Sodium synthesis in hydrogen-burning stars

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In adequately massive ($M \geq 1.5 M_\odot$) hydrogen-burning (main sequence) stars, neon depletion by the $^{22}$Ne($p, \gamma$)$^{23}$Na reaction will enrich the core with sodium by a substantial factor (5-6). In the atmospheres of carbon-poor F-K supergiants the Na excess correlates with the $^{12}$C/$^{13}$C ratio.

1. The abundances of the C, N, O group of nuclides in stellar atmospheres have long been recognized to be of crucial importance for astrophysics; their properties are a major source of information on many aspects of stellar structure and evolution. Boyarchuk and Lyubimkov, however, have recently pointed out that apart from the C, N, O abundance anomalies that have been so widely discussed in the literature, the atmospheres of type F to K supergiants seem also to be enriched in Na. The sodium excess may reach a full order of magnitude. Our aim in this letter is to show that sodium ought to be included in the short list of elements whose abundance in the atmosphere of a star offers clues about the conditions in its interior.

2. Figure 1 reproduces Boyarchuk and Lyubimkov's summary of the data currently available on the Na abundance in F-K supergiants. Each point represents a separate star; the different symbols signify determinations by different authors, and these are in reasonably good mutual agreement. In this diagram the ordinate is the sodium excess [Na/H], defined as the difference between the logarithmic Na/H ratios (by number of atoms) for a given star and the sun; the abscissa is the logarithm of the star's surface gravity g. We see a rather definite correlation here: the Na excess tends to increase toward lower g values.

Boyarchuk and Lyubimkov's conclusion regarding the Na excess was only preliminary, because in determining the Na abundances no allowance was made for non-LTE effects. Their role cannot be adequately assessed without calculating the populations of the atomic levels in detail by simultaneously solving the steady-state and radiative-transfer equations. But in September 1986 Lyubimkov informed us that I. Gubeny of the Ondřejov Observatory near Prague had recently performed appropriate calculations and had established that the equivalent widths of the (nonresonance) sodium lines from which the Na abundance is determined are little affected by departures from equilibrium. That finding demonstrates

![Image](https://via.placeholder.com/150)

FIG. 1. Correlation between the sodium excess in the atmospheres of F-K supergiants and the gravitational acceleration, after Boyarchuk and Lyubimkov. Various authors' data are indicated by differing symbols.

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TABLE I. Depletion Time Scales $\tau_p$  

<table>
<thead>
<tr>
<th>$M/M_\odot$</th>
<th>$\tau_p ~ 10^3$ yr</th>
<th>$\rho_c ~ g/cm^3$</th>
<th>$\log \tau_p$, yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.4</td>
<td>28.2</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td>2.1</td>
<td>29.1</td>
<td>6.4</td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td>30.2</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>10.4</td>
<td>30.2</td>
<td>0.74</td>
<td></td>
</tr>
<tr>
<td>13.5</td>
<td>29.3</td>
<td>2.77</td>
<td></td>
</tr>
<tr>
<td>17.0</td>
<td>29.0</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>20.7</td>
<td>1.19</td>
<td></td>
</tr>
</tbody>
</table>

b. It will be demonstrated that the Na excesses observed in F-K supergiant atmospheres correlate with the $^{12}$C/$^{13}$C isotope ratio: the lower the ratio, the higher the Na abundance. This property, combined with the deficiency (rather than excess) of carbon encountered in such stars, is a persuasive argument that the excess sodium is synthesized not during the helium-burning phase but earlier, while the hydrogen is being consumed.

3. The chain of Ne–Na cycle reactions is analogous to that for the simple carbon-nitrogen cycle: 

$^{20}$Ne(p, $\gamma$)$^{21}$Na + $\beta^-$ + $^{21}$Ne(p, $\gamma$)$^{23}$Na + $\beta^-$ + $^{23}$Ne(p, $\alpha$)$^{20}$Ne.

These reactions cannot play any appreciable role in the energy output of main-sequence stars, for the rates are too slow. Nevertheless, the reaction rates may turn out to be sufficient for perceptible nucleosynthesis to take place by the Ne–Na chain even while the star is on the main sequence. After the $10^3$ carbon–nitrogen cycles required for the hydrogen in the core of an $M \geq 1.5 M_\odot$ star to be consumed, the Na/Ne and $^{22}$Ne/$^{24}$Ne ratios will have changed greatly — a circumstance that has not hitherto been adequately recognized.

It would hardly be feasible to start out by including a solution of the system of nucleon-cycle kinetic equations directly in a detailed computer program for calculating stellar evolution. Instead, as a first step we have solved the equations of $^{24}$Ne, $^{22}$Ne, $^{23}$Ne, $^{24}$Na nucleosynthesis kinetics (all the other nuclides have negligible abundances) assuming that the temperature and density have constant values $T$, $\rho$ corresponding to the environment at the main-sequence center. The reaction rates adopted are those given by Fowler et al.9 and Harris et al.10; the latest revision11 leaves the data in these reviews unchanged. For the $f$-factors, which measure the contribution of poorly studied resonances, we henceforth take $f = 0.1$. There are two such resonances: one at $E_p = 93.5$ keV, for the reaction $^{22}$Ne(p, $\gamma$)$^{23}$Na, and the other at $E_p = 38.5$ keV, for $^{23}$Na(p, $\alpha$)$^{20}$Ne. The calculations were repeated, however, for $f = 0$ and $f = 1$; these control results indicate that the cross-section uncertainties have no effect at all on our basic conclusions.

Initial relative abundances of the neon and sodium isotopes have been adopted from Cameron’s recent survey.8 By number of atoms the proportions are: $^{20}$Ne, 86.9%; $^{21}$Ne, 0.3%; $^{22}$Ne, 10.6%; $^{23}$Na, 2.8%. Temperatures and densities have been taken in accord with calculations by Ill7 as representative of the centers of chemically homogeneous stars (H, He, and heavy-element abundances X = 0.70, Y = 0.27, Z = 0.03) with masses $M \geq 1.5 M_\odot$.

Our salient result is the following. In all main-sequence stars that are generating energy by the C, N, O cycle the $^{22}$Ne in the convective core will be depleted during the main-sequence stage and will be converted into $^{23}$Na, a process which should raise the sodium abundance by a factor of 5–6 (for the assumed initial chemical composition). The reaction $^{22}$Ne(p, $\alpha$)$^{23}$Na, whose high rate is essentially responsible for the Na enhancement, is a resonance reaction. The dominant contribution to its rate comes from a resonance at $E_p = 30$ keV, as has recently been established.8

Thus the Na excess observed in carbon-poor supergiants (evidently due to depletion of C by the C, N, O cycle) may be viewed as an indication that the material from which stars develop, at least those of the latest generation, cannot be poor in the $^{22}$Ne isotope. At any rate the $^{22}$Ne abundance should be well above the initial sodium abundance in the protostellar matter.

In Fig. 2 we compare the time scales for depletion of the H, $^{22}$Ne, and $^{13}$C at the center of main-sequence stars of differing mass. In all stars having $M \geq 1.5 M_\odot$, sodium is synthesized from $^{22}$Ne approximately 1 dex faster than the hydrogen is consumed. As we are treating the atmospheric Na abundance as a probe for studying the processes in the stellar interior, it is important to keep in mind that, by Fig. 2, sodium enrichment of the

![FIG. 2. Average lifetime of $^1$H, $^{22}$Ne, and $^{13}$C nuclei at the center of a main-sequence star as a function of stellar mass.](image-url)

TABLE II. Equilibrium Percentage Isotopic Abundances (Ne + Na = 100)

<table>
<thead>
<tr>
<th>$\tau_p ~ 10^3$ yr</th>
<th>$^{23}$Ne</th>
<th>$^{24}$Ne</th>
<th>$^{22}$Ne</th>
<th>$^{23}$Na</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>80.0</td>
<td>0.2</td>
<td>0</td>
<td>30.8</td>
</tr>
<tr>
<td>40</td>
<td>34.8</td>
<td>0.2</td>
<td>0.2</td>
<td>65.0</td>
</tr>
<tr>
<td>50</td>
<td>31.9</td>
<td>0</td>
<td>2.9</td>
<td>65.2</td>
</tr>
</tbody>
</table>


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material at the expense of $^{22}$Ne requires a much longer exposure than is needed to establish an equilibrium $^{12}$C/$^{13}$C ratio (although less time than required for significant consumption of the oxygen). It is worth noting that the ratio of the $^{22}$Ne and $^{12}$C depletion times due to protons is independent of density and is perfectly definite, since the cross sections of the reactions involved are well determined.

Table I gives the logarithmic time scales $\tau$ (in years) for consumption of $^{24}$Ne, $^{22}$Ne, $^{23}$Na by proton reactions as a function of stellar mass. We see that large amounts of $^{22}$Ne will burn and the Ne–Na cycle will approach equilibrium during the main-sequence stage only in very massive stars.

The equilibrium abundance of each of the nuclei is given in Table II for the temperature range of interest. We may conclude from these results that the energy-producing layers of very massive stars should be even more heavily enriched with sodium during the main-sequence phase than is the case for medium-mass stars.

4. It will now be clear that if the outer layers of supergiants (and red giants) are enriched with material which in the main-sequence phase has undergone nuclear transformations deep enough for $^{22}$Ne to be burned, then the atmospheres ought to display a low $^{12}$C/$^{13}$C ratio. One would therefore anticipate a correlation between the Na excess and the $^{12}$C/$^{13}$C ratio in the atmospheres of supergiants (and giants) poor in C (and rich in N). Such a correlation indeed exists: the corresponding data for the stars represented by points in Fig. 1 and having known $^{12}$C/$^{13}$C ratios are plotted in Fig. 3.

It would be very interesting to determine $^{12}$C/$^{13}$C for the following supergiants, all of which have large and reliable Na excesses (published estimates of the [Na/H] abundance are parenthesized): $\delta$ CMa (1.20), HR 4337 (1.01), $\rho$ Cas (0.96), $\mu$ Per (0.68).

One comparing Figs. 1 and 3 we see that for the sample of F–K supergiants under discussion, the $^{12}$C/$^{13}$C ratio seems to correlate with $\log g$. This relationship is brought out explicitly in Fig. 4.

5. The foregoing arguments suggest it would be worthwhile making a more comprehensive analysis of the correlations between the Na abundance and the C, N, O nuclides, not restricted solely to type F–K supergiants. It also would certainly be desirable carefully to investigate the Ne–Na nucleosynthetic chain during the hydrogen-burning phase (both in the core and in the shell source) on the basis of detailed evolutionary calculations for model stars of differing mass. Such a program is now under way at the Leningrad University Observatory.

We finally wish to point out that nucleosynthesis by the Ne–Na cycle has recently been included for the first time in evolutionary model calculations for massive ($M \geq 40 \, M_\odot$) stars. While the authors of this paper mention briefly that $^{22}$Ne will be depleted in such stars during the H-burning phase so as to raise the $^{23}$Na abundance by a factor $>5$, they do not extend this conclusion to less massive stars and make no attempt to tie it in with the observations.

We are indebted to L. S. Lyubimkov for stimulating this investigation.

6A. G. W. Cameron, loc. cit., p. 23 [Russ. tr., p. 33].