RELATIONS BETWEEN THE PHOTOSPHERIC MAGNETIC FIELD AND THE EMISSION FROM THE OUTER ATMOSPHERES OF COOL STARS. I. THE SOLAR Ca II K LINE CORE EMISSION

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ABSTRACT

Observations of a solar active region complex and its surroundings are used to establish a quantitative relation between the Ca II K line core intensity and magnetic flux density. We transform the Ca II K line core intensity to a Ca II H + K line core flux density, as measured by the Mount Wilson HK photometer, to facilitate a comparison of solar and stellar data. In this process, we derive a new absolute calibration for the Mount Wilson Ca II H + K fluxes for G-type dwarfs.

The minimum Ca II K flux, found in the centers of supergranulation cells in quiet regions on the Sun, is identical to the minimum flux that is observed for solar-type stars. The Ca II K flux from small pixels on the Sun saturates at a level comparable to the flux level from the most active solar-type dwarfs.

The nonlinear trend between the Ca II H + K line core excess flux density $\Delta F_{\text{Ca II}}$ (expressed in ergs cm$^{-2}$ s$^{-1}$) and the absolute value of the magnetic flux density $|B|$ (in gauss) is described by $\log \Delta F_{\text{Ca II}} = 0.6 \log |B| + 4.8$. This mean trend holds for pixels with sizes from 2'.'4 x 2'.'4 up to 14'.'4 x 14'.'4. The scatter about this mean relation is large and intrinsic. We discuss models that explain the nonlinearity of the mean trend and the large intrinsic scatter about it. The data suggest the existence of a characteristic length scale of $\approx 15$' in the photospheric magnetic flux density.

The solar data define a relation that is similar to the relation between stellar hemisphere-averaged magnetic flux densities and Ca II H + K excess flux densities.

Subject headings: Ca II emission — stars: chromospheres — stars: late-type — Sun: magnetic fields

I. INTRODUCTION

a) The Ca II H and K Lines as Diagnostics of Solar Activity

The Ca II H and K resonance lines are often used in the study of solar and stellar magnetic activity. Emission peaks in the cores of the Ca II H and K lines reveal outward temperature rise that cannot be caused by radiative heating. Emission lines from highly ionized species show that the temperature continues to rise to well over 1 million kelvins in the corona. Magnetohydrodynamic effects have been suggested as the source of the nonradiative heating of the outer stellar atmosphere.

Direct evidence for a relation between the radiative and magnetic flux densities is provided by the Sun. A qualitative correspondence between the magnetic field strength and the brightness of the Ca II H, K plage follows directly from a comparison of spectroheliograms and magnetograms (Leighton 1959; Howard 1959; see also Fig. 1): bright patches in Ca II H, K spectroheliograms overlie areas of strong magnetic field. In fact, much of the information about the structure of the photospheric magnetic field, with the exception of the field polarity, may be obtained from the study of Ca II H, K spectroheliograms.

Skumanich, Smythe, and Frazier (1975, hereafter SSF) used observations of a quiet solar region to establish a quantitative relation between the absolute value of the mean magnetic flux density $\langle B \rangle$ (in gauss) and the intrinsic field strength $B$ per pixel of 2'.'4 and the Ca II K line intensity $R_C$ relative to the continuum (in a 1 Å passband centered on the emission core). They proposed a linear relation between $\langle B \rangle$ and $R_C$:

$$\langle B \rangle = 2800R_C - 200 \text{ G}. \quad (1)$$

The scatter about relation (1) is appreciable. SSF note that this linear relation holds for magnetic flux densities between ~25 and 120 G, while nonlinear effects are seen outside this range of flux values.

This paper extends the study of the relationship between the Ca II K line core intensity and the magnetic flux density from quiet regions to solar active regions. We used the main spectrograph of the McMath solar telescope at Kitt Peak to observe an active region complex. Section II reviews the instru-
Fig. 1.—Spectroheliograms and magnetogram of the active region complex (McM 19672, 19673, 19675) on 1985 October 22, close to disk center. Shown are (a) the intensity in the Ca II K line wing, (b) the magnetogram, and two versions of the Ca II K spectroheliogram with different sensitivity: (c) shows the full range of flux values, (d) saturates at an intermediate flux to show details in the surrounding network. The area scans cover 4.0 Mm (in right ascension) by 2.9 Mm (in declination). The small square shows the size of a $14.4' \times 14.4'$ box (6 x 6 resolution elements).
b) Stellar Magnetic Activity: Radiation Flux Densities

Wilson (1978) pioneered the analogy in magnetic activity between the Sun and solar-type stars in his Ca II H + K survey of cool stars. During the past two decades, the Ca II H + K line-core emission has been shown to be well-correlated with other diagnostics of stellar activity, such as the coronal soft X-ray emission and the transition region emission in the ultraviolet (see, for instance, Schrijver 1983, 1986 [pp. 33–56], 1987a). Since the Ca II H and K lines are observed with ground-based telescopes, the Ca II H + K emission plays an important role in the study of stellar activity. Long-term monitoring programs have revealed rotational modulation (Vaughan et al. 1981; Noyes et al. 1984) and activity “cycles” (Wilson 1978; Baliunas 1986), phenomena also observed for the Sun.

The diagrams of Ca II H + K fluxes versus stellar color show a color-dependent lower limit. Schrijver (1986 [pp. 33–56], 1987a) argues that the lower limit flux is nonmagnetic in origin, and identifies two contributions (see also Schrijver, Dobson, and Radick 1988). The major contribution originates in the Ca II H + K line wing. The second component is a line core emission that appears to be nonmagnetic in origin (the so-called basal flux); this component may be related with acoustic heating. The lower limit flux for a solar-like star ($B - V = 0.66$) is $F_{\text{Ca} \, \text{H, min}} = 1.34$ in arbitrary units—see Rutten 1984—which differs from an absolute calibration by a constant factor; see Appendix, § I). This value of the lower limit flux is mainly determined by the giants in the sample. The smaller sample of dwarf stars suggests that the lower limit flux for solar-type dwarfs may be somewhat higher: $F_{\text{Ca} \, \text{H, min}} = 1.50$. The cause for this apparent dependence on luminosity class in the lower limit flux is not clear. Differences in the line formation in atmospheres of dwarfs and giants, or in acoustic heating efficiency may play a part. Alternatively, dwarfs may not remain on the main sequence long enough to spin down sufficiently—a result of magnetic braking—to reach their lower limit flux level. In this case even the least active dwarfs could still retain some activity that would raise their Ca II H + K flux slightly above the true minimum flux level.

Radiative fluxes in various emission lines from stellar outer atmospheres have been shown to be highly correlated. Schrijver (1983, for instance), established a tight power-law relation between the coronal soft X-ray flux and the chromospheric Ca II H + K excess flux (obtained by subtracting the empirical lower-limit flux from the observed stellar fluxes). The relation is nonlinear with an exponent of $\approx 1.7$, and is valid for giants and dwarfs, ranging in spectral type from early-F through mid-K (Schrijver 1983; Walter and Schrijver 1987). The nonlinearity of the relation between coronal and chromospheric flux densities poses a problem. Even though coronal and chromospheric emissions are not emitted by the same regions locally, a very good correspondence is observed between emissions from solar active regions as a whole. Hence, linear relations would be expected between atmospheric radiative losses if these were simply proportional to the solar activity.

III. The Magnetograph-Spectroheliograph

We used the main spectrograph of the McMath solar telescope at Kitt Peak (Livingston 1968). We briefly describe this Kitt Peak Magnetograph/Spectroheliograph. The basic design follows that of Babcock’s (1953) instrument. The spectrograph

This power law fits the solar and hemisphere-averaged stellar data, provided $<B> < 300$ G. The Ca II H + K excess flux density appears to saturate when the average stellar magnetic flux density exceeds $<B> \approx 300$ G, equivalent to a photospheric filtering factor for flux tubes of $\approx 20\%$ for a G-type dwarf.
Fig. 2—(a) Stellar soft X-ray flux density $F_X$ vs. the hemisphere-averaged absolute magnetic flux density $\langle B \rangle$ (figure from Saar and Schrijver 1987). The data for the Sun at minimum and maximum activity are connected by a dashed line (estimated by Saar and Schrijver 1987). The solid line represents eq. (2a). The mean value for solar active regions is shown for comparison (from Schrijver 1987b), with a typical range. The X-ray flux of one of the brightest nonflaring regions on the Sun, the compact active-region loop (CARL), is also shown. Symbols: circles, G dwarfs; squares, K dwarfs; triangles, M dwarfs. Arrows mark upper limits. (b) The Ca II H + K flux density $\Delta F_{\text{H+K}}^*$ measured with the Mount Wilson HK photometer, in arbitrary units) above an empirical lower limit vs. the hemisphere-averaged absolute magnetic flux density $\langle B \rangle$. The chromospheric emission of M-type dwarfs has been multiplied with a deficiency factor $\gamma$ (see Schrijver and Rutten 1987) which corrects for the color-dependent decrease in chromospheric emissions of dwarfs later than spectral type M0. The solid line segment on the left represents eq. (2b), while the less steep line segment on the right is an approximation to the most active stars in the figure. The horizontal line segment shows the maximum value observed in a solar active region. The dashed line represents the mean trend of eq. (9b) derived from solar observations. Symbols as in (a).

is coupled to the solar image by a 2'4 x 2'4 image slicer. A double exit-slit arrangement is placed at the focal plane of the spectrograph, observing the Zeeman-sensitive Fe I 5233 Å line (effective Landé factor $g = 1.3$). A feedback system drives a tipping glass plate that keeps the line centered on the exit slits, thus correcting for local velocity fields and solar rotation. The Doppler compensation system responds to step changes in velocity in less than 0.1 s. In addition to the two phototubes positioned on the line wings, a third phototube is centered on the line core.

The difference between the left-handed and right-handed circularly polarized signals in the line wings is interpreted as arising from the polarized $e$ components of a field of view filled homogeneously with a unipolar magnetic flux (the calibration is described in § IV). The method yields the correct magnetic flux if the line splitting is small compared to the line width. The aperture may, however, contain mixed polarities. We assume this is not commonly the case within a large bipolar region with the 2'4 x 2'4 resolution used here. We estimate the instrumental uncertainty to be $\approx 8$ G (we use an integration time of 40 ms; see Livingston 1968; SSF).

Two additional phototubes measured the Ca II K line core intensity $I_c$ and the intensity $I_w$ in the Ca II K line wing (7.39 Å to the red), simultaneously with the magnetic signal. A broad-band violet filter was placed over these two phototubes to reduce the contamination of the signal by other orders, and other forms of stray light. The change in absorption of this broad-band filter over the 7 Å separating the Ca II K core and wing channels is negligible.

The drives on the mirror system were used to rock the solar image back and forth over the spectrograph entrance, while a slow drift was introduced perpendicular to the rocking direc-
tional velocity (1.8 km s\(^{-1}\)). The calibration is valid only for
is the synodic equatorial rotation
where \(P\) is the angle of the solar rotation axis with the scan
equivalent to an effective magnetic flux density

The total magnetic flux (including both polarities) in the active
value of 130 G observed at present is somewhat high, it is
filling factor in the photosphere of some 10%. Although the
density of an active region is 100 + 20 G, implying a magnetic
range of values derived by Schrijver (1986 [pp. 167-189],
sunspots). This average magnetic flux density lies within the
insula of values of magnetic flux densities near the region
these values are derived for the plages, excluding the
contour: 1.9 x 10\(^{16}\). The total Ca \(\text{II}\ K\) plage area given
(19
UT), when the complex was near disk center. The raster
h
Two trailing active regions also had spots and pores in the
following parts (Fig. 1a). The regions developed on a position
McMath 19665 was observed 1 month earlier. The area of
the leading bipolar region (McMath 19672) was approxi-
mately twice that of the two other regions.

The active region complex was observed on 1985 October 22
(19\(^{th}\) UT), when the complex was near disk center. The raster scan (completed in \(\approx 25\) minutes) covered 390" \times 540", or
5
of the responses.

Part of the stray light from other optical paths in the spec-
trograph was determined by closing the mirror feeding the Fe \(\text{i} \lambda 5233\) magnetograph. This contribution was subtracted from
the Ca \(\text{II} \ K\) line core and line wing signals, as was the dark
current determined by blocking the sunlight at the entrance of
the spectograph.

We used a relative calibration for the Ca \(\text{II} \ K\) line core
intensity, which we later replaced by an absolute calibration.
The measured Ca \(\text{II} \ K\) line core intensity \(I_c\) is therefore divided
by the line wing intensity \(I_w\). The wing channel was not suffi-
ciently far from the core to be completely unaffected by activity,
resulting in a weak correlation of \(I_c\) and \(I_w\). As a result the
plage is faintly visible in the line wing spectroheliogram. The
line wing intensity increases by 3% when the Ca \(\text{II} \ K\) intensity
increases by a factor of 5. The effects of instrumental noise are
therefore very important in the wing channel, which may be
part of the cause for the poor correlation between \(I_c\) and \(I_w\).

In order to minimize the effects of an activity-dependent
wing intensity, the core intensity was not divided by the wing
intensity corresponding to the same pixel, but instead by an
average wing intensity for pixels near the endpoints of the same
raster line (i.e., in the quiet region surrounding the active
region complex). This procedure neglects the center-to-limb
variation in the line wing signal, but this is only a small effect
since the scanned area was located at disk center and the size of
the scanned area was smallest in the direction of the individual
raster scans (in declination): the variation within the scanned area
(near disk center) would be at most 2%.

SSF express their results as an intensity ratio \(R_c\) for the Ca \(\text{II} \ K\) line core: \(R_c\) is the ratio of the line core intensity to the
continuum intensity. The relation between the core relative
intensity \(R_c\) and the core wing intensity ratio is given by

\[
R_c = \frac{I_c}{I_w} \frac{C_w}{C_c}, \quad R_w = 0.40 \frac{I_c}{I_w}, \tag{4}
\]
Fig. 3.—(a) Contour map of the Ca II K spectroheliogram. The contour level corresponds to 50 G, computed using eq. (9a). Spots are shown in black. The dots identify data points in the top left of the main panel of Fig. 4, i.e., to pixels with a relatively strong Ca II K emission with a relatively small mean magnetic flux density. The square shows a 14′4 x 14′4 box (6 x 6 resolution elements). (b) Comparison of the 50 G magnetogram contour (solid) and the corresponding contour in the Ca II K spectroheliogram (dashed).
with \( R_c \) the line wing intensity relative to the continuum at disk center, and \( C_c \) and \( C_w \) the intensities of the extrapolated continuum over the line core and line channels (\( C_w/C_c \approx 1.00 \)). We use \( R_c = 0.50 \), from the flux atlas of Kurucz et al. (1984), assuming—as above—that the limb darkening is the same for the continuum and the line wing signal. This assumption is supported by the solar intensity atlas of Minnaert, Mulders and Houtgast (1940) which yields a wing to continuum ratio of \( R_c = 0.41 \).

b) From Relative Intensity to Absolute Flux Density

The observed intensities \( I \) are converted to flux densities \( F \) by using

\[
F = \pi I \left( \frac{I(r)}{I(0)} \right) \frac{2\pi dr}{\int_1^2 2\pi dr} = \pi I \Delta, \tag{5}
\]

with \( I(r)/I(0) \) the relative limb darkening curve. The limb-darkening curves for core line and core lines are taken from White and Suemoto (1968). We assume that the line wing limb darkening equals that of the continuum. The constant \( \pi \) equals 0.87 for the line core and 0.71 for the line wing, so that

\[
\frac{F_c}{F_w} = 1.23 \frac{I_c}{I_w}, \tag{6a}
\]

where the indices for the flux density \( F \) have the same meaning as for the intensity \( I \). We assume that this relation holds for pixels in both active and quiet regions.

The conversion of the relative line core intensity to a calibrated line core flux density is performed using the irradiance atlas from Kurucz et al. (1984). If no stray light remains in the line entrance signal for a quiet region after the corrections described in § Va, the flux density in this wing channel corresponds to \( 4.1 \times 10^8 \) ergs cm\(^{-2} \) s\(^{-1} \) at the solar surface so that, with equation (6a),

\[
F_c = 5.0 \times 10^8 \frac{I_c}{I_w}. \tag{6b}
\]

The irradiance atlas of Kurucz et al., obtained at solar minimum for a very quiet Sun, yields a ratio \( F_c/F_w = 0.228 \), or (with eq. [6a]) \( I_c/I_w = 0.185 \). The average for the regions surrounding the active region complex we observed is approximately \( I_c/I_w = 0.18 \). The agreement shows that the remaining stray light after the corrections described in § Va is indeed negligible.

c) The Comparison of Solar and Stellar Ca II H + K Measurements

The comparison of the solar data with the Mount Wilson Ca II H + K line core flux observations of cool stars requires a relative calibration between the two data sets. The Mount Wilson Ca II H + K flux measurements have been obtained with the HK photometer (Vaughan, Preston, and Wilson 1978) at the 60 inch (1.5 m) telescope at the Mount Wilson observatory. This instrument measures the flux in two triangular 1.09 A FWHM passbands, centered on the H and K line cores. The calibration of the observed signal requires a correction of the observed flux for the instrumental emission profile. We derive this correction by assuming that the line profile can be described by two components: (a) the absorption profile of a star of minimal activity, and (b) an added emission core of variable strength but with a fixed line profile. We use the solar Ca II K plage emission profile (Oranje 1983) to characterize the typical stellar line core profile. The Appendix describes the calibration procedure in detail (and includes the derivation of a new absolute calibration for the Mount Wilson Ca II H + K fluxes).

The resulting relation between the excess flux in the emission profile of the K line in the 1 A KPN0 passband, \( \Delta F_c \) (ergs cm\(^{-2} \) s\(^{-1} \)), and the flux \( F_{\text{Ca} \, \text{K}} \) in arbitrary units in the emission profile folded with the triangular Mount Wilson passband (which is indicated by an asterisk throughout this paper) reads:

\[
\Delta F_c = 8.4 \times 10^5 \frac{\Delta F_{\text{Ca} \, \text{K}}(\text{arb. units})}{F_{\text{Ca} \, \text{K}}} \tag{7}
\]

where \( \Delta \) indicates that a minimum profile is subtracted, leaving only a pure plage emission profile as derived by Oranje (1983).

The flux in the 1 A KPN0 passband that corresponds to the minimal Mount Wilson flux \( F'_{\text{Ca} \, \text{Kmin}} \) is calculated in § II of the Appendix. For the empirical lower limit flux \( F'_{\text{Ca} \, \text{Kmin}} \) (see § Ib) one finds a core wing intensity ratio \( I_c/I_w(F'_{\text{Ca} \, \text{Kmin}}) = 1.34 \) (see § Ib) one finds a core wing intensity ratio \( I_c/I_w(F'_{\text{Ca} \, \text{Kmin}}) = 1.34 \) or, with equation 4, a relative line core intensity \( R_c = 0.049 \). This is also the lowest value for \( R_c \) observed by SSF. If the value for the minimal Mount Wilson flux of 1.50 derived for the subsample of dwarfs is used (see § Ib)), \( I_c/I_w(F'_{\text{Ca} \, \text{Kmin}}) = 1.50 \) = 0.159, or \( R_c = 0.064 \). This latter value is close to the average cell-center emission found by SSF: \( R_c = 0.063 \).

The values of 0.123 and 0.159 for the minimum intensity ratio \( I_c/I_w \) derived from stellar data are in fair agreement with the minimal intensity ratio 0.145 (see Fig. 4) observed in the solar active region complex.

VI. MAGNETIC FLUX DENSITY AND Ca II K INTENSITY

The magnetic flux density and the Ca II K intensity can be compared pixel by pixel. We exclude pixels in pores and in the umbrae and penumbral of the sunspots, because of the intrinsically different behavior and the increased importance of stray light; we exclude pixels that have an intensity in the Fe I 5233 line core of less than 88% of the average intensity of the corresponding raster scan.

The total magnetic flux is underestimated if the polarity of the field changes within a pixel. Since the corresponding Ca II K intensity measures the magnetic flux without information on the sign of the flux, the correlation between the Ca II K intensity and the absolute magnetic flux can be drastically reduced over polarity inversions (neutral lines). The elimination of pixels near polarity reversals does not affect the mean relation significantly, so we did not apply this selection criterion. The elimination of points over polarity inversions does affect the mean relation, but only at low flux densities due to the disappearance of points in the upper left-hand corner of the main panel of Figure 4.

Figure 4 relates the observed intensity ratio \( I_c/I_w \) and the magnetic flux density for \( 2^\prime.4 \times 2^\prime.4 \) pixels. The scatter in the diagram is surprisingly large. This large scatter may be partly caused by the method of observation. First, small-scale polarity mixing may yield a net magnetic flux density that is much smaller than the total flux density. Second, the short integration time per pixel (required in order to cover the large area) results in an appreciable statistical noise. We feel that these sources of scatter are not responsible for the large scatter.
Fig. 4—The Ca II K core wing intensity ratio $I_c/I_w$ vs. the absolute value of the magnetic flux density $|\langle B \rangle|$, for 2 x 2 square pixels. Only 10% of the data are plotted, and an even smaller percentage in the box in the lower left corner, to avoid crowding. The solid lines outline the area of peculiar points, shown by dots in Fig. 3a. The dashed line represents eq. (9a). Four histograms on the left show the distribution of magnetic flux densities for pixels with comparable Ca II K intensities. Five histograms on top of the main figure show the distribution of Ca II K intensity ratios for pixels within some intervals in magnetic flux density. The bars associated with each of the histograms show the interval in $|\langle B \rangle|$ for which the histogram was determined.

in Figure 4: the large range of $I_c/I_w$ at a given mean magnetic flux density $|\langle B \rangle|$ is largely intrinsic (see § VIIb). Note that the 50 G contour in the magnetogram and the corresponding contour in the Ca II K spectroheliogram agree quite well (Fig. 3b), despite the large scatter in the relation between Ca II K intensity and magnetic flux density.

The data in Figure 4 (see also Table 1) suggest that the mean trend between the Ca II K excess flux density and the magnetic flux density is a power-law relation with an exponent smaller than unity. This mean trend is described by

$$\log \frac{\Delta I}{I_w} = (0.6 \pm 0.1) \log \langle B \rangle - 2.1 , \quad (9a)$$

where $\Delta$ indicates that a constant minimum value is subtracted from the Ca II K signal. Section Vc shows that the appropriate constant, the Ca II K lower limit intensity ratio, is $I_c/I_w = 0.13$. If the value of the lower limit is changed by 20%, the exponent in equation (9a) changes by 10%. Hence the slope of equation (9a) is not very sensitive to the value of the subtracted lower limit flux.

The relation (9a) can be transformed to arbitrary flux units as measured at Mount Wilson (using equation [8]):

$$\log \Delta F_{\text{Ca} II}^{\star} \text{(arb. units)} = 0.6 \log \langle B \rangle - 1.3 , \quad (9b)$$

and in calibrated flux units (see § I of the Appendix):

$$\log \Delta F_{\text{Ca} II}^{\star} \text{(ergs cm}^{-2} \text{s}^{-1}) = 0.6 \log \langle B \rangle + 4.8 , \quad (9c)$$

A reduction of the spatial resolution of the spectroheliogram and magnetogram by combining pixels into larger boxes within which the absolute values of flux densities are averaged (hereafter called "degradation of the image") increases the correlation in the scatter diagrams of Figure 4 markedly (compare Figs. 4 and 5). The correlation coefficient reaches its highest value when boxes of 6 x 6 times the original pixels are used. The power-law index of the relation does not change as the image resolution is degraded to pixels with a size of up to 6 x 6...
The minimum solar Ca II H, K flux is associated with very low values of the mean magnetic flux density (not significantly different from zero). This suggests that the stars with the lowest observed Ca II H + K emission are indeed extremely inactive, as suggested in recent literature.

We note another parallel between solar and stellar data. The flux-color diagram for G-type (solar-type) dwarfs suggests an upper limit in surface-averaged stellar Ca II H + K excess fluxes with a value near $\Delta F_{\text{Ca II}} \approx 3$, or equivalently $\Delta F_{\text{Ca II}} \approx 5 \times 10^6$ ergs cm$^{-2}$ s$^{-1}$ (e.g., Rutten and Schrijver 1987; Saar and Schrijver 1987). Figure 4 shows a saturation of the observed intensity ratios $I_i/I_w$ near a value corresponding to $\Delta F_{\text{Ca II}} \approx 3$, roughly the same as the maximum stellar value. The interpretation of the observed maximum level in the Ca II intensity ratios as a true saturation is supported by the shape of the distribution functions of the Ca II intensity ratio in narrow bins in $\langle FB \rangle$ (Fig. 4). At very low magnetic flux densities $\langle FB \rangle$ above 500 G saturation becomes apparent: the distribution functions become strongly asymmetrical, with a maximum intensity around $I_i/I_w = 0.6$, or $\Delta F_{\text{Ca II}} \approx 3$. Possible explanations for the saturation in the Ca II H + K (Saar and Schrijver 1987) include the following: (1) the magnetic heating reaches its maximum efficiency at this level; (2) the large optical thickness of the lines causes the Ca II H, K emission to saturate when the area filling factor of flux tubes approaches unity at the altitude where the Ca II line core is optically thick.

b) The Relation between Radiative and Magnetic Flux Densities

Between the two extremes in the Ca II K emission, we find a nonlinear trend between the Ca II K and magnetic flux densities for a solar active region complex and its surroundings. The mean relation between the Ca II K excess flux (obtained by subtracting the minimum observed stellar flux for solar-type stars from the solar fluxes) and the magnetic flux can be described by a power-law relation with an exponent of $0.6 \pm 0.1$ (eq. [9]). The data of SSF for a quiet solar region agree with this relation (Fig. 5b).

Strongly deviating pixels are found in the upper left corner of Figure 4 (i.e., relatively intense Ca II K emission over low or intermediate magnetic flux density). Some of these peculiar pixels (shown in Fig. 3b) are located near polarity changes. Possibly small-scale energetic phenomena, a large inclination of the magnetic flux tubes, or subaperture mixing of opposite polarities are responsible for the peculiar behavior of these pixels. Probably many of the peculiar pixels are associated with Ellerman bombs or Moustaches (Ellerman 1917; Severny 1957; Bray and Loughhead 1974). Note that all of the strongly deviating pixels lie inside the crude outer perimeter of the Ca II plage (Fig. 3a), where polarity inversions on subresolution scales are probably relatively rare.

The intrinsic scatter about the mean trend (eq. [9]) is large. Part of the scatter is caused by intrinsic fluctuations in the Ca II K line intensity, such as the chromospheric equivalent of the photospheric 5 minute oscillation, while the magnetic flux may remain unchanged. Liu and Sheeley (1971) and Liu (1974), for example, show that intensity fluctuations in the Ca II K line core are typically some 30% (half-amplitude). The horizontal...
Fig. 5.—(a) The Ca II K core wing intensity ratio $I_{core}/I_{wing}$ vs. the absolute value of the magnetic flux density $\langle fB \rangle$ after a degradation of the resolution to 14''4 x 14''4 (6 x 6 resolution elements). All data, but for data within the box, have been plotted. Eq. (9a) is represented by the dashed line. (b) The Ca II K excess intensity ratio (i.e., after subtraction of a minimal line core intensity ratio $I_{core}/I_{wing} = 0.13$, derived from stellar data, see § 1b) vs. absolute value of the magnetic flux density $\langle fB \rangle$. The power-law fit of eq. (9a), with exponent 0.6, is shown. The open circles mark the binned data of Skumanich, Smythe, and Frazier (1975; their Table 3).

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wavelengths of these oscillations (8'-12') are much larger than the apertures used by Liu (slit width 0'.52) and by us (2.4 square pixels) so that only a small reduction in amplitude should be expected due to spatial averaging by the use of a larger aperture. These Ca II K line core fluctuations account for a large fraction of the scatter in Figure 4, but we believe that additional intrinsic sources of scatter are required to explain the full amount of observed scatter. Section VIIc discusses a model that can explain both the scatter about the mean relation and its nonlinearity.

The relative scatter about the relation is largely independent of the magnetic flux density (Fig. 5b). The large intrinsic scatter in the solar data shows that there is no simple relation between radiative and magnetic flux densities on the scale of 2.4' x 2.4'. One may argue in favor of the use of a coarser spatial resolution, in particular a resolution scale on which the mean photospheric flux-tube density shows only random variations.

We propose that the existence of a characteristic length scale explains why the mean relation remains unchanged when the spectroheliogram and magnetogram are degraded to a coarser resolution, and why the scatter about the relation between Ca II K intensity and magnetic flux density is drastically reduced when one uses a resolution of 14.4' x 14.4' (see Fig. 5b). (The scatter will, of course, be reduced simply because the amplitude of the few-minute oscillation is decreased by the averaging.)

We note that the resulting mean relation is almost the same as when the Ca II K intensity is averaged for all pixels within a narrow range of magnetic flux values (see Table 1). The table shows that at low magnetic flux densities the computed averages of the Ca II K intensity ratio are somewhat higher than those predicted by equation (9a) because of the nonlinearity of the relation. At magnetic flux densities above ~400 G saturation effects become noticeable in \( \langle I_c/I_\omega \rangle \).

c) A Model for the Relation between \( \langle B \rangle \) and \( F_{Ca II} \)

If the atmospheric heating were simply proportional to the number of flux tubes within a resolution element (assuming, for simplicity, that they all have the same size), one would expect a well-defined, linear relation between the Ca II H + K and magnetic flux densities. A simple geometrical model can explain both the curvature of and the scatter about the observed mean relation between \( \Delta F_{Ca II} \) and \( \langle B \rangle \) (eq. [9]). Assume (1) that the Ca II H + K and magnetic flux densities are simply proportional if the flux tubes are isolated from each other and (2) that the heating mechanism is insensitive to the topology of the magnetic field. The supergranulation and granulation flows will push some tubes near enough to each other that the free fanning with height of the field lines in the flux tubes is restricted. Hence the effective area of the chromosphere will be reduced, yielding a lower Ca II H + K emission than expected from an isolated flux tube. The Ca II H + K flux density suffers increasingly from this saturation as the mean flux-tube density (i.e., the mean magnetic flux density) increases. This effect results in a curved relation between Ca II H + K and magnetic flux densities, which can be described by a power law with an exponent smaller than unity. The large scatter in Ca II H + K at a given magnetic flux density is attributed in this model to differences in the geometrical distribution of flux tubes within the pixel and the corresponding differences in flux-tube interaction and their Ca II H + K emission.

This purely geometrical model, relying only on the finite amount of available surface area, may not agree with stellar data. One such indication is that preliminary results show that the relation between stellar C IV and magnetic flux densities has the same slope as the relation for Ca II H + K (Saar 1987b), but without any obvious saturation. This points to changes in the temperature-density structure of flux tubes with increasing activity in addition to changes in their geometry. There are other, theoretical reasons why the interaction of closely packed flux tubes should affect the internal structure of the tubes, and even the heating mechanism. This leads to an alternative model in which the change in the flux-tube geometry, caused by the proximity of other flux tubes, affects the heating efficiency and the temperature-density structure (compare calculations by Ulmschneider, Muchmore, and Kalkofen 1987). This model recognizes that the expansion of flux tubes with height is more strongly inhibited as the flux tube density increases, with a correspondingly stronger effect on the heating efficiency and loop structure. As in the purely geometrical model discussed above, the scatter about the mean relation between radiative and magnetic fluxes is attributed to differences in the spatial distribution of flux tubes within the resolution element. The observed saturation in the Ca II K line core owing to the finite available surface area comes into this model only as an additional effect.

In conclusion, we propose that the nonlinearity of the relation between chromospheric radiative flux densities and mean magnetic flux densities and the scatter about the mean trend can be explained as a combined effect of geometrical filling of the Ca II chromosphere by the flux tubes, the dependence of the heating efficiency on flux-tube packing, and large differences in the patchiness of flux tubes within observational resolution elements.

Note that the apparent linearity of the relation between X-ray flux densities (Fig. 2a) and magnetic flux densities may imply that the chromosphere and the corona are heated by different mechanisms, but the evidence is certainly not conclusive.

d) Comparison of Solar and Stellar Data

Equation (9b), relating the Ca II K excess emission and the magnetic flux density, is derived for small pixels (with sizes between 2.4' and 14.4') on the solar surface. There is no a priori reason why the same relation should hold for hemisphere-averaged flux densities at different times in the activity cycle or for solar-type stars of different levels of magnetic activity. To our surprise, we find that the relation (9b) for small elements on the solar surface is essentially equal to relation (2b) derived from hemisphere-averaged stellar data, both in power-law index and in constant of proportionality (see Fig. 2b). The same appears to be the case for the relation between X-ray and magnetic flux densities (see Fig. 2a); the active region data agree with the crudely determined stellar relation.

In order to understand why the solar and stellar relations are the same, one must derive a relation for the hemisphere-averaged fluxes at different levels of activity by using the spatially resolved solar data. This requires a convolution of the magnetic and Ca II K flux densities with the distribution function for these quantities over the stellar surface. This falls beyond the scope of the present paper, and is treated by Schrijver and Harvey (1988). Two major conclusions can nevertheless be drawn: (a) radiative losses from individual small areas on the solar surface behave like the hemisphere-averaged radiative losses of cool stars (showing the same extremes and the same dependence on the mean magnetic flux density), and (b) the invariance of the solar equation (9b) to
surface averaging puts severe restrictions on the distribution function of field strengths over the solar surface throughout the activity cycle (Schrijver and Harvey 1988), and over stellar surfaces for stars of different levels of activity.

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APPENDIX

THE CALIBRATION OF THE MOUNT WILSON Ca II H+K MEASUREMENTS

I. THE MOUNT WILSON Ca II H+K EXCESS FLUX DENSITY

The Mount Wilson Ca II H+K flux measurements have been obtained with the HK photometer (Vaughan, Preston, and Wilson 1978) on the 1.5 m telescope at the Mount Wilson observatory. These measurements yield a flux index \( S = 19.2 \left( F_H + F_K \right) / \left( F_R + F_V \right) \), where \( F_H + F_K \) is the flux in two 1.09 Å FWHM triangular passbands centered on the H and K line cores, and \( F_V + F_R \) the flux in two 20 Å FWHM continuum passbands on either side of the H and K doublet (stray light contributions are neglected). The factor 19.2 in the expression for \( S \) is the product of an arbitrary factor 2.4, used at Mount Wilson to convert an observed count to units of flux. The fluxes are given per unit wavelength.

Rutten (1984) revised the constants, from which we derive for the Sun (\( B - V = 0.66 \)):

\[ F_{\text{Ca II}} \text{(arbitrary units)} = 10.85 S. \]  
(A1)

The effective transmission coefficient for the H and K flux windows of the Mount Wilson photometer depends on the line core profile. The effective transmission coefficient for the solar plage emission profile derived by Oranje (1983) of the triangular 1 Å FWHM transmission profile of the Mount Wilson photometer is \( T_{\text{eff}} = 0.83 \). The effective transmission coefficient of the plage emission profile for the 1 Å bandpass of the KPNO spectrophotograph is \( T_{\text{eff}} = 1.0 \), because this passband transmits virtually the entire plage profile. Oranje (1983) derived the plage emission profile neglecting small changes in the line wing intensities. The line wings are in fact somewhat sensitive to magnetic activity, but the response is weak (see, for instance, §Va) and occurs at wavelengths where the transmission of the HK photometer is much less than unity, so that \( T_{\text{eff}} = 1.0 \) is a good approximation of the true value.

The combination of the transmission coefficients with equation (A1) and the expression for \( S \) yields the relation between the excess flux in the emission profile of the K line in the 1 Å KPNO passband, \( \Delta F_e \left( \text{ergs cm}^{-2} \text{s}^{-1} \right) \) and the flux \( \Delta F_{\text{Ca II}} \) (in arbitrary units) in the emission profile folded with the triangular Mount Wilson passband:

\[ \Delta F_e = 0.0048 \left( F_R + F_V \right) \frac{T_{\text{eff}}}{T_A} \Delta F_{\text{Ca II}} = 8.5 \times 10^4 \Delta F_{\text{Ca II}}, \]  
(A2)

with \( r \) the ratio of the flux in the K line to the summed flux in the H and K lines (\( r = 0.56 \), because the K line emission is \( \approx 1.2 \) times that in the H line; Linsky 1970). The two 20 Å windows contain \( (F_R + F_V) = 2.62 \times 10^6 \text{ ergs cm}^{-2} \text{s}^{-1} \) at the solar surface (Oranje 1983).

Although the relative calibration of equation (A2) suffices for our present purpose, an absolute calibration for \( \Delta F_{\text{Ca II}} \) can also be derived, as illustrated below. The total excess flux in the H and K lines is \( 1/r \) times the excess flux in the K line. Hence the conversion factor in equation (A2) should be multiplied by \( 1/r \) to obtain the conversion factor to transform the H+K line core emission to absolute units. The conversion factor is 1.52 \( \times 10^6 \) ergs cm \(^{-2} \) s \(^{-1} \) for stars other than the Sun, provided that the shape of their plage emission profile is the same as the solar plage-emission profile used here (this is clearly not true for giants because of the Wilson-Bappu effect). This conversion factor differs from the value 1.29 \( \times 10^6 \) given by Rutten (1984) for two reasons: (1) we treat the emission core (corresponding to the excess flux) and absorption profile separately, and (2) we correct the line flux for effects of the triangular passband profile of the Mount Wilson HK photometer. The correction factor that we derive is consistent with a result from Rutten (1984), who compared fluxes measured by the 1.09 Å FWHM bandpass with fluxes measured by a 1.50 Å FWHM bandpass with a flat transmission profile in the central 1 Å. This consistency supports a posteriori the assumption that the Ca II H, K emission profiles do not differ substantially in shape for dwarf stars of different color.

II. THE LOWER LIMIT FLUX IN Ca II H+K

The flux in the 1 Å KPNO passband that corresponds to the minimal Mount Wilson flux \( F_{\text{Ca II, min}} \) may be calculated by using an equation similar to equation (A2), replacing \( \Delta F_{\text{Ca II}} \) by the flux in the minimum profile and using the appropriate transmission coefficients \( T_A \) and \( T_c \) and line strength ratio \( r \). The empirical lower limit flux for stars (1.34, see §1b), together with the corresponding absorption profile calculated by Oranje (1983), yields an effective transmission for the Mount Wilson bandpass of \( T_A = 0.33 \). The
flux in the central 1 Å (as measured by the KPNO spectrometer) amounts to only \( T_r = 0.24 \) of the flux in the 2 Å wide bandpass of the Mount Wilson triangular 1 Å FWHM transmission profile. The minimum profiles in the H and K lines are assumed identical, so that \( r = 0.5 \). Hence, for \( F_{Ca II, \text{min}} = 1.34 \) (see § 1b): \( F_c = 6.1 \times 10^5 \) ergs cm\(^{-2}\) s\(^{-1}\). With equation (6a) one finds a core wing intensity ratio \( I_c/I_w (F_{Ca II, \text{min}} = 1.34) = 0.123 \), and with equation (4) a relative line core intensity \( R_c = 0.049 \). If the value for the minimal Mount Wilson flux derived for the subsample of dwarfs is used (1.50, see § 1b), the transmission coefficient \( T_r \) equals 0.38, while \( T_c = 0.32 \), so that \( I_c/I_w (F_{Ca II, \text{min}} = 1.50) = 0.159 \), or \( R_c = 0.064 \).

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