THE QUASI-RIGID ROTATION OF CORONAL MAGNETIC FIELDS

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Received 1987 May 29; accepted 1987 September 23

ABSTRACT

Assuming that the coronal magnetic field can be approximated by a current-free extension of the photospheric field, we use spherical harmonic analysis and numerical simulations to study its rotational properties. In the outer corona, we find that the rotation rate is determined by three principal factors:

1. "Coronal filtering": the coronal rotation rate depends on the phase velocities and relative growth (decay) rates of only the low-order harmonic components of the photospheric field.

2. Global averages of the photospheric rotation rate: without ongoing source eruptions, the contributions of these low-order harmonics to the coronal rotation rate are given by latitudinal averages of the photosphere's angular velocity, weighted by the distribution of unsheared nonaxisymmetric flux.

3. Ongoing source eruptions: the eruption of large new sources of flux can produce erratic phase shifts in the low-order modes, leading to sudden fluctuations in the coronal rotation rate.

These principles allow us to understand the observationally inferred rotational properties of the outer coronal field. The overall rigidity of the rotation profile reflects the tendency for the photosphere's nonaxisymmetric flux to be concentrated toward lower latitudes, where the rotational shear is small; increased curvature and asymmetry occur during the rising phase of the sunspot cycle because of the presence of higher latitude flux. The coronal rotation rate shows a progressive acceleration due to the equatorward migration of sunspots, with the 27 day equatorial period being approached toward sunspot minimum as the decaying photospheric flux becomes localized near the equator.

Subject headings: Sun: corona — Sun: magnetic fields — Sun: rotation

I. INTRODUCTION

It is observationally well established that coronal structures rotate more rigidly and, away from the equator, more rapidly than the underlying photosphere. In the low corona, this behavior was strikingly apparent in the rotation of XUV and X-ray coronal holes observed by Skylab (Timothy, Krieger, and Vaiana 1975; Wagner 1975; Bohlin 1977). Quasi-rigid rotation has also been found from autocorrelation analyses of Fe xiv green-line intensities (Antonucci and Svalgaard 1974) and, at heights of up to 1.5 $R_\odot$, the underlying photosphere. In the low corona, this behavior was strikingly apparent in the rotation of XUV and X-ray coronal photospheric field (Hoeksema 1984; Hoeksema and Scherrer 1987). Some of the other properties of coronal rotation that were reported in the above studies included the following: (i) a tendency for the degree of rigidity to increase with height (Parker, Hansen, and Hansen 1982; Parker 1986). Further confirmation of this behavior is provided by potential field extrapolations of the observed photospheric field (Hoeksema 1984; Hoeksema and Scherrer 1987). Some of the other properties of coronal rotation that were reported in the above studies included the following: (i) a tendency for the degree of rigidity to increase with height (Parker, Hansen, and Hansen 1982; Parker 1986); (ii) an acceleration of the rotation rate over the course of the sunspot cycle (Antonucci and Svalgaard 1974; Fisher and Sime 1984; Parker 1986). Further confirmation of this behavior is provided by potential field extrapolations of the observed photospheric field (Hoeksema 1984; Hoeksema and Scherrer 1987). Some of the other properties of coronal rotation that were reported in the above studies included the following: (i) a tendency for the degree of rigidity to increase with height (Parker, Hansen, and Hansen 1982; Parker 1986); (ii) an acceleration of the rotation rate over the course of the sunspot cycle (Antonucci and Svalgaard 1974; Fisher and Sime 1984; Parker, Hansen, and Hansen 1982); (iii) asymmetries between the northern and southern hemispheres (Parker, Hansen, and Hansen 1982; Hoeksema and Scherrer 1987); and (iv) the dominance of two values of the synodic rotation period near 27 and 28 days (Hoeksema and Scherrer 1987).

To date, no convincing interpretation of the quasi-rigid rotation of coronal magnetic fields has been given. According to Billings (1966, p. 211) and Pneuman (1971), individual loop structures connecting high and low latitudes enforce a more rigid rotation above the photosphere, but such a mechanism fails to explain why open fieldline regions also rotate rigidly. Another frequently encountered idea is that large-scale or long-lived magnetic features, both on the photosphere and in the corona, are anchored to deeper layers within the Sun that rotate more rigidly than the surface plasma (see, e.g., Stenflo 1977; Zirker 1977). In a recent paper (Sheeley, Nash, and Wang 1987, hereafter Paper I), we showed that the rigid rotation of photospheric field patterns is in fact a natural consequence of latitudinal flux transport by supergranular diffusion, very likely assisted by a meridional bulk flow. This mechanism also partially explains the behavior of long-lived magnetic features in the low corona, such as the coronal holes observed at X-ray and XUV wavelengths (see Paper I). At greater heights, however, the magnetic field differs fundamentally from that near the photosphere, becoming increasingly dominated by global structure representing lower order multipoles of the photospheric field. In the present investigation, we show how the rotational properties of the field in the outer corona are related to photospheric source activity and the flux transport processes discussed in Paper I. (The rotation of coronal holes will be discussed in detail in a subsequent paper.)

Our general approach will follow that of Hoeksema and Scherrer (1987) in relating the coronal and photospheric fields by means of the potential field approximation. However, we are able to supplement synoptic observations of the photospheric field with numerical simulations using the flux transport algorithm of Sheeley, DeVore, and Boris (1985). This allows us to study the way in which the coronal rotation rate depends on the eruption of new flux and on the redistribution of this flux over the photosphere via

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differential rotation, supergranular diffusion, and meridional flow (§ III). In order to provide a physical basis for interpreting the simulations, we first establish some general properties of coronal rotation by means of spherical harmonic analysis (§ II).

II. GENERAL PRINCIPLES

a) The Photospheric Field and its Harmonic Components

Our objective is to derive the rotation rate of the outer coronal field as a function of latitude and time, based on the known properties of the photospheric field. The general formalism for describing this underlying field will therefore be outlined first. The evolution of the radial component of the large-scale photospheric field, as a function of elapsed time \( t \), will be assumed to obey the flux transport equation

\[
\frac{\partial B}{\partial t} = -\omega(\theta) \frac{\partial B}{\partial \phi} + \frac{\kappa}{R_\odot} \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial B}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 B}{\partial \phi^2} \right) - \frac{1}{R_\odot \sin \theta} \frac{\partial}{\partial \theta} \left[ B(\theta) \sin \theta \right] + S
\]

(see Leighton 1964; DeVore, Sheeley, and Boris 1984). Here spherical polar coordinates \((r, \theta, \phi)\) are employed, with the photosphere located at \( r = R_\odot \) and the azimuthal coordinate \( \phi \) measured westward from the Sun’s central meridian. \( B(R_\odot, \theta, \phi, t) \) denotes the radial component of the surface field, \( \omega(\theta) \) is the angular rate at which the photospheric plasma and individual elements of flux rotate as a function of colatitude \( \theta \), \( \kappa \) is the diffusion constant, \( v(\theta) \) is the meridional flow velocity, and \( S(R_\odot, \theta, \phi, t) \) is a source term representing the eruption of new flux.

In general, the solution of equation (1) may be decomposed in terms of spherical harmonics \( Y_{lm}(\theta, \phi) \) (see e.g., Jackson 1975, pp. 84–100):

\[
B(R_\odot, \theta, \phi, t) = \sum_{l=1}^{\infty} \sum_{m=0}^{l} \left[ A_{lm}(t) Y_{lm}(\theta, \phi) + A_{lm}^*(t) Y_{lm}(\theta, \phi) \right].
\]

(2)

Here an asterisk denotes a complex conjugate, and the coefficients are evaluated from

\[
A_{lm}(t) = \int B(R_\odot, \theta, \phi, t) Y_{lm}^*(\theta, \phi) d\Omega,
\]

(3)

where the integration is over solid angle \((d\Omega = \sin \theta \, d\theta \, d\phi)\). The coefficients \( A_{lm}(t) \) are complex quantities for \( m \neq 0 \). (When \( m = 0 \), the right-hand side of eq. [3] should be divided by 2; however, such axisymmetric modes do not contribute to the rotation rate.) For future convenience, we define real amplitudes \( b_{lm}(t) \) and phases \( \beta_{lm}(t) \) of the nonaxisymmetric harmonic components by means of the relations

\[
A_{lm}(t) = b_{lm}(t) \exp[-im\beta_{lm}(t)].
\]

(4)

Because the photospheric field is dominated by bipolar magnetic regions with longitudinal extents \( \Delta s \lesssim 10^5 \) km, high-order multipoles having \( l \sim \pi R_\odot / \Delta s \gtrsim 22 \) contribute significantly to the expansion given in equation (2). Fortunately, as we now discuss, the contribution of such harmonics can be entirely neglected when considering the outer coronal field.

b) The Coronal “Filter”

In order to extrapolate the photospheric field into the corona, we adopt the potential field model introduced by Schatten, Wilcox, and Ness (1969) and Altschuler and Newkirk (1969) and discussed more recently by Hoeksema (1984). Here it is supposed that no significant electrical currents flow in the region between the photosphere and a spherical “source surface” located at \( r = R_s \), where the magnetic field is required to be purely radial. Thus, by solving Laplace’s equation inside the shell \( R_e < r < R_s \), subject to the equipotential condition at \( r = R_s \) and the requirement that the radial component of the field reduce to its known behavior at \( r = R_e \), one obtains, for the radial components of the field within the shell,

\[
B(r, \theta, \phi, t) = \sum_{l=1}^{\infty} \sum_{m=0}^{l} c(r) \left[ A_{lm}(t) Y_{lm}(\theta, \phi) + A_{lm}^*(t) Y_{lm}(\theta, \phi) \right],
\]

(5)

where

\[
c(r) = \left[ \frac{l + 1}{(l + 1) + l(R_s / R_e)^{2l+1}} \right] \left( \frac{R_e}{r} \right)^{l+2} + \left[ \frac{l(R_s / R_e)^{l+2}}{(l + 1) + l(R_s / R_e)^{2l+1}} \right] \left( \frac{r}{R_e} \right)^{l+1}.
\]

(6)

At the photosphere, \( c(R_\odot) = 1 \), whereas at the source surface the reduction factor becomes

\[
c(R_s) = \frac{(2l + 1)(R_s / R_e)^{l+2}}{(l + 1) + l(R_s / R_e)^{2l+1}}.
\]

(7)

The radius of the source surface will henceforth be taken to be \( R_s = 2.5 R_\odot \). This value was arrived at semiempirically by Altschuler and Newkirk (1969) using eclipse photographs and by Hoeksema (1984), who compared the predicted and measured polarity of the interplanetary magnetic field.² From equation (7), we see that the contribution of the \( l = 22 \) multipole (for example) is reduced at the source surface by a factor \( c_{22}(R_s) \approx 6 \times 10^{-10} \) relative to its photospheric value. Since the amplitudes of the coefficients \( A_{lm} \) in equation (5) vary much more slowly with \( l \) (see Fig. 15 in Altschuler et al. 1977), only the lowest order harmonics of the photospheric field can be expected to survive at \( r = 2.5 R_\odot \): the high-order modes important at the photosphere are effectively filtered out.

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Coronal Magnetic Fields

No. 1, 1988

In this study, our main focus will be on the properties of coronal rotation in the vicinity of the source surface. However, our analysis remains applicable at lower heights, provided a sufficient number of terms are retained in the multipole expansion: the "filter" becomes less efficient when \( r < R \), since \( c(r) \) then declines less steeply with \( l \).

c) The Coronal Rotation Rate

The fact that the outer coronal field is dominated by a relatively small number of low-order multipoles in itself suggests that the corona must rotate more rigidly than the photospheric field. This is clearly the case in the limit where only one harmonic mode \((l, m)\) is present, since a single mode—e.g., a pure dipole \((1, 1)\) field—rotates rigidly by definition. More generally, the dominance of a few low-order multipoles up to some \( l \approx l_{\text{max}} \) means that only some weighted average of the rotational properties of the photospheric field, over an angular scale exceeding \( \pi/l_{\text{max}} \), is transmitted into the corona; smaller scale variations are filtered out.

As shown in Paper I, the rotational phase velocity measured by cross-correlating an arbitrary magnetic field \( B(r, \theta, \phi, t) \) at successive times can be expressed as

\[
\omega_c(r, \theta, t) = -\frac{\left( \frac{\partial B}{\partial \phi} \right)_t}{\int_0^{2\pi} \left( \frac{\partial B}{\partial \phi} \right)_t d\phi} .
\]  \(8\)

(Although this equation can be applied to any scalar function that is periodic in \( \phi \), we continue to identify \( B \) with the radial component of the magnetic field.) Thus we can determine the coronal rotation rate by extrapolating the known photospheric field outward by means of equations (5)-(6), and inserting the result into equation (8). For this purpose, it will be convenient to write the nonaxisymmetric \((m \neq 0)\) component of the radial field in the form

\[
B(r, \theta, \phi, t) = \sum_{l=1}^{\infty} \sum_{m=1}^{l} a_{lm}(r, t) P_l^m(\cos \theta) \cos m(\phi - \beta_{lm}(t)) ,
\]  \(9\)

where

\[
a_{lm}(r, t) = 2\frac{2l + 1}{4\pi} \frac{1}{(l + m)!} \int_0^{2\pi} c(r, \theta, t) P_l^m(\cos \theta) \cos m(\phi - \beta_{lm}(t)) d\phi
\]  \(10\)

and the \( P_l^m(\cos \theta) \) are the associated Legendre functions, defined as in Abramowitz and Stegun (1968, p. 334).\(^2\) (Axisymmetric modes do not contribute to the rotation rate given in eq. [8]).

Substituting the expansion (9) into equation (8) and performing the required integrations over \( \phi \), we obtain

\[
\omega_c(r, \theta, t) = \frac{\sum_{l=1}^{\infty} \sum_{m=1}^{l} \beta_{lm} w_{lm}}{\sum_{l=1}^{\infty} \sum_{m=1}^{l} \sum_{l'=1}^{\infty} w_{l'm}} + \frac{\sum_{l=1}^{\infty} \sum_{m=1}^{l} (1/m)[(\partial a_{lm}/\partial r_m) - (\partial a_{lm}/\partial a_m)] w_{l'm} \tan m(\beta_{lm} - \beta_{l'm})}{\sum_{l=1}^{\infty} \sum_{m=1}^{l} \sum_{l'=1}^{\infty} w_{l'm}}
\]  \(11\)

Here a dot indicates a time derivative, and we define

\[
w_{lm}(r, \theta, t) = a_{lm} q_{lm} \cos m(\beta_{lm} - \beta_{l'm})
\]  \(12\)

\[
a_{im}(r, t) = m a_{lm}(r, t) P_l^m(\cos \theta)
\]  \(13\)

Equation (11) expresses the instantaneous rotation rate of the total field as a weighted average of the phase velocities \( \beta_{lm} \) of its harmonic components, plus a weighted average of quantities that represent differences between the growth rates of harmonics having the same \( m \) but differing \( l \).

If we momentarily ignore all mode-coupling terms in equation (11), we obtain

\[
\omega_c(r, \theta, t) = \frac{\sum_{l=1}^{\infty} \sum_{m=1}^{l} \beta_{lm} q_{lm}^2}{\sum_{l=1}^{\infty} \sum_{m=1}^{l} q_{lm}^2}
\]  \(14\)

Equations (13) and (14) show that the contribution of each individual mode to the net rotation rate is weighted in proportion to its harmonic power, \( a_{lm}^2 \propto b_{lm}^2 c_l^2 \), which declines steeply with increasing \( l \) since \( c_l^2 \propto r^{-2l-4} \). The weighting factors are also proportional to \( [P_l^m(\cos \theta)]^2 \) and thus vary with latitude. Clearly, if a single mode were to dominate at all latitudes, the coronal field would rotate rigidly with the phase velocity of that mode. Otherwise, the rotation rate \( \omega_c \) will be independent of latitude only if the phase velocities of all the contributing modes are the same.

If the terms in equation (11) proportional to \((\partial a_{lm}/\partial r_m) - (\partial a_{lm}/\partial a_m)\) do not vanish, however, rigid rotation will not be obtained even if the individual phase velocities are the same. These terms, which couple pairs of modes having equal \( m \) but differing \( l \), arise because a change in the amplitude of one mode relative to that of the other will produce a longitudinal phase shift in the net field, unless the two modes happen to be exactly in phase with each other, i.e., \( \beta_{lm} = \beta_{l'm} \).

To illustrate these ideas further, we suppose that the coronal field consists solely of dipole \((l = 1)\) and quadrupole \((l = 2)\) contributions. This case is of particular interest because harmonic analysis based on our numerical simulations and on magnetograph measurements (§ III; see also Hoeksema 1984) suggests that the magnetic field at \( 2.5 R_\odot \) was indeed dominated by its dipole.\(^3\)

\(^2\) A more refined model of the coronal field would allow the source radius to be a function of both time and angular position. However, unless \( R_0 \rightarrow R_\odot \), we would not expect the inclusion of such variations to alter our basic conclusions concerning the coronal rotation rate.

\(^3\) This definition incorporates the phase factor \((-1)^m\) into the function \( P_l^m(\cos \theta) \).
and quadrupole components during most of sunspot cycle 21. Thus, including only the (1, 1), (2, 1), and (2, 2) modes, writing the associated Legendre functions in terms of colatitude \( \theta \), and canceling out a common factor of \( \sin^2 \theta \), we find

\[
\omega_i(r, \theta, t) = \frac{a_{11} \beta_{11} + 36a_{22} \beta_{22} \sin^2 \theta + 9a_{21} \beta_{21} \cos^2 \theta + 6a_{11} a_{21} \gamma \cos (\beta_{11} - \beta_{21}) \cos \theta}{a_{11} + 36a_{22} \sin^2 \theta + 9a_{21} \cos^2 \theta + 6a_{11} a_{21} \cos (\beta_{11} - \beta_{21}) \cos \theta},
\]

where

\[
\gamma = \frac{1}{2} (\beta_{11} + \beta_{22}) + \frac{1}{2} \left( \frac{a_{11}}{a_{21}} - \frac{a_{21}}{a_{11}} \right) \tan (\beta_{11} - \beta_{21}).
\]

According to equation (15), the (2, 2) mode will have its greatest influence on the rotation rate at low latitudes (where \( \sin^2 \theta \approx 1 \)), whereas the (2, 1) mode will have its largest effect at high latitudes (where \( \cos^2 \theta \approx 1 \)); the contribution of the dipole field alone is independent of latitude. The rotation profile will be symmetric across the equator only if \( a_{21} = 0 \) or \( a_{11} = 0 \): the coupling between the (2, 1) and (1, 1) modes is seen to yield phase-dependent contributions which are proportional to \( \cos \theta \), and which therefore change sign across the equator. More generally, such north-south asymmetries are produced by the interaction between modes of the same \( m \) but with \( l \) differing by an odd number.

In summary, although the filtering out of higher order harmonics clearly removes the small-scale latitudinal variations contributed by these modes, the degree of rigidity of the coronal rotation profile depends on the amplitudes and phase velocities of the surviving modes. Thus, in the above example, if neither \( a_{11} \) nor \( a_{22} \) nor \( a_{21} \) can be neglected, a necessary requirement for rigid rotation is that the phase velocities of these modes be the same: \( \beta_{11} = \beta_{22} = \beta_{21} \). Even if this condition is satisfied, however, the rotation profile can still be affected by changes in the relative amplitudes of the (1, 1) and (2, 1) modes.

In the Appendix we describe other specific examples, involving sectoral fields subjected to the differential rotation of the photosphere. For a field that consists initially of a single harmonic, we demonstrate explicitly that the coronal rotation profile is always more rigid than the underlying photospheric rotation, and in fact becomes asymptotically completely rigid at the equatorial rate. This is because the phase velocities of the lower order multipoles of such a configuration all approach the equatorial rate after a few windup times, while the relative growth rates also vanish.

**d) Transport Equations for the Harmonic Amplitudes and Phases**

In the preceding subsection we showed that the rotation rate of the coronal field, \( \omega_i(r, \theta, t) \), can be expressed in terms of the phases \( \beta_{im}(t) \) and amplitudes \( b_{im}(t) \) of the lowest order harmonic components of the photospheric field. We now show that the time derivatives of these quantities can be written as weighted averages of the rates of eruption and transport of photospheric flux.

From equation (4), one sees that

\[
\left( \frac{b_{im}}{b_{im}} \right) - i m \beta_{im} = \frac{A_{im}}{A_{im}} \left[ \int \frac{\partial B(R_\odot, \theta, \phi, t) / \partial t}{B(R_\odot, \theta, \phi, t)} Y_m^{*}(\theta, \phi) d\Omega \right].
\]

(Since \( a_{im}/a_{im} = b_{im}/b_{im} \) by eq. [10], the harmonics of the coronal and photospheric fields evolve at the same rates and may be considered interchangeable for the present purposes.) Inserting the expression for \( \partial B/\partial t \) given by equation (1) and taking the real and imaginary parts of equation (17), we obtain

\[
\frac{b_{im}}{b_{im}} = m \text{ Im} \langle \omega \rangle_{im} - \frac{\kappa}{R_\odot} k(l + 1) - \text{ Re} \left( \frac{v}{R_\odot} \frac{\partial Y_m^*}{\partial \theta} \right)_{im} + \text{ Re} \left( \frac{\langle S \rangle}{B_{im}} \right)_{im},
\]

\[
\beta_{im} = \text{ Re} \langle \omega \rangle_{im} + \frac{1}{m} \text{ Im} \left( \frac{v}{R_\odot} \frac{\partial Y_m^*}{\partial \theta} \right)_{im} - \frac{1}{m} \text{ Im} \left( \frac{\langle S \rangle}{B_{im}} \right)_{im},
\]

where

\[
\langle \omega \rangle_{im} = \left[ \omega(\theta) B(R_\odot, \theta, \phi, t) Y_m^{*}(\theta, \phi) d\Omega \right] / \left[ B(R_\odot, \theta, \phi, t) Y_m^{*}(\theta, \phi) d\Omega \right],
\]

\[
\left( \frac{v}{R_\odot} \frac{\partial Y_m^*}{\partial \theta} \right)_{im} = \left[ \frac{\partial B(R_\odot, \theta, \phi, t)}{B(R_\odot, \theta, \phi, t)} Y_m^{*}(\theta, \phi) d\Omega \right] / \left[ B(R_\odot, \theta, \phi, t) Y_m^{*}(\theta, \phi) d\Omega \right],
\]

\[
\left( \frac{\langle S \rangle}{B_{im}} \right)_{im} = \left[ S(R_\odot, \theta, \phi, t) Y_m^{*}(\theta, \phi) d\Omega \right] / \left[ B(R_\odot, \theta, \phi, t) Y_m^{*}(\theta, \phi) d\Omega \right].
\]

Equations (18a) and (18b) give the growth (or decay) rates and phase velocities of the harmonic components of the field in terms of weighted averages involving the photospheric differential rotation rate \( \omega(\theta) \), diffusion rate \( \kappa \), meridional flow velocity \( \nu(\theta) \), and source function \( S(R_\odot, \theta, \phi, t) \). It may be noted that diffusion contributes only indirectly to the phase velocity equation (18b), through its effect on the photospheric flux distribution \( B(R_\odot, \theta, \phi, t) \).

The terms involving \( \langle S/B \rangle_{im} \) are due to concurrently emerging flux. If this new flux is injected exactly in phase with the background field in the given harmonic, \( \langle S/B \rangle_{im} \) is real and only the amplitude of the harmonic changes, in accordance with equation (18a); conversely, if the phase difference is \( \pm \pi/2 \), \( \langle S/B \rangle_{im} \) is imaginary and only the phase changes.
CORONAL MAGNETIC FIELDS

It will be instructive to compare the sizes of the various terms contributing to equations (18a) and (18b). For this purpose, we define $t_d \equiv R_C^2/v_{eq}$ as the global diffusion time scale for the photospheric flux, $t_w \equiv 2\pi/[\omega(n/2) - \omega(0)]$ as the windup time scale, $t_s \equiv R_C/v_0$ as the time scale for meridional flow to transport flux to the poles at a peak speed $v_0$, and $\tau_\phi$ as the time scale for new sources to regenerate the large-scale photospheric field. Then in the equatorial rotation frame [where $\omega(\theta) \rightarrow \omega(\theta - \omega(n/2)$ and $\beta_{lm} \rightarrow \beta_{lm} - \omega(n/2)$], the approximate magnitude of the four terms on the right-hand side of equation (18a) may be written, in the order in which they appear, as

$$\frac{m}{\tau_w}; \frac{l(l+1)}{\tau_d}; \frac{m}{\tau_s}; \frac{1}{\tau_f}. \tag{22a}$$

Similarly, for equation (18b), we have

$$\frac{1}{\tau_w}; \frac{1}{\tau_s}; \frac{m}{\tau_f}; \frac{1}{m\tau_s}. \tag{22b}$$

If the parameters are assigned the nominal values $\kappa \sim 300 \text{ km}^2 \text{s}^{-1}, v_{eq} \sim 10 \text{ m s}^{-1}, \omega(n/2) \approx 13.3 \text{ day}^{-1}$, and $\omega(0) \approx 9.46 \text{ day}^{-1}$ (see eq. [28] below), then $t_d \sim 51 \text{ yr}, t_s \sim 2.2 \text{ yr}$, whereas $t_w \sim 3 \text{ months}$. We conclude that, in the absence of concurrent source eruptions (see eq. [28] below), then $\tau_\phi$ tends to have a greater immediate effect on the coronal rotation rate (by producing large, erratic phase shifts in the rotating patterns of photospheric field). If the latitudinal transport processes were somehow to be "switched off," the phase velocities of these patterns would instantly collapse back to the intrinsic rate of differential rotation, $\omega(\theta)$. In contrast, the rotation rate of the outer coronal field would only be affected on the time scale for transport processes to alter the global flux distribution.

It should be stressed that diffusion and meridional flow do contribute indirectly to the amplitudes and phase velocities of the low-order harmonics, since they determine the long-term evolution of the photospheric field $B(R_C, \theta, \phi, t)$ and the way in which the photospheric rotation rates are weighted in equations (23a) and (23b) at a given time. This indirect effect can be thought of as arising through the coupling between low- and high-order modes by differential rotation, as may be seen by writing

$$\langle \omega \rangle_{lm} = \frac{1}{A_{lm}} \int \sum_{m'} \sum_{l=\pm n} A_{l'm'} \int \omega(\theta) Y_{lm}^*(\theta, \phi) \phi Y_{lm}(\theta, \phi) d\Omega. \tag{24}$$

The integral on the right vanishes unless $m' = m$ and $l' = l \pm 2n$, where $n$ is an integer and the symmetry property $\omega(\theta) = \omega(\pi - \theta)$ is assumed; thus, differential rotation couples modes having the same parity and sector number. [The same selection rules apply to mode coupling by meridional flow, provided $\omega(\theta) = -\omega(\pi - \theta).$] In the absence of diffusion, power will be transferred in both directions, i.e., from high to low $l$, and vice versa. However, if diffusion is present, the high-order modes will be damped on the relatively short time scale $t_d/l(l+1)$, and the result will be a unidirectional flow of power out of the low-order modes, pumped by differential rotation. The large-scale field will thus decay after a few windup times, unless it is replenished by new sources.

It will normally be simpler to interpret the rotational behavior of the low-order modes in terms of the properties of the photospheric flux distribution, as implied by the form of equation (19), rather than in terms of the interaction between individual modes. We adopt this spatial approach in the following subsections.

e) The Latitudinal Weighting Function

According to equations (23a) and (23b), the growth rates and phase velocities of the low-order harmonics contributing to the coronal field can be obtained directly from the weighted averages $\langle \omega(\theta) \rangle_{lm}$ defined by equation (19), provided there are no ongoing
source eruptions. In that case, the real part of \( \langle \omega(\theta) \rangle_m \) yields the phase velocity \( \beta_m \), whereas its imaginary part gives the growth or decay rate of the given mode.

We now examine the properties of these weighted averages. For this purpose it is convenient to rewrite equation (19) as

\[
\langle \omega(\theta) \rangle_m = \frac{\int_0^{2\pi} \omega(\phi) B_m(\theta, \phi, t) \, d\phi}{\int_0^{2\pi} B_m(\theta, \phi, t) \, d\phi},
\]

where we introduce the latitudinal weighting function

\[
g_m(\theta, t) \equiv \sin \theta P_m^m(\cos \theta) \int_0^{2\pi} B(R_0, \theta, \phi, t) e^{-im\phi} \, d\phi.
\]

The integral in equation (26) represents the \( m \)th coefficient in a Fourier series expansion of the photospheric field and may be expressed in the form \( B_m(\theta, t) \exp \left[ -im \delta_m(\theta, t) \right] \). With this notation, the weighting function becomes

\[
g_m(\theta, t) = \sin \theta P_m^m(\cos \theta) B_m(\theta, t) \exp \left[ -im \delta_m(\theta, t) \right].
\]

For the low-order harmonics of interest here, the associated Legendre functions vary relatively slowly with \( \theta \). Consequently, the weighting function will attain its largest amplitude near the colatitude \( \theta = \theta_{\text{max}} \) where the Fourier amplitude \( B_m(\theta, t) \) peaks. Provided the phase factor \( \delta_m(\theta, t) \) varies reasonably slowly in the neighborhood of \( \theta_{\text{max}} \), this latitudinal zone will provide the dominant contribution to the weighted average given in equation (25). However, if the phase gradients are locally very steep, so that \( g_m(\theta, t) \) undergoes rapid latitudinal oscillations near \( \theta_{\text{max}} \), the net contribution to \( \langle \omega(\theta) \rangle_m \) from this region may be greatly reduced.

When an initial configuration consisting of a single Fourier component of the form \( B(R_0, \theta, \phi, t) = f(\phi) \cos(m\phi) \) is subjected to differential rotation, its latitudinal phase gradients increase with time according to \( \delta_m(\theta, t) = \omega(\theta)t \). As illustrated in the Appendix, the region providing the main contribution to \( \langle \omega(\theta) \rangle_m \) then shifts from the vicinity of \( \theta_{\text{max}} \), where the peak of the initial flux distribution is located, to the equator or the poles, where \( \omega(\theta) = 0 \). Thus the low-order modes start out with phase velocities \( \beta_m \approx \omega(\theta_{\text{max}}) \), but eventually rotate at either the equatorial or the polar rate. From a physical viewpoint, the initial sectoral field is wound into an increasing number of latitudinal bands of alternating polarity, having their narrowest spacing at midlatitudes where the rotational shear \( \omega(\theta) \) is largest. The winding process transfers power to high-order multipoles, which are filtered out in the corona. Whether the equatorial or the polar asymptote dominates depends on the relative amounts of unsheared flux remaining in those regions.

If diffusion is present, it will accelerate the approach to the asymptotic state by preferentially annihilating flux and reducing the amplitude \( B_m \) where the field has become tightly wound. This is the spatial analogue of the process mentioned in § IIb, whereby diffusion rapidly damps the power which differential rotation pumps into high-order modes. The decay process has recently been analyzed in detail by DeVore (1987).

We conclude that it is the latitudinal distribution of the "unwound," nonaxisymmetric flux that determines the phase velocities of the low-order harmonics of the photospheric field, and hence the rotation rate of the coronal field as a whole.

**f) The Source Latitudes and the Role of Latitudinal Transport Processes**

The bulk of the Sun's nonaxisymmetric flux is in reality concentrated in the sunspot latitudes, which extend more than 30° northward and southward of the equator. Except near sunspot minimum, the average age of this flux is observed to be at most of the order of a windup time \( t \), so that the peak in the distribution of "unwound" flux must be located in or near this region. Thus the coronal field and its harmonic components will rotate at rates comparable to those of the sunspot belts during most of the sunspot cycle.

The rigidity of the coronal rotation profile reflects the fact that the photosphere's rotational shear is relatively small at low latitudes: this reduces the spread in the values of the harmonic phase velocities.

The synodic rotation rate of individual photospheric features, as determined empirically by Snodgrass (1983), is described by the formula

\[
\omega(\theta) = 13.38 - 2.30 \cos^2 \theta - 1.62 \cos^4 \theta \text{ deg day}^{-1}.
\]

(A bipolar magnetic region centered at a latitude of 8°, for example, rotates with a 27.0 day period, whereas its period would be 28.0 days at a latitude of 26.5. The maximum rotational shear occurs at a latitude of 54.5 in each hemisphere.) Starting at an average latitude of \( \sim 30° \), sunspot eruptions migrate equatorward over the course of the sunspot cycle, as depicted by the Maunder butterfly diagram. The centroid of the distribution of unwound flux thus shifts progressively toward lower latitudes. This shift will be accompanied by an average acceleration of the coronal rotation rate during the sunspot cycle.

Of course, magnetic flux elements do not remain fixed at the latitudes where they first appear but are transported over the photosphere by supergranular diffusion, possibly supplemented by a meridional bulk flow. This redistribution of flux in latitude will have its greatest effect on the weighted averages \( \langle \omega(\theta) \rangle_m \) when source activity diminishes toward the end of the sunspot cycle. Supergranular diffusion spreads the sunspot-belt flux both poleward and equatorward. If source eruptions cease, however, the shearing and decay of the large-scale nonaxisymmetric field at midlatitudes would cause the peak of the distribution of unwound flux to shift toward the equator. Eventually the remnant flux would become narrowly localized about the equator, and the coronal field and its harmonic components would rotate rigidly at approximately the equatorial rate of the photosphere.

Observational evidence for a poleward bulk flow on the Sun has been presented by Duvall (1979), Howard (1979), and LaBonte and Howard (1982). The effect of such a flow on a given initial sectoral field in the presence of differential rotation is examined in the
Appendix, where we show that the harmonic phase velocities asymptotically approach either the polar or the equatorial rate. If the initial flux distribution is broadly peaked around the equator, where the flow speed is assumed to vanish, the equatorial asymptote dominates because the flux that reaches the poles is too tightly wound to contribute significant power to the low-order modes. Conversely, for narrow initial flux distributions peaked outside the equator, the polar asymptote generally wins.

The continued injection of new flux by sources will prevent the peak of the "unwound" flux distribution from migrating all the way to the poles or to the equator. If meridional flow is indeed present on the Sun, a balance between the poleward transport of fresh flux from the sunspot belts and its windup at higher latitudes may be established during times of significant source activity. This would shift the "center of gravity" of the nonaxisymmetric flux from the centroid of source activity toward higher latitudes. This spreading of the flux beyond the sunspot regions would reduce the harmonic phase velocities and increase the power in those modes that are intrinsically weighted toward higher latitudes (e.g., the harmonic $Y_{21}$, $\propto \sin 2\theta$). As a result, the coronal field would not only rotate more slowly than it would when diffusion alone is present, but its rotation profile would display a greater degree of curvature because a much wider range of photospheric rotation rates is "sampled" by the contributing modes. Furthermore, this curved profile would only be symmetric across the equator if the midlatitude flux distribution happens to be nearly symmetric between the two hemispheres.

III. SIMULATIONS

a) Procedure

Our objective here is to model the rotation of the outer coronal field numerically over the course of a sunspot cycle. First, we determine the time evolution of the radial component of the photospheric field, using the algorithm described by Sheeley, DeVore, and Boris (1985) and by DeVore (1986). Our code integrates the flux transport equation (1) numerically, depositing doublet sources during the computation according to their empirically determined strengths, positions, and times of eruption. The photospheric differential rotation rate $\omega(\theta)$ is assumed to obey the Snodgrass formula (28). We perform the simulations over the time interval 1976 August–1986 December (although no sources are entered after 1986 April), employing a grid consisting of 128 cells uniformly spaced in longitude and 64 cells uniformly spaced in latitude to represent the entire photosphere. (In computations involving meridional flow, the resolution is doubled in each direction to reduce the amount of numerical diffusion.) Applying the potential field model in longitude and 64 cells uniformly spaced in latitude to represent the entire photosphere. (In computations involving meridional flow, the resolution is doubled in each direction to reduce the amount of numerical diffusion.) Applying the potential field model described in § IIb, we extrapolate the field from the photosphere to the source surface located at $r = 2.5 R_\odot$, including the first nine multipoles ($l = 1$–9) in the expansion (5). The resulting instantaneous maps of the source surface field $B(R_\odot, \theta, \phi, t)$, obtained at consecutive intervals of 27.275 days (the approximate Carrington period), are then cross-correlated to yield the coronal rotation rate as a function of colatitude and time, $\omega_c(R_\odot, \theta, t)$. Finally, yearly averages for the period 1977–1986 are obtained from these monthly profiles. It should be remarked that the appearance of some of the individual monthly profiles may deviate considerably from the annual averages displayed below.

b) The Effect of Diffusion

Figure 1 shows the yearly averaged rotation profiles of the simulated coronal field, based on a photospheric model in which the supergranular diffusion rate is assigned the value $\kappa = 300$ $\text{km}^2 \text{s}^{-1}$, meridional flow is absent, and doublet sources are deposited until 1986 April. As in Paper I, this choice of conditions will be designated the "reference" case. The quasi-rigid behavior seen in Figure 1 is in stark contrast to the rotation of the underlying photospheric field, which closely follows the Snodgrass curve given in equation (28) except at high latitudes (see Paper I). The average coronal rotation period is seen to vary from year to year, ranging from 28 days to 26.9 days, the equatorial period of the photospheric field. The rotation is most rigid late in the cycle, when a period in the vicinity of 27.0 days is approached at all latitudes. During earlier phases, the rotation profiles show a greater degree of curvature and north-south asymmetry. As will be demonstrated below (see Fig. 4), this can be attributed to the presence of slowly rotating, high-latitude flux.

Next, we study the contribution of individual harmonic components to the coronal rotation rate. Figure 2 shows the result of cross-correlating separately the dipole component ($l = 1$; dotted lines), the quadrupole component ($l = 2$, including both the $m = 1$ and $m = 2$ contributions; dashed lines), and the combined dipole and quadrupole components (solid lines) of the coronal field as computed above. Comparing Figures 1 and 2, we conclude that the dipole-plus-quadrupole rotation rate matches that of the total field at low latitudes throughout the simulated cycle, whereas the dipole rate alone generally does not, particularly in 1978 and 1982. During 1978 and 1984, in fact, the phase velocity of the dipole component exceeds even the equatorial rate of the photosphere. These apparently anomalous values of the phase velocity are caused by the appearance of large new active regions during periods when the dipole component of the background field is relatively weak. The effect is described by the term $-m^{-1} \text{Im} \langle S/B \rangle_m$ in equation (23b), which dominates if the time scale $\tau_m$ for sources to inject power into the given mode is much less than the windup time $\tau_w$. In that case, the phase $\beta_m$ will shift abruptly to that of the new source or complex of sources. Since the magnitude of the phase shift varies inversely as the azimuthal mode number $m$, the phase velocities of the lowest order modes will tend to show the largest short-term variations.

The rotation profiles of the combined dipole and quadrupole fields displayed in Figure 2 do not accurately reproduce the curvature seen in the profiles of the total field during the rising phase of the cycle (Fig. 1). Some of this high-latitude curvature must therefore be contributed by higher order multipoles with $l \geq 3$.

Next we examine the distribution of power among the lowest order harmonics of the simulated coronal field as a function of time. For this purpose, we express the nonaxisymmetric component of the source-surface field as superpositions of its harmonic components

$$B(R_\odot, \theta, \phi, t) = \sum_{l=1}^{\infty} h_l = \sum_{l=1}^{\infty} \sum_{m=1}^{l} h_{lm},$$

(29)
FIG. 1.—Yearly averaged rotation profiles of the coronal field at 2.5 \( R_\odot \), computed on the basis of a "reference" photospheric model with diffusion rate \( \kappa = 300 \text{ km}^2 \text{s}^{-1} \), no meridional flow, and doublet sources deposited from 1976 August to 1986 April. The differential rotation rate adopted for the photosphere, eq. (28), is plotted (thin curve) for comparison. The coronal rotation profiles become increasingly rigid as they shift toward the equatorial period of the photosphere near the end of the sunspot cycle.
Fig. 2—Yearly averaged rotation profiles obtained by cross-correlating separately the dipole component (l = 1, dotted lines), the quadrupole component (l = 2, dashed lines), and the combined dipole and quadrupole components (l = 1 plus l = 2, solid lines) of the "reference simulation" coronal field of Fig. 1. The large displacements of the dipole component during 1978 and 1982 are due to strong, concurrent source eruptions.
and write the mean-square value of \( B(R_s, \theta, \phi, t) \) over the source surface as

\[
\langle B^2_R \rangle = \frac{1}{4\pi} \int B^2(R_s, \theta, \phi, t) d\Omega = \sum_{l=1}^{\infty} \sum_{m=1}^{l} \langle h_{lm}^2 \rangle.
\]

Using equations (4) and (5) for \( B(R_s, \theta, \phi, t) \) and performing the integration over solid angle, we deduce that

\[
\langle h_{lm}^2 \rangle = \frac{2}{4\pi} \left[ c(R_s) b_{lm}(t) \right]^2,
\]

which we recognize as the power per unit solid angle in the single harmonic component \((l, m)\). Similarly, \( \langle h_{l}^2 \rangle \) represents the total power per unit solid angle in a given multipole \( l \) (summed over all nonaxisymmetric modes \( 1 \leq m \leq l \)) of the source-surface field.

Figure 3 shows the evolution of the rms amplitudes \( \langle h_{l}^{1/2} \rangle \), \( \langle h_{2}^{1/2} \rangle \), and \( \langle h_{3}^{1/2} \rangle \) during the course of the simulation of Figure 1. Consistent with our discussion of Figure 2, the quadrupole \((l = 2)\) component is seen to be especially strong during 1978 and 1982, and the horizontal dipole \((l = 1)\) dominates during the late phase of the cycle when the rotation becomes rigid. Also displayed in Figure 3 are the rms amplitudes of the \((2, 1)\) and \((2, 2)\) modes individually, i.e., the quantities \( \langle h_{21}^{1/2} \rangle \) and \( \langle h_{22}^{1/2} \rangle \). It may be noted that the main contribution to the quadrupole field throughout the cycle is provided by the equatorially weighted \((2, 2)\) mode \((Y_{22} \propto \sin^2 \theta)\). This is expected since relatively little flux is present at midlatitudes, where the \((2, 1)\) mode has its strongest weighting \((Y_{21} \propto \sin \theta)\).

In § II\(f\), we anticipated that the equatorward migration of sunspot activity over the sunspot cycle should be accompanied by an acceleration of the average coronal rate, as observed in the simulation of Figure 1. However, this effect alone cannot account for the extreme rigidity of the rotation profile during the last years of the cycle, or for the close approach to the 26.9 day equatorial period at a time when sources continue to erupt over a fairly wide latitudinal range. At this late stage, the main contribution to the low-order harmonics of the field is provided by the flux from earlier source eruptions, which becomes localized about the equator because of the rapid windup and decay of the field at midlatitudes. (The tendency for the maximum of the photospheric flux...
In order to illustrate the effect of decaying flux, we performed two additional experiments. First, the computation of Figure 1 was repeated, but no doublet sources were deposited after 1979 December. After several windup times (before the end of 1981), the coronal field had largely decayed and was rotating rigidly near the equatorial rate. (The results of this computation are not displayed here.)

Next we again reran the simulation of Figure 1, but this time omitted all sources that erupted equatorward of latitude ±25° during the cycle. During the rising phase of the cycle, the coronal rotation profiles (see Fig. 4) display a substantial degree of curvature and asymmetry. This can be attributed to the fact that most of the nonaxisymmetric photospheric flux is initially located at midlatitudes, where the gradients in the photospheric rotation profile given in equation (28) are relatively steep: thus the coronal field "samples" a relatively wide range of rotation rates. Until 1984, the latitudinally averaged coronal rotation period decreases slowly from over 29 days to 28 days, corresponding to photospheric rotation in the latitude range ~25°–35°. After 1984, however, the coronal rotation becomes very rigid and rapidly approaches a rate near 27.0 days. Since no sources were deposited at latitudes below 25°, this can only be explained if the flux from decaying doublet sources has diffused to the equator and now contributes more to the low-order harmonics of the field than the few concurrent high-latitude sources do. We conclude that diffusion will act to make the coronal rotation profile rigid near the equatorial rate of the photosphere, unless new flux is injected sufficiently rapidly by sources at higher latitudes.

In Figure 4, the kink in the rotation profile for 1983 can be traced to the eruption of two strong doublet sources near 30°S latitude, which produced large instantaneous phase shifts during Carrington rotation 1735 and injected considerable power into the antisymmetric (2, 1) mode. The new sources disrupted the background field to the extent that the coronal rotation rates became ill-determined in a range of latitudes just above the equator. The resulting temporary discontinuity between the northern and the slowly rotating southern hemisphere is reflected in the annually averaged rotation curve.

c) The Effect of Meridional Flow

We now set \( \kappa = 0 \) but include a poleward meridional flow with a latitude dependence:

\[
v(\lambda) = -v_0 \left( \frac{\sin \lambda}{\sin \lambda_0} \right) \left( \frac{\cos \lambda}{\cos \lambda_0} \right)^3,
\]

where the maximum flow speed \( v_0 \), attained at latitude \( \lambda_0 = 30° \) in each hemisphere, is arbitrarily taken to be 10 m s\(^{-1}\). In this simulation, doublet sources are deposited on the photosphere only until the end of 1981. The yearly averaged rotation rates of the coronal field are displayed as a function of latitude by the solid curves in Figure 5. As compared with the reference case of Figure 1, the more pronounced curvature and asymmetry of the rotation profiles, as well as the substantially slower rotation rates, are striking. As in the simulation of Figure 4, this behavior can be attributed to the greater proportion of photospheric flux at higher latitudes. Although the flux drifts poleward on the flow time scale \( T_f \approx R_\odot/v_0 \approx 2 \text{ yr} \), the continued presence of sources through the end of 1981 keeps the coronal field rotating at periods in the range ~28–32 days. Thus, a quasi-equilibrium is established between the eruption of new flux in the sunspot belts, on the one hand, and its poleward transport and windup by differential rotation, on the other. A similar process was described by Sheeley and DeVore (1986b) to account for the 28 to 29 day recurrence patterns of the mean photospheric field.

When sources are switched off at the end of 1981, the continued windup of the midlatitude flux results in a temporary "bimodal" behavior, in which the coronal field rotates at the equatorial rate at low latitudes but tends toward the polar rate at high latitudes. By 1986, however, the center of gravity of the "unwound" flux moves to the poles, and the corona ultimately rotates rigidly at the polar rate. This contrasts with the asymmetric behavior of the underlying photospheric field, which rotates progressively more rigidly at the equatorial rate when sources are turned off (see Paper I). There it is the local phase velocity of the field at each latitude, rather than a global average of \( \alpha(\theta) \) weighted by the distribution of unwound flux, that determines the rotation profile.

Figure 6 shows the rms amplitudes of the lowest order modes during the above simulation (thick lines). For comparison, the thin lines indicate how the amplitudes evolve when sources continue to be deposited until 1986 April. In contrast to the "reference" case (see Fig. 3), the power in the (2, 1) mode is comparable to that in the (2, 2) mode. This is due to the efficient transport of flux to midlatitudes, where the (2, 1) mode has its maximal weighting \( Y_{21} \propto \sin \theta \). When sources are switched off, the power in the low-order, nonaxisymmetric modes dies out after \( \sim 2 \text{ yr} \).

In § IIC, we pointed out that the interaction between the (2, 1) and (1, 1) modes will give rise to north–south asymmetries in the rotation rate (as expressed by eq. [15]). We now illustrate this effect by cross-correlating separately the sum of the (1, 1) and (2, 1) components of the coronal field calculated here. The resulting yearly averaged rotation curves, indicated by dotted lines in Figure 5, indeed display pronounced asymmetries across the equator. Some of the sharp edges appearing in these profiles occur where a horizontal neutral line is temporarily formed, and the rotation rates become ill-defined (see the discussion in relation to Fig. 7 below). By contrast, the sum of the (1, 1) and (2, 2) components yields the smooth, symmetric profiles represented by the dashed lines in Figure 5.

d) The Decay of Single Sources

In order to study the long-term effect of strong individual sources on the coronal rotation rate, we deposited two equal-strength doublet sources at the same longitude but in different hemispheres, and allowed them to decay under the action of differential rotation with the profile (28) and supergranular diffusion at a rate \( \kappa = 300 \text{ km}^2 \text{s}^{-1} \). One doublet was centered at latitude 36°N, with its leading positive pole located 10° west and 2° south of its trailing negative one. The other source was centered at latitude 61°S, directly below the northern hemisphere doublet, with its leading negative pole 10° west and 2° north of its trailing positive one.
Fig. 4.—Yearly averaged coronal rotation profiles obtained when no doublet sources are deposited on the photosphere within the latitude range $-25^\circ < \lambda < 25^\circ$. The transport parameters are the same as for the reference model of Fig. 1. Equatorward diffusion of flux causes the coronal rotation period to shift to $\sim 27.0$ days toward the end of the cycle.
Fig. 5.—Yearly averaged coronal rotation profiles derived on the basis of a photospheric model which includes meridional flow but no supergranular diffusion. The poleward flow velocity peaks at a value of 10 m s$^{-1}$ at latitude ± 30°. Doublet sources are deposited only until the end of 1981. Solid lines: rotation rate of the coronal field as a whole. Dashed lines: rotation rate of the sum of the (1, 1) and (2, 2) components of the coronal field. Dotted lines: rotation rate of the sum of the (1, 1) and (2, 1) components of the coronal field. The poleward transport of flux introduces substantial curvature into the profiles, with the interaction between the (1, 1) and (2, 1) modes producing north-south asymmetries. After sources are switched off, the rotation rates ultimately approach the polar value.
Cross-correlations of the coronal field yielded the rotation profiles displayed in Figure 7. Initially, the northern hemisphere field rotates at the 29.1 day period of the source at latitude 36°N, while most of the southern hemisphere field rotates at the 33.7 day period of the source at latitude 61°S. In the subsequent months, the flux from each source spreads to higher and lower latitudes by diffusion. However, because windup and decay occur most rapidly in the region of maximum rotational shear around \( \lambda = \pm 54^\circ \), relatively little nonaxisymmetric flux from the lower latitude doublet survives its poleward journey; likewise most of the flux that spreads equatorward from the southern hemisphere doublet is symmetrized in the midlatitude zone. Thus, after \( \sim 13 \) Carrington rotations, practically all of the unsymmetrized flux becomes localized at low latitudes and in a small region around the south pole. The polar flux subsequently decays, and eventually (by Carrington rotation \( \sim 36 \)) the coronal rotation becomes rigid at approximately the equatorial rate of the photosphere.

In Figure 7, the discontinuity that appears in the rotation profile after about eight Carrington rotations occurs where the coronal field temporarily becomes axisymmetric. At this time, the dominant \((2, 1)\) and \((1, 1)\) harmonic components give rise to a horizontal neutral line below the equator as their phases become coaligned \((\beta_{21} = \beta_{11})\). This causes the denominator of the expression for the rotation rate given by equation (15) to vanish momentarily at that latitude, until the two modes have rotated past one another. The same process gives rise to the sharp edges seen in the annually averaged rotation profiles of the combined \((2, 1)\) and \((1, 1)\) fields shown in Figure 5.

e) Comparison with Observations: A Composite Model

In order to compare our simulated coronal rotation curves with profiles derived from actual solar data, we used our potential field algorithm to extrapolate synoptic observations of the photospheric field out to the source surface at \( r = 2.5 R_\odot \), and then autocorrelated year-long sequences of the resulting coronal maps. Figure 8 shows the annual rotation profiles based on observations from the Wilcox Solar Observatory \((\text{solid lines})\) and from the National Solar Observatory at Kitt Peak \((\text{dashed lines})\). (The NSO data for 1985–1986 were not included because of possible zero-point calibration problems affecting magnetograms recorded near sunspot minimum.) The rotation profiles obtained from the two data sets are seen to be in fairly good agreement. In Figure 9, we plot the rms amplitudes of the lowest order harmonics of the coronal field derived from the WSO data \((\text{thick lines})\) and from the NSO data \((\text{thin lines})\). The time evolution of the multipoles \( l = 1–3 \) agrees with that found by Hoeksema (1984).
Fig. 7.—Evolution of the coronal rotation profile when a single strong doublet source is deposited on the photosphere at latitude 36°N and another one of equal strength is placed at 61°S. The sources decay under the influence of diffusion ($\kappa = 300 \text{ km}^2 \text{ s}^{-1}$) and differential rotation, with no other sources present. Elapsed time in Carrington rotations is indicated on each frame. The coronal field initially rotates asymmetrically according to the rates of the doublet sources in each hemisphere, but as the flux diffuses toward the equator and the high-latitude remnants are symmetrized, the rotation becomes rigid at approximately the equatorial rate. The discontinuity at Carrington rotation 8 arises when a horizontal neutral line is temporarily formed in the southern hemisphere.

Figure 8 shows the overall tendency for the coronal rotation to accelerate at all latitudes over the course of the sunspot cycle and to approach the equatorial rate during the later years. This trend is in general accord with that found in the reference simulation of Figure 1. However, the observationally derived profiles tend to show a greater amount of curvature and asymmetry, reminiscent of the meridional flow simulation of Figure 5. This suggests that the effects of supergranular diffusion and meridional flow should be combined. (A similar conclusion was reached, on somewhat different grounds, in Paper I.) Accordingly, we display in Figures 10 and 11 the result of a computation in which $\kappa = 600 \text{ km}^2 \text{ s}^{-1}$ and the meridional flow field is given by equation (32), again assuming a peak velocity of $v_0 = 10 \text{ m s}^{-1}$ at 30° latitude. The simulated profiles and harmonic amplitudes of Figures 10 and 11 now show qualitative agreement with their empirical counterparts in Figures 8 and 9, despite the fact that our particular choice of parameters is an illustrative one and does not necessarily provide the best possible fit.
FIG. 8.—Rotation profiles of the "observed" coronal field, obtained by autocorrelating year-long sequences of Carrington maps extrapolated to 2.5 $R_\odot$ by the potential field method. Solid lines: annual coronal rotation profiles for 1977–1986 derived from Wilcox Solar Observatory (WSO) measurements. Dashed lines: annual coronal rotation profiles for 1977–1984 derived from National Solar Observatory (NSO) measurements.
CORONAL MAGNETIC FIELDS

Fig. 9.—Root-mean-square amplitudes of the lowest order harmonics of the nonaxisymmetric coronal field as derived from WSO observations (thick lines) and NSO observations (thin lines), plotted as a function of time. Possible reasons for the higher levels and sharper fluctuations of the NSO amplitudes include: the much higher spatial resolution of the NSO measurements as compared with the WSO, the use of spectral lines requiring different empirical calibrations, and zero-point calibration errors in the case of the NSO data.

IV. SUMMARY AND DISCUSSION

In the present study, we have emphasized the connection between the rotation of the outer coronal field and the global properties of the underlying photospheric field. We now summarize the main factors that determine the coronal rotation rate and its evolution in time:

1. **Coronal filtering.**—The magnetic field in the outer corona is comprised of the lowest order multipoles of the photospheric field. The filtering out of the higher order modes with increasing height means that eventually only one or two multipoles dominate. The superposition of these few low-order modes can never rotate as differentially as the full photospheric field, because only global variations of the latter are represented.

2. **The latitudinal distribution of nonaxisymmetric flux.**—The rotation rate of the outer coronal field can be expressed as a weighted average of the phase velocities and relative growth rates of the surviving low-order modes, which in turn are determined by flux-weighted averages of the photosphere's differential rotation rate. If the bulk of the nonaxisymmetric, unwound flux is located within a limited range of latitudes, the low-order multipoles of the photospheric field will rotate at the rates characteristic of those latitudes, as will the coronal field.4

The latitudinal distribution of nonaxisymmetric flux over the photosphere is determined by the locations of source activity and by flux transport processes, which include differential rotation, supergranular diffusion, and meridional flow. New sunspot groups give rise to strong nonaxisymmetric fields which tend to couple the coronal rotation rate to that of the sunspot belts. A poleward flow would make the coronal field rotate more slowly and less rigidly by transporting flux to higher latitudes, where the photosphere's differential rotation rate declines relatively steeply. Without the sustained injection of new flux by sources, however, differential rotation and diffusion will combine to wind up and annihilate the nonaxisymmetric field at midlatitudes, shifting the peak of the unwound flux distribution toward the equator. As a result, the coronal field rotates at approximately the equatorial rate near

4 The coronal rotation rate should not be thought of as simply an average of the photospheric rotation rate at the footpoints of the individual field lines. The footpoint locations are determined by the total field, including its axisymmetric component, and may thus be displaced from the region where the nonaxisymmetric flux is concentrated. Physically, it is the continual reconnection of field lines that allows the coronal field to rotate at a rate differing from that of the photospheric footpoint latitudes.
Fig. 10—Yearly averaged rotation profiles of the coronal field simulated using a "composite" photospheric model, which includes diffusion at a rate $x = 600 \text{ km}^2 \text{s}^{-1}$, poleward flow with a peak speed of $10 \text{ m s}^{-1}$ attained at latitude $\pm 30^\circ$, and doublet sources deposited from 1976 August to 1986 April. Flow introduces greater curvature and asymmetry into the profiles, while diffusion causes the rotation rates to tend to the equatorial value late in the cycle.
sunspot minimum. Good agreement between our simulated and observationally derived profiles was obtained when both supergranular diffusion and a poleward flow were included in our photospheric model.

3. Ongoing source eruptions.—On short time scales, concurrent eruptions of strong sources can produce erratic phase shifts in the low-order harmonic components of the photospheric field, and thus lead to sudden fluctuations in the coronal rotation rate. Large rotational “glitches” due to individual source eruptions are most likely to occur in the years around sunspot minimum, when the background field is relatively weak.

The observed properties of coronal rotation listed in § I can be understood in the light of these results. First, as illustrated in the Appendix, the tendency for the degree of rigidity to increase with height (Parker, Hansen, and Hansen 1982) is a consequence of the rapid radial falloff of higher order multipoles, whose amplitudes vary as $c(r)$. Second, the tendency for the coronal rotation rate to accelerate during the sunspot cycle, seen in our Figure 8 and described earlier by Fisher and Sime (1984) as a sort of “torsional oscillation,” can be interpreted instead in terms of the equatorward migration of sunspot activity, reinforced by the tendency for decaying flux to be narrowly localized around the equator late in the cycle. Third, the observed north-south asymmetries of the coronal rotation rate (Parker, Hansen, and Hansen 1982; Hoeksema and Scherrer 1987; our Fig. 8) can be explained by differences in the latitudinal flux distribution between the two hemispheres. The effect of such differences is magnified at high latitudes, where the gradients in the photospheric rotation profile $\alpha(\theta)$ are large. Thus, we would expect the most pronounced asymmetries in the coronal rotation rate to occur during the rising phase of the cycle, when high-latitude sources dominate (see Fig. 8).

Fourth, the 27 and 28 day periods which Hoeksema and Scherrer (1987) found in power spectra of the source-surface field can be understood in terms of the contribution of photospheric flux at the equator and at midlatitudes, respectively. The equatorial component of the flux distribution will give rise to a spectral peak near 27.0 days, which becomes especially prominent toward sunspot minimum when the large-scale field is dominated by decaying flux around the equator. A 28 day feature occurs only in the presence of sunspot activity, when an equilibrium is established between the poleward transport of nonaxisymmetric flux from the sunspot belts and its decay via differential rotation and supergranular diffusion (cf. Sheeley and DeVore 1986b). The attainment of such an equilibrium is suggested by the composite simulation of Figure 10 and by the observationally derived rotation profiles of Figure 8, where the coronal rotation periods are seen to remain in the general vicinity of 28 days until well into the declining phase of the cycle.

In order to provide further confirmation of these ideas, we have computed power spectra of the coronal field simulated for a variety of flux transport parameters. Although we have not included these spectra in the present paper, we shall briefly mention our
results here. First, when both diffusion and meridional flow are omitted from the model, we find that the power peaks at \( \sim 27.5 \) days, a period characteristic of the sunspot latitudes. When a diffusion in the general range 300–600 km \( ^2 \) s \( ^{-1} \) is added, the peak shifts toward 27.0 days, with most of the spectral power contributed near sunspot minimum. On the other hand, when this diffusion is combined with a nominal 10 m s \( ^{-1} \) poleward flow, the power is spread over a wider spectral range and the maximum is now located near 28.0 days. If diffusion is then removed, the power spreads out toward much longer periods.

In closing, it will be useful to summarize the basic differences between the rotation of the photospheric and outer coronal fields. We recall that it is the continual migration of flux in latitude that establishes the rigidly rotating, large-scale patterns of photospheric field (see Paper I). By preventing the buildup of the large latitudinal phase gradients associated with these “barber-pole” patterns, new sources delay the onset of rigid rotation and act to maintain the “intrinsic” differential rotation rate of individual flux elements. On the other hand, the coronal field would rotate more rigidly than the photospheric plasma even in the absence of latitudinal transport mechanisms, simply because the higher order harmonic components are filtered out with increasing height. By continually injecting fresh unwound flux, sources couple the coronal rotation to that of the sunspot latitudes, which effectively determine the “intrinsic” rotation rate of the coronal field. During times of significant source activity, a poleward flow would shift the coronal rotation profile toward longer periods and decrease its rigidity by spreading the flux to higher latitudes. In contrast, the underlying photospheric field would rotate faster and more rigidly in the presence of such a flow. Finally, as the source eruption rate declines toward the end of the sunspot cycle, the windup and diffusive decay of the midlatitude flux will cause both the coronal and photospheric fields to rotate quasi-rigidly at approximately the equatorial rate.

We continue to recognize the contributions of our colleague C. R. DeVore, who developed the flux-transport code and provided many useful comments. We are also grateful to J. W. Harvey (NSO/KP) and to J. T. Hoeksema and P. H. Scherrer (WSO/Stanford), who have generously contributed photospheric field observations and valuable advice concerning their use and interpretation. Some of the calculations in the Appendix were begun while one of us (N. R. S.) was a visitor at the National Solar Observatory in Tucson. Financial and other support was provided in part by the NASA Office of Solar and Heliospheric Physics (DPR W 14429) and by the Office of Naval Research.

APPENDIX

EVOLUTION OF A SECTORAL FIELD SUBJECTED TO THE DIFFERENTIAL ROTATION OF THE PHOTOSPHERE

In § II, we showed that the coronal rotation rate can be expressed as a weighted average of the phase velocities and relative growth rates of the lowest order harmonics of the photospheric field. In the absence of newly erupting flux, these rates of change of the harmonic amplitudes and phases are in turn given by flux-weighted, latitudinal averages of the photospheric differential rotation rate. In order to illustrate these principles, we shall now work out the rotational properties of a field which evolves from a given initial configuration due to the differential rotation of the photosphere. We also consider briefly the effect of meridional flow on the asymptotic rotation rates.

The photospheric field will be assumed to have the initial sectoral form

\[
B(R, \theta, \phi, 0) = f(\theta) \cos m\phi ,
\]

where \( f(\theta) \) is a given function of colatitude and \( m \) is the sectoral mode number. Differential rotation acting alone causes this field to evolve with time according to

\[
B(R, \theta, \phi, t) = f(\theta) \cos m[\phi - \omega(\theta)t] ,
\]

which represents the solution to the flux transport equation (1) in the absence of diffusion, meridional flow, and sources, subject to the initial condition given in equation (A1). For the intrinsic rotation rate of the photosphere, we adopt a profile of the form

\[
\omega(\theta) = \omega_0 - \omega_1 \cos^2 \theta ,
\]

where \( \omega_0 \) and \( \omega_1 \) are constants.

The harmonic amplitudes and phases of the field described by equation (A2) may be obtained from the relation (see eqs. [3]–[4])

\[
b_{lm}(t)e^{-i\omega(\theta)t} = \int B(R, \theta, \phi, t)Y_{lm}^*(\theta, \phi)d\Omega = e^{-i\omega(\theta)t}N_{lm} I_{lm}(t) ,
\]

where we define

\[
N_{lm} = \pi \left[ (2l+1) \frac{(l-m)!}{(l+m)!} \right]^{1/2}
\]

Because the data samples span only 10 yr, the heights and widths of the peaks could not be determined precisely. The series of sharp, discrete peaks found by Hoeksema and Scherrer (1987) probably represents a spurious effect, because the narrow window bandwidth \( 5 \text{nHz} \sim (3 \text{ yr})^{-1} \) needed to reveal this structure implies an unacceptably large variance for their spectral estimate (see the discussion of Jenkins and Watts 1968, pp. 277–282).
and

\[ I_m(\alpha) = \int_{-1}^{1} f(x)P_m^n(x) \exp \left( imx^2 \right) dx , \]  
(A6)

with \( \alpha \equiv \omega_1 t, x \equiv \cos \theta \), and the associated Legendre function \( P_m^n(x) \) defined as in Abramowitz and Stegun (1968, p. 334). Thus, given the latitudinal flux distribution \( f(x) \), the problem of determining \( b_{lm} \) and \( \beta_{lm} \) reduces to that of evaluating the integral \( I_m(\alpha) \).

In the remainder of this Appendix, we explore the consequences of adopting two particular functional forms for \( f(x) \).

1. Case A: \( f(x) = P_m^n(x) \)

Here we suppose that the initial field consists of a single spherical harmonic, so that the latitudinal distribution is given by an associated Legendre function \( P_m^n(x) \). The integral in equation (A6) then becomes

\[ I_m(\alpha) = \int_{-1}^{1} P_m^n(x)P_m^n(x) \exp \left( imx^2 \right) dx , \]  
(A7)

which vanishes unless \( |l' - l| \) is an even integer.

The behavior of this integral near \( \theta = 0 \) can be found by expanding the exponential factor in a power series about \( x = 0 \) and integrating term by term. Retaining the first two nonzero contributions and substituting the resulting expression for \( I_m(\alpha) \) into equation (A4), we deduce that, to lowest order in \( \alpha \),

\[ b_{lm}(t) = N_{lm} \frac{m \omega_0 t}{n!} \int_{-1}^{1} x^{2n}P_m^n(x)P_m^n(x) dx , \]  
(A8a)

\[ m \beta_{lm}(t) = m \omega_0 t - \frac{\pi}{2} , \]  
(A8b)

where \( n = |l' - l|/2 \). Thus, although all of the power initially resides in the \( l = l' \) mode, differential rotation causes a progressive transfer of power to harmonics whose \( l \) differs from \( l' \) by an even integer. Since \( b_{lm}(t) \propto t^n \) near \( t = 0 \), the neighboring \( l \)-modes grow faster than the distant-\( l \) ones; and each mode starts rotating steadily at its own characteristic, nonzero rate. This generalizes to arbitrary \( l' \) the results of Sheeley (1981) for an initial dipole field.

The asymptotic behavior of the integral in equation (A7) for large \( \alpha \) may be evaluated by expanding the integrand about \( x = 0 \) and \( x = 1 \) (see Morse and Feshbach 1953, p. 611; also Sheeley and DeVore 1986a). In the limit \( \alpha \to \infty \), the equatorial solution dominates, and we obtain

\[ b_{lm}(t) \propto t^{-1/2} , \]  
(A9a)

\[ m \beta_{lm}(t) = m \omega_0 t - \frac{\pi}{4} , \]  
(A9b)

when \( l' - m \) and \( l - m \) are even, and

\[ b_{lm}(t) \propto t^{-3/2} , \]  
(A10a)

\[ m \beta_{lm}(t) = m \omega_0 t - \frac{3\pi}{4} , \]  
(A10b)

when \( l' - m \) and \( l - m \) are odd. Thus modes with even values of \( l - m \) decay more slowly than “odd-parity” ones, and, for a given \( m \), become coaligned at a phase which differs by \( \pi/(2m) \) from that approached by the odd-parity harmonics. Since the phases \( \beta_{lm} \) and the growth rates \( b_{lm}/b_{lm} \) become independent of \( \alpha \) for either parity, equation (11) of § IIc shows that the coronal field as a whole will approach a state of rigid rotation at the rate \( \omega_0 \) common to all of the modes. It should be noted that the photospheric field continues to rotate differentially according to the adopted law in eq. (A3), since its power becomes concentrated in ever higher multipoles as windup proceeds.

In the specific case that the initial field is a horizontal dipole with \( f(\theta) = P_1^0(\cos \theta) = -\sin \theta \), we can evaluate the harmonic amplitudes and phases at \( \theta = 0 \) from equations (A8a) and (A8b), and substitute the results into equation (11) to obtain the initial rotation rate of the coronal field. We find that

\[ \omega_s(r, \theta, 0) = \omega_0 - \frac{3}{4}(1 - \epsilon_{31}(r))\omega_1 - \epsilon_{31}(r)\omega_1 \cos^2 \theta , \]  
(A11)

where the factor \( \epsilon_{31}(r) \equiv c_3(r)/c_1(r) \) may be evaluated using equation (6).

Equation (A11) reveals several important properties of the initial rotation rate. First, the rotation profile becomes increasingly rigid with height, since the factor \( \epsilon_{31}(r) \) decreases from unity at the photosphere (where eq. [A11] reduces to eq. [A3]) to a value \( \epsilon_{31}(R_s) = 0.19 \) at the source surface \( R_s = 2.5 R_\odot \). Second, even though all of the power initially resides in the dipole component, \( \omega_s(r, \theta, 0) \) differs from the dipole phase velocity \( \omega_1(0) = \omega_0 \) only if \( \omega_1(5) \) is this because at \( t = 0 \) the power in the \( \phi = 1 \) mode is increasing relative to that in the \( \phi = 1 \), causing the phase of the field to shift toward that of the growing octupole component. Only in the limit \( R_s \to \infty \) would one have \( \epsilon_{31}(R_s) = 0 \), and thus \( \omega_s(R_s, \theta, 0) = \omega_1(0) \). Third, if we use the Newton and Nunn (1951) synodic parameters \( \omega_0 = 13:39 \) day\(^{-1} \), \( \omega_1 = 2:77 \) day\(^{-1} \) to evaluate the coefficients of equation (A11) at \( R_s = 2.5 R_\odot \), we find that

\[ \omega_s(R_s, \theta, 0) = 12.94 - 0.53 \cos^2 \theta \text{ deg day}^{-1} . \]  
(A12)
For comparison, we note that Hoeksema and Scherrer (1987) derived the empirical formula \( \omega_c = 13.2 - 0.5 \cos^2 \theta \) deg day\(^{-1} \) for the synodic rotation rate at the source surface.

Proceeding in the same fashion, one can show that, for a distribution of the form \( f(\theta) = P_{2}^{2}(\cos \theta) = 3 \sin^2 \theta \), the initial rotation rate of the source-surface field is

\[
\omega_{s}(r_s, \theta) = \omega_0 - 3 \int_{-1}^{1} (1 - x^2)^2 P_{1}^{2}(x) \exp(\imath \omega_{c} x^2) dx ,
\]

(A13)

where \( \epsilon_{42}(r) \equiv c_{4}(r)/c_{2}(r) \), while for a distribution \( f(\theta) = P_{2}^{2}(\cos \theta) = -(3/2) \sin 2\theta \), the initial rate is

\[
\omega_{s}(r_s, \theta) = \omega_0 - 3 \int_{-1}^{1} (1 - x^2)^2 P_{1}(x) \exp(\imath \omega_{c} x^2) dx .
\]

(A14)

Here it is significant that the \((2, 1)\) configuration initially rotates more slowly, and somewhat less rigidly, than the \((2, 2)\). The physical basis for this difference is that the peak of the \((2, 1)\) flux distribution occurs at a latitude of 45°, where the photospheric rotation rate is relatively slow and the rotational shear is largest; the \((2, 2)\) mode, by contrast, is peaked at the equator, where the function \( \omega(\theta) \) attains its maximum value. In fact, the initial rotation rate for the \((2, 2)\) configuration is even faster and more rigid than for the \((1, 1)\) (compare eqs. [A12] and [A13]), because the \((2, 2)\) distribution is more narrowly peaked about the equator.

II. Case B: \( f(x) = x^{2n}(1 - x^2)^p \)

Rather than starting out with a single harmonic mode, we now suppose that the latitudinal dependence of the field at \( t = 0 \) can be represented by a function of the form

\[
f(\theta) = [\sin^2 \theta (\cos^2 \theta)^{p}]^\theta .
\]

(A15)

This distribution peaks at a latitude \( \lambda_{\text{max}} \) given by

\[
\tan^2 \lambda_{\text{max}} = n , \tag{A16}
\]

with the peak becoming narrower as the index \( p \) increases (\( n \) and \( p \) are assumed to be nonnegative numbers). The integral in equation (A6) now becomes

\[
I_{m}(x) = \int_{-1}^{1} x^{2n}(1 - x^2)^p P_{m}^{*}(x) \exp(\imath \omega_{c} x^2) dx .
\]

(A17)

At \( t = 0 \), we may combine equations (A17) and (A4) to obtain

\[
\dot{b}_{im}(0) = 0 , \tag{A18a}
\]

\[
\ddot{b}_{im}(0) = \omega_0 - \omega_1 \int_{-1}^{1} x^{2n+2}(1 - x^2)^p P_{m}^{*}(x) \exp(\imath \omega_{c} x^2) dx ,
\]

(A18b)

provided the integral in the denominator does not vanish. The integrals may then be evaluated in terms of gamma functions, and their ratio reduced to algebraic functions involving \( np \) and \( p \). For the dipole component, for example, one obtains

\[
\dot{b}_{11}(0) = \omega_0 - \omega_1 \left( \frac{np + 1/2}{np + p + 2} \right) .
\]

(A19)

In the limit \( p \to \infty \), this becomes

\[
\dot{b}_{11}(0) \to \omega_0 - \omega_1 \cos^2 \theta_{\text{max}} , \tag{A20}
\]

where \( \theta_{\text{max}} \) is the colatitude equivalent to \( \lambda_{\text{max}} \) given by equation (A16). Comparing equations (A3) and (A20), we conclude that the dipole component of a flux distribution sharply peaked at \( \theta = \theta_{\text{max}} \) will rotate at the corresponding photospheric rate, \( \omega(\theta_{\text{max}}) \). Since the same result can be shown to hold for arbitrary \((l, m)\), the coronal field as a whole will initially rotate at the rate \( \omega(\theta_{\text{max}}) \) when the photospheric flux has a very narrow spread in latitude.

In the limit \( x \to \infty \), one may evaluate the integral (A17) by expanding the integrand about \( x = 0 \) and \( x = 1 \) (see Morse and Feshbach 1953, p. 611). Proceeding in the manner of Sheeley and DeVore (1986a), we obtain either a polar or an equatorial asymptote, depending on whether \( \lambda_{\text{max}} \) lies above or below a critical latitude \( \lambda_{c} \), defined by

\[
\tan^2 \lambda_{c} = \begin{cases} 
1 + \frac{m + 1}{p} & (l - m) \text{ and } 2np \text{ even}; \\
1 + \frac{m}{2p} & (l - m) \text{ and } 2np \text{ odd} .
\end{cases}
\]

(A21)

If \( |\lambda_{\text{max}}| < |\lambda_{c}| \), one obtains the equatorial asymptote:

\[
\dot{b}_{im} \propto \begin{cases} 
(x^{-(np + 1/2)} & (l - m) \text{ and } 2np \text{ even} , \\
(x^{-(np + 1)} & (l - m) \text{ and } 2np \text{ odd} .
\end{cases}
\]

(A22a)

\[
\dot{b}_{im} = \omega_0 ;
\]

(A22b)
whereas if $| \lambda_{\text{max}} | > | \lambda_1 |$, the polar asymptote dominates, and we have

$$b_{lm} \propto t^{-(p+1+m/2)}, \quad \beta_{lm} = \omega_0 - \omega_1 . \quad (A23a)$$

We note from equation (A21) that the critical latitude $| \lambda_1 |$ will always exceed 45°, although it will approach this value for a narrow distribution with $p \gg m$.

Using the above asymptotic expressions for the harmonic phase velocities and amplitudes in equation (11), we conclude that the coronal field as a whole will rotate either at the equatorial or the polar rate as $r \to \infty$, depending on whether $\lambda_{\text{max}}$ lies below or above the critical latitude $\lambda_c$. The physical interpretation of this result is as follows: as the photospheric field is wound up into increasingly narrow bands of alternating polarity and harmonic power is transferred to higher and higher $l$, the main contribution to the remaining power in the lower order modes is provided by the flux near the equator and poles (where the rotational shear vanishes). Thus, depending on where most of the flux is initially located, the low-order multipoles that constitute the coronal field will rotate asymptotically at either the equatorial rate $\omega_0$ or the polar rate $\omega_0 - \omega_1$.

Sheeley and DeVore (1986a) have studied the evolution of the sectoral field considered here when a poleward meridional flow of the form

$$u(\theta) = -v_0 \sin 2\theta \quad (A24)$$

is added to the differential rotation. Although they considered only the asymptotic behavior of the $(1, 1)$ mode, their results can easily be generalized to arbitrary $(l, m)$. The integral $I_{lm}(\theta)$ appearing in equation (A4) in this case takes the form

$$I_{lm}(\theta) = \int_{-1}^{1} u_{2^{m+1}}(1 - u^2)^{p} P_{ml}^q(\cos \theta)[1 + u^2(\sin \theta^2 - 1)]^{m+1} \cos \theta \, du . \quad (A25)$$

Here the dependent variables $u$ and $\theta$ are related by

$$\cos \theta = \frac{-u_{2^{m+1}}}{[1 + u^2(\sin \theta^2 - 1)]^{1/2}}, \quad (A26)$$

and the parameter $\tau_f = \tau_{w} v_0 / R$ measures the ratio of the flow time scale $\tau_f = R / v_0$ to the windup time $\tau_{w} = 2\pi / \omega_1 . \quad (A27)$

At $t = 0$, equation (A25) reduces to equation (A17), and the field initially evolves as if meridional flow were absent. In the limit $\alpha \to \infty$, we may proceed as before by expanding the integrand about $u = 0$ and $u = 1$. Again we obtain either a polar or an equatorial asymptote, depending on the relation of $\lambda_{\text{max}}$ to a critical latitude $\lambda_{\text{cf}}$, given by

$$\tan^2 \lambda_{\text{cf}} = \frac{m - 1}{2p}, \quad (A28)$$

independent of whether $l - m$ and $2np$ are even or odd. When $| \lambda_{\text{max}} | < | \lambda_{\text{cf}} |$, we obtain the equatorial asymptote

$$b_{lm} \propto e^{-2^{m+1}/\tau_f}, \quad \beta_{lm} = \omega_0, \quad (A28a)$$

and when $| \lambda_{\text{max}} | > | \lambda_{\text{cf}} |$, the polar asymptote applies:

$$b_{lm} \propto e^{-2\pi/\tau_f}, \quad \beta_{lm} = \omega_0 - \omega_1 . \quad (A28b)$$

From equation (A27), we see that for $m = 1$ the critical latitude extends all the way to the equator, so that all two-sector configurations which are not peaked exactly at the equator will eventually rotate at the polar rate $\omega_0 - \omega_1$, while decaying exponentially with time constant $\tau_{f/2}$. Moreover, even if $m > 1$, the polar asymptote will be attained provided the initial flux distribution is sufficiently narrowly peaked ($p \gg m$) away from the equator. This is because the meridional flow field eventually carries most of the flux to the poles: the polar asymptote will then dominate provided differential rotation does not transfer too much power to higher harmonics along the way. Clearly, the rate at which the flux is wound into narrower stripes increases with its initial spread in latitude and with the number of sectors present, so the polar asymptote is favored for narrow distributions with few sectors.

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