THE POSSIBLE ROLE OF MERIDIONAL FLOWS IN SUPPRESSING MAGNETIC BUOYANCY

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ABSTRACT

The equation of motion for a toroidal flux ring in a stellar convective envelope is derived. It is shown that, in the presence of an equatorward meridional flow in the convection zone, a flux ring is pushed away from the rotation axis, causing the gas inside the ring to rotate slower than the surrounding convection zone. This angular velocity difference produces a Coriolis force on the ring in the direction toward the rotation axis. Under certain conditions the ring attains an equilibrium position in which magnetic buoyancy, tension, Coriolis, and drag forces are in balance. The linear stability of this equilibrium against axisymmetric perturbations is investigated. We find that flux rings can be stable in the lower part of the convection zone where the (equatorward) flow velocity increases radially outward. The precise region of stability in the (r, θ)-plane depends on the field strength B of the flux tubes, on the entropy gradient δ = V - V_{sh} and on the details of the meridional flow pattern. The stable region shrinks with increasing field strength and disappears completely for B above 1.3 × 10^{5} G. Numerical simulations of the motions of axisymmetric flux rings are also presented. The results are in agreement with the linear stability calculations, and show that unstable flux rings rise to the surface along a path which is more or less parallel to the rotation axis. Finally, we consider the effects of longitudinal drag forces, which cause a slow migration of the flux ring toward the equator. Crude estimates indicate that the migration is sufficiently slow for the solar differential rotation to produce the toroidal flux tubes.

Subject headings: convection — hydromagnetics — Sun: interior — Sun: magnetic fields — Sun: rotation

I. INTRODUCTION

More than a decade ago, Parker (1975) pointed out that magnetic buoyancy within the solar convection zone poses a serious problem in building models of the solar dynamo. He showed that flux tubes with reasonable field strength (B > 100 G) would quickly rise to the surface, without allowing sufficient time for field amplification to occur (also see Moreno-Insertis 1983). Since then there have been several suggestions on how to circumvent this difficulty (see the review by Schüssler 1983). One suggestion by van Ballegooijen (1982) was that a toroidal flux ring can be maintained in equilibrium near the base of the convection zone due to a balance among the forces of magnetic buoyancy, Coriolis force, and the drag due to a meridional flow in the convection zone. The basic idea is explained in Figure 1, which shows a flux ring symmetric around the rotation axis. The magnetic buoyancy acts in the radially outward direction. If the flux ring is displaced toward or away from the rotation axis, conservation of angular momentum gives rise to an internal rotation which is faster or slower than the external rotation. This produces a Coriolis force which is directed toward the rotation axis for an outward displacement. The force due to magnetic tension is also in this direction. If the meridional flow at the base of the convection zone is in the equatorward direction, then drag due to this flow can presumably keep all the forces in balance. For equatorward meridional flow at the base, it thus seems possible that an azimuthal flux ring can be stored in equilibrium. However, for such an equilibrium to have any physical significance, we also require that it is stable under perturbations. Hence, the important questions are, first, whether such a stable equilibrium is in principle possible, and, second, whether it can be realized in the Sun. The answer to the first question turns out to be "yes" provided certain conditions are satisfied. But the second question probably cannot be answered definitively given our present limited knowledge of the conditions prevailing at the base of the solar convection zone.

Recent calculations by Choudhuri and Gilman (1987) show that the motions of axisymmetric flux rings within the solar convection zone can be profoundly influenced by Coriolis forces. It is our aim in the present paper to study the additional effects introduced by meridional flow in this problem. Unfortunately, our knowledge of meridional flows within the Sun, either from an observational or from a theoretical point of view, remains quite unsatisfactory. There are observational indications from Doppler measurements (Duvall 1979; Howard 1979) for a poleward flow at the solar surface with a velocity perhaps as high as 70 m s^{-1} (Beckers and Taylor 1980). However, the Doppler measurements remain somewhat uncertain because of difficulties involved in correcting for center-to-limb effects. Indirect evidence for a poleward meridional flow comes from observations of the evolution of weak, remnant magnetic field from low-latitude active regions (Howard and LaBonte 1981; Topka et al. 1982). If the observations of a poleward surface flow are real, then in order to conserve mass there must be an equatorward counterflow at some depth in the

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convection zone, presumably near the base. Because of the large density difference between the top and the bottom of the convection zone, the flow velocity at the bottom may be much smaller than that at the top, unless the equatorward flow is confined to a thin layer (thickness less than the density scale height). The theoretical picture is also somewhat unclear. There have been various attempts to explain the observed differential rotation of the solar surface using different models of the solar convection. Models which rely on a description in terms of an anisotropic eddy viscosity (Kippenhahn 1963; Köhler 1970) or latitude-dependent thermal transport (Durney and Roxburgh 1971) predict meridional flows which are equatorward at the top and poleward at the bottom, in contradiction with present observations. However, models in which differential rotation is caused by Reynolds stresses in the convection zone seem to predict meridional flows which are poleward at the surface and equatorward at the base, although they can be highly variable (Glatzmaier and Gilman 1982). In the present paper we study the effect of a steady meridional flow with a magnitude of order 10 m s⁻¹ at the base of the convection zone, directed toward the equator.

In the next section we look at the equations of motion of a toroidal flux ring, and we consider the equilibrium of such a ring. In § III we derive necessary conditions for the stability of toroidal flux rings, and we present results of stability calculations for a particular model of the meridional flow. In § IV we consider the motions of the flux rings when the rings are far from their equilibrium position or when equilibrium does not exist. The results confirm the linear stability analysis and show that in the absence of stable equilibrium the rings move toward the solar surface along a trajectory which is parallel to the rotation axis. We expect that viscosity will tend to reduce the rotational velocity difference between the flux ring and its surroundings, thus reducing the Coriolis force and altering the equilibrium. This issue is addressed in § V, where we estimate the storage time of toroidal flux rings. Finally, some implications for the Sun are discussed in § VI.

II. DYNAMICS OF TOROIDAL FLUX RINGS

Consider a thin, axisymmetric ring of magnetic flux encircling the star within the convective envelope at a radial distance \( r \) from the center and at an angle \( \theta \) relative to the rotation axis. We assume that the ring has a circular cross section with radius \( a \ll r \) and a field strength \( B \), which is uniform over the cross section and constant along the ring. In the presence of an axisymmetric meridional circulation \( v(r, \theta) \) in the convection zone, the forces (per unit volume) acting on the flux ring are the buoyancy force \( F_b \), tension force \( F_t \), Coriolis force \( F_C \), and drag force \( F_d \):

\[
F_b = \frac{B^2}{8\pi H}\hat{r},
\]

\[
F_t = -\frac{B^2}{4\pi r \sin \theta} (\hat{r} \sin \theta + \hat{\theta} \cos \theta),
\]

\[
F_C = -\rho_i r \sin \theta (\Omega_z^2 - \Omega^2) (\hat{r} \sin \theta + \hat{\theta} \cos \theta),
\]

\[
F_d = -\frac{C_d}{2\pi a} \rho_i |v_p| v_p.
\]

Equation (1) is derived from \( F_b = (\rho_l - \rho_e)g \), where \( \rho_l \) and \( \rho_e \) are the densities inside and outside the flux tube and \( g = -g\hat{r} \) is the
acceleration of gravity. Assuming pressure balance \( p_i + B^2/8\pi = p_e \) and temperature balance \( T_i = T_e \) with the surroundings, and using the ideal gas law \( p = \rho RT \), the magnitude of the buoyancy force may then be written as \( B^2/(8\pi H) \), where \( H \equiv RT \rho (g) \) is the local pressure scale height. Equation (2) gives the magnetic force associated with the curvature of the flux ring; it acts in the direction toward the rotation axis. Equation (3) gives the Coriolis force associated with the difference between the angular velocities \( \Omega_i \) and \( \Omega_e \) of the flux ring and its surroundings; if \( \Omega_i < \Omega_e \), \( F_C \) also acts in the direction toward the rotation axis. Finally, equation (4) gives the aerodynamic drag on the ring due to the meridional flow in the surrounding medium; \( \nu_p \) is the velocity of the ring with respect to its surroundings,

\[
\nu_p = \nu_i - \nu_i(r, \theta) ,
\]

and \( C_d \) is a dimensionless coefficient of order unity (Goldstein 1938; Schlichting 1979). Equation (4) gives the force per unit volume, and is derived by dividing the force per unit length along the ring, \(-\frac{1}{2}C_d a p_i |\nu_p|^2 \), by the area of the cross section, \( \pi a^2 \). The velocity of the ring is given by

\[
\nu_i = \frac{dr}{dt} \hat{r} + r \frac{d\theta}{dt} \hat{\theta} .
\]

Then the equation of motion of the flux ring is given by

\[
\mu \rho_i \frac{d\nu_i}{dt} = F_b + F_t + F_C + F_d ,
\]

where \( \mu \) is a factor of order unity describing the enhancement of the ring's inertia due to the fact that its motion imparts kinetic energy to the surrounding fluid (Lamb 1945, p. 77; Spruit 1981). For an incompressible flow around a tube with circular cross section, \( \mu = (\rho_i + \rho_e)/\rho_i \). In the present case the internal and external densities are nearly equal, so that \( \mu = 2 \).

We first consider the case in which the flux ring is stationary in the \((r, \theta)\)-plane, i.e., we look for equilibrium solutions of equation (7). Note that, unless the flux ring lies in the equatorial plane, the drag force \( F_d \) due to the meridional flow is essential for the existence of equilibrium, because the buoyance force \( F_b \) acts in the radial direction, whereas the magnetic tension and Coriolis forces are parallel to the equatorial plane. Thus the forces on the right-hand side of equation (7) can balance each other only if the drag force \( F_d \) is directed toward the equatorial plane (van Ballegooijen 1982). Assuming \( \nu_t = 0 \) and writing the external flow velocity as

\[
r \hat{r} + r \cos \theta \hat{\theta} = v_0 \hat{r} \sin \alpha ,
\]

with \( v_0 \) defining the magnitude and \( \alpha \) the direction of the meridional flow, the \( r \)- and \( \theta \)-components of equation (7) yield

\[
0 = \frac{B^2}{8\pi H} - \frac{B^2}{4\pi r} - \rho_i r \sin^2 \theta (\Omega_i^2 - \Omega_e^2) + \frac{C_d}{2\pi a} \rho_i v_0^2 \sin \alpha ,
\]

and

\[
0 = -\frac{B^2}{4\pi r} \cos \theta - \rho_i r \sin \theta \cos \theta (\Omega_i^2 - \Omega_e^2) + \frac{C_d}{2\pi a} \rho_i v_0^2 \cos \alpha .
\]

From these relations we derive the following expressions for \( (\Omega_i^2 - \Omega_e^2) \) and \( v_0^2 \):

\[
r \sin \theta (\Omega_i^2 - \Omega_e^2) = \frac{v_0^2}{2H} q ,
\]

\[
\frac{C_d}{2\pi a} v_0^2 = \frac{v_0^2}{2H} \cos \theta ,
\]

where \( v_A \equiv \sqrt{B/(4\pi \rho_i)^{1/2}} \) is the Alfvén speed and \( q \) is defined by

\[
q \equiv \frac{(1 - 2f) \sin \theta \cos \alpha + 2f \cos \theta \sin \alpha}{\sin (\theta - \alpha)} ,
\]

with

\[
f \equiv \frac{H}{r} .
\]

From equation (12) we see that \( \sin (\theta - \alpha) \) must be positive in the northern hemisphere \((\cos \theta > 0)\) and negative in the southern hemisphere \((\cos \theta < 0)\). Since the component of the meridional flow along the rotation axis is given by \(-v_0 \sin (\theta - \alpha)\), it follows that the meridional flow must be directed toward the equatorial plane, as expected.

A second requirement for equilibrium is that \( (\Omega_i^2 - \Omega_e^2) < \Omega_e^2 \), so that according to equation (11) the Alfvén speed must be smaller than some critical value,

\[
v_A < (2rH/q)^{1/2} \Omega_e \sin \theta .
\]

Hence, for a given location \((r, \theta)\) there exists a maximum field strength for which equilibrium is possible. Near the base of the convection zone the meridional flow is nearly horizontal, so we approximate \( \alpha = 0 \) for the northern hemisphere and \( \alpha = \pi \) for the

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southern hemisphere. Then equation (15) reduces to
\[ v_A < (2rH)^{1/2}(1 - 2f) \]^{-1/2}\Omega_e \sin \theta. \tag{16}

With parameter values relevant for the base of the solar convection zone (\(r_b = 4.9 \times 10^{10} \text{ cm}, H = 5.6 \times 10^9 \text{ cm}, \Omega_e = 3 \times 10^{-6} \text{ s}^{-1}, \rho_e = 0.23 \text{ g cm}^{-3}\)) we obtain a maximum Alfvén speed of \(8 \times 10^4 \sin \theta \text{ cm s}^{-1}\) and a maximum field strength of \(1.3 \times 10^5 \sin \theta \text{ G}\). Stronger fields cannot be kept at the base because of the upper limit on the Coriolis force.

The magnitude of the meridional flow necessary to hold a flux ring at the base follows from equation (12):
\[ v_0 = v_A \left( \frac{\rho_e}{C_a H \sin \theta} \right)^{1/2}. \tag{17} \]

For a flux ring at middle latitudes (\(|\cos \theta|/\sin \theta \approx 1\)) we can estimate the necessary \(v_0\) as function of flux tube radius \(a\) and field strength \(B\). The results are presented in Figure 2, which shows the values of \(a\) and \(B\) corresponding to \(v_0 = 5 \times 10^2 \text{ cm s}^{-1}\) and \(v_0 = 1.5 \times 10^3 \text{ cm s}^{-1}\). We assumed \(C_a = 0.4\), a reasonable value for high-Reynolds number flows. Note that, for a given field strength, slender tubes require a smaller meridional flow velocity than thick tubes. Flux tubes with a field strength near the maximum value (\(B_{\max} \approx 1.3 \times 10^5 \sin \theta \text{ G}\)) can be held down only if the tube radius is very small (\(a < 10^6 \text{ cm}\)). Thus the upper limit on the field strength is probably not relevant for the Sun.

III. STABILITY AGAINST AXISYMMETRIC PERTURBATIONS

Consider a flux ring in equilibrium at position \((r, \theta)\), and give the ring a small displacement \((\delta r, r \delta \theta)\). Will the ring return to its equilibrium position or will it move away from this position? To answer this question we must solve the linearized version of equation (7). The dynamical behavior of flux rings far from their equilibrium position will be considered in the next section.

To compute the perturbation of the buoyancy force, \(F_b = g(\rho_e - \rho_i)\), we assume that the flux tube moves adiabatically:
\[ \frac{\delta \rho_t}{\rho_t} = \frac{1}{\gamma} \frac{\delta \rho_e}{\rho_e}, \tag{18} \]
where \(\gamma = c_p/c_v\) is the ratio of specific heats (\(\gamma = 5/3\) at the base of the convection zone). The external density perturbation is given by
\[ \frac{\delta \rho_e}{\rho_e} = \frac{\delta \rho_e}{\rho_e} - \frac{\delta T_e}{T_e} = (1 - \nabla) \frac{\delta T_e}{T_e}, \]
where \(\nabla = d \log T_e/d \log \rho_e\) is the logarithmic temperature gradient. Since the temperature gradient is close to its adiabatic value (\(\nabla_{\text{ad}} \equiv 1 - \gamma^{-1}\)), we introduce the quantity \(\delta = \nabla - \nabla_{\text{ad}},\) which is a measure of the entropy gradient in the convection zone (\(\delta \ll 1\)).

Fig. 2.—Diagram of tube radius \(a\) vs. magnetic field strength \(B\) for azimuthal flux rings. The lines \(v_0 = \text{constant}\) indicate the meridional flow velocity necessary for equilibrium (a latitude-dependent factor is neglected; see text). The asterisks and circle indicate numerical models discussed in § IV.
Then we obtain for the density perturbation

$$\frac{\delta \rho_p}{\rho_p} = \left( \frac{1}{\gamma} - \delta \right) \frac{\delta \rho_e}{\rho_e} = -\left( \frac{1}{\gamma} - \delta \right) \frac{\delta r}{H},$$

where $H$ is the pressure scale height. We assume that the tube remains in pressure balance with its surroundings ($p_i + B^2/8\pi = \rho_i$), so that the perturbations of the internal and external pressures are related via

$$\delta p_i + \frac{B}{4\pi} \delta B = \delta p_e.$$  

Finally, we assume that the tube is initially in temperature balance with its surroundings ($T_i = T_e$). Then we obtain for the relative change of the magnitude of buoyancy force

$$\frac{\delta F_b}{F_b} = \frac{\delta g}{g} + \frac{\delta \rho_e - \delta \rho_i}{\rho_e - \rho_i} = -2\frac{\delta r}{r} + \beta \delta \frac{\delta r}{H} + 2\frac{\delta B}{\gamma B},$$

where we introduced $\beta \equiv 8\pi p_i/B^2$, and we assumed $\beta \gg 1$. We also assumed that the acceleration of gravity varies as $r^{-2}$, i.e., the layers above the flux ring contain a negligible fraction of the stellar mass. Conservation of mass ($\rho a^2 r \sin \theta = \text{constant}$) and magnetic flux ($B a^2 = \text{constant}$) yield the following relations between the perturbations of density, tube radius, and field strength:

$$\frac{\delta \rho_i}{\rho_i} + 2\frac{\delta a}{a} + \frac{\delta r}{r} + \cot \theta \delta \theta = 0,$$

$$\frac{\delta B}{B} + 2\frac{\delta a}{a} = 0.$$

Using these relations we find for the perturbation of the buoyancy force

$$\delta F_b = \frac{B^2}{8\pi H} \left\{ \left[ \left( \beta - 2f + \frac{2f}{\gamma} - \frac{2}{\gamma^2} \right) \frac{\delta r}{H} + \frac{2}{\gamma^2} \cot \theta \delta \theta \right] \hat{r} \sin \theta + \hat{\theta} \cos \theta \right\}.$$

The last term arises because of the change in direction of the buoyancy force.

Similar expressions can be derived for the perturbations of the magnetic tension, Coriolis, and drag forces. For the tension force one obtains

$$\delta F_t = -\frac{B^2}{4\pi r \sin \theta} \left\{ \left[ f - \frac{1}{\gamma} \right] \frac{\delta r}{H} + \cot \theta \delta \theta \right\} \hat{r} \sin \theta + \hat{\theta} \cos \theta.$$

Assuming there are no drag forces along the flux ring, the angular momentum of the gas within the ring is conserved ($\Omega, r^2 \sin^2 \theta = \text{constant}$). Hence, the perturbation of the internal rotation rate is given by

$$\frac{\delta \Omega_i}{\Omega_i} = -2\frac{\delta r}{r} + \cot \theta \delta \theta.$$

For simplicity, we assume that the surrounding convection zone rotates rigidly, $\delta \Omega_e = 0$. Then the perturbation of the Coriolis force is

$$\delta F_C = -\rho_i r \sin \theta \left\{ \left[ \Omega_i^2 - \Omega_f^2 \right] \left[ f - \frac{1}{\gamma} \right] \frac{\delta r}{H} + \cot \theta \delta \theta \right\} \left( \hat{r} \sin \theta + \hat{\theta} \cos \theta \right).$$

Writing the gradients of the meridional flow velocity as $A_{\phi} = \partial v_{\phi}/\partial r$, $A_{\theta} = r^{-1}\partial v_{\phi}/\partial \theta$, etc., the perturbation of the relative velocity $v_p$ is given by

$$\delta v_p = \delta v_{\phi} - \delta v_{\phi} \hat{\theta}.$$

where

$$\delta v_{\phi} = \frac{d}{dt} - (A_{\phi} \delta r + A_{\theta} r \delta \theta) + v_0 \cos \alpha \delta \theta,$$

$$\delta v_{\theta} = \frac{d}{dt} - (A_{\phi} \delta r + A_{\theta} r \delta \theta) - v_0 \sin \alpha \delta \theta.$$

To first order in the displacement,

$$|v_p| = v_0 - [\delta v_{\phi} \cos \alpha + \delta v_{\theta} \cos \alpha \sin \theta] + \cdots,$$

so the perturbation of $|v_p| v_p$ is given by

$$\delta(|v_p| v_p) = v_0 [\delta v_{\phi} \sin \alpha + \delta v_{\phi} \cos \alpha \sin \theta] + v_0 [\delta v_{\phi} \sin \alpha \cos \alpha + \delta v_{\phi} \cos \alpha \sin \theta] \hat{\theta}.$$
Then the perturbation of the drag force can be written as
\[
\delta F_d = -\frac{C_d}{2\pi a} \rho v_r \left[ \frac{1}{2} \left( f - \frac{3}{2} \frac{\delta r}{H} + \frac{1}{2} \cot \theta \delta \theta \right) \right] \left( \dot{r} \sin \alpha + \dot{\theta} \cos \alpha \right) - \frac{C_d}{2\pi a} \rho \delta \langle v_p | v_p \rangle .
\]  
\quad (31)

Inserting expressions (24), (25), (27), and (31) in the equation of motion (7), one obtains two coupled equations for \( \delta r(t) \) and \( \delta \theta(t) \). The solutions of these equations are of the form
\[
\delta r(t) \sim e^{\Gamma t},
\]  
where \( \Gamma \) is a complex number describing the growth rate and frequency of the mode. The dynamical equations can therefore be written as
\[
\begin{pmatrix} M_{rr} & M_{r\theta} \\ M_{\theta r} & M_{\theta\theta} \end{pmatrix} \begin{pmatrix} \delta r \\ \delta \theta \end{pmatrix} = 0 ,
\]  
\quad (32)

where, using equations (11) and (12),
\[
M_{rr} = -2\mu \left( \frac{H \Gamma}{v_A} \right)^2 + \beta \delta - 2f + \left( q - \frac{2}{\gamma} \left( \frac{1}{2} - f \right) + 2f \left( \frac{2}{\gamma} - f \right) \right) - x \sin^2 \theta 
\]  
\quad (33)

\[
+ \frac{\cos \theta}{\sin (\theta - \alpha)} \left[ \frac{1}{2} \left( \frac{2}{\gamma} - f \right) \right] \sin \alpha - \frac{H}{v_0} \left( \Gamma - A_{r\theta} \right) (1 + \sin^2 \alpha) + \frac{H}{v_0} A_{\theta} \sin \alpha \cos \alpha 
\]  
\quad (34)

\[
M_{r\theta} = \left( \frac{2}{\gamma} - 2f - q \right) f \cot \theta - x \sin \theta \cos \theta + \frac{\cos \theta}{\sin (\theta - \alpha)} \left[ - \frac{1}{2} f \cot \theta \sin \alpha 
\right. 
\]  
\quad (35)

\[
+ \frac{H}{v_0} A_{\theta} (1 + \sin^2 \alpha) - \frac{H}{v_0} \left( \Gamma - A_{\theta\theta} \right) \sin \alpha \cos \alpha - f \cos \alpha 
\]  
\quad (36)

\[
M_{\theta r} = \left[ 2f \left( \frac{2}{\gamma} - f \right) + \left( \frac{1}{\gamma} - f \right) q \right] \cot \theta - x \sin \theta \cos \theta 
\]  
\quad (37)

\[
+ \frac{\cos \theta}{\sin (\theta - \alpha)} \left[ \frac{1}{2} \left( \frac{3}{\gamma} - f \right) \cos \alpha - \frac{H}{v_0} \left( \Gamma - A_{\theta} \right) \sin \alpha \cos \alpha + \frac{H}{v_0} A_{\theta} (1 + \cos^2 \alpha) \right] 
\]  
\quad (38)

Here \( x \) is defined by
\[
R = \frac{8H^2 \Omega^2}{v_A^2} .
\]  
\quad (39)

The complex growth rate \( \Gamma \) is determined by the dispersion relation
\[
M_{rr} M_{\theta\theta} - M_{r\theta} M_{\theta r} = 0 .
\]  
\quad (40)

This is a fourth-order polynomial in \( \Gamma \), so in general there are four solutions corresponding to different eigenmodes of the flux ring.

\[ a \] Approximate Criterion for Instability

To get some idea of the stability conditions, we first consider the case in which the flux ring is located in the lower part of the convection zone, so that the meridional flow is nearly horizontal (\( \alpha = 0 \) in the northern hemisphere and \( \alpha = \pi \) in the southern hemisphere). Furthermore, we neglect all but the radial gradient of the flow, \( A_{\theta} \equiv \delta v_r / \delta r \). Finally, we assume that \( 2\mu (H \Gamma / v_A)^2 \ll 1 \), so that inertial effects can be neglected. Then the matrix elements (34)-(37) reduce to
\[
M_{rr} = \beta \delta - \frac{1}{\gamma} - \frac{2}{\gamma^2} - 3f + \frac{4f}{\gamma} - x \sin^2 \theta - y ,
\]  
\quad (41)

\[
M_{r\theta} = 2f \left( \frac{1}{\gamma} - 1 \right) \cos \theta - x \sin \theta \cos \theta ,
\]  
\quad (42)

\[
M_{\theta r} = 2f \left( \frac{5}{2\gamma} - \frac{3f}{2} + \frac{2f}{\gamma} + 2z \right) \cot \theta - x \sin \theta \cos \theta ,
\]  
\quad (43)

\[
M_{\theta\theta} = f - \frac{3f}{2} \cot^2 \theta - x \cos^2 \theta - 2y ,
\]  
\quad (44)
where

\[ y = \Gamma \frac{H}{v_0} \left\{ \cot \theta \right\}, \quad (44) \]

\[ z = A_{\nu} \frac{H}{v_0} \text{sgn} (\cos \theta). \quad (45) \]

Then the dispersion relation becomes

\[ 2y^2 + C_1 y + C_2 = 0, \quad (46) \]

where

\[ C_1 = x(\cos^2 \theta + 2 \sin^2 \theta) - \beta \delta - \frac{2}{\gamma} + \frac{4f}{\gamma} + \frac{8f}{\gamma} + \frac{2f}{2} \cot^2 \theta, \quad (47) \]

\[ C_2 = x \cos^2 \theta \left( \frac{3}{2\gamma} + \frac{2}{\gamma} + f + 2z - \beta \delta \right) - x f \sin^2 \theta + f \left( \beta \delta + \frac{1}{2} - \frac{2}{\gamma} - 3f + \frac{4f}{\gamma} \right) \left( 1 - \frac{3}{2} \cot^2 \theta \right) - 2f \left( \frac{1}{\gamma} - 1 \right) \left( \frac{5}{2\gamma} - \frac{3f}{2} + \frac{2x}{2} + 2z \right) \cot^2 \theta. \quad (48) \]

The two solutions of equation (46) are

\[ y^{(\pm)} = \frac{1}{2} \left[ -C_1 \pm \sqrt{C_1^2 - 4C_2} \right]. \quad (49) \]

Before analyzing equation (49) in more detail we first discuss the relative magnitude of the various quantities that enter expressions (47) and (48). Note that the entropy gradient \( \delta (= \nu - \nu_{ad}) \) appears only in the combination \( \beta \delta \), which is inversely proportional to \( B^2 \). For field strengths less than \( 10^5 \) G we have approximately \( \Omega \approx \Omega_0 \) (see § II), so that the parameter \( x \) defined in equation (38) is also inversely proportional to \( B^2 \). Hence, the ratio of \( x \) and \( \beta \delta \) is nearly independent of field strength, and the inequality \( \beta | \delta | \ll x \) corresponds to

\[ \left| \delta \right| \ll \frac{4H\delta^2}{g} \approx 3.6 \times 10^{-6}. \quad (50) \]

Mixing-length models of the solar convection zone (e.g., Spruit 1977, chap. 2) indicate that in the lower one-third of the convection zone \( \delta < 3 \times 10^{-7}, \) so inequality (50) appears to be satisfied. We now consider the case \( x \gg 1 \), which corresponds to field strengths \( B \ll 8 \times 10^4 \) G. Then the terms with \( \Delta \) in equations (47) and (48) dominate, so that

\[ C_1 \approx x(\cos^2 \theta + 2 \sin^2 \theta), \quad (51) \]

\[ C_2 \approx x \cos^2 \theta \left( \frac{3}{2\gamma} + \frac{2}{\gamma} + f + 2z - \beta \delta \right) - x f \sin^2 \theta. \quad (52) \]

We can now derive an approximate instability criterion as follows. Instability occurs if the real part of \( y^{(+)} \) or \( y^{(-)} \) is positive. Since \( C_1 > 0 \), \( \text{Re} \left[ y^{(+) \ast} \right] \) is always negative, but \( \text{Re} \left[ y^{(-) \ast} \right] \) is positive if and only if \( C_2 < 0 \). Thus, from equation (52) we find that the flux ring is unstable if

\[ \frac{3}{2\gamma} + \frac{2}{\gamma} + f + 2z - \beta \delta + f(1 - \tan^2 \theta) < 0. \quad (53) \]

As expected, we find that increasing \( \delta (= \nu - \nu_{ad}) \) has a destabilizing effect. However, increasing the velocity gradient \( z \) has a stabilizing effect; \( z > 0 \) corresponds to an equatorward meridional flow which increases radially outward (see eq. [45]). For those latitudes where the horizontal flow model applies (\( \tan \theta \sim 1 \)), the last term in equation (53) is relatively unimportant because \( f \) is rather small (\( f = H/r \approx 0.11 \) at the base). Neglecting this term and taking \( \gamma = 5/3 \), we find the following approximate criterion for instability:

\[ z < -0.81 + \frac{1}{2} \beta \delta. \quad (54) \]

For \( x \gg 1 \), the growth rate of the instability is given approximately by

\[ \Gamma \approx -\frac{v_0}{H} \left| \tan \theta \right| \frac{C_2}{C_1} \approx -\frac{v_0}{H} \left| \tan \theta \right| \left( -0.81 + \frac{1}{2} \beta \delta - z \right). \quad (55) \]

If \( C_2 > 0 \), the flux ring is stable, but, depending on the magnitude of the velocity gradient, the approach to equilibrium can be either exponential or oscillatory, depending on the sign of \( (C_1^2 - 8C_2) \). Oscillatory decay occurs when

\[ z + 0.81 - \frac{1}{2} \beta \delta > \frac{1}{16} x \cos^2 \theta (1 + 2 \tan^2 \theta)^2, \quad (56) \]
and exponential decay occurs in the opposite case. Since we assumed $x \gg 1$, oscillatory behavior occurs only if the velocity gradient $z$ is very large or if the stratification is strongly subadiabatic ($\beta \delta < -1$).

The assumption that inertial effects are negligible is justified if the terms $2\mu HT/v_{A}$ in expressions (34) and (37) are small compared to unity. This is always the case near the onset of instability, since $\Gamma \approx 0$ in that case (see eq. [55]). Thus the inertia of the flux ring has no effect on the criterion for instability; it can only change the magnitude of the growth rate.

Mixing-length models of the solar convection zone predict that in the deeper layers $\delta \sim 10^{-7}$ (e.g., Spruit 1977). Thus the term $\beta \delta$ in equation (54) becomes important when $\beta > 10^{7}$, corresponding to field strengths $B < 1.3 \times 10^{4}$ G. Due to convective overshooting, the entropy gradient actually changes sign at some level a few tenths of a pressure scale height above the base of the convecting region (van Ballegooijen 1982; Schmitt, Rosner, and Bohn 1984; Pidatella and Stix 1986). Hence, close to the base there is a so-called overshoot layer where $\delta < 0$, and within this layer the entropy gradient has a stabilizing effect on toroidal flux tubes. Since the precise entropy stratification in the lower convection zone and overshoot layer is not well known, we will neglect the effect of $\delta$ in what follows. However, it should be kept in mind that for $B < 1.3 \times 10^{4}$ G there is an additional stabilizing or destabilizing effect, depending on height above the base. Since the kinetic energy of the convection is larger than $B^{2}/8\pi$ in this regime, it is in any case questionable whether flux tubes with $B < 10^{4}$ G can exist.

b) Stability Boundary in the Meridional Plane

We now return to the general case of expressions (34)-(37); thus we allow for relatively strong fields ($x \sim 1$) and inertial effects ($\mu = 2$), but we neglect the effect of the entropy gradient ($\delta = 0$). Both the horizontal and vertical components of the meridional flow are included. The density $\rho(r)$ is approximated by

$$\rho(r) = C \left( \frac{R}{r} - 1 \right)^{n},$$

where $R$ is the stellar radius and $n = 1.5$ for an adiabatically stratified stellar envelope with a ratio of specific heat coefficients $\gamma = 5/3$. The constant $C$ follows from the density $\rho_{0}$ and radius $r_{0}$ at the base of the convection zone. The meridional flow velocity must satisfy the continuity equation, $\nabla \cdot (\rho v) = 0$; we take it to be of the form

$$v_{\phi}(r, \theta) = u_{0} \left( \frac{R}{r} \right)^{2} \left( - \frac{1}{n+1} + \frac{c_{2}}{2n+1+k+1} \xi^{2n+k} - \frac{c_{1}}{2n+1} \xi^{2n+1} \right) \sin^{m} \theta \left( (m+2) \cos^{2} \theta - \sin^{2} \theta \right),$$

$$v_{\phi}(r, \theta) = u_{0} \left( \frac{R}{r} \right)^{3} \left( -1 + \frac{c_{1}}{2n+1+k+1} \xi^{2n+1} \right) \sin^{m+1} \theta \cos \theta,$$

where $\xi(r) = (R/r) - 1$, $u_{0}$ is a measure of the velocity amplitude, and $k$ and $m$ are two free parameters which describe the radial and latitudinal dependence of the velocity, respectively ($k > 0$ and $m \geq 0$). The constants $c_{1}$ and $c_{2}$ are chosen such that $v_{\phi}(r, \theta)$ and $v_{\phi}(r, \theta)$ vanish at the base of the convection zone:

$$c_{1} = \frac{(2n+1)(n+k)}{(n+1)k} \xi_{b}^{-n},$$

$$c_{2} = \frac{(2n+1+k)n}{(n+1)k} \xi_{b}^{-(n+k)},$$

where $\xi_{b} = (R/r_{b}) - 1$. The terms involving $c_{2}$ in equations (58) and (59) cause the velocity to increase with increasing $r$ near the base of the convection zone. Therefore, in view of the above stability analysis we expect flux rings with field strengths of a few times $10^{4}$ G to be stable in this region.

To delineate the region of stability more precisely, we set up a grid in $r$ and $\theta$, and we test at each grid point whether an equilibrium is possible for a given field strength $B$ (see § II). If an equilibrium exists for the chosen $B$, we determine the internal rotation rate $\Omega$ and the tube radius $a$ using equations (11) and (12). We then evaluate the coefficients of the fourth-order polynomial (39), and solve for the complex roots $\Gamma$ which describe the growth rate and frequency of the four modes. If any $\Gamma$ has a positive real part, the flux ring is unstable, otherwise it is stable. It should be kept in mind that the tube radius $a$ is not a free parameter in this search process; $a$ is determined by the requirement that the flux ring be in equilibrium at a specified location in the $(r, \theta)$-plane.

Figure 3 presents the results for a meridional flow pattern with $k = 0.5$, $m = 0$, and $u_{0} = 10^{4}$ cm s$^{-1}$; the other parameters in the model are taken as follows: $R = 6.96 \times 10^{10}$ cm, $r_{0} = 4.87 \times 10^{10}$ cm, $\rho_{0} = 0.23$ g cm$^{-3}$, and $\Omega_{0} = 3 \times 10^{-6}$ s$^{-1}$. Figure 3a shows the velocity pattern. The maximum equatorward velocity in the model is about $1.5 \times 10^{3}$ cm s$^{-1}$ and occurs at a level of $3 \times 10^{3}$ cm above the base of the convection zone, at $\theta = 45^\circ$. Figures 3b, 3c, and 3d show where in the $(r, \theta)$-plane flux rings with $B = 1 \times 10^{4}$, $6 \times 10^{4}$, and $1 \times 10^{5}$ G are stable. We see that for $B = 10^{5}$ G the stable region extends all the way from the equator to the pole, although the actual thickness of the stable layer may be somewhat smaller when entropy gradient effects are taken into account. Apparently the equatorial region is rather stable despite the fact that the upflow in this region enhances the magnetic buoyancy of the rings. At middle latitudes the stable region extends to about $4 \times 10^{3}$ cm above the base of the convection zone, in good agreement with the prediction based on expression (54). Stability diagrams for $B \leq 4 \times 10^{4}$ G are essentially the same as Figure 3b.

As the field strength approaches its maximum value ($1.3 \times 10^{4}$ G) the stable region near the base shrinks because equilibrium is then possible only close to the equator (see § II). Note that for $B \geq 6 \times 10^{4}$ G there appears a secondary region of stability at higher latitude, in the region of downward meridional flow. We do not attach much significance to this region as a way of storing magnetic fields because the region is rather shallow and elevated above the base, i.e., it is very susceptible to fluctuations in the flow.
IV. NUMERICAL SIMULATIONS

In the previous section we showed that, if we specify the field strength $B$ of the flux ring and allow it to have any radius $a$, then we obtain an area of stability just above the base of the convection zone. If, on the other hand, we specify the radius along with the field strength, then stable equilibrium occurs only at points $(r, \theta)$ lying on a curve within the area of stability for that particular field strength. In the present section we study the motion of azimuthal flux rings with given field strength and radius, assuming the flux rings start from a point at the base of the convection zone. Such flux rings can attain equilibrium only at points on the appropriate curve of stability, or, equivalently, the curve of stability is nothing but the locus of equilibrium points for flux rings with specified field strength and radius starting from different latitudes at the base.
Choudhuri and Gilman (1987) developed a code to study the motion of axisymmetric flux rings under the influence of magnetic buoyancy, Coriolis force, and magnetic tension. We now present some calculations done with that code by extending it to include the drag due to a meridional flow. For details of how the convection zone is treated in this code the reader is referred to the original paper. We present results with the assumption that the flux rings are in thermal equilibrium with their surroundings. It has been shown that the other extreme of completely adiabatic motion gives very similar results on account of the nearly adiabatic stratification of the convection zone (Choudhuri and Gilman 1987, § 3).

We consider the motion of a flux ring in a meridional flow identical to that discussed in the previous section. In Figures 4a, 4b, and 4c we show trajectories of flux rings which start from rest at the base of the convection zone at latitudes 10°, 20°, 30°, 45°, 60°, and 75° with the same initial radius \( a = 10^8 \) cm, but with different field strengths \( B = 4 \times 10^4, 7 \times 10^4, \) and \( 10^6 \) G. These three cases are indicated by asterisks in the parameter space of Figure 2. We remind the reader that Figure 2 does not take into account the factor \(|\cos \theta|/\sin \theta\) in equation (17), and that when the flux rings eventually come to equilibrium, \( a \) and \( B \) are somewhat different from their initial values due to the stratification of the convection zone. Still, we find that the flux ring with \( B = 4 \times 10^4 \) G, which lies below the line \( v_0 = 1.5 \times 10^3 \) cm s\(^{-1}\) in Figure 2, always finds a stable equilibrium at some height before the peak value of \( 1.5 \times 10^3 \) cm s\(^{-1}\) of the meridional flow is reached. For the flux ring with \( B = 7 \times 10^4 \) G, no equilibrium is found if it starts from 45°, 60°, or 75° latitude, although it does reach an equilibrium if it starts from lower latitudes. Finally, for \( B = 10^6 \) G the flux ring keeps rising for any starting latitude. The marks on the trajectories correspond to positions of the flux rings at intervals of 50 days for \( B = 4 \times 10^4 \) G, and of 100 days for \( B = 7 \times 10^4 \) and \( 10^6 \) G.

The numerical results are in agreement with the linear stability analysis of § III. Although we see in Figure 3b that stable equilibrium is still possible at all latitudes for \( B = 10^6 \) G, it should be kept in mind that these equilibria correspond to rather small values of the tube radius \( a \), much smaller than the value \( a = 10^8 \) cm adopted in the above simulation. A look at Figure 2 would be helpful in understanding this result. To demonstrate that stable equilibrium is possible for even larger values of \( B \) provided \( a \) is small enough (in accordance with Figs. 3c and 3d), we show in Figure 4d trajectories with \( B = 4 \times 10^4 \) G and \( a = 10^6 \) cm. The marks along the trajectories indicate time intervals of 100 days. This case is represented in Figure 2 by a small circle. Since magnetic buoyancy and drag forces are rather strong in this case, the flux tubes that start from 60° latitude or less move away from the rotation axis for some time before the Coriolis force becomes sufficiently strong to establish equilibrium. It is also interesting to note that the tube which starts from 75° latitude follows a trajectory quite different from the others and eventually comes to an equilibrium considerably above the base of the convection zone. It is clear that this particular case is an example of the secondary region of stability at higher latitude found in Figures 3c and 3d and discussed in the previous section.

In all cases where an equilibrium is reached we find a nonscillatory approach to equilibrium, in accordance with equation (56). We repeated some of the calculations by eliminating the layer with \( dv_u/dr > 0 \) at the base of the convection zone. The flux rings never reached equilibrium in these cases. Thus it seems that the predictions made on the basis of linear calculations are borne out by numerical experiments.

V. THE EFFECT OF AZIMUTHAL DRAG

In the previous sections we studied the dynamics of flux rings assuming angular momentum is conserved; i.e., we neglected the effects of azimuthal drag forces on the ring. However, from equation (11) it follows that \( v_{e,\phi} = \Omega_r \sin \theta \) is smaller than \( v_{i,\phi} = \Omega_\phi \sin \theta \) for a flux ring in equilibrium. Hence, a small amount of drag will tend to speed up the ring to the external velocity, thereby increasing the angular momentum of the gas inside the ring. As a result the ring moves to a new equilibrium position. If the azimuthal drag is maintained for a time long compared with the time scale for reaching equilibrium, the ring will evolve through a series of equilibrium states. In the following we compute the resulting velocity.

The ring's angular momentum per unit mass is given by \( J = \Omega_\phi r^2 \sin^2 \theta \). The rate of change of \( J \) is equal to the torque exerted on the ring by the surrounding medium. This torque may be written as

\[
\frac{dJ}{dt} = r \sin \theta \frac{v_{e,\phi} - v_{i,\phi}}{t_f}
\]

where we introduced \( t_f \), the frictional relaxation time. For flux rings with \( v_{i,\phi}^2/(2Hr) \ll \Omega_\phi^2 \), equation (11) shows that

\[
\frac{\Omega_\phi^2 - \Omega_r^2}{\Omega_r^2} \ll 1,
\]

so we can approximate \( \Omega_\phi^2 - \Omega_r^2 \approx 2\Omega_r(\Omega_\phi - \Omega_r) \) in this case. Then equation (11) yields

\[
v_{e,\phi} - v_{i,\phi} = r \sin \theta (\Omega_\phi - \Omega_r) = \frac{v_0^2(1 - 2f)}{4\Omega_r H \sin \theta},
\]

where we assumed the meridional flow to be horizontal (\( a = 0 \)), so that \( q = 1 - 2f \) from equation (13). Since we take \( \Omega_\phi \) to be independent of \( r \) and \( \theta \), and since \( \Omega_r \approx \Omega_\phi \), the rate of change of angular momentum is given by

\[
\frac{dJ}{dt} \approx 2\Omega_r r \sin \theta \frac{d}{dt} (r \sin \theta).
\]

Equating this to expression (62) and inserting equation (64), we obtain

\[
\frac{d}{dt} (r \sin \theta) \approx \frac{v_0^2(1 - 2f)}{8\Omega_r^2 H t_f \sin \theta}.
\]
Fig. 4.—Trajectories of azimuthal flux rings in a meridional cross section of the convection zone. Initial field strength $B$ and tube radius $a$ are given by (a) $4 \times 10^3$ G, $10^8$ cm; (b) $7 \times 10^3$ G, $10^8$ cm; (c) $1 \times 10^4$ G, $10^8$ cm; (d) $4 \times 10^4$ G, $10^6$ cm.
Since the meridional flow velocity $v_0$ is a strong function of $r$ near the base of the convection zone, the motion of the flux ring will be predominantly in the $\theta$ direction. Thus the rate of change of $\theta$ is approximately

$$ \frac{d\theta}{dt} \approx \frac{v_0^2(1 - 2f)}{4\Omega^2 \sin 2\theta}. $$

Inserting parameter values appropriate for the base of the solar convection zone, we obtain for $\theta = 45^\circ$:

$$ \frac{d\theta}{dt} \approx 2.8 \times 10^{-3} B_4^2 t_f^{-1}, $$

where $B_4$ is the field strength in units of $10^4$ G. Thus, the speed at which a flux tube is transported toward the equator depends on the frictional relaxation time $t_f$.

The appropriate value of $t_f$ is difficult to estimate, since the mechanism of frictional relaxation is not known. If the friction were due to the molecular viscosity $\nu$, then the relaxation time would be given by

$$ t_f \sim \frac{a^2}{v}. $$

Using the expression for the viscosity of an ionized hydrogen plasma (Chapman 1954; Pneuman and Orrall 1986),

$$ v = 1.2 \times 10^{-16} \frac{T^{5/2}}{\rho} \text{cm}^2 \text{s}^{-1}, $$

we find $v = 3.7 \text{ cm}^2 \text{s}^{-1}$ near the base of the convection zone ($T = 2.2 \times 10^6$ K, $\rho = 0.23 \text{ g cm}^{-3}$), which yields

$$ t_f \sim 3 \times 10^{15} a^2 \text{ s}, $$

where $a$ is the tube radius in units of $10^8$ cm. Inserting this into equation (68) we find that molecular viscosity would cause the flux tube to move toward the equator at a speed of only $3 \times 10^{-11} B_4^2 a^{-2} \text{ rad yr}^{-1}$. We expect, however, that turbulent motions are present within the flux tubes, which will greatly enhance the transport of angular momentum. If the magnetic field is sufficiently strong to prevent the tubes from being strongly distorted by the fluid, then the effects of these turbulent motions could be relatively benign. For example, a slow intermingling of magnetic field lines within a flux tube would transport magnetized plasma from the "interior" to the "surface" of the tube, where it would come into closer contact with the surrounding medium and be subjected to viscous drag. As a result, the flux tube as a whole would slowly gain angular momentum, causing a slow latitudinal drift. On the other hand, if the turbulent motions are strong enough to tangle up the field, the frictional coupling with the surroundings would be too strong, preventing the azimuthal flow necessary for equilibrium. In fact, the flux tube would lose its identity altogether, and we would expect to find a more turbulent magnetic field.

We conclude that the frictional relaxation time $t_f$ depends on the nature of the turbulent motions in and around flux tubes, about which little is known. Therefore, the question whether the force balance of Figure 1 is realistic cannot be answered; further insight into the nature of magnetic and velocity fields in the convection zone is required. All we can do is to estimate for which values of $t_f$ an equilibrium would be possible. To allow sufficient time for the toroidal field to be wound up by the solar differential rotation ($\sim 5$ yr), we require that the latitudinal motion of the flux tube be less than about 0.1 rad yr$^{-1}$. This leads to a lower limit on the frictional relaxation time,

$$ t_f > 10^6 B_4^2 \text{ s}. $$

VI. DISCUSSION

The main question we are concerned with in this paper is whether flux tubes can be stored at the bottom of the convection zone due to a suppression of magnetic buoyancy by the combined effect of Coriolis force and meridional flow. It seems that such a possibility exists provided the following conditions are satisfied: (1) there has to be a meridional flow in the equatorward direction at the base of the convection zone; (2) there has to be a layer in which the gradient of the flow velocity is above a certain minimum value (see eq. [54]); (3) the flux tube must have a radius and field strength such that $v_0$ calculated from equation (17) is less than the maximum velocity within that layer; and (4) the frictional relaxation time of the tube must be larger than about $10^6 B_4^2$ s. None of these conditions seems particularly unreasonable. Whether they are realized in the Sun is an open question.

For simplicity, we restricted our discussion to a consideration of closed flux rings. The fact that most bipolar regions on the Sun emerge in accordance with Hale’s polarity law seems to suggest that the subsurface field is indeed predominantly toroidal. However, our motivation for finding a flux storage mechanism was to allow sufficient time for the solar differential rotation to amplify the field, which requires that the field must also have a poloidal component (i.e., a component in the meridional plane). Thus the flux tubes are actually spirals, winding themselves around the Sun from higher to lower latitude (Babcock 1961). We expect that a small tilt of the flux tubes with respect to the azimuthal direction does not invalidate the basic idea of our model, namely, that flux tubes have an equilibrium of the kind suggested in Figure 1. The reason is that the azimuthal velocity necessary to establish equilibrium is rather small (eq. [64]),

$$ v_{\phi} \approx 4 \times 10^2 B_4^2 (\sin \theta)^{-1} \text{ cm s}^{-1}. $$

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Hence, the time \( t_1 \) for gas inside the flux tube to circulate once around the Sun is given by

\[
t_1 = \frac{2\pi r \sin \theta}{v_{\phi} - v_{\varphi}} \approx 7.6 \times 10^6 B^{-2} \sin^2 \theta \text{ s}.
\]

(74)

This is much longer than the time it takes for the differential rotation to add an additional winding to the spiral pattern (~10 s), making it unlikely that the internal flow can significantly modify the processes by which the flux tube is formed.

One important aspect of the present model is that, with \( B \) a few times \( 10^4 \) G and meridional flows of 5-15 m s \(^{-1}\), the radii of the flux tubes must be rather small, \( a = 10^6-10^8 \text{ cm} \) (see Fig. 2). The magnetic flux contained in one such flux tube is \( 3 \times 10^{16} \text{ Mx} \), which is only a small fraction of the flux that emerges in a large active region (~3 \( \times 10^{22} \) Mx). Hence a large number of flux tubes must be present if we are to explain active regions as emerged toroidal fields. One possible model is that the toroidal field in the Sun is highly fragmented and that each individual fragment has an equilibrium of the kind suggested by Figure 1. The size of the flux tubes may be determined by a complicated process of fragmentation and coalescence of flux elements. Hence, the question whether our model applies to the Sun depends in large part on the degree of fine structure of the toroidal field, about which little is known.

In this paper we only considered stability against axisymmetric perturbations. Nonaxisymmetric instabilities of horizontal flux tubes were considered by Spruit and van Ballegooijen (1982) for the case without stellar rotation and by van Ballegooijen (1983) for the case with rotation. In both cases the flux ring was assumed to lie in the equatorial plane of the star. Van Ballegooijen (1983) took into account the effects of an azimuthal flow on the equilibrium and stability of the flux ring; however, the drag force due to a meridional flow was neglected, and only motions within the equatorial plane were considered. It was found that there are basically two types of instability. For field strengths \( B > 6 \times 10^4 \) G, the entropy stratification of the surrounding medium plays an important role, and the instability is driven by the buoyancy of the flux tube. On the other hand, for \( B < 6 \times 10^4 \) G, rotational effects dominate, and the \textit{differential rotation} plays a more important role. Flux rings were found to be unstable against nonaxisymmetric perturbations if \( \Omega(r) \) decreases radially outward; it is likely that this instability also occurs in some form in the present model, with the flux ring located at nonzero latitude. Therefore, a further study of nonaxisymmetric effects is required.

A number of ideas for suppressing magnetic buoyancy have been proposed, such as storing the flux in a stably stratified layer, using thermal shadows (Parker 1987), or using meridional flows (present paper). These models have one aspect in common: they all suggest a stable region at or near the base of the convection zone where magnetic flux can reside for some time while it is amplified by the solar dynamo. It is expected that parts of the flux tubes will occasionally come out of this stable region, either due to perturbations by convective flows in the overlying medium or due to the development of instabilities in the field (e.g., Moreno-Insertis 1986). According to conventional wisdom, the \( \Omega \)-shaped flux loops that are formed this way can rise to the surface and eventually produce the active regions. However, recent investigations by Choudhuri and Gilman (1988) show that the problem is not as straightforward as one might expect. For values of the initial field strength less than \( 10^5 \) G, the tops of the \( \Omega \)-shaped loops tend to move parallel to the rotation axis and emerge at very high latitude if the convection zone is assumed to rotate as a solid body. It is clear that more research is needed to understand the connection between the magnetic field generated by the dynamo and the emergence of active regions.

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