HELOSEISMIC OBSERVATIONS OF THE SOLAR CYCLE

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ABSTRACT

Satellite and limb photometry observations of the Sun yield a consistent picture of a changing effective temperature distribution in the photosphere. Here we use those observations to show that a large change in helioseismic splitting coefficients and mode centroid frequencies should be observed in helioseismic data obtained in 1988.

Subject headings: Sun: activity — Sun: oscillations

I. INTRODUCTION

A simple model has had reasonable success in relating the even coefficient helioseismic splitting coefficients to solar limb photometry observations (Kuhn 1988a, b, hereafter K1 and K2; 1989). This model uses the mean photospheric brightness measurements to derive a global aspherical interior effective sound speed perturbation. The corresponding acoustic frequency splitting due to this perturbation can be related to the helioseismic observations. The surface brightness measurements show that the mean brightness or latitudinal effective temperature distribution at the photosphere varies with the magnetic solar cycle. In addition this brightness variation appears to describe the bolometric solar luminosity cycle that has been observed from space (Kuhn, Libbrecht, and Dicke 1988, hereafter KLD; Willson and Hudson 1988).

Limb observations and this heuristic model allow several predictions which may shortly be tested against the growing body of helioseismic data. Repeat observations of solar oscillations, using the same techniques, but at widely separated times during the cycle, may shortly be available (Harvey 1988; Libbrecht 1988). Anticipating these results, this Letter makes several predictions based on the model which are particularly vulnerable to the new helioseismic observations.

II. SOUND SPEED PHENOMENOLOGY

The Sun’s spherical symmetry is broken by (among other effects) global magnetic, velocity, and temperature fields, each of which may contribute to the even order helioseismic frequency splitting coefficients \( b_2 \), in

\[
\nu_{\text{lim}} - \nu_{\text{eq}} = \sum_i b_i P_i (-m/l). \tag{1}
\]

Direct attempts at inverting the observed coefficients to obtain the global magnetic field (Dziembowski and Goode 1984, 1986) have not yielded physical results. This may be understandable if magnetic fields are not the dominant contribution to the even coefficients. The magnetic inversions require field strengths of order 10^6 G near the base of the convection zone to generate splittings of the observed magnitude. On the other hand, from Parker’s (1987) calculations we might expect a field of only perhaps a few kilogauss near the base of the convection zone to affect the temperature (and sound speed) structure to produce measurable splittings. Thus, a global internal magnetic field may be more easily detected by observing the aspheric stratification it produces.

This is not to say that magnetic fields do not contribute to the splitting observations. For example, it may be that the surface fibril field described by Bogdan and Zweibel (1985) is a significant contribution to the \( b_2 \). Lacking a self-consistent formalism which incorporates each of these contributions to the eigenproblem, we have attempted to parameterize all of these effects by proposing a local effective sound speed (LESS) perturbation model. In the case where the temperature dominates other contributions, this is a justifiable approximation.

The local sound speed perturbation model is naturally related to the photometric solar observations. Furthermore, these limb data (KLD) and ACRIM results (Willson and Hudson 1988) are consistent with a photosphere which has smooth 10^4 latitudinal and temporal color temperature variations. We argued (K2) that the magnitude and yearly trends in the temperature field (corresponding \( c^2 \) field) implied variations in \( b_2 \) and \( b_4 \) splitting coefficients which were observed. In that calculation we tacitly acknowledged other contributions to the splitting coefficients by using only the component of the temperature field which appeared to vary during the solar cycle. We also found that reasonable solutions for the radial structure of the temperature perturbation could not account for the systematic offset between surface temperature and splitting measurements. Perhaps a better approach is to compare the splitting and temperature data using an undetermined time-independent splitting contribution in a least squares solution for the unknown constants.

III. INTERPRETING THE SPLITTING AND LIMB OBSERVATIONS

In K2 we argued that the radial structure of the asphericity was barely constrained by the observed splittings. The minimum \( \chi^2 \) two-parameter solutions for this structure implied fractional squared sound speed asphericities at the surface that were consistent with the observed temperature-related sound speed variation. In this two-parameter radial model, the best-fit solutions required an asphericity of zero interior to a radius of between 0.8 and 0.95 \( R_\odot \) (\( \chi^2 \) was flat over this range, so the depth of the asphericity was correspondingly indeterminate). A shallower model required a larger fractional asphericity. For example, if the perturbation extends down from the photosphere only 2 \( \times 10^5 \) km, then the surface asphericity must be of order a few percent (\( \chi^2 \) is also significantly greater than in the deeper models). The splitting data is best described by a perturbation that is about 10^5 km deep and has a fractional asphericity amplitude of order 10^{-4}.

A model that assumes a constant fractional sound speed variation with depth is useful for discussing the solar cycle...
variation in the separate data sets. This was the model used in K1. Note that the difference between the splittings, calculated using the minimum $\chi^2$ solutions discussed above, and this model are small—of order 10%, because the corresponding kernel functions are peaked near the surface (cf. K1).

In KLD we argued that the surface photometry data indicated a smooth excess flux distribution (which was well described by a temperature excess) near the active sunspot latitudes in the photosphere, plus a time-independent quadrupolar flux contribution that peaked near the poles. This asphericity is not well parameterized by two terms of a Legendre polynomial expansion because the flux excess is peaked near the active latitude bands. The temperature distribution has the form of two hot bands (one in each hemisphere), $10^\circ$–$20^\circ$ in width, that drift toward the equator during the solar cycle. On the other hand, since the coefficients of the Legendre expansion of the asphericity can be directly related to the corresponding asymptotic coefficients of the Legendre expansion of the frequency splittings, this formalism is useful for relating the two data sets. Table 1 shows a multiparameter Legendre decomposition, out to $l = 8$, of the temperature excess using the data plotted in KLD (Legendre polynomials are not orthogonal on this sample domain). The errors are standard fit errors and do not reflect calibration and facular contamination uncertainties. These are probably the dominant systematics and may contribute an additional 20% error. With a constant radial dependence to the asphericity, the temperature data and the LESS model imply splitting coefficients of the form $b_{2l} = k_2 t_2$, where $k_2$, $k_4$, etc., are $-140, 110, -86, 75$ nHz K$^{-1}$, etc., for 5 minute band intermediate $l$ oscillations.

To date, only $l = 2$ and 4 splitting coefficients have been published. Figure 1 plots these against the LESS splittings. Here we have allowed for a time-independent splitting contribution by adding a constant to the LESS data points. This constant was 14 and $-66$ nHz for the $l = 2$ and 4 coefficients. The sum of these terms corresponds to an undetermined asphericity that peaks near the poles and has an amplitude of about $2 \times 10^{-4}$.

Table 1 suggests that the higher order terms ($l = 6$ and 8) should be nearly as large as the quadrupolar terms in the splitting expansion. If the signal-to-noise ratio of the splitting measurements is good enough, these higher terms should be

<table>
<thead>
<tr>
<th>Date of Observation</th>
<th>$t_0$ (K)</th>
<th>$t_2$ (K)</th>
<th>$t_4$ (K)</th>
<th>$t_6$ (K)</th>
<th>$t_8$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1983.5</td>
<td>1.19 ± 0.1</td>
<td>-1.73 ± 0.1</td>
<td>1.73 ± 0.2</td>
<td>0.79 ± 0.2</td>
<td>-0.95 ± 0.2</td>
</tr>
<tr>
<td>1984.5</td>
<td>0.68 ± 0.1</td>
<td>-0.69 ± 0.1</td>
<td>1.30 ± 0.1</td>
<td>0.56 ± 0.1</td>
<td>-0.37 ± 0.1</td>
</tr>
<tr>
<td>1985.6</td>
<td>0.48 ± 0.1</td>
<td>-0.37 ± 0.1</td>
<td>1.20 ± 0.1</td>
<td>0.05 ± 0.1</td>
<td>0.12 ± 0.1</td>
</tr>
<tr>
<td>1987.6</td>
<td>0.48 ± 0.1</td>
<td>0.22 ± 0.1</td>
<td>-0.03 ± 0.1</td>
<td>1.00 ± 0.1</td>
<td>-0.22 ± 0.1</td>
</tr>
<tr>
<td>1988.5</td>
<td>1.31 ± 0.1</td>
<td>-1.12 ± 0.1</td>
<td>-0.65 ± 0.1</td>
<td>3.03 ± 0.1</td>
<td>-1.36 ± 0.1</td>
</tr>
</tbody>
</table>

Fig. 1.—Time dependence of the helioseismic splitting measurements (references to the original observations are listed in K1 and K2), and model splittings inferred from limb photometry measurements. The most recent helioseismic data were obtained by Jefferies et al. (1988). The most recent 1988 photometric $b_2$ were computed from data described in Libbrecht et al. (1989).
detectable. This is especially true in 1988 when we should find that $b_6$ decreases dramatically while $b_2$ makes the largest positive jump since 1983.

IV. CENTROID FREQUENCY SHIFTS

The centroid frequency of all modes in a multiplet is not affected by the asphericity. To the extent that the observations equally sample the oscillation power of each $m$ value of a multiplet, there will be no solar cycle dependent frequency shift due to the $t_{2i}$ with $i \neq 0$. In fact, integrated full disk solar oscillation observations do not sample the different $m$ components of a multiplet equally. For example, integrated observations are not sensitive to modes with $l - m$ odd. Thus, in observations which do not resolve the frequency difference between multiplet components, there will be an apparent frequency shift in the centroid due to the variable asphericity. The resulting centroid shifts will be of order the splittings, as calculated above, but in detail must depend on exactly how each component contributes to the observed spectrum.

The observed $t_{0}$ in Table 1 imply a time-dependent variation in centroid frequencies even in observations that equally sample multiplet components. The average temperature change during the solar cycle leads to an overall shift in the centroid of the frequencies of a multiplet of modes. Note that solar radius changes could also produce similar frequency variations, but because we have accounted for the total solar constant variation during the solar cycle as a variation in photospheric effective temperature (KLD) without finding evidence for solar radius changes, we expect the radius induced frequency shifts to be small. Calculations by Dearborn and Blake (1980) support this conclusion. They found that fractional radius changes induced by perturbations in the convection zone are typically $10^{-2}$ of the fractional luminosity variation.

The proportionality between temperature and frequency shift, $k_{0}$, analogous to the $k_{2i}$ described above, is $270 \text{ nHz K}^{-1}$. Thus, in the constant fractional temperature LESS model, the $l = 0$ centroid mode frequencies should increase about 230 nHz between 1987 and 1988. Between 1985 and 1987, mode frequencies should have varied by less than 10 nHz. These calculations appear to agree qualitatively with the recent observations of Geliy, Fossat, and Grec (1988) and Palle, Regulo, and Cortes (1988).

V. CONCLUSIONS

We need more data. Helioseismic observations obtained during 1988 should show the most dramatic change in splitting coefficients and centroid frequency shifts since 1983. The splitting data should be analyzed with higher order Legendre polynomials. The centroid frequency variations are interesting and also apparently consistent with the limb photometry and ACRIM observations.

If the model holds up in 1988, then it will be interesting to disentangle the various possible contributions to the asphericity splittings. For example, what is the time-independent contribution, and are surface fibril fields important to the asphericity? Calculations along these lines should allow us to learn more from LESS.

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