can sometimes be obtained from a superposed power-spectrum analysis. This has been well demonstrated by Grec, Fossat, and Pomerantz (1983), who employed a nonuniform dissection to optimize the entire superposed spectrum. A similar procedure has been used here on real and artificial spectra, using segments of length $\Delta \nu$ determined by Equation (2). The parameters $\beta_l$ and $\gamma_l$ determining $\Delta \nu_l$ depend on $l$, as approximately linear functions of $l(l + 1)$. Hence, separate dissections should be used for each value of $l$, extrapolating $\beta_l$ and $\gamma_l$ linearly in $l(l + 1)$ to infer the dissection for $l = 4$; when applied to an artificial solar spectrum, based on computed frequencies, and with a frequency resolution corresponding to the 6-day time string of Grec, Fossat, and Pomerantz, this results in some improvement of the concentration of the $l = 4$ peak. The effect is substantially more pronounced for longer time series where the frequency resolution is higher. Thus application of this type of analysis to the Birmingham data would be very interesting.

Despite the relatively low resolution, we have applied the superposed spectrum analysis, with the nonuniform dissection (2), to the South Pole data. The results, illustrated in Figures 4 and 5, show no significant evidence for modes with $l = 4$. This may be because the background noise is too high. Nevertheless we have compared the observed spectrum with a synthetic spectrum based on observed frequencies of solar oscillations. Though the result, shown in Figure 6, is hardly significant, it is interesting that there are peaks in the spectrum of the South Pole data corresponding to those of the synthetic spectrum. We conjecture that had greater temporal resolution been achieved, the modes would have been visible.

Of course for most stellar observations spatially unresolved data are all that can foreseeably be acquired. For rapidly rotating stars, Doppler imaging, which uses the Doppler shift due to the line-of-sight component of the rotational velocity to map stellar longitude onto wavelength shift (e.g., Osaki, 1971; Vogt and Penrod, 1983), permits the detection of modes of somewhat higher degree than those discussed in this paper. But when rotation is too slow for this to be possible, rotational enhancement of $S_{lm}^{(e)}$ is extremely important. The observations should resemble those of the Sun, though the
rotational enhancement of the $l = 4$ modes would depend quite sensitively on the rotation rate and the inclination of the rotation axis to the line-of-sight. The relative heights of the peaks of resolved multiplets would also depend on the inclination, and indeed could be used to determine it.

The extent of the improvement to the superposed power spectrum that can be achieved by employing nonuniform dissections depends on the structure of the core of the star, which determines the quantity $\varepsilon$ in Equation (1), on the frequency interval over which the modes are excited to substantial amplitudes, and on the temporal resolution of the observations. Nevertheless, since the effort required to carry out the procedure
we have recommended is insignificant against that needed to acquire the data, it seems worthwhile always to gain that improvement.

Acknowledgements

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References