SIMULATION OF LARGE-SCALE FLOWS AT THE SOLAR SURFACE

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ABSTRACT

We take a simple analytic axisymmetric function to represent the radial outflow associated with an isolated convection plume at the solar surface. The vertical velocity can be deduced from the continuity equation. A regular cellular pattern of convection can be created by superposing a number of such sources. Our motivation for this study came from the large-scale horizontal motions observed by the SOUP instrument on Space- lab 2. We have simulated the flow pattern visible in three different regions covered by the SOUP observations. In each case a superposition of our plume functions mimics the observed mesogranular and supergranular motions remarkably well. The model can be improved by using the same procedure to represent sinks as well as sources in the observed velocity field. We use these model flows to compute the motion of passive test particles (corks) which accumulate in a network that outlines mesogranular cells. The corks appear to resemble the magnetic network observed at the time of the SOUP measurements. Detailed comparisons suggest that magnetic flux tubes are affected more by outflow from sources at the centers of mesogranules than by flow into sinks within the network.

Subject headings: convection — Sun: atmospheric motions — Sun: granulation

I. INTRODUCTION

The pattern of motion in the convective zone of a late-type star affects the magnetic field at its surface. If the field is sufficiently weak, magnetic flux will be swept away from upwellings and into regions where the horizontal flow converges. Cellular motion has been detected on the Sun with scales ranging from the photospheric granulation, with a typical diameter of 1 Mm, to supergranules with diameters of about 30 Mm. Doppler measurements of the horizontal velocity in supergranules show a cellular pattern with the flow diverging from cell centers and converging at their boundaries, which coincide with the magnetic network observed in Ca II emission (Leighton, Noyes, and Simon 1962; Simon and Leighton 1964). Simon (1967) showed that the motion of individual granules tended statistically to follow the supergranular flow pattern. More recently, such proper motions of granules, derived from the excellent seeing-free white-light images obtained by the Solar Optical Universal Polarimeter (SOUP) on SpaceLab 2, have been used to determine in detail the complex pattern of large-scale horizontal motion on the solar surface (Title et al. 1986; November et al. 1987; Simon et al. 1988; Title et al. 1989). These new results not only show supergranular flow, but also confirm the existence of smaller scale mesogranules, with diameters around 8 Mm, which had previously been detected by ground-based Doppler measurements (November et al. 1981; November 1989; Brandt 1989; Deubner 1989). Moreover, the evolution of hypothetical magnetic fields, as represented by the horizontal transport of passive test particles (corks) by the large-scale flow, follows the behavior of the actual fields measured in magnetograms obtained at Big Bear Solar Observatory (Title et al. 1987; Simon et al. 1988).

These new results have encouraged us to develop a simplified kinematic model of convection at the surface of the Sun that can be compared with observations and used to predict the evolution of magnetic fields. This model is introduced in § II. Supergranules and mesogranules show broad upwellings at the centers of the cells with narrow sinking regions at their boundaries. We therefore assume that the motion can be represented by axisymmetric plumes centered on the upwellings and that the cellular network is produced by the interactions between neighboring plumes. We adopt a simple functional form for the horizontal velocity in an isolated plume: the flow is purely radial and depends on just two parameters, which determine its magnitude and horizontal scale. The vertical component of the motion is proportional to the divergence of the horizontal velocity. Then we show that a grid of identical plumes placed on a regular square or hexagonal lattice generates a tessellated pattern of convection in the plane.

In § III we attempt to represent the observed motions derived from the SOUP data. We site our axisymmetric plumes at the centers of supergranules and mesogranules, where the divergence of the horizontal velocity attains a local maximum. The strength and radius of each plume are next obtained by numerically fitting each observed upwelling. Then the total horizontal velocity can be computed and compared with the observations. In addition, we follow the motion of passive test particles and compare the patterns formed by corks moving with our model velocity with those formed by corks that follow the actual measured flow. In § IV the same procedure is applied to sinks as well as sources and the two models are contrasted. Finally, in § V we compare the cork patterns with the observed magnetic network and discuss the...
extent to which magnetic flux tubes can be regarded as moving passively with the large-scale photospheric flow.

It must be emphasized that this treatment is confined to two-dimensional surface motion and is purely kinematic. Nordlund (1985a, b) has simulated photospheric granulation numerically, and the inferred behavior of magnetic fields in our model is consistent with his results. Three-dimensional kinematic behavior has been studied in idealized hexagonal and square configurations (Galloway and Proctor 1983; Schmidt, Simon, and Weiss 1985), while the dynamics of magnetoconvection has been extensively investigated in the Boussinesq approximation (Proctor and Weiss 1982). More recently the systematic exploration of two-dimensional compressible convection and magnetoconvection has begun (Hurlbut, Toomre, and Massaguer 1984; Hurlbut et al. 1989; Hurlbut and Toomre 1988; Hughes and Proctor 1988; Steffen and Muchmore 1988; Steffen, Ludwig, and Krüss 1989). As more powerful computers become available, fully dynamical simulations will provide increasingly realistic descriptions of solar behavior. Meanwhile, we can still improve our understanding of photospheric magnetic fields by developing simpler models.

II. THE MODEL FLOW

We first consider a single isolated plume, referred to cylindrical solar coordinates $(r, \theta, z)$, where the $z$-axis points upward. We assume that there is no swirl and that the velocity $u = (v, 0, w)$ is axisymmetric. Let the surface velocity at $z = 0$ be defined by setting $v = f(r)$, where $f(0) = 0$, $f(r) \geq 0$, and $f(r) \to 0$ as $r \to \infty$, and assume that $f(r)$ has a single maximum. In the annelastic approximation $\nabla \cdot (\rho u) = 0$, where $\rho$ is the horizontally stratified density, so that

$$w \frac{d(\ln \rho)}{dz} + \frac{\partial w}{\partial z} = -\frac{1}{r} \frac{\partial (\rho u)}{\partial r}.$$  

(1)

Following November et al. (1987), we set

$$w = \frac{c}{r} \frac{d}{dr} [f(r)],$$

(2)

where

$$\frac{1}{c} = \frac{1}{H_H} + \frac{1}{H_w}$$

and $H_H, H_w$ are the density and velocity scale heights, assumed uniform on the surface $z = 0$.

We have tried several functional forms for $f(r)$, including both polynomials and exponentials. It is convenient to choose a localized velocity, so that $v$ falls off rapidly with $r$, and we therefore take

$$f(r) = \left( \frac{Vr}{R} \right) e^{-\left( \frac{r}{R} \right)^n},$$

(3)

where $V$ and $R$ are parameters which determine the amplitude and radius of the plume and $n$ is a positive integer. For the rest of this section we set $V = R = 1$. Then

$$v(r) = re^{-r}, \quad w(r) = (2 - nr^2)e^{-r}.$$  

(4)

Thus $w(0) = 2c$, $w = 0$ when $r^* = 2/n$, and $w \to 0$ as $r \to \infty$. (Note that $w$ cannot be evaluated independently of $c$; we assume that $c > 0$.) The maximum radial velocity occurs when $r^* = 1/n$, and the downward component of the velocity is largest when $r^* = (2/n) + 1$. In Figures 1a and 1b we show the variation with radius of $v$ and $w/c$ for the cases $n = 1$ to $n = 5$. When $n = 1$, the radial velocity falls off too slowly and the peak value of the downward velocity is only $5\%$ of the upward velocity at the origin; moreover, $dw/dr$ is finite at $r = 0$. This value of $n$ is clearly unsuitable for modeling photospheric convection. As $n$ increases, the profile of the vertical velocity becomes flatter at $r = 0$, while the downward velocity increases and is more sharply localized. In this section we shall confine our attention to the two cases $n = 2$ and $n = 4$. The maximum radial velocity increases from $0.429$ at $n = 2$ to $0.551$ at $n = 4$, and occurs at $r = 0.707$ in each case; the minimum value of $w/c$ decreases from $-0.272$ at $r_2 = 1.414$ to $-0.893$ at $r_4 = 1.107$.

Next we demonstrate that a tessellated (or cellular) pattern can be generated by appropriately situated plumes. Consider first the total velocity field $(u_x, u_y, u_z)$, referred to Cartesian coordinates, produced by adding the contributions from identical plumes arranged on a square lattice with spacing $d$. If $d$ is very large, the plumes are effectively isolated; if $d$ is too small the maxima of $u_x$ are over lain by minima of adjacent plumes. For intermediate values of $d$ each plume is surrounded by a closed curve on which $u_x = 0$, and lies at the center of a square cell with $u_x < 0$ everywhere along its boundary. Thus the plane is tessellated with square cells, and there is a topological distinction between the isolated upwellings and the network of sinking fluid that encloses them.

We have computed the horizontal and vertical velocities generated by sources on a square lattice with spacing $d = 2$ and investigated the effects of varying $d$ by factors of approx-

Fig. 1.—Normalized velocities for an isolated plume. (a) Radial velocity $v$ and (b) vertical velocity $w$, shown as functions of $r$ for $n = 1, 2, 3, 4, 5$. 

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imately $\sqrt{2}$. In Figure 2 we represent the horizontal velocity ($u_x$, $u_y$, 0) by arrows in the upper half of each panel, and the vertical velocity $u_z$ by contours in the lower half. Values of $u_z/c$ at extrema are listed in Table 1. We illustrate in Figures 2a and 2b the velocity field produced with spacing $d = 2.8$ for $n = 2$ and $n = 4$. Each plume can easily be identified, and the topological distinction between rising and sinking fluid is apparent. The positions of maxima and minima of $u_z$ are indicated in each case. In Figure 2a, with $n = 2$, the upward velocity is greatest at the center of the plume, while the downward velocity is greatest at two points on the walls of the square cells. At the vertices $u_z$ is negative, attains a local maximum, and there is a saddle at the midpoints of each wall. When $n = 4$ the situation becomes more complicated, since the minimum value of $w$ for a single plume occurs at $r \approx 1.107 < 1.1d$ (cf. Fig. 1b). In Figure 2b there are again saddles at the midpoints of the sides, but the peak value of the downward velocity is found within the cells on either side of the saddles. At the vertices $u_z$ has a local maximum approaching zero. For closely packed cells the geometry of the pattern becomes more dominant, owing to interference between adjacent plumes. In Figure 2c, with $d = 1.4$ and $n = 2$, the upward and downward velocities combine to give plumes that are virtually equivalent, so that the topological distinction between rising and falling material can scarcely be discerned. In Figure 2d, with $n = 4$, the basic plume structure is disguised by the superposition of many nearby source functions, which produce strong isolated sinking plumes at the corners, surrounded by a network of rising fluid. When $n = 4$ the magnitude of the peak downward velocity in an individual plume is 45% of the upward velocity at its center, from Figure 1b. Hence the value of $u_z$ at the center of a source is drastically reduced by negative contributions from its nearest neighbors. Thus the upward velocity reaches its peak value not at the centers of the rising plumes but midway between adjacent sources.

Alternatively, we may site the plumes on a hexagonal lattice with spacing $d$. With $d = 2$ there are hexagonal cells with isolated upwellings surrounded by a network of sinking fluid. For $n = 2$ the upward velocity at the cell center is twice as large as the downward velocity at its six vertices (cf. Table 1), but for $n = 4$ the peak upward velocity is only 77% of the downward velocity at the vertices. Figures 2e–2h show the velocity patterns for $d = 2.8$ and $d = 1.4$ with $n = 2, 4$. With wider spacing the topological distinction between rising and sinking fluid is preserved. For $n = 2$, in Figure 2e, the peak downward velocity is at the vertices of the cells, but for $n = 4$, in Figure 2f, the global minima of $u_z$ are once again within the walls, on either side of midpoint saddles. The hexagonal pattern becomes more striking in Figures 2g and 2h, where the plumes are closer together. For $n = 2$ the cells are similar to those with larger values of $d$, though the magnitudes of the velocities are drastically reduced owing to cancellation from nearby sources. The interaction between close-packed plumes with $n = 4$ is particularly interesting, for the pattern is inverted, in the sense that the maximum upward velocity is found not at the plume centers but at the vertices of the hexagons, where $u_z$ is almost twice as big. Cell boundaries are usually loci of downdraft, and there are updrafts not only at the cell center but also at the boundary, with downflows in intermediate regions.

The results in Figure 2 confirm that a tessellated pattern of horizontal motion can be represented by axisymmetric sources distributed over a plane. In addition, these examples show that simple models must be used with caution: the flow pattern is governed not only by the strengths of the plumes but also by their sizes and relative proximities to each other.

### Table 1

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<th>$n = 2$</th>
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<tr>
<td>1.4</td>
<td>(c) $0.42555$</td>
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<td>(e) $1.96769$</td>
</tr>
<tr>
<td>1.4</td>
<td>(g) $0.18100$</td>
</tr>
</tbody>
</table>

Note.—Letters in parentheses refer to panels in Fig. 2, where plume centers (asterisks), peak downward velocities (crosses), and local maxima of $u_z/c$ (plus signs) are indicated.

**Fig. 2.**—Velocity for plumes on a regular lattice, illustrating differences due to geometry, spacing $d$, and degree $n$. The horizontal velocity is indicated by arrows in the upper half of each figure, with contours of the vertical velocity $u_z$ in the lower half. Plume centers are indicated by asterisks and the peak downward velocities by crosses. Local maxima of $w$ are indicated by plus signs. Solid contours indicate positive values, dashed contours negative values, and dotted contours zero. Contour spacing is indicated above the figures. Upper panels (a–d): square lattice; lower panels (e–h) hexagonal lattice. Details are given in Table 1.
dimensions $131'' \times 119''$, is illustrated in Figure 1 of Simon et al. (1988).

From this data set it is possible to compute the proper motions of individual granules using local cross-correlation techniques (November 1986; November et al. 1987; November and Simon 1988). The high quality of the distortion-free SOUP images allows flow speeds of 100–1000 m s$^{-1}$ to be measured with an accuracy of about 20 m s$^{-1}$. The velocity $u_H$ deduced from the proper motions of granules is purely horizontal. Given $u_H = (u_x, u_y, 0)$, it is possible to calculate the divergence of the horizontal velocity,

$$\Delta = \mathbf{V} \cdot \mathbf{u}_H = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y},$$

which is proportional to the vertical velocity $u_z$ (November et al. 1987). Figure 3 shows contours of $\Delta$ over the region of interest. The topology is not immediately obvious. There are many localized regions of strong positive and negative divergence (corresponding to rising and falling motion, respectively) but there seems to be a preference for isolated upwellings surrounded by downward flowing material.

In order to describe this flow field, we identify the most prominent sources (regions where $\Delta > 0$) and fit axisymmetric plumes at their centers, with appropriately chosen values for the strength ($V$) and radius ($R$) of each plume. This can be done by eye, but it is more satisfactory to fit numerically to each observed source an axisymmetric plume with the form given by equation (3). Let $\Delta_{\text{max}}$ be the local maximum of $\Delta$ in a source, and let $R_0$ be the mean radius at which $\Delta = 0$; then

$$R = \left(\frac{n}{2}\right)^{1/4} R_0, \quad V = \frac{1}{2} R \Delta_{\text{max}},$$

from equation (3). The calculations in §1 suggest that $n$ should not be too large, and we shall consider the cases $n = 1, 2, 3$ with $R/R_0 = 0.5, 1.0, 1.145$, respectively.

We have applied the model to a number of isolated sources in order to ascertain the most appropriate choice for $n$ and to see how well the model fits the observations. Figure 4 illustrates the procedure used to fit an isolated source with coordinates (37°4, 40°6) in Figure 3. The observed velocity $u_H$ is interpolated onto a grid of points around the source, and these velocities are used to calculate $\Delta$ on the grid. Figure 4a is an enlarged contour map of $\Delta$ covering 40° × 40°, with the source indicated by a plus sign. At the origin $\Delta$ attains a maximum value $\Delta_{\text{max}} = 0.027$ minute$^{-1}$. In Figure 4b values of the normalized divergence $\Delta/\Delta_{\text{max}}$ are plotted against the distance $r$ from the origin, measured in arcseconds. The values of $\Delta$ are binned and averaged in order to obtain the mean divergence at spacings of 0°4, indicated by open circles. $R_0$ is then obtained by quadratic interpolation, and the normalized divergence

$$\Delta_n(r) = \frac{\Delta}{\Delta_{\text{max}}} = \left[1 - \left(\frac{r}{R_0}\right)^n\right] \exp\left[-\frac{2}{n} \left(\frac{r}{R_0}\right)^n\right]$$

is computed and plotted as a function of $r$ for $n = 1, 2, 3$. For the source in Figure 4 $R_0 = 4721$.  

![Fig. 3.—Contours of the divergence $\Delta$ derived from the SOUP data over the $131'' \times 119''$ domain. Solid contours indicate positive $\Delta$, dashed contours negative $\Delta$, and dotted contours the zero level. The contours are spaced at intervals of 0.0065 minute$^{-1}$.](image)

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The same procedure can be followed for the radial component $v(r)$ of the observed horizontal velocity relative to the source origin. Since $u_\mu$ does not vanish at the origin, it is convenient to subtract the mean drift velocity $\bar{u}_\mu$ averaged over a $10'' \times 10''$ region centered on the source. The arrows in Figure 4c show the corrected velocity $\bar{u} = u_\mu - \bar{u}_\mu$ together with contours of the divergence $\Delta$ in this small region. The radial component $\bar{v}$ of this corrected velocity can then be calculated at each grid point, and values of the normalized radial velocity $2\bar{v}/\Delta_{\text{max}}$ are plotted against the radial distance $r$ in Figure 4d. (Note that $\Delta$ is not affected by subtracting a constant velocity from $u_\mu$, and that the expected results of averaging $\bar{v}$ and $v$ are identical.) Once again the mean values obtained at spacings of $0.4''$ are indicated by open circles. These mean values can then be compared with the normalized velocities

$$v_n(r) = r \exp \left[ -\frac{2}{n} \left( \frac{r}{R_0} \right)^n \right],$$

which are plotted in Figure 4d for $n = 1, 2, 3$.

Inspection of Figures 4b and 4d shows that the best fit to the averaged values of $\Delta(r)$ and $\bar{v}(r)$ is obtained with $n = 2$. For $r \leq R_0$ the fit is remarkably precise: the maximum in $\bar{v}$ differs by less than 3% from its predicted value, and its position differs by less than 2% from that given by equation (8). This agreement is to be expected as a consequence of the broad Gaussian windows used in the cross-correlation method by which the granule velocities were calculated. We have, however, established that the model source with $n = 2$ still fits the data when the FWHM is reduced from 5'' to 2''. For $r > R_0$ there is considerable scatter in the individual values of $\Delta$ and $\bar{v}$. This scatter, which increases with increasing $r$, is related to the presence of isolated downdrafts about 5'' from the source and to other nearby sources at distances of 8''-15''. From our model the value of $\Delta$ for a single plume reaches a minimum with $\Delta_1 = -0.135$ at $r/R_0 = 1.41$, while the observations yield a minimum with $\Delta/\Delta_{\text{max}} = -0.150$ at $r/R_0 = 1.38$, followed by a rise to a subsidiary maximum with $\Delta/\Delta_{\text{max}} = 0.109$ at $r/R_0 = 2.29$. The differences between our model and the real data can...
be ascribed to the effects of nearby sources which should be included in the model so as to provide a more accurate description of the flow in the region surrounding the source. Beyond the subsidiary maximum individual values of $\Delta$ fluctuate wildly as $r$ is increased, yielding positive and negative values greater in magnitude than $\Delta_{\text{max}}$, so that the average value of $\Delta$ has little significance for $r \geq 2R_{\odot}$. Similarly, the average value of $\bar{v}$ cannot be compared with the model when $r \geq 2R_{\odot}$, owing to the effect of radial outflows from neighboring sources.

We have applied the above procedure to a number of sources, and in each case the expressions in equations (7) and (8), with $n = 2$, provide an excellent fit to the observed mean values of the normalized divergence and radial velocity for $r/R_{\odot} \leq 1.5$. In what follows we shall therefore restrict our attention to the model flow with $n = 2$ and use it to represent an assembly of irregularly distributed sources.

We now consider in detail three separate regions within the field shown in Figure 3. These regions are magnetically quiet and were chosen to avoid the sunspot and the pores that are visible in Figure 1 of Simon et al. (1988). In each region we shall fit plumes at the centers of all sources with $\Delta_{\text{max}} \geq \Delta_{\odot}$, for some appropriately chosen value of $\Delta_{\odot}$, and compute the resulting flow $u_{\parallel}$, which can then be compared with the velocity $u_{\parallel}$ obtained from the measured granule motions. We assume that these velocities vary sufficiently slowly with time for them to be regarded as steady over the intervals with which we shall be concerned. The time-averaged velocities therefore include any effects of short-term variations such as exploding granules (Title et al. 1989) during the 28 minute observing period. In order to discover the effect of the horizontal flow on magnetic fields, we follow the motions of passive particles (or "corks") traveling with the velocity $u_{\parallel}$ or $u_{\parallel}$. Suppose that such corks are deposited at the mesh points of a uniform grid at time $t = 0$. Then they will be transported to cell boundaries and gradually move along those boundaries to accumulate in sinks where $\Delta$ is a local minimum. Thus the corks outline a network that can be compared with the magnetic network observed in the regions being studied.

The first region has dimensions $35^\circ \times 35^\circ$, and is located near the bottom right-hand corner of Figure 3. We shall refer to it as region A. Figure 5a shows the observed velocities $u_{\parallel}$, represented by arrows on a square mesh, and contours of the divergence $\Delta$ derived from them. The arrows reveal a prominent supergranule at the right-hand edge of the region together with several mesogranules to the left.

The remaining panels of Figure 5 indicate how an initially uniform weak magnetic field would be transported by this flow. We depict the motion of 1225 corks calculated at 1 minute intervals using velocities obtained by linear interpolation from values calculated on a square mesh with 1" spacing. Those corks that leave the region are ignored, so the total number gradually declines and only 1061 corks remain when $t = 16$ hr. Figures 5b–5f show the positions of the corks after 1, 2, 4, 8, and 16 hr have elapsed. A perceptible change is already apparent after 1 hr. After 2 hr the corks have moved away from the centers of mesogranules, and by 4 hr they are assembled in the lanes between the granules, and strong concentrations have formed where there are local sinks. Thereafter, the corks are gradually confined to a network so that the pattern is predominantly linear after 16 hr have elapsed. As time proceeds further, the network grows narrower and particles migrate along it until, after 48 hr (not shown), all the corks have accumulated over the centers of six major sinks.

Here we should mention that the absolute magnitude of the velocities we have used depends on the window size adopted for the local correlation tracking procedure. We have taken a Gaussian window with 5" FWHM (November and Simon 1988) which probably underestimates the velocities by a factor of 5–10 (cf. Title et al. 1989, Fig. 14). Hence the actual transport times for corks are probably 5–10 times less than those given in Figure 5. Such a compressed time scale would be consistent with results obtained by Brandt et al. (1988) using a window size less than 1".

In Figure 6 we show corresponding results obtained by fitting plumes to all sources with $\Delta \geq \Delta_{\odot} = 0.009$ minute$^{-1}$. (In addition to the nine sources within region A, we have included seven others immediately outside it, which can be located by reference to Fig. 3.) The computed velocity $u_{\parallel}$ and the associated divergence $\Delta$ are shown in Figure 6a. They reproduce the principal features of the observed flow. In particular, the lanes surrounding the supergranule on the right and the prominent mesogranule near the center are clearly visible. On the other hand, motions like the streaming motion at the bottom of the region in Figure 5a are missing in Figure 6a. The concentrated downflows (minima of $\Delta$ in Fig. 5a) are also absent in the model flow. As expected, corks moving with the computed velocity $u_{\parallel}$ are swept away from the centers of mesogranules and into the regions between them, where $\Delta < 0$. Once there, they move slowly and are only gradually sucked into a linear network. Thus there are still significant differences between the patterns in Figures 5f and 6f. The network outlining mesogranules and the supergranule is clearer for the model flow, because corks have not accumulated over sinks, but the prominent J-shaped feature in Figure 5f is not present in Figure 6f. At later times, corks moving with velocity $u_{\parallel}$ outline the supergranule and the boundary of the mesogranule at the bottom left up to $t = 64$ hr. Our overall conclusion is that the corks motions in Figures 5 and 6 are qualitatively similar, although isolated sinks are obviously more important in the observed velocity field.

We have investigated the effect of varying the cutoff level $\Delta_{\odot}$. Figure 7a shows the velocity $u_{\parallel}$ and its divergence $\Delta$ when $\Delta_{\odot} = 0.015$ minute$^{-1}$, so that only the nine strongest sources are included. (Three of these lie outside the region shown.) The supergranular flow has disappeared, and much of the right-hand half of the region is stationary: as a result, the cork pattern in Figure 7b shows a network enclosing mesogranules at the center and the bottom of the region after 16 hr, while cork positions in the supergranule are virtually unchanged. Figure 7c shows the velocity pattern with $\Delta_{\odot} = 0.006$ minute$^{-1}$: now there are 19 sources (eight of which lie outside the region). In addition to the features visible in Figure 6a, there are two small sources between the supergranule and the prominent mesogranule, which affect the network significantly after 16 hr, as can be seen from Figure 7d. No comparable structure appears in Figure 5f. We conclude that with $\Delta_{\odot} = 0.015$ minute$^{-1}$ major features of the observed flow are omitted but that for $\Delta_{\odot} = 0.006$ minute$^{-1}$ small features that may originate from noise in the observed data can have a disproportionate effect. Hence we shall choose an intermediate cutoff value, and set $\Delta_{\odot} = 0.008$ minute$^{-1}$ for the model flows described in the rest of this paper.

Figure 8a shows the measured velocity $u_{\parallel}$ and the divergence $\Delta$ for a second area (region B) with dimensions $65^\circ \times 65^\circ$ in the top left-hand part of Figure 3. This region has a more complicated pattern of motion. Figures 8b–8f show the positions of corks moving with this velocity. They are rapidly
swept to mesogranule boundaries and accumulate at sinks. After 4 hr the corks have been swept away from the sources and have already begun to accumulate at junctions in the network. By 16 hr the corks are confined to a few linear features and some isolated sinks. Some network features remain, but it is difficult to identify the mesogranules. The corresponding flow $u_H$ and its divergence $\Delta$ are shown in Figure 9a. Here again the mesogranules and supergranules are satisfactorily reproduced by the plumes, though sinks with negative $\Delta$ are not adequately represented. In this case 54 sources were required. The corks form similar patterns in Figures 8 and 9. As before, the network becomes clearly apparent after 8 hr (Fig. 9e), and the mesogranules are still outlined by corks after 16 hr (Fig. 9f).

In summary, the obvious difference between particles transported with the actual velocity $u_H$ and those moving with the model velocity $u_H^0$ is that the latter provide a more convincing image of the network enclosing individual sources. What is missing, however, are the isolated sinks that appear in the observed velocity field.
Fig. 6.—Region A: as in Fig. 5, but for corks moving with the velocity \( u_{H} \) derived from the model with sources only and a cutoff at \( \Delta_{0} = 0.009 \) minute\(^{-1}\).

(a) \( u_{H} \) and \( \Delta^{*} \); (b-f) Positions of corks after 1, 2, 4, 8, and 16 hr have elapsed.

IV. SINKS AND SOURCES

We can clearly extend the model so that it represents sinks as well as sources. Then the horizontal velocity will be a better approximation to that derived from the SOUP measurements, and the corks motions should therefore be closer to those calculated in Figures 5 and 8. In complete analogy to the procedure described in § II, we model downflows by axisymmetric sinks, with \( V \) negative in equation (3). Figure 10 shows an isolated sink with coordinates (62°6, 51°7) in Figure 3 and compares the observed flow with the model, as Figure 4 did for an isolated source. In Figure 10b values of the normalized divergence \( \Delta/\Delta_{\text{max}} \) where \( \Delta_{\text{min}} < 0 \) is the minimum value of \( \Delta \) at the center of the sink, are plotted against \( r \), the distance from the sink. Averaged values are indicated by circles, and the curves show the values predicted by equation (7) with \( n = 1, 2, 3 \). Once again, the predicted values of \( \Delta/\Delta_{\text{max}} \) with \( n = 2 \) are in excellent agreement with the measured values for \( r \leq 1.5 \) \( R_{0} \). Beyond that distance \( \Delta \) is affected by neighboring sources visible in Figure 10a, and the scatter in local values of \( \Delta \) becomes extremely large. As before, we determine the average velocity \( \bar{u} \) over a 10" × 10" region surrounding the sink and obtain the corrected velocity \( \tilde{u} = u_{H} - \bar{u} \), which is shown in Figure 10c.
Fig. 7.—Effects of varying the cutoff \( \Delta_0 \): Motion of corks in region A traveling with velocity \( \mathbf{u}_h \). Left-hand side (\( \Delta_0 = 0.015 \) minute \(^{-1} \)): (a) \( \mathbf{u}_h \) and \( \Delta \); (b) cork positions after 16 hr. Right-hand side (\( \Delta_0 = 0.006 \) minute \(^{-1} \)): (c) \( \mathbf{u}_h \) and \( \Delta \); (d) corks after 16 hr.

Next we repeat the calculations illustrated in Figs. 6 and 9, again setting \( n = 2 \) but now including sinks as well as sources. As before, we impose a cutoff \( \Delta_0 \) and fit rising plumes at the centers of all sources with \( \Delta_{\text{max}} \geq \Delta_0 \); in addition, we fit sinking plumes at the centers of all sinks with \( \Delta_{\text{min}} \leq -\Delta_0 \) and then compute the horizontal velocity \( \mathbf{u}_h \) produced by all the plumes. (This procedure assumes that sources and sinks are sufficiently far apart that each can be fitted individually without taking account of neighboring plumes.) We have investigated the effect of varying \( \Delta_0 \) and again find that the appropriate cutoff value is \( \Delta_0 = 0.009 \) minute \(^{-1} \).

Figure 11 shows results obtained for region A, which should be compared with those in Figures 5 and 6. From Figure 11a we see that the velocity \( \mathbf{u}_h \) and the corresponding divergence \( \Delta' \) provide a better description of the original velocity \( \mathbf{u}_h \) than was obtained, using only sources, with \( \mathbf{u}_h \) in Figure 5a. In particular, the flows toward the sinks at bottom left and bottom right in Figure 5a are more adequately represented. The corks in Figures 11b–11d form patterns that closely resemble those in Figures 5b–5d, with strong accumulations over sinks after 4 hr have elapsed. Nevertheless, after 8 and 16 hr, in Figures 11e and 11f, the detailed structure differs from that with the velocity \( \mathbf{u}_h \) in Figures 5e and 5f.

In Figure 12a the velocity \( \mathbf{u}_h \) and divergence \( \Delta' \) for region B clearly provide an improved approximation to \( \mathbf{u}_h \) and \( \Delta \) in Figure 5a than was achieved by \( \mathbf{u}_h \) and \( \Delta' \) in Figure 9. This time, corks moving with velocity \( \mathbf{u}_h \) form patterns closer to those in Figure 8 for times up to 8 hr, when the cellular network is very clear. For \( t = 16 \) hr there is certainly a stronger qualitative similarity between Figures 8 and 12 than there is between either of them and Figure 9, owing to the increasing prominence of sinks as particles coagulate above them. On the other hand, there is no detailed agreement between Figures 8f and 12f.

In Figure 13 we compare the positions of test particles after much longer intervals have elapsed. Figures 13a and 13b show corks moving with the measured velocity \( \mathbf{u}_h \) after 32 and 48 hr, while Figures 13c and 13d and Figures 13e and 13f show corresponding positions of corks moving with velocities \( \mathbf{u}_h \) and \( \mathbf{u}_h \) respectively. By this stage all three patterns look different and are evolving very slowly. The model with sources only retains more linear structure than the actual SOUP flow, while the model with sources and sinks is too effective at concentrating corks above isolated downflows.

Our general conclusion from these comparisons is that the
simple model of § III is surprisingly successful in reproducing the main features of the original flow. Adding sinks as well as sources naturally produces a more accurate approximation, but the improvement in predicting cork positions is less significant than might have been expected if sinks and sources were equally important. In this paper we have allowed an initial distribution of test particles to evolve without introducing new corks to compensate for the accumulation of particles at down drafts. In the real Sun new flux is constantly emerging into the photosphere. It is clearly possible to extend the lifetime of the network by introducing new corks, as has been done by Title (1988). We plan to investigate such extensions to our models in the future.

V. DISCUSSION

We have demonstrated that the qualitative features of two- dimensional kinematic flows on the surface of the Sun are satisfactorily represented by sources of radial outflow with the form given by equation (3) with $n = 2$. In particular, the cellular pattern produced by interactions among these plumes is consistent with observed mesogranular and supergranular flow.
Moreover, this simple model provides an adequate description of the evolution of the magnetic network.

In § III we showed how this model could be used to represent the flow derived from proper motions of granules measured by the SOUP instrument on Spacelab 2. The observations show a topological distinction between regions of positive and negative divergence $\Delta$, and the values of $|\Delta|$ are on average higher over sources than they are over sinks. Nevertheless, there are many examples of isolated sinks in Figure 3, and some of them are strong. In § IV we therefore extended the model to include downward flows in the same way as upwellings. This procedure allowed us to construct approximations to the observed surface flow that were significantly more accurate. We have compared the motions of passive test particles (corks) moving with the observed horizontal velocity $u_h$ with the velocity $u'_h$ given by the simple model with sources only and with the velocity $u''_h$ given by a model with sources and sinks. The initial evolution of the pattern is similar for all three velocities as corks are swept away from sources at the centers of mesogranules and supergranules. After a few hours the corks are confined to a linear network with strong concentrations over isolated sinks for the cases with $u_h$ and $u'_h$. For times around 4–8 hr $u''_h$ is clearly a better approximation than $u'_h$. For times greater than 16 hr the cork pattern for $u_h$ preserves
the network, while the patterns for $u_H$ and $u_H'$ are almost entirely reduced to isolated points. On the other hand, the isolated cork concentrations in Figures 13b and 13f do not actually coincide. Although $u_H'$ provides a pattern qualitatively closer to that for the observed flow, corks do not necessarily congregate over the centers of the principal sinks, and the agreement is less precise than might have been expected.

One obvious discrepancy between our models and the observed flow arises from the fact that our axisymmetric plumes have no swirl, so the horizontal velocities $u_H$ and $u_H'$ are irrotational. The vertical component of the vorticity of the observed flow, $\omega = \partial u_H / \partial x - \partial u_H / \partial y$, can be calculated in the same way as the divergence (November and Simon 1988). It is shown in Figure 14. Apparently there are isolated regions of positive and negative vorticity in roughly equal numbers, and $|\omega|$ has values comparable to $|\Lambda|$. We have found a few examples of sources or sinks with associated swirl. For instance, the source in Figure 4 coincides with a region of negative vorticity. Comparison of Figure 14 with Figure 3 reveals, however, that there is no significant overall spatial correlation between vortices and sources or sinks.

It is clearly possible to model local concentrations of vorticity in the same way as sources and sinks by introducing an axisymmetric azimuthal velocity $u_\phi = g(r)$ with

$$g(r) = \frac{VR}{2r} \left[ 1 - e^{-\omega(r)} \right],$$

so that

$$\omega = \frac{1}{r} \frac{d}{dr} \left[ rg(r) \right] = \frac{V}{R} e^{-\omega(r)}.$$

Such a procedure might produce a more accurate representation of the observed velocity $u_H$, but it would not be instructive unless plumes and vortices coincided. Where the occasional strong vortex coincides with a sink or source, as in the flow observed by Brandt et al. (1988), such a description could be useful.
So far we have considered only kinematic aspects of large-scale photospheric motion. Nothing has been said about the dynamics of the underlying flow or the nature of the cellular pattern below the surface. We presume that photospheric motion is dominated by hot plumes which expand as they rise through a stratified layer and spread out when they impinge upon a stably stratified region. The structure of isolated buoyant plumes has been studied in other contexts (Turner 1973; Scorer 1978) and related to solar granules (Musman 1972). In the granulation adjacent plumes interact to give broad upwellings at cell centers, while cold fluid sinks in narrower downflows at cell boundaries, producing the observed topological distinction between upward and downward motion as described in § II. The existence of strong sinks in the SOUP data suggests that such a picture may be an oversimplified description of mesogranular and supergranular convection. Moreover, numerical experiments and simulations show that motion below the surface is dominated by rapidly sinking plumes that break away from an unstable thermal boundary layer (Stein and Nordlund 1989; Cattaneo, Hurlburt, and Toomre 1989). Previous calculations had shown that small tornadoes, with locally enhanced helicity, may develop in
three-dimensional compressible convection (Graham 1977; Nordlund 1985b), and conservation of angular momentum can lead to bath-plug vortices at sinking plumes. Thus a successful kinematic description of the apparent cellular velocity field in the solar photosphere may be a poor or misleading guide to the real structure of subphotospheric convection.

In this paper we have, however, had a different underlying purpose, namely, to explore the connection between the magnetic network and outflows from photospheric sources. We have demonstrated that the observed outflows from plumes at the solar surface are adequately represented by simple kinematic models. Now we turn to the relationship between cork patterns and the observed magnetic field. We noted in §§ III and IV that corks moving with the velocity $u'_m$ calculated using only sources tend to preserve a network structure, while corks traveling with the actual velocity $u_m$ (or with velocity $u'_m$) accumulate over sinks at junctions in the network. The computed patterns can be compared with actual magnetograms obtained at Big Bear Solar Observatory (Simon et al. 1988). In Figure 15 we show the observed magnetic fields in region B, which was illustrated in Figures 8, 9, 12, and 13. The magnetic network did not change significantly over the 9 hr observing period. The contour levels indicate the magnitude of the field without regard to its polarity and correspond to notional field
strengths of 25 and 85 G. Superposed on the contours are the positions of corks moving with velocities $u_n$, $u_{np}$, and $u_p$ after intervals of 8 and 16 hr have elapsed. It is immediately apparent that the test particles do indeed accumulate in regions with significant magnetic fields, as demonstrated by Simon et al. (1988, Figs. 2c and 2d). The cork patterns are also correlated with facular emission in white-light images.

We have made a quantitative comparison between cork positions and magnetic fields. This is somewhat complicated by the fact that the seeing at Big Bear Solar Observatory was relatively poor at the time of the observations. Hence weaker field regions were distorted and smeared out so that much of the signal disappeared into the noise. This was exacerbated by the long integration times needed to obtain a decent magnetic signal in quiet Sun regions. Furthermore, the alignment, orientation, and scaling of the two data sets were completely different. Thus it was difficult to register the two sets of images with great precision, especially where the field signal was weak. We have therefore adopted the following procedure. We first choose a reference level $B_{\text{min}}$ for the field strength. Let $m$ be the number of corks that lie within 1 pixel (1.86") of a point where the magnitude of the magnetic field $|B| \geq B_{\text{min}}$ at time $t$. At
Fig. 14.—Contours of the vorticity $\omega$ derived from the SOUP data over the 131" $\times$ 119" domain. Solid contours indicate positive $\omega$, dashed contours negative $\omega$. The contour spacing is 0.0065 minute $^{-1}$, as in Fig. 3.

$t = 0$ there are $N$ corks distributed uniformly on a square grid in the region where the field was measured and $m = m_0 \gg 1$. So we take the probability that a random cork lands in a magnetic region to be $p = m_0/N$; i.e., $p$ measures the fractional area of the image covered by magnetic field. If the corks are randomly distributed, $m$ will then follow a binomial distribution with mean $m_0$ and variance $\sigma^2 = m_0(1 - p)$. At some later time $t$ we expect the distribution of $m$ to be approximately normal, in which case the number of standard deviations from the mean is given by $s = (m - m_0)/\sigma$.

For region B we have $N = 1083$. In Table 2 we list values of $m$ and $s$ for the observed velocity $u_H$, and the model velocities $u_H$ (sources only) and $u_H$ (sources and sinks) at $t = 8$ and 16 hr with the notional field magnitude $B_{\text{min}} = 25$ G that corresponds to the lower contours in Figure 15. The values of $s$ are so large that the observed correlation cannot be ascribed to chance. (Since we have not allowed for corks that leave the region, these values of $s$ are conservative.) We have repeated the same procedure for $B_{\text{min}} = 85$ G, corresponding to the upper contour level in Figure 15, and also for $B_{\text{min}} = 15$ G, where magnetic data are contaminated by noise. Results for $t = 8$ hr are given in Table 2. Again there is clear statistical evidence for a correlation between corks and magnetic fields. We note also that for $B_{\text{min}} = 15$ and 25 G the velocity $u_H$ provides a better correlation than our model flows, but for $B_{\text{min}} = 85$ G the models do better and the best correlation is obtained with $u_H$.

The statistical analysis confirms that the familiar relationship between supergranules and the magnetic network extends to mesogranules too. Figure 15 also allows us to make a more detailed comparison between mesogranules and magnetic structures. At the upper right is an area of strong field including a small pore at (69°, 95°). Note that the magnetogram does not cover the upper left-hand corner. It is apparent that the observed magnetic fields are not confined to isolated sinks. Instead the magnetic pattern provides an incomplete network that outlines several mesogranules. The clearest examples are the two mesogranules centered at (71°, 88°) and (63°, 76°) in Figure 8a, which were labeled 1 and 2 by Simon et al. (1988). As expected, the cork patterns generated with the actual velocity $u_H$ coincide more nearly with magnetic features than do the patterns generated with the velocity $u_H$ derived from sources only. For instance, the prominent feature at (43°, 60°) is overlain by corks in Figure 15a but not in Figure 15c. That is

| $B_{\text{min}}$ (G) | $m_0$ (G) | $t$ (hr) | $u_H$ | $u_H$ | $u_H$
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<td>572</td>
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<td>16</td>
<td>666</td>
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<td>436</td>
<td>8</td>
<td>604</td>
<td>10.4</td>
<td>597</td>
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* Number ($m$) of corks within 1 pixel of a region with field strength $|B| \geq B_{\text{min}}$ and number of standard deviations ($s$) from the mean $m_0$. 

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simply because \( u_H \) is an inaccurate representation of \( u_H \). On the other hand, the cork pattern generated with the velocity pattern \( u_H \) (derived from sinks as well as sources) in Figure 15c is less well correlated with magnetic features, since too many particles have accumulated at the sinks.

A more significant aspect of Figure 15 is the qualitative similarity between the observed magnetic network and the cork patterns in Figures 15c and 15d. If magnetic flux tubes were passively transported, they would inevitably accumulate at sinks within the network after an interval of 9 hr had elapsed. Then we might expect to see patterns like those in Figures 13a and 13b. In fact, the network is preserved for periods that are long compared with the mesogranular time scale. This implies that the individual flux tubes that compose the network follow the outflows to cell boundaries without being dragged to sinks as readily as passive particles.

Such behavior can be explained only by considering the dynamical interaction between subphotospheric convection and magnetic fields (Galloway and Weiss 1981), which lies beyond the scope of this paper. As a first stage it will be neces-
sary to model the hydrodynamics of mesogranular and supergranular motion and to establish the connection between them (cf. Stein and Nordlund 1989). Then the motion of individual flux tubes can be followed (cf. Schmidt, Simon, and Weiss 1985) in order to establish where they intersect the surface. As further observations become available and more sophisticated models are developed, we hope to use detailed motions of magnetic flux tubes to probe subphotospheric flows.

The agreement between our simple model and the observations confirms that radial sources can be used to represent surface motions on the Sun and to explore the resulting evolution of weak magnetic fields. Of course it may be preferable to use the actual velocity fields, rather than those derived from a model, where observations are available, as they are for large-scale horizontal flows. However, the success of our simple approach suggests that the same technique can also be applied to small-scale photospheric flows, and we plan to use it to study the evolution of intergranular magnetic fields under the combined effects of mesogranular and granular convection.

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