HELIOSEISMLOGICAL SPLITTING MEASUREMENTS AND THE NONSPHERICAL SOLAR TEMPERATURE STRUCTURE

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ABSTRACT

Helioseismological observations continue to yield new information on the structure of the solar interior. Measurements of the splitting of the frequency degeneracy of modes of a given spherical harmonic order already provide a measure of the internal solar radial differential rotation. We show here how the even-order terms may be related to an internal nonspherical solar temperature structure. In addition, the apparent inconsistency of the different splitting measurements may be understood from the time dependence observed in the limb observations, which, as shown in this Letter, imply a corresponding temporal variation in the splitting data.

Subject headings: Sun: interior — Sun: oscillations

I. INTRODUCTION

The Sun's acoustic spectrum is observationally known, at frequencies near 3300 µHz, to about 1 part in 10^6, but is consistent with the best spherically symmetric model Sun to only about 1 part in 10^3 (see Duvall et al. 1988; Christensen-Dalsgaard 1982). Model frequencies depend on a radial quantum number, n, and the spherical harmonic degree, l, of the corresponding eigenmode. Observed and calculated spectral differences have not been reconciled to the level demanded by the precision of the observed frequencies. Despite model uncertainties, definite conclusions regarding the interior solar structure are possible using current data. For example Christensen-Dalsgaard et al. (1985) determined the average sound speed versus depth in the Sun. From such an approximate spherical model the nonspherical Sun may be inferred from the m dependence of the eigenfrequencies using perturbative calculations, since the tangential variations are small compared to the radial gradients of the important physical quantities in the Sun.

We expect solar rotation, magnetic fields, global circulation or temperature fields, and a nonspherical shape of equipotential surfaces in the Sun to split the m degeneracy of its vibrational modes, while incomplete knowledge of the radial structure of the sound speed, buoyancy frequency, and density structure does not affect splitting estimates at the level of the observational errors. In this sense an inversion of the splitting data requires fewer unknown parameters than, for example, the inversion of eigenfrequencies to get the radial structure of the interior sound speed (see Gough 1987). Brown (1987) has discussed how the radial and latitudinal variation in the solar rotation may be inferred from splitting data. A convenient description of the m-dependence is to take

$$\delta \omega_{nml} = \omega_{nml} - \omega_{n,0} = 2\pi l \sum_i a_i P_i(-m/l), \quad (1)$$

where \(P_l\) are the Legendre polynomials of degree \(l\), and \(\omega_{nml}\) is the measured angular frequency of the eigenmode of degree \(l\), azimuthal order \(m\), and radial order \(n\). Then, with reasonable assumptions on the rotation gradient and magnetic fields, the dominant contributions to \(a_1\), \(a_3\), etc., are from the solar rotation.

The even-order terms are smaller than the odd expansion coefficients in equation (1). Brown (1987) noted a l/l dependence in his data which is important. Table 1 summarizes the observational results (define \(b_i = Q_i\)). The date of the observations and \(b_i\) are indicated if enough data were given to calculate the l dependence (the indicated error in \(b_i\) is the 1 play standard error from my unweighted fit of \(b_{i,l}\) to the published \(a_i\)). Note that the \(b_4\) terms show less yearly variability than the \(b_2\) coefficient and that there is a clear secular trend to decreasing \(b\) values between the earliest measurements in 1981 and the last available data in 1986.6.

The even-order terms may be due to a solar structural asymmetry due to, for example, magnetic fields (Dicke 1982) or the solar shape (Gough and Taylor 1984). We can rule out the Sun's shape as the cause of a \(a_2 > 0\) since this requires a prolate limb profile which strongly disagrees with the observations (Dike, Kuhn, and Libbrecht 1987). A well-placed interior magnetic field might produce this result, but this Letter suggests another explanation—a nonspherical sound speed variation. Since the surface temperature field is directly observed, we will describe the sound speed variation in terms of an interior temperature field which can be directly related to the photometric limb measurements. It should be noted that such a temperature variation might also be related to, for example, magnetic changes in the convective flux, e.g., a "thermal" shadow of a deep toroidal magnetic field (Parker 1987). Thus, the temperature signal must in turn be directly related to, as yet, unspecified Reynolds or magnetic stresses.

II. EFFECTS OF A NONSPHERICAL SOLAR SOUND SPEED

A global latitudinal variation in the sound speed which is symmetric about the equator will lead to nonzero \(\delta_{24}\) (an antisymmetric, azimuthally constant perturbation leads to no first-order m-dependent frequency shift). Since the temperature varies as \(c^2\), and since a \(P_2\) and \(P_4\)-dominated lim temperature variation has been observed, it is interesting to see how this will affect the measured splitting coefficients, \(b_2\) and \(b_4\).

Table 1 also summarizes the amplitude, in K, of the \(P_2\) and \(P_4\) terms (labeled \(t_2\) and \(t_4\)) that best describe the limb temperature modulation. The errors are 1 σ fit errors and are rela-
then the eigenvalues of equation (2) are shifted by an amount 
model sound speed of the form 
direct integration of the oscillation equations.
approximate but easily solved (it can be readily reduced to a
symmetric tridiagonal matrix eigenvalue problem). This for-
marginal decreasing trend between 1983.6 and 1987.6. We
these quantities. By considering high-degree p-modes we may
spherical harmonic order, l, and some radial derivatives of
amplitude of the P
2
spatial term changes with a several year time scale
the rotation-sensitive a
2
+1
coefficients). The lack of an m-
dependent term indicates that if modes of different m value
in unresolved spectra then there will be no bias in the mean eigenfrequencies, \( \omega_m \), from such a tem-
perature perturbation. The surface temperature \( T_s = 5700 \) K
enters equation (3) because the radial function \( f(r) \) is normal-
ized to a surface value of unity.

### III. RELATING FREQUENCY SHIFTS AND TEMPERATURE PERTURBATIONS

To relate the fractional temperature and frequency shift amplitudes \( \tau_2i \) and \( \beta_2i \) we need to evaluate the radial term \( I_{nl} \) in equation (3). The \( n \) eigenvectors of the discretized version of equation (2), \( V_{nl}^{(l)} \) for \( i = 1 \ldots n \) at a fixed \( l \), are orthogonal with respect to a weighting function \( W_l \) so that
\[
\delta_{ij} = \sum_k V_{nk}^{(l)} V_{ik}^{(l)} W_l
\]
It is not difficult to show that if \( r_2i \) \( f_s/T_s \) is the relative variation in \( c^2 \) at radius \( r_l \) of the \( P_{2i} \) coefficient, then
\[
I_{nl}(f_s) = \sum_k V_{nk}^{(l)} W_l V_{lk}^{(l)} f_s = \sum_k I_{nlk} f_s,
\]
where the \( l \) dependence of \( W_{nl} \), \( f_s \), and \( V^2 \) is implicit. Thus, \( I_{nl} \) is a radial averaging kernel for the observed frequency splitting. The observed splitting coefficients are effectively averaged over \( n \) at fixed \( l \). Figure 1 plots the corresponding averaged \( I_{nl} = \langle I_{nlk} \rangle \) for \( l = 5 \) and \( l = 60 \) and \( k = 1, \ldots, 64 \). These curves show the approximate sensitivity of the \( l = 5 \) and \( l = 60 \) splitting coefficients to the radial structure of the relative temperature perturbation. Given the limited splitting data in Table 1, a complete inversion for the radial temperature profile is not possible. Instead we will consider (1) how much of a
radially constant relative temperature perturbation is needed
to yield the observed \( \beta_2i \) and \( \beta_2i \) coefficients. If the perturbation expansion results are negligibly changed by using the eigenfunctions of a direct integration of the oscillation equations. The perturbation expansion results are negligibly changed by using the eigenfunctions of a direct integration of the oscillation equations.

If we consider a small angular and radial variation in the model sound speed of the form
\[
c^2(r) = c^2_0[r + \sum_i f_2i(r)P_{2i}(\cos \theta)/T_0],
\]
then the eigenvalues of equation (2) are shifted by an amount \( \delta \omega_{nl} \) given by
\[
\frac{2 \delta \omega_{nl}}{\omega_{nl}} = \sum_i \sqrt{\frac{4 \pi}{4i + 1} C(l, m, 2i, \tau_2i) I_{nl}[f_2i(r)]/T_0},
\]
where \( I_{nl}[f_2i(r)] \) is a radial integral over \( y_{nl}, f_2i(r) \), and functions of the nonconstant coefficients and \( C(l, m, 2i) \) is an angular integral of a triple product of spherical harmonics that may be cleanly expressed in terms of Wigner 3J coefficients (Edmunds 1957). For \( l > 2i \) we get approximately
\[
C(l, m, 2i) = -0.3 \times P_2/2i and C(l, m, 4i) = 0.3 \times P_2/2i.
\]
Thus a \( P_2 \) or \( P_4 \) spatial term indicates that if modes of different \( m \) value are weighted equally in unresolved spectra then there will be no bias in the mean eigenfrequencies, \( \omega_m \), from such a temperature perturbation. The surface temperature \( T_s = 5700 \) K enters equation (3) because the radial function \( f(r) \) is normalized to a surface value of unity.
bation with depth, i.e., \( f_k = f = 1 \), yields \( I_{nl} = 1 \), and from equation (3) we obtain approximately
\[
b_2 = -0.14t_2, \quad b_4 = 0.11t_4,
\]
where the \( b \) coefficients are expressed in \( \mu \text{Hz} \) and \( t_2 \) and \( t_4 \) are the corresponding surface \( P_2 \) and \( P_4 \) temperature field coefficients in \( K \) (comparable to the observed coefficients in Table 1). Note that this temperature model allows no \( l \) dependence to the \( b_{2l} \) splitting coefficients, which is consistent with the observation that the measured \( a_{2l} \) show a \( 1/l \) dependence (implying constant \( b_{2l} \)).

IV. CONCLUSIONS

Ignoring for the moment the secular variation in the data of Table 1, we consider the mean \( t_2 \) and \( t_4 \) from the sixth and seventh columns. Assuming the above temperature model, we can estimate the magnitude of the even-order splitting coefficients from equation (5). The mean \( t \) values suggest that \( b_2 \approx -60 \text{ nHz} \) and \( b_4 \approx 130 \text{ nHz} \). In fact the mean \( b_4 = 82 \text{ nHz} \) and \( b_2 = 96 \text{ nHz} \). Both coefficients are of the correct magnitude, and the dominant \( P_4 \) temperature term agrees to within the measurement uncertainties. Undoubtedly, the internal temperature structure is more complicated than is suggested by this simple model, and we have ignored the temporal variations suggested by the measurements. Also, the relation between splitting and surface temperature is approximate because the limb observations measure the brightness temperature at a slightly different height in the solar atmosphere than the oscillation model. Nevertheless, the rough correspondence suggests that this simple model is an appropriate approximation.

Other qualitative conclusions follow from Table 1 and equation (5). Notice that \( t_2 \) was increasing during the solar cycle from 1983 to 1987. The negative coefficient in equation (5) predicts that \( b_2 \) should decline during this period—as it did. Also \( t_4 \) was nearly constant or only slightly decreasing during this time. The positive coefficient in equation (5) suggests that \( b_4 \) should do the same. In fact, the fractional decrease in \( b_4 \) was much smaller than the corresponding change in \( b_2 \), as expected.

The temperature kernels in Figure 1 show that between \( l = 5 \) and \( l = 60 \) the splitting data are sensitive to the internal temperature structure outside about 0.2 solar radii—although all of the kernels are peaked toward the surface and mostly sensitive to the temperature structure in the top part of the convection zone. Nevertheless, we can begin to ask how the depth dependence of the temperature asphericity affects the splitting data from a simple "forward" calculation. For example, we will consider a model with no temperature asphericity below a fixed radius, but with a smoothly rising perturbation that matches the surface temperature results. Thus we take \( f_k = 0 \) at some inner radius \( r_0 \) and assume \( f_k \) rises linearly to \( f_k = 1 \) at the outer boundary. Assuming a surface temperature of \( t_4 \approx 1 \text{ K} \), it follows that to obtain \( b_4 \approx 80 \text{ nHz} \) requires that \( r_0 \approx 0.2 R_\odot \) for \( l = 5 \) modes. The corresponding \( l \) dependence to the derived \( b_4 \) between \( l = 5 \) and \( l = 60 \) in this model is only about 50%—probably not enough for the radial profile to be constrained by the observed splitting data (yet). This suggests that the mean asphericity in the Sun's temperature distribution extends beneath the photosphere.

Using a simple aspherical solar internal sound speed model the apparent discrepancy between different observations of the even-order solar oscillation splitting coefficients may be resolved and reconciled with the independent photometric limb temperature measurements. The data suggest that the nonspherical interior temperature structure may extend deep into the convection zone with an amplitude of nearly \( 10^{-3} \) of the angle-averaged temperature at a given radial distance from the center. The data support the notion that this interior structure varies over a several year time scale.

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