BEAM-DRIVEN RETURN CURRENT INSTABILITY AND ANOMALOUS PLASMA HEATING IN SOLAR FLARES

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(Received 18 February 1987; in revised form 2 March, 1988)

Abstract. We consider the problem of ion-acoustic wave generation, and resultant anomalous Joule heating, by a return current driven unstable by a small-area thick-target electron beam in solar flares. With a prescribed beam current evolution, \( j_b(t) \) (and, therefore, a prescribed return current \( j_r(t) = -j_b(t) \)), and using an approximate local treatment with a two component Maxwellian plasma, and neglecting energy losses, we demonstrate the existence of two quite distinct types of ion-acoustic unstable heating regimes. First, marginally stable heating occurs when the onset of instability occurs at electron-ion temperature ratios \( T_e/T_i > 4.8 \). Secondly, there exists a 'catastrophic' heating regime for which marginally stable evolution is impossible, when the onset of instability occurs at \( T_e/T_i < 4.8 \).

For the marginally stable case, we solve the electron and ion heating equations numerically and find that rapid anomalous Ohmic heating occurs in a substantial plasma volume. This large hot plasma emits thermal bremsstrahlung hard X-rays (\( \gtrsim 20 \text{ keV} \)) comparable to, or exceeding, the nonthermal bremsstrahlung which would have been emitted by the beam in a conventional thick target, large area, collisional scenario without anomalous effects. This means that, contrary to the usual assumption, onset of return current instability need not turn off hard X-ray production by a beam, though changing its source from direct to indirect. Indeed with small beam areas, this indirect mechanism can result in a higher hard X-ray bremsstrahlung efficiency than in a conventional collisional thick target.

The catastrophic heating regime, for which we expect much larger wave levels, is discussed qualitatively, and preliminary results cited of an alternative approach, incorporating an equation directly describing the electrostatic wave energy level. Which of these two regimes will pertain in any particular case depends (discontinuously) on the beam and atmospheric parameters and we suggest that this effect may manifest itself in the distinctive temporal behaviour of X-ray flares.

1. Introduction

It is now commonly believed that electron beams, propagating downwards in the solar atmosphere, play a major role in the production, by collisional bremsstrahlung, of hard X-ray bursts during the impulsive phase of solar flares (see review by Brown and Smith, 1980) and that they may also be responsible for a considerable amount of flare atmospheric heating. In addition, it has long been recognized (e.g., Hoyng, Brown, and van Beek, 1976) that the large electron flux demanded by the observed X-ray photon flux in such an interpretation requires that a beam-neutralizing return current by set up. Ohmic dissipation of this return current increases the energy loss of the beam electrons, which also lose energy in direct Coulomb collisions with the ambient plasma, and as a result the non-thermal bremsstrahlung efficiency is reduced while plasma heating is locally increased. In the case where the return current is stable against generation of electrostatic waves (i.e., with classical Spitzer resistivity), Emslie (1980, 1981) found this

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reduction in nonthermal bremsstrahlung efficiency to be significant but not large. However, if the return current becomes unstable to generation of electrostatic waves we must expect anomalous enhancement of resistivity. The effect on hard X-ray emission will be twofold:

(a) The electric field required to drive the return current will increase, and so further reduce the lifetime of the beam electrons and the non-thermal bremsstrahlung from them.

(b) Enhanced plasma heating will take place and thereby increase the thermal bremsstrahlung output.

The possibility that a narrow channel electron beam might produce more thermal bremsstrahlung through return current plasma heating than direct non-thermal bremsstrahlung has already been suggested by Brown and Hayward (1982) and by Spicer and Sudan (1984). However, Brown and Hayward (1982) considered only Spitzer resistivity while Spicer and Sudan (1984) estimated only the temperature of the anomalously heated plasma and not its emission measure.

The purpose of the present paper, then, is to investigate quantitatively the effect of this return current instability, and corresponding anomalous resistivity, on solar hard X-ray production. This paper is an extension of a preliminary study of this process (Cromwell, McQuillan, and Brown, 1986) in two ways: firstly, we have examined the variation with plasma parameters more thoroughly and, secondly, we have included the physical effect of direct collisional heating by the beam.

In Section 2 we present a simple model of the beam-return current system, together with electron and ion-heating equations for the background plasma, and we compare our model with the Duijveman, Hoeyng, and Ionson (1981) analysis of anomalous flare plasma heating driven by a prescribed electric field. In Section 3, the dimensionless forms of the heating equations are derived, the important parameters of the problem are discussed, and the results for the classical heating phase as a function of these parameters are presented. In Section 4, we introduce the hypothesis of marginal stability to allow us to solve the electron and ion heating equations when the return current is ion-acoustic (i.a.) unstable. It turns out that meaningful marginally stable solutions only exist in a finite regime, which we interpret in terms of the existence of two distinct types of i.a. unstable heating:

Type (i): a well behaved marginally stable heating for which we present detailed numerical results; and

Type (ii): a ‘catastrophic’ heating, for which the marginal stability hypothesis fails, which we discuss qualitatively.

In Section 5, we derive formulas for the thermal and non-thermal emissivities and, for type (i) marginally stable heating, calculate and compare the thermal and non-thermal hard X-ray signatures of a beam. Finally, in Section 6, we discuss the implications of our results and make some suggestions for improvements and extensions of this work.
2. Theoretical Model

We consider a warm 2-component Maxwellian plasma (initially isothermal, \( T_e = T_i = T_0 \)), fully-ionized, of homogeneous hydrogen with density \( n_p \), into which we inject an electron beam characterized by density \( n_b(t) \) and single (mean) particle energy \( E_0 \) at injection. We emphasize that \( E_0 \) is intended, for simplicity, to represent a mean beam energy rather than a physical delta function distribution which, if real, would, on plasma time-scales, be smeared out by quasi-linear relaxation (Emstlie and Smith, 1984; McClements, 1987). Current neutralization (see, e.g., Brown and Bingham, 1984) demands that a return current be set up in the plasma on timescales much shorter than heating timescales such that the total current, i.e., plasma current \( j_p(t) \) plus beam current \( j_b(t) \), is zero. We, therefore, have \( j_p(t) = - j_b(t) \) which implies

\[
v_d(t) = -(n_b(t)/n_p)v_b, \tag{1}
\]

where \( v_b = (2E_0/m_p)^{1/2} \) is the injected beam velocity and \( v_d(t) \) is the drift velocity of the plasma electrons with respect to the plasma ions. In this paper we concentrate purely on the ion-acoustic instability of the return current itself, assuming the beam to have been quasi-linearly relaxed with mean energy \( E_0 \) and neglecting any nonlinear interplay between beam and return current instability (c.f. Rowland and Vlahos, 1985).

We adopt a typical large solar flare value for the peak electron injection rate \( F_0 \) (s\(^{-1}\)), and for simplicity allow this to be attained through linear increase in \( n_b \) from zero over typical beam rise times \( t_r \) (\( \sim 20 \) s). However, we choose the beam area, \( A \), to be well below the upper limit set by hard X-ray images in order to ensure that unstable return current drift velocity thresholds are exceeded: viz.,

\[
F_0 = 10^{36} \text{ s}^{-1} ; \quad A = 10^{16} \text{ cm}^2 \tag{2}
\]

(see Hoyng, Brown, and van Beek, 1976; and Duijveman, Hoyng, and Machado, 1982) and, therefore, our prescribed \( j_p(t) \) is

\[
j_p(t) = 5 \times 10^{10} t/t_r \text{ statamps cm}^{-2}. \tag{3}
\]

The heating equations for the plasma electrons and ions, neglecting convective, thermal conduction, and radiation losses (cf. Duijveman, Hoyng, and Ionson, 1981 and later discussion), are

\[
\frac{(3/2)\eta_p k (dT_e/dt)}{} = \frac{(3/2)\eta_p k (T_i - T_e)}{\tau_{eq}} + 2\pi e^3 \ln \Lambda n_p j_p/E_0 + \eta_{cl/j_p}^2 + \frac{\chi_{i.a.}}{\eta_{i.a.}} n_{i.a.} j_p^2, \tag{4a}
\]

\[
\frac{(3/2)\eta_p k (dT_i/dt)}{} = \frac{(3/2)\eta_p k (T_e - T_i)}{\tau_{eq}} + (1 - \chi_{i.a.}) n_{i.a.} j_p^2, \tag{4b}
\]

where

\[
\tau_{eq} = (9/128\pi)^{1/2} (m_e m_p/\ln \Lambda n_p e^4) (kT_e/m_e)^{3/2}
\tag{5}
\]

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is the electron-ion equilibration time, and
\[ \eta_{\text{cl}} = (8/9)^{1/2} m_e^{1/2} e^2 \ln A(kT_e)^{-3/2} \] (6)
is the classical resistivity (Spitzer, 1962).

\( X_{\text{a.i.}} \) and \( (1 - X_{\text{a.i.}}) \) are the fractions of the anomalous Ohmic power dissipation \( \eta_{\text{a.i.}} J_p^2 \) absorbed by the electrons and ions, respectively, as a result of collisions with ion-acoustic waves (Tange and Ichimaru, 1974), and \( \eta_{\text{a.i.}} \) is the extra contribution to resistivity due to the presence of these waves.

The second term on the right-hand side of Equation (4a) represents direct Coulomb collisional heating of the background plasma by the beam. Collisional heating by the beam electrons is in fact an inhomogeneous process: it varies with beam particle energy and, therefore, with position. In order to avoid this complication, and keep our equations homogeneous in space, we have, however, used the approximation of a fixed collisional heating rate corresponding to the injection energy \( E_0 \).

As we mentioned in the Introduction, the electron beam is decelerated by the Coulomb collisions with the ambient plasma and also by the electric field that drives the return current. Assuming that the electron time of flight is short compared to the other time-scales of the problem, the beam has a ‘stopping length’, \( s \), due to collisional and Ohmic losses (Brown and Hayward, 1982) given by
\[ s = E_0 / (e \eta_p + 2 \pi e^4 \ln \Lambda n_p / E_0) , \] (7)
where \( \eta \) is the total resistivity of the plasma.

Since \( j_p \) is increasing with time (Equation (3)), and we expect that the resistivity will be changing with time, we anticipate (from Equation (7)) that the stopping length of the beam will also be varying considerably with time. It is important to realize, therefore, that the heating Equations (4a) and (4b) apply, for all time, only to the plasma layer of thickness \( s_{\text{min}} \), the smallest depth of beam penetration into the atmosphere, because it is only this region which is heated continuously throughout the simulation.

Although, in a magnetized plasma, ion-cyclotron waves have a lower threshold drift speed than ion-acoustic waves for \( T_E / T_i < 8 \) (Kindel and Kennel, 1971), they do not contribute greatly to the anomalous resistivity (Papadopoulos, 1977; Pritchett, Ashour-Abdulla, and Dawson, 1981). For this reason, and to gain some physical insight, we consider here the problem of an unmagnetized plasma with only ion-acoustic waves being generated. (In fact early results for ion-cyclotron waves in the magnetized case do not radically change our conclusions; Cromwell, 1987). This assumption and its implications will be discussed further in Section 6.

The anomalous resistivity \( \eta_{\text{a.i.}} \) has the property that \( \eta_{\text{a.i.}} = 0 \) if the plasma drift speed \( v_d < v_{\text{crit}} (T_e, T_i) \), the critical drift speed for ion-acoustic waves. When \( v_d \geq v_{\text{crit}} \), ion-acoustic waves are generated, \( \eta_{\text{a.i.}} \) increases due to wave-particle collisions and as a result the Ohmic heating term \( \eta_{\text{a.i.}} J_p^2 \) rises leading to increased plasma heating. It should be emphasized that our situation is fundamentally different from the Duijveman, Hoyng, and Ionson (1981) model where, instead of a prescribed plasma current \( j_p(t) \), the electric field \( E(t) \) is prescribed. Specifically, their Ohmic heating term, \( E^2 / \eta \), is reduced by any
turbulent increase in resistivity $\eta$, a fact which explains their unspectacular heating rates. In other words, with a prescribed electric field, anomalous resistivity leads to reduced, rather than increased, plasma heating.

The solution of Equations (4a) and (4b) for $T_e(t)$ and $T_i(t)$ can be obtained numerically in a straightforward manner as long as $v_d$ remains less than $v_{\text{crit}}$ throughout, but, as soon as $v_d \geq v_{\text{crit}}$ we require an additional equation for $\eta_{\text{h,a}}$. In the next section we will investigate in some detail the classical heating regime, throughout which $v_d < v_{\text{crit}}$, and in Section 4 we will use the hypothesis of marginal stability to determine $\eta_{\text{h,a}}$ when $v_d \geq v_{\text{crit}}$.

3. Classical Heating

Before proceeding any further, we introduce for convenience the following dimensionless variables:

$$x = T_e/T_0, \quad y = T_i/T_0, \quad \tau = t/\tau_{\text{eq}}(0), \quad z = \eta_{\text{h,a}}/\eta_{\text{cl}}(0), \quad (8)$$

where $T_0$, $\tau_{\text{eq}}(0)$ and $\eta_{\text{cl}}(0)$ are the initial values of temperature, electron-ion equilibration time and classical resistivity respectively.

With respect to these new variables, and using Equation (3) to substitute for $j_p(t)$, our heating equations (4a) and (4b) become

$$\frac{dx}{d\tau} = \left(y - x\right)/x^{3/2} + A\tau + B\tau^2(1/x^{3/2} + \chi_{\text{i.a.}}z), \quad (9a)$$

$$\frac{dy}{d\tau} = (x - y)/x^{3/2} + B\tau^2(1 - \chi_{\text{i.a.}})z, \quad (9b)$$

where

$$A = \frac{538T_0^2}{(n_p^2\tau_r E_0)} \quad (10a)$$

and

$$B = 3.5 \times 10^{33}(T_0/n_p^2\tau_r)^2 \quad (10b)$$

(we have assumed that the Coulomb logarithm $\ln \Lambda = 20.0$) with initial conditions $x = 1$ and $y = 1$ at $\tau = 0$.

For the remainder of this section, we will restrict our attention to classical heating for which $v_d < v_{\text{crit}}(T_e, T_i)$, the critical drift speed for ion-acoustic waves. The function $v_{\text{crit}}(T_e, T_i)$ was first calculated by Fried and Gould (1961) and we will take the following approximate analytic form for it:

$$v_{\text{crit}} = 1.2(T_i/T_e) (kT_e/m_e)^{1/2}, \quad (11)$$

i.e., in dimensionless variables,

$$v_{\text{crit}} = 1.2(kT_0/m_e)^{1/2}y/x^{1/2}. \quad (12)$$

In the classical heating regime $z = 0$, and from Equations (9a), (9b), (10a), and (10b), it is clear that the classical heating is completely specified as a function of dimensionless...
time $\tau$ by two parameters $T_0/(n_p^2 t_r)$ and $T_0/E_0$. Furthermore, it is also clear, using Equation (5), that the classical heating is completely specified as a function of absolute time $\tau = \tau_{eq}(0) \tau$ by three parameters:

$$T_0/(n_p^2 t_r), \quad T_0/E_0 \quad \text{and} \quad T_0^{3/2}/n_p.$$ 

As we shall find in the next section, the value of $T_e/T_i$ at the onset of ion-acoustic instability turns out to be crucial for the subsequent anomalous evolution of the plasma. We have, therefore, solved Equations (9a) and (9b) numerically for various values of the parameters $T_0/(n_p^2 t_r)$ and $T_0/E_0$ relevant to the flaring corona, starting at $\tau = 0$ and stopping when the condition $v_d > v_{\text{crit}}$ is first encountered, i.e., stopping the simulation when the i.a. instability is reached.

In Figure 1, we have graphed the value of $T_e/T_i$ at onset of i.a. instability for $T_0/(n_p^2 t_r)$ in the range

$$5 \times 10^{-18} < T_0/(n_p^2 t_r) \quad (\text{K cm}^6 \text{ s}^{-1}) < 5 \times 10^{-15},$$

![Fig. 1. The electron-ion temperature ratio at onset of ion-acoustic instability as a function of $T_0/n_p^2 t_r$ (K cm$^6$ s$^{-1}$) and $T_0/E_0$ (K keV$^{-1}$).](image-url)
and
\[ T_0/E_0 = 5 \times 10^4, \quad 2 \times 10^5, \quad \text{and} \quad 5 \times 10^5 \quad (\text{K keV}^{-1}), \]
respectively.

In Figure 2, we have also graphed the dimensionless time \( \tau \) taken to arrive at the onset of ion-acoustic instability for the same range of \( T_0/(n_p^2 t_r) \) and \( T_0/E_0 \).

![Graph showing the dimensionless time \( \tau \) as a function of \( T_0/n_p^2 t_r \) and \( T_0/E_0 \).](image)

Fig. 2. The dimensionless time taken to reach ion-acoustic instability as a function of \( T_0/n_p^2 t_r \) (K cm\(^6\) s\(^{-1}\)) and \( T_0/E_0 \) (K keV\(^{-1}\)).

Figures 1 and 2, in conjunction with Equation (5) can be used to give the \( T_e/T_i \) value at the onset of ion-acoustic instability and the time \( t \) (s) taken to reach instability for any classical heating within the parameter range which we have explored.

E.g., with \( T_0 = 5 \times 10^6 \) K, \( n_p = 10^{11} \) cm\(^{-3}\), \( t_r = 10 \) s and \( E_0 = 25 \) keV, we have \( T_0/(n_p^2 t_r) = 5 \times 10^{-17} \) K cm\(^6\) s\(^{-1}\) and \( T_0/E_0 = 2 \times 10^5 \) K keV\(^{-1}\). Using Figure 1 these tell us that the \( T_e/T_i \) value at onset of turbulence will be \( \simeq 10 \), and using Figure 2 that
this will be reached at dimensionless time $\tau \approx 3.0$, which, using Equation (5), is at time $t = \tau_{eq}(0) \tau \approx 4.2$ s.

4. Marginal Stability

We return now to the problem of finding a value for $z = \eta_{i.a.}/\eta_{cl}(0)$ when the plasma drift speed exceeds the critical drift speed for ion-acoustic waves. A rigorous treatment would require the determination of the wave spectrum $W_\lambda$ and of the electron and ion-distribution functions self-consistently. To circumvent the complexities of this procedure, however, we make the hypothesis of marginal stability.

This approach has been successfully applied to many problems in anomalous plasma transport, such as shock structures and Tokamak temperature profiles, as described by Manheimer and Boris (1977). The marginal stability concept relates to driven systems where there is a balance between two competing mechanisms – the external driver and the natural tendency of the plasma to stabilize itself. Rapid oscillations occur around, and close to, the boundary between stability and instability, i.e., a state of marginal stability is attained.

In our present problem, the marginal stability hypothesis amounts to the assumption that as soon as $v_d$ rises to the level $v_{crit}$, where the fastest growing mode has zero-growth rate, then the system maintains $v_d = v_{crit}$, constraining the system to evolve along the marginal stability curve. Physically this means that the rapid anomalous heating raises $v_{crit}$ such as to just keep the system at wave growth onset. Using Equation (12), we therefore set

$$v_d = 1.2(kT_0/m_e)^{1/2} y/x^{1/2}$$  \hspace{1cm} (13)

and rearranging this we have

$$y = 1/1.2(m_e/kT_0)^{1/2} v_d x^{1/2}.$$  \hspace{1cm} (14)

Now, since $v_d (= j_p(t)/n_e)$ is a prescribed function of $t$, Equation (14) gives us an extra relationship between $x$ and $y$ in marginal stability. This relationship, together with the functional form of $\chi_{i.a.}$, is, in principle, enough information to continue the solution of Equations (9a) and (9b) into the anomalous regime. However, it turns out that there is a contradiction which can arise in the marginal stability treatment in some parameter regimes. Its origin is most easily demonstrated by retaining only the anomalous ion-acoustic terms in Equations (9). Retaining only these terms, Equations (9) become

$$\frac{dx}{d\tau} = B \tau^2 \chi_{i.a.} z,$$  \hspace{1cm} (15a)

$$\frac{dy}{d\tau} = B \tau^2 (1 - \chi_{i.a.}) z.$$  \hspace{1cm} (15b)

With the following analytic form for $\chi_{i.a.}$ (Tange and Ichimaru, 1974):

$$\chi_{i.a.} = 1 - c_s/v_{crit}$$

(where $c_s$ is the ion-sound speed), and using expression (12) to substitute for $v_{crit}$ this
becomes
\[ \chi_{\text{i.a.}} = 1 - (1/1.2) (m_e/m_p)^{1/2} x/y, \] (16)
i.e.,
\[ \chi_{\text{i.a.}} = 1 - x/(51.4y). \] (17)

Now, using \( v_d = f_p(t)/n_p e \), the marginal stability relationship (14) can be written in the form
\[ y = C \tau x^{1/2}, \] (18)
where \( C \) is a constant.

Differentiating this expression w.r.t. \( \tau \) and using Equations (15) to eliminate \( dx/d\tau \) and \( dy/d\tau \), the resulting equation can be rearranged to give the anomalous resistivity \( z (= n_{\text{i.a.}}/n_{\text{el}}(0)) \) explicitly as
\[ z = (Cx^{1/2}/B \tau^2) \times 1/(1 - \chi_{\text{i.a.}}(1 + y/2x)). \]

Looking at this expression we see that \( z \) is \( +ve \) or \( -ve \) according to whether \( 1 - \chi_{\text{i.a.}}(1 + y/2x) \) is \( +ve \) or \( -ve \), or, substituting for \( \chi_{\text{i.a.}} \) from Equation (17), whether \( 1 - (1 - x/(51.4y))(1 + y/2x) \) is \( +ve \) or \( -ve \). It is easy to show that this last expression is \( +ve \) for \( x/y > 4.8 \) and \( -ve \) for \( x/y < 4.8 \). Therefore, we see that the marginal stability hypothesis leads to negative anomalous resistivity if \( T_e/T_i (= x/y) < 4.8 \) at onset of ion-acoustic instability. Now, if we add Equations (15a) and (15b) we find that \( d(x + y)/d\tau = B \tau^2 z \), and so we see that negative resistivity implies a net cooling of the background plasma. This is obviously unphysical. Thus setting \( v_d = v_{\text{crit}}, \) and using the associated \( \chi = \chi_{\text{i.a.}}(T_e/T_i), \) is not consistent with \( \eta > 0, \) for \( T_e/T_i < 4.8 \).

The failure of marginal stability in this regime is, we believe, directly related to the neglect of the rate of change of plasma wave energy density, \( W, \) in the energy equations (Equations (4)). Underlying the marginal stability hypothesis is the assumption that \( dW/dt = 0 \) only over negligibly short times compared to the heating time-scale. If this is not true then \( W \) will become much larger and then the plasma would be even more rapidly heated, on time-scales far shorter than the beam rise time \( \tau_\beta, \) until eventually \( dW/dt = 0. \) To confirm this interpretation we have conducted numerical simulations (Cromwell, 1987) on beam-driven ion-acoustic wave growth. These simulations show that, for \( T_e/T_i > 4.8, \) the waves are rapidly switched on and off, as the plasma oscillates about the marginal stability curve, and that the values of \( W \) and \( dW/dt \) remain very low; marginal stability is, therefore, applicable here. However, for \( T_e/T_i < 4.8, \) non-negligible values of \( W \) and \( dW/dt \) are indeed attained and very rapid heating occurs. In these cases, the marginal stability concept is no longer applicable because the form of the driver is incompatible with the zero wave growth rate at these \( T_e/T_i \) and must be replaced by an alternative method (i.e., a proper wave growth analysis).

We, therefore, conclude that ion-acoustic instability, driven by a prescribed plasma current, may lead to two quite distinct types of anomalous plasma heating:
Type (i): if $T_e/T_i > 4.8$ at onset of instability we have a rather well-behaved heating throughout which the plasma is in a state of marginal stability.

Type (ii): if $T_e/T_i < 4.8$ at onset of instability, marginally stable evolution is impossible, and we anticipate that the plasma drift speed will exceed the marginally stable condition (13) and, as a result, the plasma will enter a 'catastrophic' heating regime in which the ion-acoustic wave levels and the resulting Ohmic heating rate will reach much larger values than the (small) marginally stable values.

This bifurcation of the plasma behaviour may have interesting observational ramifications and it is, therefore, important to relate its occurrence to the parameters of the problem.

Specifically, looking at Figure 1, we see that, from within the range of parameters $5 \times 10^{-18} < T_0/n_p^2 r, (\text{K cm}^6 \text{s}^{-1}) < 5 \times 10^{-15}$, marginally stable heating is only possible for

$$T_0/E_0 = 5 \times 10^4 \text{ K keV}^{-1} \quad \text{if} \quad T_0/n_p^2 r \leq 2.3 \times 10^{-16} \text{ K cm}^6 \text{s}^{-1}, \quad \text{or} \quad (19a)$$

$$\left(\frac{T_0}{10^6 \text{ K}}\right) / \left(\frac{E_0}{100 \text{ keV}}\right) = 5 \quad \text{if} \quad \left(\frac{T_0}{10^6 \text{ K}}\right) / \left(\left\{\left(\frac{n_p}{10^{11} \text{ cm}^{-3}}\right)^2 (t_r/10 \text{ s})\right\}\right) \leq 23;$$

$$T_0/E_0 = 2 \times 10^5 \text{ K keV}^{-1} \quad \text{if} \quad T_0/n_p^2 r \leq 4.1 \times 10^{-16} \text{ K cm}^6 \text{s}^{-1}, \quad \text{or} \quad (19b)$$

$$\left(\frac{T_0}{10^6 \text{ K}}\right) / \left(\frac{E_0}{100 \text{ keV}}\right) = 20 \quad \text{if} \quad \left(\frac{T_0}{10^6 \text{ K}}\right) / \left(\left\{\left(\frac{n_p}{10^{11} \text{ cm}^{-3}}\right)^2 (t_r/10 \text{ s})\right\}\right) \leq 41;$$

$$T_0/E_0 = 5 \times 10^5 \text{ K keV}^{-1} \quad \text{if} \quad T_0/n_p^2 r \leq 7.8 \times 10^{-16} \text{ K cm}^6 \text{s}^{-1}, \quad \text{or} \quad (19c)$$

$$\left(\frac{T_0}{10^6 \text{ K}}\right) / \left(\frac{E_0}{100 \text{ keV}}\right) = 50 \quad \text{if} \quad \left(\frac{T_0}{10^6 \text{ K}}\right) / \left(\left\{\left(\frac{n_p}{10^{11} \text{ cm}^{-3}}\right)^2 (t_r/10 \text{ s})\right\}\right) \leq 78,$$

where $[ ]$ expressions are in scaled units.

More generally, Figure 1 also reveals that, with a fixed $T_0/E_0$, marginally stable heating obtains only for small values of $T_0/n_p^2 t_r$, and, with a fixed $T_0/n_p^2 t_r$, marginally stable heating obtains only for large values of $T_0/E_0$.

Therefore, for example, with a fixed initial plasma temperature $T_0$, we conclude that catastrophic heating will set in for large electron injection energies ($E_0$), small plasma densities ($n_p$) and small rise times of the beam ($t_r$).

If we choose typical flaring coronal values for temperature and density, viz. $T_0 = 5 \times 10^6 \text{ K}$ and $n_p = 10^{11} \text{ cm}^{-3}$, and we take $E_0 = 25 \text{ keV}$, then (19b) tells us that the rise time of the beam $t_r$ must be greater than $\approx 1.2 \text{ s}$ for marginally stable ion-acoustic heating to be possible, and conversely if $t_r \leq 1.2 \text{ s}$ we may expect catastrophic heating.

Having described qualitatively the two types of heating possible, we will now give some detailed numerical results for two cases of marginally stable heating:

Case (a): $T_0 = 5 \times 10^6 \text{ K}$, $n_p = 10^{11} \text{ cm}^{-3}$, $E_0 = 25 \text{ keV}$, and $t_r = 10 \text{ s}$ which is comfortably within the marginally stable regime.
Case (b): $T_0 = 5 \times 10^6$ K, $n_p = 10^{11}$ cm$^{-3}$, $E_0 = 25$ keV, and $t_r = 2$ s which is rather closer to onset of the catastrophic regime.

We solve Equations (9a) and (9b) starting from $x = y = 1.0$ at $\tau = 0$, and using the marginal stability condition (14) when the ion-acoustic threshold is reached.

Case (a)

Figure 3(a) shows the evolution of the plasma in the $(v_d/v_e, T_e/T_i)$ plane, where $v_e = (kT_e/m_e)^{1/2}$ is the electron thermal speed. Initially, classical Ohmic heating and beam collisional heating cause an increase in $T_e/T_i$ from its starting value of 1.0, while the drift velocity rises, until the i.a. marginal stability curve is reached. The system is then constrained by condition (13) to evolve along the marginal stability curve. Figure 4(a) shows the variation of the normalized resistivity with time, ion-acoustic turbulence switching on after about 4.2 s. Figure 5(a) shows the variation of $T_e$ and $T_i$ throughout the simulation; the increase in the heating rate at about 4.2 s, due to the ion-acoustic waves, is clearly displayed. The stopping length of the beam is shown.

Fig. 3a.

Fig. 3. Evolution of the plasma in the $(v_d/v_e, T_e/T_i)$ plane. The dashed line is the ion-acoustic marginal stability curve.
in Figure 6(a) from which we see that $s_{\text{min}}$, the smallest depth of beam penetration into the plasma is $\approx 1.4 \times 10^9$ cm. The temperatures obtained after say 10 s, $T_e \approx 120T_0 \approx 6 \times 10^8$ K and $T_i \approx 12T_0 \approx 6 \times 10^7$ K (see Figure 5(a)), therefore only relate to the plasma layer of thickness $s_{\text{min}}$, because as we pointed out earlier, it is only this region which is heated continuously during the simulation.

In Figure 7(a), we have graphed the ratio collisional heating/Ohmic heating for the electrons against time, i.e., $A \tau / B \tau^2 (1/x^{3/2} + x_{\text{i,a}}^{-2})$ against time (see Equation (9a)).

This graph clearly shows that the beam collisional heating is $> \text{Ohmic heating rate}$ throughout the 20 s of the simulation. We, therefore, conclude that even in this anomalous regime, for the parameter set of case (a), collisional and Ohmic heating are equally important (cf. Emslie, 1980, 1981).

Case (b)

Figures 3(b) to 7(b) contain the graphs, corresponding to Figures 3(a)–7(a), for the parameter set (b) which represents heating which is closer to the onset of the catastrophic regime. Figures 3(b) and 4(b) when compared to 3(a) and 4(a), confirm the results of Figures 1 and 2, i.e., they demonstrate that, with increasing values of $T_0/\rho_i^2 t_e$, the $T_e/T_i$
value at onset of instability and also the dimensionless time taken to arrive at instability are reduced. Figures 4(a) and 4(b) also reveal that the closer the heating is to being catastrophic the larger is the anomalous jump in resistivity (i.e., the larger is the marginally stable wave level) at onset of instability. Looking at Figure 5(b), we see that the large values of anomalous resistivity obtained for this case lead to even greater electron and ion heating than Case (a), and in fact, after 10 s, we find if our equations remained valid that the temperature would be $T_e = 1600T_0 \approx 8 \times 10^9$ K and $T_i \approx 200T_0 \approx 10^9$ K. However, Figure 6(b) reveals that $s_{\text{min}} \approx 2.7 \times 10^8$ cm for this case, and so a smaller volume of plasma is heated in this case. In reality at such extreme temperatures our equations would break down because of cooling processes and relativistic corrections. This is even more the case at $t = 20$ s. Finally, Figure 7(b) shows that beam collisional heating is much less important than the Ohmic heating for the parameter set of Case (b).

In comparing Case (a) and Case (b), an interesting parameter to look at is the percentage of the beam kinetic energy which has been converted into plasma heating inside the volume bounded by $s_{\text{min}}$. This value turns out at the end of our runs to be
\( \eta / \eta_{cl}(o) \)

Fig. 4b.

\( \approx 83\% \) for Case (a) and \( \approx 40\% \) for Case (b). In other words about 17\% (Case (a)) and 60\% (Case (b)) of the beam kinetic energy has gone into plasma heating outside the volume bounded by \( s_{\text{min}} \). We, therefore, conclude that our simple treatment of a homogeneously heated volume will give the right order of plasma heating for cases which are well within the marginally stable regime, but the closer we go to the catastrophic regime the more energy will be deposited outside \( s_{\text{min}} \), and so the less accurate our description of the thermal X-ray emission will be. However, our treatment below of this emission is an underestimate in so far as we consider only the emission measure from the region inside \( s_{\text{min}} \), though this may be offset somewhat by our neglect of cooling processes which may reduce the temperature inside \( s_{\text{min}} \).

5. Thermal and Non-Thermal Radiation Signatures

In this section, we take the two preceding sets of results for the marginally stable heating, and calculate the thermal bremsstrahlung from the resulting rapidly heated plasmas. For comparison, we also calculate the conventional collisional thick-target non-thermal bremsstrahlung from the same beam but with Coulomb collisional losses only (as would
Fig. 5a-b. The rise in electron \((T_e)\) and ion \((T_i)\) temperatures, normalized to the initial temperature \((T_0)\), with time \(t\) (s).

be relevant to a beam with larger area. We examine the ratio of these as a function of time throughout the 20 s of the simulations.

The thermal emissivity from the volume of \(V = A s_{\text{min}}\) is

\[
\langle dJ/d\varepsilon \rangle_T = V n_p \int_\varepsilon^\infty v(E) \frac{dn}{dE} dQ/d\varepsilon(\varepsilon, E) dE ,
\]

where \(dn/dE\), the number density of plasma electrons per unit energy range, is calculated assuming a Maxwellian distribution function, \(v(E)\) is the velocity of an electron with energy \(E\), and \(dQ/d\varepsilon(\varepsilon, E)\) is the bremsstrahlung cross section differential in photon energy. For comparison purposes it will be adequate to use Kramer's cross section

\[
dQ/d\varepsilon(\varepsilon E) = Q_0 m_e c^2 / \varepsilon E ,
\]

where \(Q_0 = (8/3)x r_e^2\) in the usual notation.

The non-thermal emissivity (Brown, 1971) from the beam is

\[
\langle dJ/d\varepsilon \rangle_{NT} = \int_\varepsilon^\infty F(E) v(\varepsilon, E) dE ,
\]
where $F(E) = F_b \delta(E - E_0)$, $F_b$ is the number of electrons injected per second, and $v(\varepsilon, E)$, the number of photons per unit energy $\varepsilon$ emitted by an electron with initial energy $E$, is given by

$$
   \nu(\varepsilon, E) = \int_{\varepsilon - \varepsilon}^{E_0 = E} \frac{dQ/d\varepsilon(\varepsilon, E_0)}{n_p v(E_0)/(|dE_0/dt|)} dE_0.
$$

The energy loss equation for beam losses due to Coulomb collisions alone (i.e., neglecting even Spitzer resistive losses to the beam through the return current) is

$$
   \frac{dE_0}{dt} = -K n_p v(E_0)/E_0,
$$

where $K = 2\pi e^4 \ln \Lambda$ in the usual notation.

Using all the above, then, we find the ratio of (anomalous) thermal to (purely collisional) non-thermal thick-target emissivity is

$$
   \frac{(dJ/d\varepsilon)_T}{(dJ/d\varepsilon)_{NT}} =
   \frac{(8/\pi m_ekT_e)^{1/2} Kn_p^2 A_{\text{min}} \exp(-\varepsilon/kT_e)/F_b(E_0 - \varepsilon)}.
$$
Using the marginally stable results of the last section, together with our specified beam injection rate $F_b = 10^{36} \, t \, t_s^{-1}$ (see Section 2), Figures 8(a) and 8(b) show the variation of the thermal to non-thermal emissivity ratio, Equation (25), throughout the 20 s of our simulations for a representative* hard X-ray photon energy of $\varepsilon = 20$ keV, for Case (a) and Case (b), respectively. One must be careful in interpreting these figures. The thermal component of the hard X-rays is that emitted only from the plasma volume bounded by $s_{\text{min}}$. Clearly, by using $s_{\text{min}}$ in Equation (25), we are ignoring the plasma heating which

* Our choice of $\varepsilon = 20$ keV for $E_0 = 25$ keV is not arbitrary but should perhaps be explained since it might be thought that $(dJ/dE)_{\nu T}$ could be made arbitrarily small by choosing $\varepsilon$ arbitrarily close to $E_0$ (Equation (2.5)). Our choice is based on the fact that for a single electron of injection energy $E_0$, the nonthermal yield is (Equations (23) and (24)) $\nu(\varepsilon, E) = Q_0(E_0 - \varepsilon)/(K\varepsilon)$. Thus if we consider the conventional thick target case with an electron injection spectrum $dF/dE = AE^{-\delta}$ it follows readily that the injected electron energy $E_0$ which maximizes the yield $\nu$ at energy $\varepsilon$ is given by $d/dE[(E/\varepsilon - 1)E^{-\delta}] = 0$ or $E_0 = (\delta(\delta - 1)\varepsilon = 1.25\varepsilon$ for typical $\delta = 5$. We have, therefore, chosen $\varepsilon = 20$ keV, $E_0 = 25$ keV in the monoenergetic case to mimick most closely the usual thick target power law in terms of efficiency, while avoiding the complications of inhomogeneity required to fully incorporate a power-law injection spectrum in our present analysis.
has already taken place outside $s_{\text{min}}$, and we are, therefore, underestimating the thermal emissivity. Even neglecting the emission from outwith the minimum volume, however, Figures 8(a) and 8(b) also reveal that after a few seconds of anomalous resistive heating, the thermal emissivity has risen to approximately 1.3 times (Case (a)) and 0.18 times (Case (b)) the non-thermal thick-target emissivity (purely collisional).

Thus, when one considers a fixed beam injection rate $F_b$, and reduces the beam area $A$ until the return current goes i.a. unstable, the beam length and non-thermal bremsstrahlung are greatly reduced as is usually assumed (e.g., Hoyng, Knight, and Spicer, 1978), but enhanced thermal bremsstrahlung from the rapidly heated plasma is at least comparable to the collisional thick target emission which could be directly produced by the same beam if its return current were stable (over a larger area $A = 10^{18} - 10^{19}$ cm$^2$).

6. Discussion and Conclusions

We have presented the first quantitative analysis of the problem of ion-acoustic wave generation by a prescribed, beam-driven, return current using simplifying approximations in order to keep the problem tractable and results intelligible. Our results
demonstrate that there exist two quite distinct types of ion-acoustic unstable heating. The first is marginally stable heating regime where the onset of instability occurs at $T_e/T_i > 4.8$, and the second is a ‘catastrophic’ heating where the onset of instability occurs at $T_e/T_i < 4.8$ and a marginal stability treatment is impossible. Further, using typical flaring corona parameter values, i.e. $T_0 = 5 \times 10^6$ K, $n_p = 10^{11}$ cm$^{-3}$, and $E_0 = 25$ keV, together with $t_r = 10$ s (Case (a)), we have shown that marginally stable plasma evolution leads to rapid heating, by both anomalous Ohmic dissipation of the return current and direct beam collisions, of a substantial plasma volume; viz. (after 10 s)

$$T_e = 6 \times 10^8 \text{ K}, \quad T_i = 6 \times 10^7 \text{ K} \quad \text{in} \quad V = 1.4 \times 10^{25} \text{ cm}^3.$$

This hot plasma emits so much thermal bremsstrahlung that, contrary to previous expectations, the unstable beam-plasma systems may actually produce more hard X-rays (thermally) than does the beam (non-thermally) in the purely collisional thick-target regime relevant to larger injection areas (see Figure 8(a)).
The results we presented in Sections 4 and 5 are representative of many results we have obtained using different values of the parameters $T_0/n_e^2t_e$ and $T_0/E_0$. Specifically, we found that the closer the heating is to being catastrophic, then
(i) the larger are the subsequent values of anomalous resistivity, therefore,
(ii) the larger are the values of $T_e$ and $T_i$ attained,
(iii) the smaller is the volume of plasma heated to these temperatures,
(iv) the less important is the beam collisional term compared to the Ohmic terms,
(v) the smaller is the peak thermal to thick-target non-thermal emissivity ratio, and
finally,
(vi) the smaller is the percentage of the beam kinetic energy which is converted into plasma heating inside the volume bounded by $s_{\min}$.

In our analysis we have used a number of simplifying assumptions, the effects of which require fuller investigation in future treatments of this important problem. We now reiterate these.

Firstly, we have neglected the effect of a magnetic field and the possibility of ion-cyclotron waves being generated. It is known that for $T_e/T_i < 8$, the threshold drift speed for ion-cyclotron waves is lower than the threshold drift speed for ion-acoustic waves. Although ion-cyclotron waves saturate quickly and so produce less rapid heating
Fig. 8a–b. The ratio of thermal to collisional non-thermal thick-target emissivity as a function of time \( t \) (s), for an emitted photon energy of \( \varepsilon = 20 \text{ keV} \).

...themselves, they preferentially heat ions (i.e., \( \chi_{\text{ic}} < 0.5 \)) so that the net effect of these waves (Cromwell, 1987) is a reduction in the \( T_e/T_i \) value at onset of ion-acoustic instability (if it should occur), making the occurrence of catastrophic ion-acoustic heating more likely.

In order to study the time evolution of the plasma in this catastrophic regime, we are also at present investigating (Cromwell, 1987) an alternative approach in which we adopt a wave energy equation of the form

\[
\frac{dW}{dt} = \gamma W,
\]

where \( \gamma \) is the self-consistently calculated linear growth rate for ion-acoustic waves, together with a saturation level \( W_{\text{sat}}(T_e, T_i) \), at which the linear growth of the wave energy \( W \) is switched off. If \( T_e/T_i < 4.8 \) at onset of instability, we argued earlier that for a very short time \( dW/dt \) would act as a sink for the beam energy, thereby explaining the paradoxical negative resistivity we encountered in applying the marginally stable assumption (\( \dot{W} = 0 \)) to this regime.
With a value so found for the wave level $W$, we are then able to calculate the anomalous resistivity directly (see Hasegawa, 1974), and so to continue the calculation into the anomalous regime. This alternative approach should, of course, also be applicable when $T_e/T_i > 4.8$, and should produce a mean (oscillating) $\dot{W} = 0$ in this case. Early results (Cromwell, 1987) indeed appear to so vindicate the present analysis in the marginally stable regime and show interesting new phenomena in the catastrophic regime.

Secondly, we have seen in Figures 6(a) and 6(b), that the stopping length of the beam varies considerably throughout the simulations, but, as we have said before, the model presented here only allows us to calculate the uniform plasma heating in the volume bounded by $s_{\text{min}}$. In order to describe the heating outside $s_{\text{min}}$ we will require a treatment of the combined spatial and temporal dependence. When this extension is being undertaken it is also planned to incorporate the effect of thermal conduction which requires a spatial dependence to describe it adequately. We also plan to replace the constant beam collisional heating term $2\pi e^3 \ln \Lambda n_p j_p / E_0$ in Equation (4a), with the correct energy and, therefore, position, dependent form $2\pi e^3 \ln \Lambda n_p j_p / E$. One may wonder if the results we have obtained and the conclusions we have reached in this paper are sensitive to the form of the beam collisional heating term we have used. In this
respect, it should be pointed out that we have run our numerical simulations using a collisional term which is twice as large as the term in Equation (4a), and we have found that, although the detailed numerical values change somewhat, the conclusions remain valid.

Thirdly, our neglect of thermal conduction in this problem is not as serious an omission as one may initially suspect, because, in the anomalous regime, the thermal conduction is reduced by the same factor as the electrical resistivity is increased. The shortest conductive cooling time attainable by classical conduction is \( s_{\text{min}}/(kT_e/m_i)^{1/2} \) or about 0.15 s for the parameters of Case (a) (Figures 5(a) and 6(a)) at 10 s. However, ion-acoustic waves typically increase the conductive cooling time to around \( s_{\text{min}}/(kT_e/m_i)^{1/2} \approx 43 \) times longer (Brown, Melrose, and Spicer, 1979) \( \approx 6.4 \) s. Convective time-scales are longer still, being \( s_{\text{min}}/(kT_i/m_i)^{1/2} \approx 22 \) s for the parameters of Figures 5(a) and 6(a) at \( t = 10 \) s. Consequently, for heating which is well within the marginally stable regime, although our final temperatures will be reduced somewhat, we do not expect any radical change in our conclusions when convective and anomalous conduction are included, at least for times \( \lesssim 10 \) s. However, as we approach the still hotter catastrophic regime (e.g., Case (b)), these loss terms will become more important and, in fact, in the catastrophic regime itself, it will be vital to include both these and possibly relativistic effects also.

In conclusion, we believe that the essential result presented here of enhanced Ohmic return current dissipation leading to rapid plasma heating to hard X-ray temperatures will remain unaltered. The simple model we have described has yielded useful information and clearly demonstrated the importance of return current instability on the hard X-ray signature of solar flares.

Finally, our analysis is based on the assumption of Maxwellian velocity distributions, and quasi-study response of the temperatures to the changing beam input. This assumption may break down in two important ways. Firstly, at high heating rates, the distribution functions may not have time to relax. Secondly, our arguments concerning ‘conductive cooling’, and also thermal bremsstrahlung, ignore the uncertain processes of production and escape of Maxwellian tail particles (Brown, Melrose, and Spicer, 1979; Smith and Brown, 1980). To treat these a much more complex kinetic treatment of the whole problem will be needed.

Acknowledgements

We gratefully acknowledge the financial support of a S.E.R.C. Research Grant and Studentship, and valuable discussions with Prof. P. A. Sweet, Prof. A. E. Laing, and Dr A. L. MacKinnon.

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Proceedings of SMM Workshop on Rapid Fluctuations in Solar Flares, NASA CP 2449.