A HIGH-ENERGY SOLAR FLARE BURST COMPLEX AND THE PHYSICAL PROPERTIES OF ITS SOURCE REGION

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Abstract. We discuss a solar flare microwave burst complex, which included a major structure consisting of some 13 spikes of 60 ms FWHM each, observed 21 May, 1984 at 90 GHz (3 mm). It was associated with a simultaneous very hard X-ray burst complex. We suggest that the individual spikes of both bursts were caused by the same electron population: the X-bursts by their bremsstrahlung, and the microwave bursts by their gyrosynchrotron emission. This latter conclusion is based on the evidence that the radio turnover frequency was \( \lesssim 150 \text{ GHz} \). It follows that the emission sources were characterized by an electron density of about \( 10^{11} \text{ cm}^{-3} \), a temperature of \( 5 \times 10^{8} \text{ K} \) and a magnetic field of about 1400–2000 G. They had a size of about 350 km; if the energy release is caused by reconnection the sources of primary instability could have been smaller and in the form of thin sheets with reconnection speed at a fraction of the Alfvén velocity and burst-like energy injections of \( \approx 10^{27} \text{ erg} \) during about 50 ms each. The energized plasma knots lost their injection energy by saturated convective flux (collisionless conduction) in about 30 ms.

1. Introduction

Recent comparisons of hard X-ray bursts and microwave radio bursts emitted simultaneously during solar flares have shown how to resolve the previously assumed discrepancy between the numbers of electrons needed to explain both phenomena (see review by Holt and Ramaty, 1969): only one electron population and virtually the same numbers of electrons are needed to explain both phenomena. Differences in presently accepted numbers of electrons are not larger than about a factor ten, a number that should be compared to the values \( 10^3 \) to \( 10^4 \) assumed earlier (cf. Batchelor et al., 1985, and review by De Jager, 1986).

The above statement applies to hard X-ray bursts in – roughly – the 30–300 keV range and to microwave radiation of centimeter wavelengths. In this paper we will investigate an X-ray burst complex with relatively hard spectra and the corresponding microwave burst complex, which was observed in the mm-wavelength region. We will find that, in this case as well, a source model exists in which the same numbers of electrons are responsible for the emission of the microwave and the hard X-ray bursts. It will appear that such bursts are emitted in relatively small, dense plasma knots (by coronal standards). We will show that the primary site of instability of that burst complex was also located in these knots.
2. Millimeter-Wave Bursts in a Solar Flare

A complex of bursts observed simultaneously at 90, 30, and 7 GHz on 21 May, 1984 was described by Kaufmann et al. (1985). The time development of these bursts and of the simultaneously observed X-ray bursts in the 24–108 keV and 108–219 keV bands are reproduced here as Figure 1. Particularly remarkable is the burst complex $A$: it is very pronounced at 90 GHz and practically invisible at 30 and 7 GHz, as well as at 15 GHz and the lower microwave frequencies observed at Sagamore Hill Radio

Fig. 1. (a) shows the solar burst on 21 May, 1984 at 13h26m UT, as observed by HXRBS on the Solar Maximum Mission in X-rays at 24–108 keV and at 108–219 keV (above), and in three frequencies in the mm/cm range observed at the radio observatory at Itapetinga (below). (b) shows a 4 s time-expansion of structure $A$ of (a), in hard X-rays (above) and at 3.3 mm (below). From Kaufmann et al. (1985).
Observatory (Cliver, 1984). The microwave burst complex correlated with the most energetic X-ray burst of that flare, which produced significant flux beyond 100 keV. This is unusual because only \( \approx 10\% \) of the flare observed with the Hard X-Ray Burst Spectrometer (HXRBS) aboard the Solar Maximum Mission produced detectable flux beyond \( \approx 100 \) keV (A. L. Kiplinger, private communication). Another aspect of burst complex \( A \) is that it appears to consist of a large number of spikes, at least thirteen. The average e-folding rise time of the spikes is 30 ms; the decay time is about the same: the spikes are virtually symmetric. One proposed interpretation of these observations is that the radio-bursts are due to synchrotron emission by ultra-relativistic electrons, with a turnover frequency in the far infrared, while the X-rays are produced by inverse Compton-scattering on the synchrotron-emitting photons (Kaufmann et al., 1986; McClements and Brown, 1986).

In this paper we will test other possibilities using current models. The first one assumes that microwave and X-ray emissions are both produced by bremsstrahlung of the same thermal electron population. The second considers the microwave emission produced by gyro-synchrotron radiation.

Analysis of the X-ray spectrum and intensities of the burst complex \( A \) with standard techniques (cf. Crannell et al., 1978), assuming thermal bremsstrahlung, yields

\[
\text{temperature: } T = 5 \times 10^8 \text{ K} ;
\]

\[
\text{emission measure: } Y = 3 \times 10^{44} \text{ cm}^{-3} .
\]

We wish to compare these results with those derived from the radio data. It can easily be shown that the assumption that the radio burst is due to the mechanisms of free-free radiation or bremsstrahlung leads to an inconsistency, because it requires an emission measure of order \( 10^{52} \text{ cm}^{-3} \) (cf. Ramaty and Petrosian, 1972), larger by a factor \( 10^8 \) than the value derived from the hard X-ray spectrum. In passing we note that, assuming the same volume as the one assumed for the X-rays, this would require an electron density \( n_e \) of order \( 10^4 \) times larger to account for the radio data than for the hard X-ray data. This negative result is in agreement with McClements and Brown (1986).

Let us, therefore, assume that the millimeter burst is due to the much more efficient mechanism of gyro-synchrotron emission from quasi-thermal electrons. Since there are no polarization measurements of these bursts, we have to rely on spectral information. For the optically thick part of the spectrum, with frequencies \( f < f_i \) (the turnover frequency), the flux is approximately (Dulk et al., 1979)

\[
F \approx 1.36 \times 10^{-2} T_8 l_8^2 f^2 \text{ s.f.u.} ,
\]

where \( T_8 \) is the temperature in units of \( 10^8 \) K, \( l_8 \) the cubic root of the volume in units of \( 10^8 \) cm, and \( f \) is the frequency in GHz. The observations show that in the spectral range studied the flux \( F \) can be represented by

\[
F = 8.6 \times 10^{-3} f^2 \text{ s.f.u.}
\]

Substitution of Equation (2) in (1) yields

\[
T_8 l_8^2 = 0.63 .
\]
With \( T_8 = 5 \), as derived from the X-ray data, one obtains \( l_8 = 0.35 \), hence, \( l = 350 \text{ km} \). Combining these results with the emission measure \( Y \) derived from the X-ray data, assuming for the volume \( V = l^3 \), one arrives at an electron density \( n_e = 8 \times 10^{10} \text{ cm}^{-3} \).

For the turnover frequency \( f_t \) of the radio spectrum, i.e., the frequency where the plasma changes from optically thick to optically thin, Dulk et al. (1979) and Dulk and Marsh (1982) derived simplified approximate formulae, describing the calculated gyrosynchrotron spectrum under conditions typical for flares:

\[
f_t \approx 2.3 T_8^{0.7} B_2 \text{ GHz}.
\]

A more accurate expression, valid for \( 10^8 < T < 10^9 \text{ K} \) is

\[
f_t \approx 2.2(n_{10} l_8/B_2)^{0.1} (\sin \theta)^{0.6} T_8^{0.7} B_2 \text{ GHz},
\]

where \( B_2 \) is the magnetic field in units of \( 10^2 \text{ G} \).

Let us assume \( f_t \approx 100 \text{ GHz} \), and that \( 45^\circ \leq \theta \leq 90^\circ \) (reasonable in the case of a disk flare like this); then from Equation (4) or (5) with \( T_8 = 5 \): \( B \gtrsim 1400 \text{ G} \).

We, therefore, arrive at the conclusion that the hard X-ray and radio data for the burst complex \( A \) yield consistently

\[
T = 5 \times 10^8 \text{ K}, \quad n_e = 10^{11} \text{ cm}^{-3}, \quad B \gtrsim 1400 \text{ G}, \quad l = 350 \text{ km}.
\]

The model of Dulk et al. (1979) is only valid for the harmonics 10–100. For \( B = 1000 \text{ G} \) it holds if \( f_t = 28–280 \text{ GHz} \). If the chromospheric fields \( B \) are \( \lesssim 2000 \text{ G} \), we obtain from Equation (4): \( f_t \lesssim 150 \text{ GHz} \) which is consistent with the observations. Hence, \( 1400 \lesssim B \lesssim 2000 \text{ G} \).

Under our assumptions of a quasi-thermal source population, the above result identifies the source regions as small plasma knots with rather strong magnetic fields and electron densities near \( 10^{11} \text{ cm}^{-3} \).

The electron density in the source region cannot be much larger than \( 10^{11} \text{ cm}^{-3} \) because of the cut-off at the plasma frequency (e.g., \( f_p \approx 16 \text{ GHz} \) at a density of \( 3 \times 10^{12} \text{ cm}^{-3} \)) and the Razin-suppression (since the Lorentz factor of the electrons is only \( \gamma = 1.13 \) the conventional extremely relativistic estimate cannot be used). Also, the source is probably not located at the chromospheric height which corresponds to a density of \( 10^{11} \text{ cm}^{-3} \). The reason is the severe attenuation by free-free absorption in the cooler overlying chromospheric layers. The absorption coefficient is (Ginzburg, 1970)

\[
\kappa = \frac{f_p^2 v_{ei}}{f^2 N c} \text{ cm}^{-1},
\]

where \( f_p \) is the electron plasma frequency; \( f \), the observing frequency; \( v_{ei} \), the electron-ion collision frequency; \( N \), the refractive index; and \( c \), the velocity of light. Since the source is only 350 km in scale while the scale height of the surrounding chromosphere is 1000 km, we use \( 10^{11} \text{ cm}^{-3} \) as an estimate for the density of the overlying material, a thickness of \( 10^3 \text{ km} \) and a temperature of \( 10^4 \text{ K} \). The optical depth for free-free
absorption at 90 GHz is then \( \tau \approx 60 \), which becomes prohibitive for the microwaves to escape. Thus, a chromospheric source conflicts with the observations, and the source must be located in the corona.

It is of interest to note in this connection that Batchelor (1986) applied the method of Crannell \textit{et al.} (1978) and Batchelor \textit{et al.} (1985) (hence, the same method as applied by us in the first part of the present analysis) to the burst complexes \( B \) and \( C \) of Figure 1, and found for these

\[ T = 7.5 \text{ to } 7.9 \times 10^8 \text{ K}, \quad B > 1000 \text{ G}, \quad n_e \approx 10^{11} \text{ cm}^{-3}, \]

values very similar to ours. These bursts, although they had longer rise times than burst complex \( A \) (\( \sim 100 \text{ ms} \)) had nearly the same physical parameters and, therefore, may have originated at the same location as burst complex \( A \). In the rest of this paper we restrict ourselves to an additional discussion of the burst complex \( A \).

3. Inferences from the Time Profiles of the Spikes

The time profiles of the spikes have rise and decay times of 30 ms. One may ask whether the decay reflects the energy losses of the burst sites. Radiation losses are inaccurate. The characteristic times for radiation losses, both in gyrosynchrotron (radio) and in bremsstrahlung (X-rays) (cf. Rybicki and Lightman, 1979), are much longer than the observed decay times. But these small burst sites with their high temperatures will lose their energy by 'conduction', as was shown by Smith (1986). We will show below that the bursts lose their energy in a time, comparable to the observed decay times of about 30 ms.

Conduction at right angles to the field lines is only important if a high level of magnetic fluctuations is present (Achterberg and Kuijpers, 1984) or at the very location of reconnection. Since it will be shown that the reconnection area may be smaller than the source, we assume one-dimensional transport. Let \( z \) be the coordinate along the field lines.

For the above source classical conduction would imply a conductive flux

\[ w = - \kappa \frac{dT}{dz} \approx 8 \times 10^{16} \text{ erg cm}^{-2} \text{ s}^{-1}. \]  

(7)

On the other hand, if the (Maxwellian) electron gas expands freely along the magnetic field one obtains an energy flux

\[ w = nkT v_{\text{se}} (2/\pi)^{1/2} \approx 1.5 \times 10^{14} \text{ erg cm}^{-2} \text{ s}^{-1}, \]  

(8)

where \( v_{\text{se}} \approx (kT/m_e)^{1/2} \). Since (8) present the maximum flow of energy possible and since its value is much less than the result (7) we infer that conduction by collisions plays no role, because the collisional mean free path is much larger than the source dimension.

The precise value of the 'conductive' flux in the collisionless case at most equals (8) in the case of free streaming. However, the effects of a return current which gives rise to a retarding electric field or to unstable plasma waves reduce the flux to a value below (8) (Brown and Bingham, 1984). The expression for the saturated convective flux which
is best in agreement with laboratory experiments is given by Smith (1986):

\[ W = nkT 5c_s = 0.117v_{te}nkT , \]  

(9)

where \( c_s \equiv (kT/m_i)^{1/2} \) is the ion-acoustic velocity and \( m_i \) the ion mass.

For the application of the conductive losses to the present case we assume a simple model for the source region. To that end we take a triangular temperature profile in the \( z \)-direction of the flaring knot. If the average temperature is \( \overline{T} \), the maximum temperature is \( 2\overline{T} \). If the characteristic length of the region is \( l \), the base length is \( 2l \).

We consider a column in the \( z \)-direction with a cross-section of 1 cm\(^2\). The thermal energy content of such a column with vertical extension \( l \) and with \( T \) varying linearly from zero to \( 2\overline{T} \) is then

\[ E_{th} = 3nlk\overline{T} . \]  

(10)

The time needed to lose this energy by `conduction' on both sides of the source is

\[ \tau_c = 0.5E_{th}/W , \]  

(11)

which, with Equation (9), implies \( \tau_c = 0.052 \text{ s} \), similar to the observed decay times of the spikes.

This result has two consequences:

(1) Since the decay times of the individual spikes are equal to \( \tau_c \) (within observational uncertainties), the decay time-profiles of the spikes actually represent the conductive cooling of the plasma knot, while the time profiles of the rising branches (\( e \)-folding time also \( \approx 30 \text{ ms} \)) represent the energization time-profile, convolved with the profile of the conductive loss process. Hence, the energization process must have a characteristic time \( \gtrsim 30 \text{ ms} \) but certainly \(< 60 \text{ ms} \). We should note, of course, that the agreement found here, depends on the correctness of the temperature derived, and of the expressions used for the collisionless heat flux.

(2) Since the energy losses are described by (9) only if the electrons are hot while the ions are relatively cool we conclude that primary energization occurs in the electrons and not in the ions. If the ions were to have a comparable high temperature a shock would be generated with a velocity above the sonic speed parallel to the magnetic field and the fast magneto-acoustic speed in a direction perpendicular to it. Then the loss-time would have been much longer. Note that the electron-ion collision time (0.65 s) and the electron-ion temperature equalization time (0.65\( m_i/m_e \) s) are much longer than the spike duration and, therefore, not inconsistent with the existence of a collisionless hot electron gas.

We thus conclude that the area of initial energization is most probably small as compared with the observationally derived size (350 km) of the radiating area. One may speculate that a reasonable upper limit may be 50 to 100 km.

Another way of estimating the physical conditions in the area of energization follows considerations of Loran and Brown (1985), with two changes. As in their paper we assume that the primary energization is due to field line reconnection, and that the conversion of magnetic energy to other forms of energy proceeds on a time-scale longer
than the Alfvén time-scale. Let $L$ be the largest dimension and $\lambda L$ ($\lambda \leq 1$) be the sheet thickness. Unlike Loran and Brown, we make the reasonable assumption that the external Alfvén travel time $L/v_{A\text{ext}}$ in the low-beta environment is less than the internal Alfvén travel time across the reconnecting sheet $\lambda L/v_A$. Then the reconnection time is

$$\tau = \frac{\lambda L}{v v_A},$$

where $\tau$ is the conversion time, $\lambda L$ the size of the area at right angles to the reconnection surface, and $v < 1$. We now find

$$\tau = \frac{0.5 \lambda}{v} n_{10}^{1/2} L_8 B_2 \quad \text{s},$$

(12)

where $n_{10}$ is $n_e/10^{10}$, $B_2 = B/10^2$, etc. Our Equation (12) differs by a factor $\lambda$ from Equation (1) in Loran and Brown. Assuming further that the escape rate of the electrons cannot exceed the limit set by Equation (9), the escaping flux is $F_\epsilon \leq 0.117 v n_{\epsilon A}$, where $A = \lambda L^2$ is the escape area (instead of our factor 0.117, Loran and Brown use the value 0.2). One then finds a minimum value for the magnetic field strength

$$B_{\text{min}} = 170 F_{35}^{1/2} T_7^{-1/4} \tau^{-1} \lambda^{1/2} v^{-1} \quad \text{G},$$

(13)

where $F_{35}$ is the escape rate of electrons in $10^{35} \text{ s}^{-1}$. The crucial difference of our result (13) from Equation (4) of Loran and Brown is the appearance of the factor $\lambda^{1/2}$ in the numerator instead of in the denominator.

For $F$ we write $n_{i}^{1/3}/0.03 = 1.4 \times 10^{35} \text{ s}^{-1}$. Introducing the known quantities in Equation (13), with $\tau = 0.03 \text{ s}$ and $B_{\text{min}} = 1400 \text{ G}$, one obtains

$$\lambda^{-1/2} v \approx 1.8,$$

which does not pose a problem for the reconnection process if the sheet is sufficiently thin ($\lambda < 1$).

We may next speculate about the density in and the location of the source. The electron density found ($10^{11} \text{ cm}^{-3}$) is characteristic for chromospheric regions, but there are a few cases in the literature where reconnection at coronal locations demands electron densities in the reconnection region of the same order (see De Jager et al., 1983).

We think that the initial electron density at the source of reconnection was not much higher than $10^{11} \text{ cm}^{-3}$, because the energization time is comparable to the loss time. Note also that the total (hot and cold) electron density is fixed by the ion density; the ion plasma with its slow expansion speed cannot have expanded appreciably during a spike duration.

Another question is whether the initial energization occurs in the same area; in other words, are the energetic radiations emitted at the spot where the particles are accelerated or does particle energization occur elsewhere? Some observations suggest that for phenomena of lower energy, e.g., the $\sim 10$ keV flare bursts, the initial energization normally occurs in coronal regions, at an average height of about $10^4 \text{ km}$ above the
photosphere, while the X-radiation is mostly emitted at chromospheric levels, as a result of thick-target beam interaction (Duijveman et al., 1982). The radio emission, on the other hand, can originate at all levels in the interacting flux tubes that cause the flare (Kundu, 1984). It seems to us that this situation does not apply to the high-energy X-ray and millimeter burst discussed here. The main reason is that the energized plasma responsible for the emission lives very briefly ($\approx 0.03 \, \text{s}$) and can only progress over a distance of about $400 \, \text{km}$ during the bursts. Thus it is impossible for the sites of energization and of radiation to be significantly different. Further, a non-thermal interpretation for the X-rays is inconsistent with the gyro-synchrotron interpretation at mm-wavelength, as has been shown by McClements and Brown (1986); in that case the required high electron density ($> 5 \times 10^{12} \, \text{cm}^{-3}$) would lead to Razin suppression in the radio spectrum which is not observed.

We arrive at the same conclusion from the following argument: a substantially different location of the radio source from the X-ray source would imply a larger altitude and dimension ($> 350 \, \text{km}$) and a smaller field strength for the radio source. Since the radio spectrum is optically thick and the turnover frequency is at $90 \, \text{GHz}$ or larger, Equation (4) then implies a larger effective temperature and Equation (1), therefore, a fortiori a larger flux than is observed.

For the thermal X-ray bremsstrahlung interpretation more reasons exist, implying that a beam of electrons or protons from ‘above’ cannot be responsible for the hot electron plasma. For, if a beam of energetic electrons was the source of the hot electron plasma then the energy of the electrons in the beam should in any case be larger than $\frac{3}{2}k(5 \times 10^8 \, \text{K}) \approx 65 \, \text{keV}$. But the column stopping distance of electrons of such an energy is (Thompson et al., 1986)

$$D_e \approx \frac{E^2}{(2\pi e^4 \ln \Lambda)} \approx 2 \times 10^{22} \, \text{cm}^{-2} ;$$

hence, the source region should in that case have an extension of

$$\phi = D_e / n_e \approx 2 \times 10^{22} / n_e > 2 \times 10^{11} \, \text{cm} ,$$

which disagrees with the observed size.

A proton beam, which is stopped primarily by electrons for $T < 5 \times 10^8 \, \text{K}$, cannot be responsible for various reasons. First

$$D_p \approx (m_e / m_p) D_e \approx 10^{20} \, \text{cm}^{-2} ,$$

hence, a $65 \, \text{keV}$ proton would be stopped after $D \approx 10^{20} \, \text{cm}^{-2}$, i.e., in $350 \, \text{km}$ for $n_e \approx 3 \times 10^{12} \, \text{cm}^{-3}$. Such a compact chromospheric loop would have an ambient density near its top at an altitude of $400 \, \text{km}$ of $10^{11} \, \text{cm}^{-3}$. For an ambient temperature of $10^4 \, \text{K}$ and a scale height of $1000 \, \text{km}$ the optical depth $\tau$ for free-free absorption at a frequency of $90 \, \text{GHz}$ would be $\tau \approx 60$, prohibitive for detecting the microwaves. Also, the required beam density would be too large: assuming energy injection during $\Delta t = 0.05 \, \text{s}$, the beam density should be

$$n_b > 9 \times 10^{26} \, \text{erg} / (0.05 \, \text{s} \, \phi^2 E_b v_b) \approx 6.7 \times 10^{11} / (E_b / 65 \, \text{keV})^{3/2} \, \text{cm}^{-3} .$$
Since it is required that \( n_p \ll n_e \) a beam of protons from 'above' cannot provide the energy.

These negative results confirm that heating should occur 'at the spot', and not by beams of electrons or protons.

4. Conclusions

We have shown for a solar millimeter radio burst complex and the associated hard X-ray bursts (the '100 keV bursts') that they are most probably emitted by the same fairly dense region of the corona, a region characterized by \( n_e \approx 10^{11} \text{ cm}^{-3} \), electron temperature \( T \approx 5 \times 10^8 \text{ K} \), \( B \gtrsim 1400 \text{ G} \), and diameter \( l \approx 350 \text{ km} \). Such regions very rapidly lose their excess energy by a saturated convective electron flux, in about 0.05 s. The radio emission occurs in discrete fairly symmetric spikes with durations of \( \approx 0.06 \text{ s} \). The observed decline of the time profiles of the spikes represents the 'conductive' cooling rate, and the total profile represents the time profile of the energy injection or energization process, convolved with the cooling time profile.

Since the sites of energization and of (X-ray or radio) emission appear to be virtually identical (in view of the short conduction loss times, during which the heated particles cannot run far away), we conclude that the mm-100 keV bursts we investigated emit at the unusually high frequency of 90 GHz because the primary energization (and, hence, the primary energy release) occurs in a small-scale dense region with the properties enumerated above, particularly the high \( B \) value.

The total energy deposit per spike is \( E = 3NkT \), where \( N = n_e l^3 \). With \( T = 5 \times 10^8 \text{ K} \), \( n_e = 10^{11} \text{ cm}^{-3} \), \( l = 3.5 \times 10^7 \text{ cm} \), one obtains \( E = 9 \times 10^{26} \text{ erg} \).

Note that for the fairly energetic bursts discussed here the parameters derived differ considerably from those derived customarily. Batchelor et al. (1985) who applied a similar method to a number of hard X-ray bursts found average values \( n_e \approx 10^9 \), \( B = 100 \text{ G} \), \( l \approx 10^4 \text{ km} \). Obviously, the problem may be reversed, and the question may be asked: what would happen if an energy \( E = 9 \times 10^{26} \text{ erg} \) was injected into a small (say diameter \( \approx 50 \text{ km} \)) coronal volume, where \( n_e = 10^{11} \), with \( B = 1400 \) to 2000 G, while the time profile of the injection had a characteristic width of the order of some 50 ms. On the basis of the preceding discussion we may state that in the time during which the heated volume loses its heat by conduction (0.03 to 0.05 s) the heated area would have acquired a size of a few hundred km. The heated plasma knot would emit hard X-rays by thermal bremsstrahlung, and microwave bursts by gyro-synchrotron radiation in the mm wavelength region, as observed.

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References