THE REBOUND SHOCK MODEL FOR SOLAR SPICULES: DYNAMICS AT LONG TIMES

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ABSTRACT

The spicule model due to Hollweg is extended and developed. The dynamics is emphasized here; radiative and ionization losses, heat conduction, and nonshock heat input, are not included. In the model, a series of rebound shocks results in chromospheric material with spicule-like properties below a raised transition region. The shocks result from a single impulsive energy source in the photosphere. We find that at long times, the model approaches a new hydrostatic equilibrium with the transition region remaining raised, and with a region of shock-heated chromosphere below it. We investigate the variation of the properties of the model in response to different values for the magnitude and location of the source, and to different initial transition region heights. We conclude that the model is capable of generating structures with properties consistent with observations of spicules (with the exception of temperature) when only the dynamics is considered.

Subject headings: hydrodynamics — shock waves — Sun: atmosphere — Sun: transition region

1. INTRODUCTION

Solar spicules have been observed for over a century, and yet fundamental questions regarding their time evolution, energetics, and the nature of their source remain unanswered. That they play a key role in the dynamics of the entire solar atmosphere is virtually certain. This is attested to by the fact that their total upward mass flux is two orders of magnitude greater than that of the solar wind (e.g., Beckers 1972). The numbers for the total energy flux in spicules are not as dramatic (Campos 1984), but may still be large enough to play a role in the energy balance of the solar atmosphere. Moreover, when viewed individually, the energy requirements of spicules are comparable to the energy requirements of other regions of the solar atmosphere, such as the quiet chromosphere or corona.

Several general features of spicule dynamics have been noted from observations (see, e.g., Athay 1976; Beckers 1968; Michard 1974). They typically have upward velocities of some 25 km s⁻¹, and there is evidence that the motion is non-ballistic. There is also evidence for complex velocity structure within spicules (see below). Their lifetimes are 5–10 minutes. They reach a range of maximum heights, at times exceeding 10,000 km. A key unknown is the mechanism by which the material which makes up the spicule returns to the solar surface. Observationally, a spicule appears to either fade from view or, at times, return along the same trajectory over which it evolved.

Recent nonlinear models of spicules explain the observed spicule properties with varying degrees of success. These theories all involve identifying material below a raised transition region (TR) as the spicule. Hollweg, Jackson, and Galloway (1982) consider the role of nonlinear Alfvén waves and fast shocks in lifting the TR. Their model yields density profiles which resemble those of spicules, but the calculated temperatures are much too low due to the nearly adiabatic cooling of the chromospheric material below the raised TR. This low-temperature problem also occurs in the spicule model of Suematsu et al. (1982). They consider the role of a single upward propagating slow shock in lifting the TR. The shock results from a sudden pressure enhancement at the base of a magnetic flux tube with constant cross section. The model due to Hollweg (1982) overcomes the temperature problem by utilizing a train of acoustic shocks to heat the chromospheric gas beneath the uplifted TR. The shock train results from a single initial impulsive energy source at the base of the model (at photospheric levels). In linear theory, the impulse response (see the Appendix) of the lower solar atmosphere consists of a wavefront followed by an oscillating wake as has been known for a long time (e.g., Morse and Feshbach 1953; Stein and Schwartz 1972; Rae and Roberts 1982). The wake arises as follows. Some of the material displaced upward by the initial wavefront falls due to gravity. The momentum of this falling material results in compression of the material in the atmosphere below it. The compressed material then "rebounds," generating a new wave. This process repeats several times, giving rise to the oscillating wake. Nonlinearly, the wavefront and wake steepen into a shock train. We thus refer to the shocks resulting from the wake as "rebound shocks." The shocks are channeled along a strong magnetic field of varying cross section and repeatedly interact with the TR, resulting in an upward velocity of the TR which is roughly constant, and in substantial chromospheric heating. This is a significant departure from the model of Suematsu et al. (1982) which considers only a single shock interaction and thus induces ballistic motions of the TR and insufficient heating. The model due to Hollweg (1982) will subsequently be referred to as the "rebound shock model."

Rebound shocks are also discussed by Suematsu (1985) in his study of the formation of fibrils by a pressure pulse on a magnetic loop.

Some observers, using Doppler techniques, have noted velocity variations in spicules (e.g., Zirker 1967; Weart 1970). Their observations often include multiple variations in the line-of-sight component of velocity. Multiple variations in the sign of the line-of-sight component have also been observed. Such observations may be consistent with time-dependent acceleration induced along the axis of the spicule in association with the wake, as predicted by the rebound shock model.

In this paper, the rebound shock model is extended and further developed. We first address the question of the long-time behavior of the model. The original goal of this investiga-
REBOUND MODEL FOR SOLAR SPICULES

The new equilibrium can be roughly characterized as consisting of three layers. In a representative example where the initial conditions are the same as those given in Hollweg (1982), these layers include (1) a region of nearly undisturbed chromospheric gas below some 1200 km; (2) an intermediate region of shock heated chromospheric gas which we identify with the spicule; and (3) nearly undisturbed coronal gas at heights greater than approximately 18,800 km.

Further investigations in this paper involve the behavior of the model's characteristics as the input parameters are varied. Specifically, the variation of the TR's upward velocity, and of the spicule's final density, temperature, and height as functions of the magnitude of the initial impulse, its location, and the initial TR height is studied. We find that a variety of final spicule characteristics can result from different input parameters. This is an important result in view of the low velocities and densities obtained by Hollweg (1982) in his model. We conclude that the low values he obtained are a consequence of the initial parameters chosen and do not reflect an intrinsic failing of the rebound shock model. In particular, reasonable heights, densities and velocities can be obtained if the initial energy source is located a few hundred km below the height where \( \tau_{5000} = 1 \), and if the initial height of the TR is lower than in the models of Hollweg (1982).

As in the case of the Hollweg paper, the calculations here are very idealized; the effects of nonshock heating, heat conduction, radiation, and ionization are not included. Thus we concentrate on the dynamics of the system. The results should be of value in understanding the dynamics of the solar atmosphere in various circumstances, but certainly a true model of spicules will have to await a more complete calculation including the above-mentioned factors. However, we will nonetheless be cavalier with the use of the term "spicule" when describing the model results.

The following section reviews the rebound shock model, upon which the subsequent results are based. We also study the energetics of the model. Sections III and IV are devoted to the system's long time behavior and the variation of the system's characteristics as the input parameters are changed. The results are further discussed in § V. Also included is an Appendix in which we discuss analytically the linear theory of the response of a strong magnetic flux tube to an initial impulse; we include this Appendix because existing discussions in the literature are incomplete.

II. REBOUND SHOCK MODEL

We begin with a review of the rebound shock model of spicules (Hollweg 1982). The model studies the evolution of a perturbed fluid guided by a magnetic flux tube. Only vertical motions are considered. The flux tube is taken to be rigid. Hollweg pointed out that this approximation is valid for heights in excess of \( \approx 10^4 \) km above the \( \tau_{5000} = 1 \) solar surface. These heights are the most important regarding spicule formation, and so the rigid flux tube assumption was employed in the original model, and will be employed throughout this paper.

The basic equations solved are the MHD mass, momentum, and energy conservation equations expressed in the following form:

\[
\frac{\partial}{\partial t} (pA) + \frac{\partial}{\partial z} (pvA) = 0 ,
\]

\[
\frac{\partial}{\partial t} (pvA) + \frac{\partial}{\partial z} (pv^2 A) = A \left( \frac{\partial p}{\partial z} - \rho g \right) + F \rho Ah(z, t) ,
\]

\[
\frac{\partial}{\partial t} (EvA) + \frac{\partial}{\partial z} (EvaA) = -pvAg - \frac{\partial (pvA)}{\partial z} + F \rho Ah(z, t) ,
\]

\[
E = \frac{1}{2} \rho v^2 + \frac{p}{\gamma - 1} ,
\]

where \( \rho \) is the density, \( v \) is the vertical velocity, \( p \) is the scalar thermal pressure, and \( g \) is the acceleration of gravity taken to be \( 2.7 \times 10^4 \) cm s\(^{-2}\). A perfect gas is assumed with \( \gamma = 5/3 \). The variable \( z \) is height in the solar atmosphere; the height where \( z = 0 \) in our models coincides roughly with \( z = 0 \) in the HSRA/VAL model solar atmosphere (Gingerich et al. 1971; Vernazza, Avrett, and Loeser 1973, hereafter VAL). However, the ambient chromosphere in our model is isothermal, but it provides a reasonable fit to the height variation of chromospheric density in the HSRA/VAL model. The factor \( A \) describes the cross sectional area of the magnetic flux tube. Figure 1 displays \( [A(z)/A(0)]^{1/2} \) in the range \(-880 \) km < \( z < 6600 \) km, where the greatest area variation occurs. At higher heights, \( A(z) \) remains constant. Our Figure 1 represents the area factor used by Hollweg (1982) and is a correction to Figure 1 which appears in that paper. Above \( z = 0 \), the cross sectional area expands by a factor of approximately 150, rather than 200 as stated in Hollweg (1982). Consequently, a field strength of 1500 G at \( z = 0 \) corresponds to a coronal field strength of 10 G. The reader is referred to the original Hollweg (1982) paper for additional details of the initial solar atmosphere in the model. At time \( t = 0 \), an (unphysical) quasi-impulsive vertical body force localized to the vicinity of a few grid points near \( z = 0 \) is imparted to the flux tube. This body force is represented by the final terms in equations (2) and (3). The localized spatial extent and the quasi-impulsive time variation of the force are contained in \( h(z, t) \). In our models, the time variation of \( h \) is one-half of a sinusoidal cycle lasting 90.26 s. The factor \( F \) is the amplitude of the acceleration of the body force. (The effects of changing the location of the body force and the value of \( F \) are investigated in § IV.)

Equations (1)–(3) are solved numerically using SHASTA—a fully nonlinear, flux-corrected transport code due to Boris and Book (1973, 1976). The code published by Boris (1976) was used with the Eulerian grid option for all the original model calculations, as well as for the calculations in this paper. Flow-through boundary conditions are also used for all calculations. These conditions are implemented by insisting that the value of a given quantity beyond the boundary equals the value of that quantity just inside the boundary. The value on the boundary is the average of these two and is therefore the same as the value just inside the boundary. These boundary conditions allow for the free flow of plasma out of the numerical domain. We have tested these conditions by launching sound waves at the upper and lower boundaries. The energy reflected by the
boundaries was always found to be less than a few percent of the incident energy.

In the model calculations, the gravitational acceleration is constant throughout the region which represents the solar atmosphere. However, in the presence of gravity, the flow-through boundary conditions would allow material to continuously flow downward through the system, thus precluding a static equilibrium. We have circumvented this problem by introducing two regions, above and below the model atmosphere, in which the gravitational acceleration is smoothly tapered to zero. Tests have been applied to ensure that these regions do not reflect more than a few percent of any incident energy, and that they are not sources of any significant drifts or energy fluxes. (In all figures in this paper, the regions of varying gravity are excluded.)

The actual numerical computations used a convenient set of dimensionless variables. However, in presenting the results we have converted everything back to physical (dimensional) quantities. This accounts for the unusual units which appear in all figures.

The response of the system to the imparted impulse at \( t = 0 \) is an upward propagating wave front, followed by an oscillating wake. Due to the nonlinear nature of the calculation, the wave front and wake evolve into a train of upward propagating rebound shocks. The shock train interacts with and repeatedly lifts the TR, which is modeled as a contact discontinuity. The TR is regarded as the top of the spicule. Figure 2 shows the height of the TR as a function of time. The series of dashed lines indicates the trajectories of the rebound shocks. Each subsequent shock prevents the TR from following the ballistic path instigated by the previous shock. The result is a relatively smooth (compared to ballistic) upward trajectory of the TR at a velocity of some 16 km s\(^{-1}\). Figure 2 is based on the system response to a body force localized in space to a few grid points near \( z = 110 \) km. The parameters used in producing Figure 2 are the same as those used to produce Figure 3 of Hollweg (1982). The slight differences in TR height versus time in the two figures is accounted for by the inclusion of the force term in the energy equation (last term in eq. [3]); this term was inadvertently omitted in the Hollweg (1982) work. Also, we represent the initial TR by smooth variations in pressure and density, rather than by discontinuous jumps as in Hollweg (1982).

The kinetic \( (E_{\text{kin}}) \), thermal \( (E_{\text{ther}}) \), and gravitational \( (E_{\text{grav}}) \) energies in the flux tube, normalized to the cross sectional area at the top, \( A_{\text{top}} \), are formulated as follows:

\[
E_{\text{grav}}(t) = A_{\text{top}}^{-1} \int_{z_1}^{z_2} A(z) \delta p(z, t) \phi(z) dz + F_g(t),
\]

where \( \phi \) is the gravitational potential (taken to be zero at \( z = 0 \)),

\[
E_{\text{ther}}(t) = (\gamma - 1)^{-1} A_{\text{top}}^{-1} \int_{z_1}^{z_2} A(z) \delta p(z, t) dz + F_t(t),
\]

and

\[
E_{\text{kin}}(t) = 0.5 A_{\text{top}}^{-1} \int_{z_1}^{z_2} A(z) \frac{\partial v(z, t)}{\partial z}^2(z, t) dz + F_s(t).
\]

The quantities \( \delta p, \delta \rho, \) and \( \delta v \) represent the density, pressure, and velocity deviations from their initial values. The limits \( z_1 \) and \( z_2 \) are in the region of uniform gravity. Now any wave motions, shocks, or flows induced by the source (located in a localized region between \( z_1 \) and \( z_2 \)) will yield a nonzero energy flux out of the region of integration. The terms \( F_s(t), F_t(t), \) and \( F_d(t) \) are added to compensate for these fluxes and are given by

\[
F_s(t) = A_{\text{top}}^{-1} \sum_{i=1}^{2} (-1)^i A(z_i) \int_0 t' \delta v(z_i, t') \phi(z_i) dt',
\]

\[
F_t(t) = A_{\text{top}}^{-1} (\gamma - 1)^{-1} \sum_{i=1}^{2} (-1)^i A(z_i) \int_0 t' \delta v(z_i, t') \phi(z_i) dt',
\]

\[
F_d(t) = A_{\text{top}}^{-1} \gamma (\gamma - 1)^{-1} \sum_{i=1}^{2} (-1)^i A(z_i) \int_0 t' \delta v(z_i, t') \phi(z_i) dt'.
\]
and
\[
F_k(t) = 0.5A_{top}^{-1} \sum_{i=1}^{2} (-1)^i A(z_i) \int_0^t \rho(z_i, t') \delta v(z_i, t') dt'.
\] (10)

The quantities given by equations (5)-(7) are plotted in Figure 3 for the case where \(z_1 = 0\) and \(z_2 = 19,400\) km. The two large-amplitude oscillations represent the gravitational and thermal energies per cross sectional area, and the light solid curve represents the kinetic energy per cross sectional area. The thermal and gravitational energies oscillate out of phase with each other, and both, being first-order quantities, dominate the second-order kinetic energy term. Also indicated in Figure 2 is the time behavior of the sum \(E_{\text{kin}}(t) + E_{\text{grav}}(t) + E_{\text{ther}}(t)\) denoted by \(E_{\text{tot}}(t)\). Note that \(E_{\text{tot}}(t)\) is conserved to within a few percent. We thus feel confident in extending the

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**Fig. 2.** Height of transition region vs. time (solid line) in the rebound shock model of spicules. The dashed lines represent trajectories of the rebound shocks. Figs. 2-8 are all derived from the “reference model” described in § II.

**Fig. 3.** Energy balance in the rebound shock model as a function of time, as described by eqs. (5)-(10). The dotted line represents the thermal, the dashed line the gravitational, and the light solid line the kinetic energy normalized to the cross sectional area at the top of the flux tube in the model. Compensation has been made for fluxes out of the region of calculation (see text). The heavy solid line represents the sum of the other three quantities.
numerical calculation to study the long-time development of the model.

III. LONG TIME BEHAVIOR

The original work on the rebound shock model conducted by Hollweg (1982) concluded with the question of the model spicule's fate unresolved. The TR was still moving up after 18 minutes, at which time the calculation concluded. We now proceed to address the model's long-time behavior.

The principal result is depicted in Figure 4, which is identical to the solid line in Figure 2, but continued out to \( t = 75 \) minutes. The TR continues to rise until a maximum height is reached—in this case, some 14,500 km. This occurs after about \( t = 26 \) minutes. At later times, the TR remains raised, oscillating about the average maximum height in response to continued interaction with the upward propagating train of progressively weaker rebound shocks.

The same general features are apparent in Figure 5, which shows density as a function of height. Each profile corresponds to the results at a different time, the lowest being at \( t = 180.54 \) s. Each subsequent plot occurs 180.54 s after the one immediately below it.

The spicule is approaching a new hydrostatic equilibrium. This is indicated most clearly by the energy versus time plot in Figure 6. This figure is the time extended version of Figure 3, except here \( z_2 \), the upper limit of integration in equations (5)-(7), is at \( z = 34,100 \) km. The amplitudes of all the energy oscillations approach zero at long times. Note that \( E_{\text{tot}}(t) \) is conserved to within some 20%; this is a measure of the inaccuracy of our long-time calculations.

As discussed in the Introduction, the new equilibrium state consists of three layers: (1) nearly undisturbed chromosphere, (2) an intermediate layer which we associate with the spicule, and (3) nearly undisturbed corona. These layers are clearly visible in the temperature versus height plot of Figure 7. The plot is at the time indicated by the arrow in Figure 4, i.e., \( t = 63.2 \) minutes. In this example, the spicule region has a temperature of \( 5-7 \times 10^4 \) K if the molecular weight is 0.5. The heating is a consequence of the rebound shocks.

Ultimately, the shock heating explains why the spicule remains raised: a portion of the energy from the initial impulse is deposited into the spicule as thermal energy via the shock heating. Since the model contains no further dissipation mechanism for this energy, the thermal energy remains in the spicule and acts to balance the gravitational energy as hydrostatic equilibrium is approached.

We have verified that, at long times, the density of the intermediate layer varies with height very nearly in the manner expected for a gas in hydrostatic equilibrium with the temperature shown in Figure 7. There are, however, some small deviations from hydrostatic equilibrium due to the motions which are present even at long times. For example, the top of the spicule oscillates with a velocity amplitude of some 16 km s\(^{-1}\) (see Fig. 4), which is small compared to the sound speed of 40 km s\(^{-1}\). The velocities at lower heights are even smaller.

Examination of plots of density versus time at a given constant height reveals that after the short time scale oscillations are averaged out, there are no large systematic drifts (\( \leq 10\% \)) in average density after the TR has passed that given height (Fig. 8a). Density fluctuations of around a factor of 2 on short time scales (a few hundred seconds) are seen at early times in response to the still relatively strong rebound shocks, but at later times the amplitude of the fluctuations decreases due to the progressive weakening of the rebound shocks. The temperature versus time plot at the same height (Fig. 8b) shows some similar features. The change in average temperature from the time the TR passes until the end of the calculation is about 50%; the time average temperature is about \( 5 \times 10^4 \) K. The temperature increases slowly due to the shock heating.

Figure 8 shows that the amplitudes of the fluctuations of density and temperature change with time. Most notable are the changes which occur in both Figures 8a and 8b at...
$t \approx 900$ s. This amplitude change corresponds to the passage of the TR at the height for which the plot was calculated, $z = 11,200$ km. Figure 8a also shows a discontinuous change in the amplitude of the density fluctuations at $t \approx 1985$ s. (A change in the amplitude of the temperature fluctuations is also present in Fig. 8b, but it is less apparent.) We have found that these amplitude changes correspond to a decrease in the strength of the rebound shocks. We do not know why this transition occurs.

IV. SYSTEM RESPONSE TO INPUT PARAMETER VARIATIONS

Of key interest is the variation of the spicule velocity, and the physical properties of the system in its new equilibrium state, as various input parameters are changed. For comparison purposes, the model run by Hollweg (1982) discussed in §§ II and III will be referred to as the “reference model.”

Table 1 shows the results of varying $F$, the amplitude of the initial acceleration in equations (2) and (3). This amplitude is
Fig. 7.—Temperature vs. height at time indicated by arrow in Fig. 4 (t = 63.2 minutes). The TR has achieved its new height. Three layers are evident in the profile: chromosphere and photosphere below z = 1200 km; an intermediate shock-heated region associated with the spicule; corona above approximately z = 18,800 km.

given in column (1), normalized to the corresponding amplitude in the reference model (denoted by subscript RM). Negative values of $F$ denote an acceleration in the negative direction; the initial force of the reference model was applied in the positive direction. Note that negative values of $F$ are as effective in producing spicules as positive values. In the second column of Table 1, a representative density of the spicule material after the maximum height is achieved is given. The value was determined at the midheight of the spicule. This height was determined from the temperature versus height profile at late times using a visual estimate. Column (3) gives the spicule temperature at the midheight. The average final height, column (4), was obtained from TR height versus time plots similar to that of Figure 4.

The approximate average upward velocity of the TR is tabulated in the final column. This velocity was determined in a fashion analogous to that used by Hollweg (1982); i.e., the quoted figures represent a time-averaged upward velocity of the TR during roughly the first 15 minutes of the spicule development.

A comparison of the columns of Table 1 reveals some trends. The terminal height of the TR increases as the magnitude of the input force increases. Similarly, the temperature of the spicule increases with the input force magnitude. Although all of the temperature values in the Table are substantially greater than those observed in spicules, the amount of this energy which would be dissipated by other processes is not taken into account by the model. Further discussion of this point is included in the following section.

These trends reflect the fact that the increased input energy distributes itself in such a manner that the magnitudes of $E_{\text{grav}}$ and $E_{\text{ther}}$ both increase. The magnitude of $E_{\text{kin}}$ also increases, but Table 1 indicates that the final densities, which are already low in the reference model case compared to observations, decrease as $F$ is increased. This density decrease occurs because roughly the same amount of material is spread out over a larger height when $F$ is increased. However, since the magnitude of $E_{\text{kin}}$ increases with $F$, the wave velocities must be correspondingly larger. The increased velocities lead to stronger shocks which in turn produce higher TR velocities. The trend of increased TR velocity with increased force amplitude is borne out by Table 1.

The Table 1 results are useful vis-à-vis solar dynamics, but none of the specific cases conforms very closely to the properties of observed spicules. The densities and total mass contents of the model spicules are particularly low; observationally, spicule densities are of the order of $10^{-13}$ g cm$^{-3}$. In order to produce high enough densities, the trends of Table 1 indicate that the input force amplitude would have to be somewhat lower than the reference model case, but such a model would have unacceptably low TR velocities.

The densities are increased substantially when the initial TR height is decreased. Table 2 gives the results for three cases of this type. The amount of reduction of the TR height from the reference model height of 2200 km is given in column (2) in terms of ambient (i.e., at $t = 0$) chromospheric scale heights, $H$; for these models, the ambient isothermal scale height $H =$

### Table 1

<table>
<thead>
<tr>
<th>$F_0/F_{\text{RM}}$ (1)</th>
<th>$\rho$ (g cm$^{-3}$) (2)</th>
<th>$T$ (K) (3)</th>
<th>Final Height (km) (4)</th>
<th>TR Velocity (km s$^{-1}$) (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (reference model)</td>
<td>$5.1 \times 10^{-15}$</td>
<td>60,000</td>
<td>14,500</td>
<td>16</td>
</tr>
<tr>
<td>1.5</td>
<td>$4.5 \times 10^{-15}$</td>
<td>64,000</td>
<td>21,000</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>$2.3 \times 10^{-15}$</td>
<td>78,000</td>
<td>22,900</td>
<td>20</td>
</tr>
<tr>
<td>$-1$</td>
<td>$3.2 \times 10^{-15}$</td>
<td>57,000</td>
<td>12,800</td>
<td>16</td>
</tr>
<tr>
<td>$-2$</td>
<td>$2.6 \times 10^{-15}$</td>
<td>77,000</td>
<td>20,200</td>
<td>19</td>
</tr>
</tbody>
</table>

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Fig. 8.—Density (a) and temperature (b) as a function of time at \( z = 11,200 \) km. At times less than approximately 90 s, the atmosphere at this height is basically coronal in nature. At later times, the TR has moved to higher heights; the densities and temperatures are then characteristic of the intermediate layer in Fig. 7.

\( \frac{p_0}{\rho_0 g} = 147 \) km. The remaining columns of Table 2 are determined in the same manner as the corresponding ones in Table 1. All other initial parameters remain unchanged. The spicule density increases in all cases, because the density on the lower side of the TR is increased when the initial height of the TR is lowered. Additional consequences are lower final TR heights and lower upward TR velocities. This behavior results from the increased spicule density. The same amount of energy imparted to the denser column of material will not raise the material as high as in the lower density case. Also, investigation of the distribution of energy in the model via plots similar to Figure 6 reveals that the same portion of the energy goes into kinetic energy in both the Table 1 and the Table 2 cases. Thus, since the density is higher in the latter case, the wave velocities must be lower than in the former. The result is weaker shocks and a lower upward TR velocity.

All the results presented so far were deduced with the initial force at a height of 110 km. There is, however, no \textit{a priori}
TABLE 2
MODEL RESPONSE TO INITIAL TR HEIGHT VARIATION

<table>
<thead>
<tr>
<th>$F_d/F_{RM}$ Location, $z$(km)</th>
<th>TR Height Reduced by</th>
<th>$\rho$(g cm$^{-3}$)</th>
<th>$T$(K)</th>
<th>Final Height (km)</th>
<th>Final Velocity (km s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.69$H$</td>
<td>1.4 x $10^{-14}$</td>
<td>56,000</td>
<td>119,000</td>
<td>11</td>
</tr>
<tr>
<td>1.00</td>
<td>2.3$H$</td>
<td>6.3 x $10^{-13}$</td>
<td>11,000</td>
<td>3900</td>
<td>6</td>
</tr>
<tr>
<td>2.00</td>
<td>2.3$H$</td>
<td>5.9 x $10^{-14}$</td>
<td>50,000</td>
<td>8800</td>
<td>9</td>
</tr>
</tbody>
</table>

reason to rely on this height value, and the consequences of varying the source location are also of interest. We have investigated the consequences of varying the source location, and the results appear in Table 3. The source location is given in column (2) in km above the $r_{5000} = 1$ surface. The initial height of the TR is the same as in the reference model. The other columns in the table are the same as the corresponding columns in the previous tables.

For a constant input force, the spicule temperature and density show no clear trend as the source location is lowered. (To see this, note also that the third row of Table 1 can be compared with the first row of Table 3, and the first row of Table 1 can be compared with the second through fourth rows of Table 3 in studying spicule parameter variations with initial source location.) However, the final TR height and TR velocity both increase with the decrement of source location height.

V. DISCUSSION

The goal of this paper has been to present some new dynamical aspects of the spicule model due to Hollweg (1982). The model utilizes a series of rebound shocks to lift the TR and underlying chromosphere. The shocks result from a single quasi-impulsive source. The material below the raised TR is found to possess spicule-like properties. Chief among our new results involves the behavior of the model at long times; the TR, and therefore the putative spicule, remains raised and a new three-layer hydrostatic equilibrium condition is approached.

The calculations were continued moving the source location below the surface. The most interesting results from the standpoint of comparing with observed spicule properties were obtained when the initial TR height was also reduced. Our results, given in Table 4, include the variation of the input source location for the case where the TR is lowered to 2.3$H$ below that of the reference model case. All the column headings are the same as those in Table 3. The trends in TR height and velocity noted for the Table 3 results continue to hold true for the models runs in Table 4. The results tabulated in the fourth row of Table 4 give densities ($6 x 10^{-14}$ g cm$^{-3}$) and velocities (24 km s$^{-1}$) which compare relatively well with those of observed spicules ($\sim 10^{-13}$ g cm$^{-3}$ and $\sim 25$ km s$^{-1}$, respectively).

From the results tabulated in Tables 1-4, the following generalizations of the behavior of the rebound shock spicule model can be made.

A.—The final spicule densities (1) decrease as $|F|$ increases, (2) decrease as the initial TR height increases, and (3) show no clear trend as the source height increases.

B.—The upward velocity of the TR (1) increases as $|F|$ increases, (2) increases as the initial TR height increases, and (3) decreases as the source height increases.

C.—The terminal TR height (1) increases as $|F|$ increases, (2) increases as the initial TR height increases, and (3) decreases as the source height increases.

D.—The final raised spicule temperature (1) increases as $|F|$ increases, (2) increases as the initial TR height increases, and (3) shows no clear trend as the source height increases.

There may, however, be additional heating mechanisms available. Sterling and Hollweg (1984) suggested a method by which Alfvén waves trapped in a resonant cavity could lead to UV-emitting temperatures inside the cavity. The resonance cavity is formed by the spicule, with regions of differing Alfvén speeds (the chromosphere and photosphere below and the corona above) forming the Alfvén wave reflecting regions. A turbulent dissipation mechanism inside the spicule Alfvén resonance cavity was assumed. We note that the three-layer system which evolves out of the rebound shock model may be

TABLE 3
MODEL RESPONSE TO INPUT SOURCE LOCATION VARIATION

<table>
<thead>
<tr>
<th>$F_d/F_{RM}$ Location, $z$(km)</th>
<th>$\rho$(g cm$^{-3}$)</th>
<th>$T$(K)</th>
<th>Final Height (km)</th>
<th>Final Velocity (km s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.00</td>
<td>550</td>
<td>4.6 x $10^{-15}$</td>
<td>62,000</td>
<td>11,200</td>
</tr>
<tr>
<td>1.00</td>
<td>330</td>
<td>3.6 x $10^{-13}$</td>
<td>63,000</td>
<td>11,200</td>
</tr>
<tr>
<td>1.00</td>
<td>65</td>
<td>4.3 x $10^{-13}$</td>
<td>55,000</td>
<td>15,000</td>
</tr>
<tr>
<td>1.00</td>
<td>0.0</td>
<td>7.6 x $10^{-15}$</td>
<td>77,000</td>
<td>31,700</td>
</tr>
</tbody>
</table>

TABLE 4
MODEL RESPONSE TO INPUT SOURCE LOCATION VARIATION WITH LOWERED INITIAL TR HEIGHT

<table>
<thead>
<tr>
<th>$F_d/F_{RM}$ Location, $z$(km)</th>
<th>$\rho$(g cm$^{-3}$)</th>
<th>$T$(K)</th>
<th>Final Height (km)</th>
<th>Final Velocity (km s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.0</td>
<td>3.6 x $10^{-14}$</td>
<td>52,000</td>
<td>12,000</td>
</tr>
<tr>
<td>1.00</td>
<td>-220</td>
<td>4.8 x $10^{-14}$</td>
<td>57,000</td>
<td>17,200</td>
</tr>
<tr>
<td>1.00</td>
<td>-330</td>
<td>4.9 x $10^{-14}$</td>
<td>67,000</td>
<td>21,200</td>
</tr>
<tr>
<td>1.00</td>
<td>-440</td>
<td>6.2 x $10^{-14}$</td>
<td>73,000</td>
<td>24,200</td>
</tr>
</tbody>
</table>
The return of the spicule material could conceivably be affected by the preferential radiation of thermal energy from the bottom portion of the spicule. Relatively uniform heating of the spicule could account for the ultimate tendency of the spicule to fall to lower heights; just the opposite would be expected. The return of the spicule material may then return near the spicule’s base, and the elevated spicule could no longer be supported. The spicule material may return in balance of the spicule indicated by Figure 6 would no longer hold near the spicule’s base, and the elevated spicule could no longer be supported. The spicule material may then return in the form of the UV downflows observed in the TR (e.g., Pneuman and Kopp 1978; Gebbie et al. 1981; Roussel-Duprè and Shine 1982; Athay et al. 1983; Dere, Bartoe, and Bruecker 1984). We again emphasize, however, that these considerations are wholly speculative at this point.

Also subject to speculation is the actual source of the spicules. Granular buffeting (Roberts 1979) of magnetic flux tubes has been suggested as a possible mechanism. An estimate of the velocities needed to generate spicules from photospheric motions has been made by Hollweg (1982). He found that the energy contained in 2 km s$^{-1}$ velocities between $z = 0$ and $z = 300$ km is sufficient to generate a spicule with $\rho = 10^{-13}$ g cm$^{-3}$ and a height of 8900 km. If the same velocities were imparted at lower levels, where the density is greater, even more energetic spicules could be produced. This was the case in our models: higher final TR heights and larger upward TR velocities were found when the source height was lowered (see Table 4). However, it is still unknown whether the required energy source exists at low levels, or what its nature is.

By running the rebound shock model with various input parameters, we have found it possible to produce a variety of spicules. The case given in the fourth row of Table 4 source location – 440 km; height 1860 km) gives densities and velocities closest to those observed. But we see no reason to favor this particular case over any of the others. We can say, however, that the rebound shock model is capable of generating spicules consistent with observations (with the exception of the temperature) when only the dynamics is considered.

Without the inclusion of the additional energy loss mechanisms of ionization and radiation, our study of the rebound shock model can in no way be construed as being complete; the objective was to study some of the principal dynamics associated with the model. As an indicator of where future, more complete work on the model may lead, we note the results of the numerical work on Alfvenic pulses in the solar atmosphere by Mariska and Hollweg (1985) which included the effects of heat conduction and radiation. In that study, Alfvenic pulses nonlinearly drove acoustic-gravity waves propagating along a magnetic flux tube. These acoustic-gravity waves were weaker than those in the rebound shock model, however, since they resulted from higher order processes. Consequently, spicule-like features did not develop in the Mariska and Hollweg work, even in the cases without heat conduction and radiation. Moreover, in the Mariska and Hollweg study, the source of the acoustic-gravity waves was propagating upward at the local Alfven speed, in contrast to the present work, where the source is at a fixed height. We believe that this qualitative difference also accounts for the absence of spicule-like features in Mariska and Hollweg’s study. Nonetheless, the TR was found to undergo significant motions. The effect of the addition of radiative losses and heat conduction was to decrease the amplitude of these TR motions by about a factor of 2. The Mariska and Hollweg results therefore suggest that the inclusion of additional loss mechanisms into the rebound shock model will lead to some differences in the details of the results, but probably not so significantly different as to discount the studies of the dynamics presented here. However, the effects of ionization have not been addressed in any study to date, and it is thus not clear to what extent ionization will affect our results.

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APPENDIX

LINEAR EVOLUTION OF ACOUSTIC GRAVITY WAVES ON STRONG FLUX TUBES

A problem of direct relevance to the rebound shock model of spicules is the linear evolution of acoustic gravity waves on strong magnetic flux tubes. This problem can be solved analytically and gives insight into the fully nonlinear rebound shock model.

The flux tube is taken to be vertical and to have constant cross section. The initial atmosphere is isothermal and in hydrostatic equilibrium under a constant gravitational acceleration, $g$; there is no TR. The linearized MHD mass, momentum, and energy equations are then

$$\frac{\partial}{\partial t} \delta \rho + \rho_0 \frac{\partial}{\partial z} \delta v + \delta v \frac{\partial \rho_0}{\partial z} = 0 , \quad (A1)$$

$$\rho_0 \frac{\partial}{\partial t} \delta v = - \frac{\partial}{\partial z} \delta p - g \delta \rho + f(z, t) , \quad (A2)$$

and

$$\frac{\partial}{\partial t} (\delta p - c_s^2 \delta \rho) + \rho_0 \delta v \frac{\partial}{\partial z} \left( \frac{\rho_0}{\rho_0'} \right) = 0 . \quad (A3)$$

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The subscript "0" denotes zeroth-order quantities and the prefix "δ" denotes first-order perturbation quantities. The initial vertical velocity, $v$, is taken to be zero. The gas pressure is $p$, the density is $ρ$, $z$ is height, and $c_s^2 = γP_0/ρ_0$ is the sound speed squared, with $γ$ the ratio of specific heats. The term $f(z, t)$ is an explicit representation for some volume force which ultimately drives the acoustic gravity waves. Combining equations (A1)-(A3) yields the wave equation

$$\frac{δ^2Q}{δt^2} = c_s^2 \frac{δ^2Q}{δz^2} - ω_{ac}^2 Q + ρ_0(z)^{-1/2} \frac{δ}{δt} f(z, t),$$

(A4)

where $ω_{ac} = γg/(2c_s)$ is the acoustic cutoff frequency. Also,

$$Q = ρ_0(z)^{1/2} δv.$$

(A5)

Equation (A4) is a Klein-Gordon equation with an additional driving force term. This equation may be solved for a variety of input forces, $f(z, t)$. We choose

$$f(z, t) = f_0 δ(z)H(t),$$

(A6)

where $f_0$ is a constant, $δ(z)$ is the Dirac-delta function in $z$, and $H(t)$ is the Heaviside step function. With the initial conditions

$$\frac{δQ}{δt}(z, 0) = Q(z, 0) = 0,$$

(A7)

equation (A4) may be solved via Laplace transforms. The solution is

$$Q(z, t) = \left[ \frac{f_0}{2c_s^2 ρ_0^{1/2}(0)} \right] \left[ \frac{ω_{ac}}{c_s} \left( c_s^2 t^2 - z^2 \right)^{1/2} \right] H(t - \frac{z}{c_s})$$

(A8)

for $z > 0$. We note that this solution is the same as that obtained by Rae and Roberts (1982). Their solution therefore is not the response localized to an impulsive force, but rather corresponds to a force which is localized at $z = 0$, turns on at $t = 0$, and remains on at all subsequent times. The force (A6) is only implicitly included in the calculation of Rae and Roberts via their derivative initial condition on $Q$ (Rae and Roberts 1982, eq. [16]).

A form of $f(z, t)$ of more relevance to the rebound shock model is a pulse which turns on at time $t = 0$, remains constant for a short time, $b$, and then turns off:

$$f(z, t) = a δ(z)[H(t) - H(t - b)],$$

(A9)

where $a$ is the pulse amplitude. Since equation (A4) is linear, the theorem for superposition of solutions holds. We therefore solve (A4) using the two terms on the right-hand side of equation (A9) separately. Then using equation (A8), the solution to the pulse

![Graph showing the linear response of MHD fluid on a strong magnetic flux tube to an initial acceleration pulse as given by eq. (A10) at t = 9 minutes. The pulse was turned on for a time b = 27 s and has evolved into the "pulse region" between z = 1.9 and z = 2.0 on the horizontal axis scale. The "reference model" values for the sound speed and acoustic cutoff frequency, $c_s = 8.12$ km s$^{-1}$ and $ω_{ac} = 2.77 \times 10^{-2}$ s$^{-1}$, were chosen.]

Fig. 9—Linear response of MHD fluid on a strong magnetic flux tube to an initial acceleration pulse as given by eq. (A10) at t = 9 minutes. The pulse was turned on for a time b = 27 s and has evolved into the "pulse region" between z = 1.9 and z = 2.0 on the horizontal axis scale. The "reference model" values for the sound speed and acoustic cutoff frequency, $c_s = 8.12$ km s$^{-1}$ and $ω_{ac} = 2.77 \times 10^{-2}$ s$^{-1}$, were chosen.
problem can be written down:

$$\frac{2Cs\rho_0^{1/2}(0)}{a} Q(z, t) = J_0(0) \left( \frac{\omega_{ac}}{c_s} \left( \frac{z^2}{c_s^2} + t^2 - z^2 \right)^{1/2} \right) H(t - \frac{z}{c_s}) - J_0 \left( \frac{\omega_{ac}}{c_s} \left[ c_s^2 \left( t - b \right)^2 - z^2 \right]^{1/2} \right) H \left( t - b - \frac{z}{c_s} \right)$$

(A10)

for $z > 0$.

Figure 9 plots the solution (A10) at time $t = 9$ minutes as a function of height, $z$. For numerical values of the parameters we have chosen the values for the “reference model,” i.e., $c_s = 8.12$ km s$^{-1}$ and $\omega_{ac} = 2.77 \times 10^{-2}$ s$^{-1}$. We have taken $b = 27$ s. Apparent in the figure is a wave front at $z = 4400$ km followed by the “pulse region” at the front of the solution between $z = 4180$ km and $z = 4400$ km, which was launched between time $t = 0$ and $t = b$. The wake at lower heights results from a superposition of the two Bessel functions in the solution (A10). The most general features of the solution are the same as those of the Rae and Roberts solution in that they both have a wave front followed by the oscillating wake. This wake is still capable of steepening nonlinearly into a train of rebound shocks. Consequently, the description of the nonlinear behavior of the solution in relation to the rebound shock model of spicules in Hollweg (1982) remains valid.

It is interesting to note the time development of the pulse region of the solution. This is depicted in Figure 10. (The example in Fig. 10 is of the same solution as that of Fig. 9.) In the first frame, Figure 10a, the solution is shown at an early time, $t = 2.26$ minutes. The pulse region of the solution is positive and shows little structure. Oscillations appear in the region at a later time, $t = 18$ minutes (Fig. 10b). At even later time, $t = 72.2$ minutes (Fig. 10c) internal structure dominates the region. As time advances, there is a piling up of oscillations in the “pulse region.” This behavior can also be seen from equation (A10) by noting the value of the first term at $z = c_s(t - b)$ (which is the height at which the second term becomes nonzero at time $t$). For large $t$, and thus large $z$, this value varies as $J_0(\omega_{ac}(2b)^{1/2})$, indicating that an increasing number of oscillations will occur in the “pulse region” as time increases. Note, however, that for the models presented in this paper, only one or two oscillations accumulate in the “pulse region” before the initial wavefront encounters the transition region (recall that there is no TR affecting the results of Figs. 9 and 10).

It is also interesting to examine the behavior of the wake well behind the pulse region, i.e., at $z < c_s(t - b)$ and $\omega_{ac}(t - b) \gg 1$. From the asymptotic behavior of the Bessel functions, it is readily deduced that the two Bessel functions in equation (A10) can combine to reinforce or partially cancel the distant wake, depending on the value of $b$. Reinforcement occurs when $\omega_{ac} b \approx \pi, 3\pi$, etc., for $b \approx 113, 340$ s, etc., if $\omega_{ac} = 2.77 \times 10^{-2}$ s$^{-1}$. On the other hand, partial cancellation can occur when $\omega_{ac} b \approx 2\pi, 4\pi$, etc., i.e., for $b \approx 227, 454$ s, etc. None of these special cases occurred in the models considered in this paper.

![Figure 10a](image-url)
Fig. 10b

Fig. 10c
REFERENCES

Boris, J. P. 1976, NRL Memorandum Rept. 3237.
Morse, P. M., and Feshbach, H. 1953, Methods of Theoretical Physics (New

Joseph V. Hollweg and Alphonse C. Sterling: Space Science Center, Institute for the Study of Earth, Oceans and Space, University of New Hampshire, Durham, NH 03824