ON THE GENERATION OF MAGNETOHYDRODYNAMIC WAVES IN A STRATIFIED AND MAGNETIZED FLUID. II. MAGNETOHYDRODYNAMIC ENERGY FLUXES FOR LATE-TYPE STARS

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ABSTRACT

Magnetohydrodynamic (MHD) wave energy fluxes for late-type stars are calculated, using previously obtained formulae for the source functions for the generation of MHD waves in a stratified, but otherwise uniform, turbulent atmosphere; the magnetic fields in the wave generation region are assumed to be homogeneous. In contradiction to previous results, we show that in this uniform magnetic field case there is no significant increase in the efficiency of MHD wave generation, at least within the theory's limits of applicability. The major results are that the MHD energy fluxes we calculate for late-type stars are less than those obtained for compressible modes in the magnetic field-free case, and that these MHD energy fluxes do not vary enough for a given spectral type to explain the observed range of UV and X-ray fluxes from such stars. We therefore conclude that MHD waves in stellar atmospheres with homogeneous magnetic fields in the wave generation region cannot explain the observed stellar coronal emissions; if such MHD waves are responsible for a significant component of stellar coronal heating, then nonuniform fields within the generation region must be appealed to.

Subject headings: hydromagnetics — stars: atmospheres — stars: late-type — turbulence

I. INTRODUCTION

The problem of wave generation by turbulent motions in the outer convective zones of stars is central to the heating of stellar chromospheres and coronae. Beginning with Biermann (1946) and Schwarzschild (1948), and continuing with the work of Lighthill (1952) and Proudman (1952), the initial focus was on the generation of acoustic waves, which was thought to be dominated by quadrupole emission. The effects of stratification, and the dependence of results on the turbulence spectra, were investigated in the 1960s (cf. Stein 1967), and predictions of the energy fluxes for the Sun and other late-type stars were constructed in the following decade (cf. de Loore 1970; Renzini et al. 1977; Ulmschneider 1979). These calculations were carried out immediately before the flights of the International Ultraviolet Explorer (IUE) and the Einstein Observatory, whose results (in the UV and soft X-ray, respectively) established in gross outline the observational requirements for any comprehensive theory for the heating of the hot outer layers of stars (Linsky 1981; Vaiana et al. 1981). These observational results made plain the inadequacy of the simpler coronal heating theories, which did not take into account effects such as stratification and the presence of magnetic fields (which undoubtedly vary considerably throughout the H-R diagram); and it was not surprising that these deficiencies were soon remedied. Thus, Bohn (1980, 1984) made comprehensive calculations for the expected acoustic energy fluxes for stars throughout the H-R diagram, following Stein's approach, and Stein (1981) and Ulmschneider and Stein (1982) considered the effects of magnetic fields on the generation of waves in stellar surface layers (following the earlier work of Kulsrud 1955, Osterbrock 1961, Parker 1964, and Kuperus 1965, who argued that the efficiency of wave generation could be substantially increased by the presence of magnetic fields). These latter calculations are all based on the assumption of a uniform background magnetic field in the wave generation region.

In this paper we present a different point of view, and show that there is no significant increase in the efficiency of the generation of MHD waves in an atmosphere in which the background magnetic field is unstructured, at least as long as the theory (based on the original Lighthill approach) is not pushed beyond the limits of its validity. We adopt the expressions for the MHD energy fluxes obtained by Musielak and Rosner (1987, hereafter Paper I) and apply these formal expressions to evaluate the wave energy fluxes for the Sun and other late-type stars. Our approach (valid as long as magnetic field gradients are negligible) extends Stein's calculations by including magnetic field effects, and treats the generation of fast, slow, and Alfvén waves separately.

The plan of our paper is as follows: the computational procedure is described in § II; the MHD energy fluxes for late-type stars are presented and discussed in § III; and the concluding remarks are to be found in § IV.

II. THE COMPUTATIONAL PROCEDURE

In order to calculate the MHD energy fluxes for late-type stars, a model for the stellar convective zone must be provided, and the various free parameters entering into the calculations and into the turbulence spectrum have to be specified. In the following, we deal with each of these problems in turn.

a) Modeling of Stellar Convective Zones

The model convective zone used by us is based on the envelope model of Paczyński (1969, 1970), which was originally
developed in order to calculate the interior structure of late-
type giants. The envelope code has been recently modified
(Paczynski 1984) to include the effect of pressure ioniza-
tion and to treat the simultaneous ionization of H i, He i, and He ii
in a self-consistent manner (see Ratcliff 1987 for more details).
This code allows us to calculate the structure of convective
zones for main-sequence stars as well as for the giants. The Los
Alamos opacity tables (Huebner et al. 1977) originally used by
the code have been extended according to Alexander, Johnson,
and Rypma (1983) to include the effects of molecules and
grains (an essential modification for stars in the lower effective
temperature range). The standard mixing-length theory for
convection, as described by Cox and Giuli (1968), was used to
calculate the average convective velocity \( \bar{v}_c \),

\[
\bar{v}_c = \frac{\rho \alpha Q(\nabla - \nabla')}{{a_1}^3 {H_p}},
\]

the average convective flux \( F_c \),

\[
F_c = \rho \bar{v}_c C_p T(\nabla - \nabla'),
\]

and the convective efficiency,

\[
\Gamma = \frac{\nabla - \nabla'}{\nabla - \nabla_{\text{ad}}} = \frac{C_p \alpha^2 \sigma^4}{a_3} \frac{\nabla - \nabla}{\nabla_{\text{ad}}},
\]

where \( a_1, a_2, \) and \( a_3 \) are equal to 8, 2, and 24, respectively;
the notation used here is identical to that of Cox and Giuli (1968).
The mixing-length parameter \( l \) is assumed to be equal to the
local pressure scale height \( H_p \) in all our calculations; although
the results do depend upon this assumption, we shall not
concern ourselves with varying \( l/H_p \), since the focus of our
study is comparison of calculations with and without magnetic
fields for fixed \( l/H_p \). These well-known equations are displayed
here merely to show the numerical constants we chose in our
definition of the standard mixing-length theory; this point is
not always obvious in the literature (see, for example, de Loore
1970 or Fontaine, Villeneuve, and Wilson 1981 for discus-
sions).

b) Choosing the Free Parameters

To calculate the MHD energy spectra, we have used equa-
tions (4.8)-(4.14) given in Paper I. These equations depend on
the physical parameters describing the structure of the convec-
tive zone, as well as on the turbulence spectrum. In addition,
there are three entirely free parameters: the wave frequency,
the strength of the magnetic field, and the angle between
the magnetic field and the wave propagation vector; we postpone
discussion of the turbulence spectrum to the next subsection,
and instead now show how these free parameters entering into
our calculations have been chosen.

In the approach presented in Paper I, the background mag-
netic field is assumed to be uniform, but can be oriented at an
arbitrary angle \( \theta \) with respect to the vertical; our results are
also valid for small (compared with the density) gradients in
the magnetic field, so that we can treat the magnetic field as
"locally" uniform and derive a local dispersion relation (using
the WKB approximation). This local dispersion approach
restricts our results to high-frequency waves, e.g., waves with
frequencies well above the acoustic cutoff frequency, so that in
this approximation all gradients in the propagation operator
can be neglected. An important further advantage of this
approach is that we were not forced to solve an eigenvalue
problem numerically, and hence were able to obtain analytical
formulae for the MHD energy fluxes; however, to keep the
approach self-consistent, only the energy fluxes generated in
high-frequency waves can be estimated. It should also be noted
that our approach is further circumscribed by the requirement
that the sound speed \( V_s \) be larger than the Alfvén velocity \( V_A \).

In the calculations presented in this paper, we have assumed
that the background magnetic field energy density is 0.2 of the
total local internal energy density (e.g., gas pressure plus local
kinetic energy, the latter usually being a small contribution); in
order to test this assumption, we have determined the depen-
dence of our results on variations in magnetic field strength;
this will be discussed further in § III. The angle dependence
of the energy fluxes has been considered by allowing the angle \( \theta \)
between the wave propagation vector and the local magnetic
field vary between 0° and 45°. Finally, the lower limit for the
frequency of the waves considered here has been set at 2.5
times the acoustic cutoff frequency (i.e., well into the domain
of applicability of our approximations.)

Let the spectra be partitioned into two classes: (a) those that
are frequency independent at all frequencies (i.e., the Keplerian
form; e.g., we adopt the Gaussian form

\[
E(k, \omega) = E(k)\delta(\omega/ku)
\]

(see Batchelor 1953; Kraichnan 1957). In addition, if the range
of wavenumbers is sufficiently large that an inertial range
develops, then standard dimensional analysis shows that the
turbulence energy spectrum for intermediate eddies (i.e., for
eddies with spatial scales in the inertial range) has the Kolmo-
gorov form (Kolmogorov 1941)

\[
E(k) = \frac{u_t^2}{k_t^5} \left( \frac{k}{k_t} \right) \delta(\omega/ku)
\]

where \( k_t \) is the maximum of the spectrum occurs close to \( k = 2n/l \),
and where \( u_t \) and \( l_t \) are the characteristic turbulent velocity and
length scale, respectively.

As for the frequency factor, we simply assume its functional
form; e.g., we adopt the Gaussian form

\[
\delta(\omega/ku) = \frac{2}{\pi^{1/2} ku} \exp \left[ -\left( \frac{\omega}{ku} \right)^2 \right].
\]

Stein (1967) and Bohn (1980) considered three alternative
forms for this frequency factor (as well as three different forms
for the turbulent energy spectrum), one of which is identical
with that used here; both authors treated the case of stratified
and magnetic field–free atmospheres. It is not our aim simply
to repeat these calculations for magnetized stratified atmos-
pheres; rather, we wish to calculate the MHD energy fluxes
for a particular energy spectrum and frequency factor, and
then to compare our results with the acoustic energy fluxes
obtained in the same manner. Since this basic comparison
should not be very sensitive to the precise functional form
of the spectrum or the frequency factor, we will restrict attention
to only the one case discussed here. Thus, in all of our calculations, we assumed that for both magnetized and non-magnetized atmospheres the turbulence is described by a Kolmogorov energy spectrum (eq. [2.5]) and a Gaussian frequency factor (eq. [2.6]), and further assumed that the characteristic turbulent length scale \( l_t \) is equal to the local density (pressure) scale height. The dimensionless form of the turbulence energy spectrum evaluated for two distinct wavevectors \( k_1 \) and \( k_2 \) (see eqs. [4.14a] and [4.14b] in Paper I) can then be obtained from equation (2.5), and can be written as follows:

\[
E(\beta_{1,2}) = \frac{1}{2\pi} \left( \frac{\beta_{1,2}}{2\pi} \right)^{-5/3},
\]

where \( \beta_1 = k_1 l_t = p, \beta_2 = k_2 l_t = q \), and \( \beta_{1,2} \geq 2\pi \). The dimensionless Gaussian frequency factor \( G(p, q, \omega) \) can be evaluated using equation (2.6), leading to the result

\[
G(p, q, \omega) = \frac{4}{\pi^{1/2} (p^2 u_p^2 + q^2 u_q^2)^{1/2}} \exp \left[ -\frac{\omega^2}{p^2 u_p^2 + q^2 u_q^2} \right],
\]

where the dimensionless turbulent velocities are defined by

\[
u_{p,q} = u_{1,2} = 0.745 \left( \frac{2\pi}{\beta_{1,2}} \right)^{1/3}.
\]

III. THE MAGNETOHYDRODYNAMIC ENERGY FLUXES

Using the Paczynski code and the wave flux generation formalism developed in Paper I, we computed a series of envelope models. Each model entailed an inward integration (in the plane-parallel approximation) of the source function, down to a point where MHD energy generation becomes negligible; that is, the MHD energy fluxes were calculated according to equation (4.8) of Paper I, and for values of the free parameters varying within the ranges described in § II above. The results of these calculations for the Sun and for late-type stars are presented in this section. We show the energy spectra for compressible and incompressible MHD waves, discuss the dependence of the results on the free parameters, and compare our results with those previously obtained.

a) Dependence of Wave Energy Flux Results on the Free Parameters

The energy spectra for fast, slow, and Alfvén MHD waves are given in Figure 1; the spectra are obtained for \( \theta = 0^\circ \), and for a Mach number \( M = 0.2 \); for this angle, both the slow and the Alfvén wave are incompressible. We allowed the lower wave frequency limit to vary somewhat, and carried out calculations for values of 2.0, 2.5, and 3.0 times the local acoustic cutoff frequency; the results do depend somewhat on this lower frequency limit, as shown in Table 1. We shall henceforth only present calculations with a value for the lower wave frequency limit 2.5 times the local acoustic cutoff frequency. Under these conditions, the fast wave becomes purely longitudinal and is generated via quadrupole emission only; however, both slow and Alfvén waves are purely transverse, and a dipole type of emission is responsible for their generation. The energy spectra presented in Figure 1 show that the efficiency of quadrupole emission is much higher than that of dipole emission as long as the wave frequency is well above the acoustic cutoff; this result reflects the strong dependence of the dipole emission on the acoustic cutoff.
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b) Energy Fluxes for Late-Type Stars

We next consider the variation of wave generation efficiency as a function of stellar atmosphere properties—the stellar effective temperature and the value of the surface gravity. The results of the necessary calculations are shown for the total wave luminosities in Figure 4, and for the individual component modes for one value of the surface gravity in Figure 5. Figures 4 and 5 demonstrate that for late-type stars the efficiency of acoustic wave generation is higher than that for MHD waves, and that the fast wave flux dominates in the total MHD energy flux. Both acoustic and fast waves are generated significant decrease reflects a strong dependence of the slow wave velocity on the magnetic field. Figure 3 also shows that for strong magnetic fields (i.e., for \( V_s \approx V_A \)), the slow wave energy flux no longer decreases; this reflects the fact that for such strong magnetic fields the mode becomes "more compressible," and the contribution from quadrupole emission becomes important.

### Figure 2

Wave luminosities for the Alfvén mode, slow mode, fast mode, and their sum, and for the pure acoustic mode (all scaled by the solar bolometric luminosity) plotted against the angle \( \theta \) between the magnetic field and the wavevector; the assumed gravity is that of the Sun.

The significant increase of the slow energy flux with \( \theta \) is caused, first, by the decrease in the slow phase velocity when \( \theta \) increases (leading to higher Mach number) and, second (and more important), by the increasing dominance of the compressible component in the structure of the wave. This latter effect leads to an increasing contribution from quadrupole emission, which, in our formalism, is a very efficient source of emission.

Figure 3 shows the dependence of the MHD wave energy fluxes on the strength of the local magnetic field, again compared with the corresponding purely acoustic result. Our results show that the energy flux decreases when the magnetic field increases, mainly because of the decrease in the efficiency of generation of slow and fast waves. The results are obtained for \( \theta = 25^\circ \) and for magnetic field energy densities in the range 0.2–0.8 times the local gas pressure (the latter restriction is necessary in order to maintain the constraint that \( V_s > V_A \)). As shown in Paper I, both fast and slow modes are partially longitudinal and partially transverse for \( \theta = 25^\circ \); however, as long as \( V_s \) is larger than \( V_A \), the fast wave is predominantly compressible, whereas the incompressible component dominates in the structure of the slow wave. Thus, the efficiency of fast-wave generation decreases slightly as the Mach number for the fast wave decreases. The same effect is responsible for the decreasing efficiency of slow wave generation, but in this case the

![Figure 2](https://example.com/figure2.png)

**Figure 2**—Wave luminosities for the Alfvén mode, slow mode, fast mode, and their sum, and for the pure acoustic mode (all scaled by the solar bolometric luminosity) plotted against the angle \( \theta \) between the magnetic field and the wavevector; the assumed gravity is that of the Sun.

![Figure 3](https://example.com/figure3.png)

**Figure 3**—Wave luminosities for the Alfvén mode, slow mode, fast mode, and their sum, and for the pure acoustic mode (all scaled by the solar bolometric luminosity) plotted against the strength of the background magnetic field; the assumed angle between the wavevector and the ambient magnetic field is \( \theta = 25^\circ \).
Fig. 4.—Wave luminosities for pure acoustic modes (dashed curves) and for the sum of all MHD waves (solid curves), scaled by the stellar bolometric luminosity, plotted for stars with (a) log $g = 4.0$ and 5.0, and (b) log $g = 2.0$ and 3.0, as functions of stellar effective temperature. The assumed angle between the wavevector and the ambient magnetic field is $\theta = 25^\circ$. 
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Fig. 5.—Individual contributions to the total wave luminosities for pure acoustic modes (uppermost dashed curve) and for all MHD (fast, slow, and Alfvén) modes, scaled by the stellar bolometric luminosity, plotted for stars with $\log g = 4.0$ as functions of stellar effective temperature (the total MHD wave luminosity is indicated by the solid curve). The assumed angle between the wavevector and the ambient magnetic field is $\theta = 25^\circ$.

Figure 6 presents a comparison of our results with the acoustic energy fluxes obtained by Bohn (1980, 1984); in both cases, the calculations are made for the same (Kolmogorov) energy spectrum and the same (Gaussian) frequency factor. The figure shows, first, that compressible modes dominate incompressible modes even in a (uniformly) magnetized stratified fluid; and, second, that the acoustic fluxes obtained upon using Paczynski's envelope code are substantially higher than those given by Bohn (1980). This second result reflects differences in the convective zone models rather than differences in the approach of estimating the energy fluxes; thus, this discrepancy reemphasizes the often forgotten fact that uncertainties in the convection zone velocity structure within the region of dominant wave generation can lead to extremely large uncertainties in the predicted wave fluxes.

Figure 6 also gives the range of observed mean stellar surface X-ray fluxes seen by the Einstein Observatory (adapted from the compilation of Rosner, Golub, and Vaiana 1985); these results (obtained by using the observed range of X-ray luminosities, and dividing by the appropriate stellar surface area, for each individual spectral type) show that the predicted wave fluxes lie roughly in the observed range. However, because of the large uncertainties in the absolute value of the calculated fluxes alluded to above, and because we have neglected radiative damping (which changes the compressional component of MHD wave energy fluxes significantly after propagation through a stellar photosphere), we do not regard this agreement as a particularly strong confirmation of theory. Instead, we call attention to the large range of emission levels at any given effective temperature. The present theory is inherently incapable of accounting for this range, since it predicts a single (i.e., unique) value for the dominant flux (the acoustic wave flux) at any given spectral type; and since the range of variation in the MHD wave energy fluxes at any given spectral type (produced by appealing to variability of the ambient magnetic field; see Fig. 3) is insufficient to explain the observed range in X-ray emission levels.

Thus, the range of observed stellar coronal emission levels remains to be accounted for; one natural explanation is that the mean surface wave flux is in fact determined by the mean filling factor of filamentary magnetic fields in the surface layers.
IV. CONCLUSIONS

The results presented in this paper show that MHD wave energy fluxes at frequencies well above the acoustic cutoff (i.e., in the frequency domain in which our calculations are valid) cannot significantly exceed the acoustic wave energy fluxes as long as the background magnetic field is homogeneous and insufficiently strong to control the turbulent fluid motions. These restrictions may well be fatally flawed as far as applying them to stars like the Sun is concerned; although we have no good observational or theoretical basis for deciding what the structure is of magnetic fields in the regions where waves are generated, we do know that the surface magnetic fields are far from uniform, and that the magnetic field at the surface undoubtedly does strongly affect fluid motions, at least locally. Nevertheless, our results are useful in correcting impressions left by previous calculations that the presence of magnetic fields in and of itself leads to a strong enhancement in the efficiency of wave generation.

We further note that previous calculations (see papers quoted in § I) attempted to estimate the generation efficiency of MHD waves for the case of a strong uniform magnetic field, i.e., one well above the weak field limit ($V_s > V_A$) with which we are concerned here, and argued on qualitative grounds that as the background uniform magnetic field grows in strength, the increasing anisotropy of the turbulent motions will enhance monopole emission, and hence lead to negligible contributions from quadrupole and dipole emission. This effect—if really present—would lead to a significant increase in the efficiency of MHD wave generation as compared with the field-free case, contrary to our findings in the weak field limit. The best argument one can instead put forward for the strong field case runs as follows: For such strong magnetic fields, the fast wave becomes predominantly transverse, and propagates with the Alfvén velocity. Thus, as the magnetic field strength increases, the Alfvén velocity also increases (resulting in a decrease of the magnetic Mach number), and the efficiency of both quadrupole and dipole emission for fast and Alfvén waves decreases in spite of the fact that the magnetic energy density may dominate over the kinetic energy density of the turbulent motions (see Paper I). The ratio of these energy densities is, however, relevant for monopole emission, and does lead to a linear increase of the wave energy flux with the magnetic field strength. The argument for the variation in efficiency of slow mode generation is somewhat different. Since the slow wave is longitudinal and propagates with the sound speed, the slow mode phase velocity does not depend on the magnetic field, and monopole emission can enhance the generation efficiency of these waves. Both of these qualitative arguments thus do suggest that increasing efficiency of monopole emission can lead to an enhancement of MHD wave generation in the strong field limit. However, whether this is in fact the case requires direct calculation; certainly present theory is inadequate to the task of calculating the wave energy fluxes, or even to estimating the efficiency of the individual source emission contributions. Thus, any such extrapolations must be treated as only (very plausible) speculations.

It should not be surprising that our results show quadrupole emission to dominate in the generation of fast waves; this occurs mainly because the compressible component dominates in the structure of these waves for all possible directions of the...
magnetic field (with respect to the vertical). In the same limit, slow waves are predominantly transverse, and hence the generation efficiency is dominated by dipole emission; the results presented in § III show, however, that if the angle between the magnetic field and the vertical increases, the phase velocity decreases and the wave becomes "more compressible," leading to higher efficiency for wave generation. Finally, the purely transverse Alfvén waves are generated only by the dipole emission mechanism, which is a less efficient source of emission than the quadrupole mechanism (mainly because of our restriction to the high wave frequency limit).

Finally, we emphasize again that a simplenided comparison of the calculated MHD energy fluxes obtained above with the total radiative losses from the outer atmospheres of late-type stars is inherently flawed if the structure of magnetic fields within the wave generation region is not uniform and if the magnetic field strength in this region is not small (when compared with the kinetic equipartition field strength): since the observational data show that solar (and probably also stellar) surface magnetic fields are strongly inhomogeneous and locally strong (cf. Stenflo 1978; Harvey 1977; Robinson, Worden, and Harvey 1980), it is likely, though by no means certain, that in the region of wave generation one may expect the magnetic field to have a spatially intermittent "flux-tube" structure, rather than being homogeneous. In this eventuality, it would be flux-tube waves that carry the wave energy away from the convective zone, and would lead to heating of the outer atmospheric layers (cf. Spruit and Roberts 1983). As pointed out in § III, this picture is consonant with the idea that it is the magnetic flux-tube filling factor that largely determines the actual mean level of coronal heating in late-type stars. Thus, a more realistic assessment of the relevance of MHD waves must be based on wave flux generation calculations which take the flux-tube structure into account; this sort of calculation can now also be carried out (Musielak, Rosner, and Ulmschneider 1988).

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