Magnetically Collimated Winds from Accretion Disks

Takashi Sakurai

Tokyo Astronomical Observatory, University of Tokyo, Mitaka, Tokyo 181

(Received 1986 December 25; accepted 1987 August 27)

Abstract

A steady, axisymmetric wind from a magnetized accretion disk is studied by applying a numerical scheme developed for the stellar wind modeling (Sakurai 1985; AAA 40.064.036). As in the magnetic stellar winds, the collimation of the wind toward the rotation axis is seen in the wind from a disk. The driver of the collimation is the toroidal magnetic field which develops in the wind due to the rotation of the disk. Magnetic winds from rotating disks can therefore naturally lead to the formation of collimated jets, which are found in star-forming regions and in extragalactic radio sources.

Key words: Accretion disks; Magnetohydrodynamics; Stellar winds.

1. Introduction

A system made of the magnetic field anchored in a rotating object is found in a variety of astrophysical settings. The solar wind is one of such examples. One important effect of the solar magnetic field is the magnetic braking, which is believed to be responsible for the present slow rotation of the sun (Weber and Davis 1967). A similar process is expected in stellar winds and stellar magnetic fields, as was originally proposed by Schatzman (1962). Hartmann and MacGregor (1982) and Draine (1983) studied magnetic winds from protostars, in an attempt to account for the vigorous activity of young stars. The case in which the central object is not a spherical star but an accretion disk has been suggested as a possible model for extragalactic radio sources (Blandford and Payne 1982) and for bipolar flows from star-forming regions (Uchida and Shibata 1985; Pudritz and Norman 1986).

In a previous paper (Sakurai 1985; referred to as paper I), we developed a computational method to find the magnetic configuration of steady axisymmetric winds. The method was applied to a stellar wind with a simple magnetic configuration (the split monopole geometry). An important aspect found there is that the magnetic field, which is assumed to be nearly radial in the vicinity of the star, tends to bend toward the rotation axis of the star due to the magnetic pinching force of spiraling field lines. This effect was observed, though very small, in the solar wind and was subsequently explained by using a perturbation analysis applied to an unperturbed spherical wind (Nerney and Suess 1975).

The motivation for the present paper is to show that the same scheme works in a disk geometry and to suggest this as a mechanism for the formation of collimated flows
or jets) seen in star-forming regions and in extragalactic radio sources. A major
difference between the present study and that of paper I is in the rotation profile of the
central object. While a rigid rotation was assumed in the stellar wind model of
paper I, a differential (such as Keplerian) rotation of the accretion disk is incorporated
in the present model. Little change is introduced in the other aspects of the model
for practical reasons. As in the stellar wind case, a wind is assumed to start from the
“atmosphere” of the disk, and the infalling flow in the disk itself is not included in the
wind model.

This paper is organized as follows. In section 2 we specify the distribution of
mass and magnetic flux in the disk. In section 3 the equation to be solved and the
numerical method for the solution are presented. The computed model is shown in
section 4 and its implication is discussed in section 5.

2. Construction of a Disk Model

In the stellar wind model of paper I, physical variables such as the magnetic field
strength, temperature, and rotation frequency are uniform on the stellar surface. In
the disk geometry these quantities are generally functions of the radial distance. Al-
though the computational method can apply to any model, it is necessary from both
astrophysical and practical points of view to restrict our analysis to a particular model
that is specified by a small number of parameters.

2.1. Magnetic Field Distribution

First we will focus on the distribution of the magnetic field threading the disk. In
order for the magnetic field distribution to be in a steady state, the inward transport
of the magnetic field by the infalling disk material must be balanced by the outward
diffusion of the magnetic field. The magnetic diffusivity in the disk might be due to
the turbulence in the disk, as is presumed in the so-called alpha-disk model (Shakura
and Sunyaev 1973). The inflow of the disk material may be controlled either by the
angular momentum transport in the disk due to turbulent viscosity as in the alpha-
disk model, or by the magnetic braking (Blandford and Payne 1982; Pudritz and
Norman 1986). In considering a large uncertainty in the key processes mentioned
above, we will not try to determine a self-consistent magnetic field distribution but
rather adopt a simple ad hoc model.

Hereafter we will use the cylindrical coordinates ($\bar{\omega}$, $\varphi$, $z$) where $z=0$ is the plane
of the disk, $\bar{\omega}$ is the distance from the rotation axis, $\varphi$ is the azimuthal angle, and the
polar angle $\theta$ is defined by $\tan \theta = \bar{\omega}/z$. Blandford and Payne (1982) showed that the
equation of axisymmetric winds allows a self-similar solution, if the $B_z$-component on
the disk scales as $B_z \sim \bar{\omega}^{-b/4}$. On the other hand, if the magnetic field is sufficiently
strong near the disk, the magnetic field there may be approximated by the current-
free field, which has no toroidal component. The current-free magnetic field with
$B_z \sim \bar{\omega}^{-\mu}$ on the disk is examined in the Appendix. Figure A1 shows the magnetic
field lines for various values of $\mu$, including the case $\mu=5/4$. The “collimation” seen
in the current-free field lines comes from the accumulated effect of the magnetic flux
in the outer part of the disk. An extreme case is the uniform magnetic field in the
$z$-direction ($\mu=0$), which is ideally collimated. The similarity solution of Blandford and Payne (1982) is also reproduced in figure A1 for comparison. For the same boundary condition (i.e., $\mu=5/4$), the current-free field shows more collimation than does the similarity solution. The reason for this is discussed in section 5.

Since our aim is to show that the collimation can take place due to the toroidal magnetic field, it is not convenient to adopt such a magnetic field configuration that shows the collimation in the absence of the toroidal component. Keeping this in mind, we will use the same magnetic configuration as in paper I, namely the split monopole geometry. In the case of the stellar wind, the magnetic field is uniform and positive/negative in the northern/southern hemisphere, respectively. A discontinuity in the magnetic field (a current sheet) exists in the equatorial plane (hence the name “split monopole”). In the case of the disk studied here, the magnetic field in the volume $z>0$ is due to the magnetic monopole placed below the disk plane (at the depth $d$) as shown in figure 1. Namely,

$$B_{\theta}^{(0)} = \Phi \bar{\omega} \left[ \bar{\omega}^2 + (z+d)^2 \right]^{-3/2},$$

$$B_{z}^{(0)} = \Phi (z+d) \left[ \bar{\omega}^2 + (z+d)^2 \right]^{-3/2} .$$

(1)

Here $\Phi$ is a constant representing the total magnetic flux. The mirror symmetry with respect to $z=0$ is also assumed. The deviation of the magnetic field from this initial radial geometry after the introduction of the rotation of the disk (see section 4) is attributed to the effect of the rotation-induced toroidal magnetic field.

Our disk model does not have an edge but extends to infinity, contrary to the model by Bisnovatyi-Kogan and Blinnikov (1972). For computational convenience,

![Diagram of the model](image)

**Fig. 1.** The geometry of our model. The left half depicts the magnetic field configuration in the physical space, whereas the right half shows the way it is handled in the computation. A magnetic monopole and a point mass are located at the depth $d$. The (artificial) edge of the disk is at $\bar{\omega} = \bar{\omega}_d$.  

© Astronomical Society of Japan • Provided by the NASA Astrophysics Data System
however, we will identify the outer radius of the disk \( \overline{w}_d \) as in figure 1, and we will look for solutions in the volume covered by the field lines coming from within this radius. The magnetic field in our model is supposed to originate from the disk. The central object may have its own magnetic field and the interplay of the two magnetic systems would lead to a more complex field geometry (Ghosh and Lamb 1979; Kaburaki 1986), but this is beyond the scope of the present paper.

2.2. Gravitational Field

Next we will specify the gravitational field of the system, which is a combination of the gravity due to a point mass \( M_\odot \) at the center and the gravity of the disk itself whose total mass is \( M_d \). A simple model for the gravity of the disk would be due to a fictitious point mass placed below the disk plane, as we did for the magnetic field. The surface density of the disk in this case is given by

\[
\Sigma = \frac{M_d}{2\pi d (\overline{w}^2 + d^2)^{-3/2}}. \tag{2}
\]

This is a special case of the mass distributions studied by Brandt and Belton (1962). In the following we assume that the disk is massive enough \( (M_d \gg M_\odot) \) so that the gravitational potential is approximated by

\[
\phi = -G M_d [\overline{w}^2 + (z + d)^2]^{-1/2}. \tag{3}
\]

If the disk is in equilibrium under the action of gravitational, centrifugal, and magnetic forces, but the action of the pressure force is negligible, the rotational velocity of the disk will be given by

\[
Q^2(\overline{w}) = f_k GM_a (\overline{w}^2 + d^2)^{-3/2},
\]

\[
f_k = 1 - \frac{\phi^2}{GM_d^2}. \tag{4}
\]

The rotation of the disk in our model is slower than the Keplerian rotation by the factor \( f_k \). For an equilibrium to be possible, the magnetic flux \( \Phi \) should not exceed \( G^{1/2} M_d \). If \( \Phi \) is close to the upper limit, the disk is mostly suspended by the magnetic force. If \( \Phi \sim 0 \), the disk is supported by its rotation (Gillis et al. 1974).

By assuming that the wind is cold (the thermal pressure is negligible) and corotates with the disk with the Keplerian velocity \( (f_k = 1) \), we can evaluate the net force along the magnetic field \( F_\theta \), which is the difference between the inward-directed gravity and the outward-directed centrifugal force. If the gravity is due to the central point mass alone, it is known that this net force is directed outward near the disk, if the field line is tilted by more than 30° from vertical (Blandford and Payne 1982). In this portion of the disk the wind can be purely centrifugal. In our model, however, we obtain

\[
F_\theta = \frac{GM_d}{\overline{w}^2 + (z + d)^2} \left[ \frac{\overline{w}^2}{\overline{w}^2 + (z + d)^2} \left( \frac{z + d}{d} \right)^3 - 1 \right], \tag{5}
\]

and near \( z = 0 \), we find \( F_\theta < 0 \). Therefore, the net force along the magnetic field is always directed inward near the disk. This is because the \( z \)-component of the gravity
does not vanish at the disk in our model. Therefore, our model is not purely centrifugally driven, and the effect of thermal pressure cannot be neglected near the disk. The wind far from the disk, however, will always have the character of centrifugal winds, if the disk rotates rapidly. [The parameter \( \omega \) in equation (16) is the measure of whether the rotation is rapid or slow.]

3. Method of Solution

3.1. Basic Equations

The basic equations for the axisymmetric, steady, polytropic winds were derived in paper I, and here we will only collect the necessary expressions. The axisymmetry and the divergence-free condition lead to the expression for the magnetic field \( \mathbf{B} \) as

\[
\mathbf{B} = \mathbf{F} a \times \nabla \varphi + \bar{\omega} B_\varphi \nabla \varphi ,
\]

where \( a(\bar{\omega}, z) \) is the magnetic stream function and \( B_\varphi(\bar{\omega}, z) \) is the toroidal component of the magnetic field. The velocity which satisfies the continuity equation is given by

\[
v = \frac{\alpha B}{\rho} + \bar{\omega} \Omega \nabla \varphi ,
\]

where \( \rho \) is the density, and \( \alpha \) and \( \Omega \) are functions of \( a \). The pressure \( p \) is given by the polytropic relation

\[
p = K \rho^\gamma,
\]

where \( \gamma \) is a constant and \( K \) is a function of \( a \). The \( \varphi \)-component of the equation of motion leads to

\[
\bar{\omega} \left( v_\varphi - \frac{B_\varphi}{4\pi \alpha} \right) = \Omega \bar{\omega}_A^2 ,
\]

where \( \bar{\omega}_A \) is a function of \( a \). From equations (7) and (9) we can show that

\[
v_\varphi - \bar{\omega} \Omega = \frac{4\pi \alpha^2 \Omega (\bar{\omega}_A^2 - \bar{\omega}^2)}{\bar{\omega} (4\pi \alpha^2 - \rho)} .
\]

Therefore \( \bar{\omega} = \bar{\omega}_A \) at the point where \( \rho = \rho_A \equiv 4\pi \alpha^2 \). The latter condition is rewritten as \( v_\varphi = B_\varphi^2/(4\pi \rho) = \) poloidal Alfvén speed squared, and \( \bar{\omega}_A \) is called the Alfvén radius.

The equation of motion along the magnetic field can be integrated and leads to the Bernoulli equation:

\[
\frac{1}{2} v_\varphi^2 + \frac{1}{2} (v_\varphi - \Omega \bar{\omega})^2 + \frac{\gamma}{\gamma - 1} \frac{p}{\rho} + \phi - \frac{1}{2} \Omega^2 \bar{\omega}^2 = E ,
\]

where \( \phi \) is the gravitational potential, \( E(a) \) is the energy constant, and \( v_\varphi \) is the poloidal component of the velocity. Finally the equation of motion across the magnetic field is
\[ P \cdot \left( \frac{\alpha^2}{\rho} - \frac{1}{4\pi} \frac{V a}{\omega^2} \right) = \rho \left( E' - \frac{1}{\gamma - 1} \frac{pK'}{\rho K} + \omega^2 \Omega' \right) + \frac{B^2}{\rho} \alpha \alpha' \]
\[ + D \left[ \frac{D}{4\pi} \omega \omega' \alpha' - \alpha^2 \omega \omega' \omega' - \alpha^2 \omega' \Omega' \right] \cdot \frac{\alpha}{\omega} \frac{\omega}{(\alpha^2 - \rho)} \]
(12)

where the prime denotes \( d/da \).

The stream function \( a \) is constant along the magnetic field and therefore labels the poloidal field lines. For the monopole magnetic field we adopted in equation (1) \( a = \Phi \sin^2 (\theta/2) \). Although the stream function \( a \) deviates from this initial state in the final solution, the boundary value of \( a \) should be fixed to this initial setting. In other words the footpoints of the field lines are fixed, and the field lines are labeled by the location \( \omega_0 \) of the footpoints on the disk. Hence \( \Omega(a) \) is specified by equation (4), because \( \omega = \omega_0 = d \tan \theta \) and \( a = \Phi \sin^2 (\theta/2) \) on the disk. Of the remaining four functions \( K, E, \alpha, \text{and } \omega \Lambda \), we can specify \( K \) and \( E \), while \( \alpha \) and \( \omega \Lambda \) are determined automatically in the solutions.

### 3.2. Dimensionless Form of the Basic Equations

We will introduce the scaling in the position vector \( r = (\omega, z) \) and other variables as

\[ \frac{r}{r_{\Lambda e}} = \hat{r}, \quad \frac{\alpha}{\alpha_{\Lambda e}} = \hat{\alpha}, \quad \frac{\Phi}{\Phi_{\Lambda e}} = \hat{\Phi}, \quad \frac{\rho}{\rho_{\Lambda e}} = y \quad (\rho_{\Lambda e} \equiv 4\pi \alpha_{\Lambda e}^2), \]
\[ E \left( \frac{G M_a}{r_{\Lambda e}} \right) = \hat{E}, \quad \phi \left( \frac{G M_a}{r_{\Lambda e}} \right) = \hat{\phi}. \]
(13)

The reference length \( r_{\Lambda e} \) and density \( \rho_{\Lambda e} \) are taken from the Weber–Davis (1967) model for the magnetic stellar wind (also paper I). The Weber–Davis (1967) model is defined as the solution to equations (7)–(9) and (11) with \( \theta = \pi/2 \) (equator) and with the undisturbed radial magnetic field. After this scaling, the Bernoulli equation (11) is written as

\[ E(a) = \frac{1}{2} \beta \alpha^2 \left( \frac{V a}{\omega^2} \right)^2 + \frac{1}{2} \omega \left[ \frac{\alpha^2}{\alpha^2 - y} - \frac{\omega}{\omega} \right]^2 \omega^2 \]
\[ + \frac{\theta}{\gamma - 1} y^{\gamma - 1} - \phi, \]
\[ \equiv \hat{H}(r, y), \]
(14)

where all the carets are dropped. Three dimensionless parameters appear in this equation, namely,

\[ \Theta(a) = \frac{\gamma K(a) \rho_{\Lambda e}^{-1} r_{\Lambda e}}{G M_a}, \]
(15)

\[ \omega(a) = \frac{\omega(a) \gamma r_{\Lambda e}}{G M_a}, \]
(16)

and
\[ \beta = \frac{\Phi^2}{4\pi \rho_\infty GM_a r_{\infty}^2} . \]  

Equation (12) is also cast into a dimensionless form as

\[
\begin{align*}
F \cdot \left( \frac{\alpha^2}{\omega^2} - 1 \right) \frac{\nabla a}{\omega^2} & = \frac{y}{\beta} \left[ \frac{E' - \frac{\Theta'}{\gamma(y-1)} + \frac{1}{2} \omega' \omega^2}{\omega} \right] + \frac{\alpha' \gamma \nabla a}{y} \omega^2 \left[ \frac{\alpha' \gamma \omega^2}{\omega} - \frac{\omega'}{2\omega} \right] \\
+ \frac{\omega}{\beta} \frac{\nabla \left( \omega^2 \right)}{\omega^2 (\alpha^2 - y)} & \left[ \frac{\alpha' \gamma \omega^2}{\omega} - \frac{\omega'}{2\omega} \right] \left( \alpha^2 - \omega^2 \right) - \alpha^2 \omega^2 \gamma \omega^2 - \frac{\omega'}{2\omega} \alpha^2 \left( \omega^2 \right) \frac{\omega^2}{\alpha^2 - y} \right].
\end{align*}
\]

(18)

The Bernoulli equation (14) determines the profile of dimensionless density \( y \). Two X-type critical points appear when plotting the function \( H \) in the \((r, y)\)-plane. These are the slow/fast-mode critical points, denoted by \( r_s \) and \( r_f \) respectively, where the flow speed matches the speed of the magnetohydrodynamic slow/fast modes. The values of \( \alpha \) and \( \omega \) are determined in such a way that the \( y \)-profile passes through the two critical points. The constant \( \beta \) is determined in the Weber–Davis (1967) scaling as will be described in section 3.3.

The cross-field force-balance equation (18) is singular at the Alfvén point \(( \gamma = \alpha^2 \)). Therefore a certain condition must be imposed there in order to select a regular solution. This condition and the inner boundary condition on \( a \) (imposed at the surface of the disk) are sufficient to determine the solution. On the side boundary (along the field line starting from the edge of the disk), one might assume a rigid boundary condition. However, we have adopted a more natural boundary condition that the volume beyond the boundary field line contracts or expands homologously in responding to the displacement of this field line. The force-balance equation is not exactly satisfied in the region beyond this field line.

### 3.3. The Weber–Davis (1967) Scaling

The Weber–Davis (1967) model in a dimensionless form is obtained by substituting \( \Theta = \pi/2 \), \( (F a)^2 = 1/r^2 \), \( \alpha = 1 \), and \( \omega = \omega_\infty \) into equation (14). The Alfvén point is at \( r = 1 \) and \( y = 1 \). By specifying the two parameters \( \Theta = \Theta_\infty \) and \( \omega = \omega_\infty \), the \( y \)-profile as well as the values of \( \beta \) and \( E \) \((= E_\infty)\) are determined. The relation between the dimensionless \((\Theta_\infty, \omega_\infty)\) and the actual boundary conditions is found by calculating the two functions \( f_1 \) and \( f_2 \):

\[
\begin{align*}
f_1 &= E_\infty \omega_\infty^{-1/3} = \left( \frac{C_{s*}^2}{\gamma - 1} + \psi_{s*} - \frac{1}{2} \Omega r^2 \psi_{s*} \right) (GM_\infty \omega)^{-2/3}, \quad (19a) \\
f_2 &= (\beta r \omega_\infty^{-4/3})^{1/2} = C_{s*} V_{s*}^{-1} (\Omega r_*)^{-2/3}, \quad (19b)
\end{align*}
\]

Here \( C_{s*} \) and \( V_{s*} \) are the sound and Alfvén speeds, respectively, defined by \( C_{s*} = \gamma p_\infty/\rho_\infty \) and \( V_{s*} = B_{t*} / (4\pi \rho_\infty) \). The asterisks denote the boundary values given on the disk plane. Figure 2a shows the values of \( f_1 \) and \( f_2 \) as functions of \( \Theta_\infty \) and \( \omega_\infty \). The values of \( \Theta_\infty \) and \( \omega_\infty \) are found from the intersection of the two curves. Figure 2b shows the values of slow/fast-mode critical radii \( r_s \) and \( r_f \). As the centrifugal character of the wind strengthens (large \( \omega_\infty \) and small \( \Theta_\infty \)), the three critical points tend to be largely
Fig. 2. (a) The parameter space of the Weber–Davis (1967) model. Two dimensionless parameters $\Theta_\alpha$ and $\omega_\alpha$, which measure the effects of the thermal pressure and the centrifugal force respectively, characterize the model. The lower right portion is the region of thermal winds, and the upper left is the regime of centrifugal winds. No wind solutions exist in the lower left portion due to the insufficient thermal and centrifugal forces. The solid and dashed contours show the values of two functions $f_1$ and $f_2$ [see equations (19a, b)]. These functions can be evaluated by using dimensional physical parameters, and this figure is then used to obtain the corresponding dimensionless parameters $\Theta_\alpha$ and $\omega_\alpha$. (b) The location of critical points in the Weber–Davis (1967) model. The slow- (fast-) mode critical radii $r_s$ ($r_f$) are shown by the solid (dashed) contours, respectively. The unit of length is the Alfvén radius.
separated from each other \( r_s \ll r_A \ll r_f \). Due to computational restriction we cannot go beyond, say, \( r_f \sim 10 r_A \).

In order to specify the values of \( \Theta(a), \omega(a), \) and \( E(a) \) for any value of \( a \) between the \( z \)-axis and the side boundary, we will assume simple functional forms for them as follows:

\[
\Theta(\tilde{\omega}_d) = (\Theta_e - \Theta_o) \left( \frac{\tilde{\omega}_d}{\omega_d} \right)^{2} \left[ 1 + \left( \frac{\tilde{\omega}_d}{d} \right)^{2} \right]^{1/2} + \Theta_o , \quad (20a)
\]

\[
\omega(\tilde{\omega}_d) = \omega_o \left( \frac{\tilde{\omega}_d}{\omega_d} \right)^{2/2} = f_1 \left( \tilde{\omega}_d + d \right)^{-3/2} , \quad (20b)
\]

\[
E(\tilde{\omega}_d) = (E_e - E_o) \left( \frac{\tilde{\omega}_d}{\omega_d} \right)^{2} \left[ 1 + \left( \frac{\tilde{\omega}_d}{d} \right)^{2} \right]^{1/2} + E_o . \quad (20c)
\]

Here \( \Theta_o, \Theta_e, \omega_o, E_o, \) and \( E_e \) are constants, and \( \tilde{\omega}_d \) is the radial location of the footpoint of the field line, which can be used to label the line of force. The distribution of the physical variables on the disk can be found by using equations (19a, b) and the definitions (15)–(17).

3.4. Method of Solution: Summary

The solution will be found by the following procedure:

(a) Specify parameters \( \Theta_e, \Theta_o, \omega_o, \) and \( E_o \).

(b) Evaluate \( E_o \) and \( \beta \) in the Weber–Davis (1967) model.

(c) Functions \( \Theta(a), \omega(a), \) and \( E(a) \) are defined by the interpolation formula (20a–c).

(d) Equations (14) and (18) are solved by iteration. Functions \( \alpha(a) \) and \( \tilde{\omega}_A(a) \) are automatically determined.

We repeat procedures (a)–(d) by gradually increasing \( \omega_o \), starting from a very small value. Dimensionless solutions thus obtained can be connected to physical solutions by virtue of equations (15)–(17) and (19a, b).

4. Results

As an example, we will show the solution for the parameter setting \( d=0.1, \tilde{\omega}_d = 0.4, \gamma=1.2, \Theta_e=0.275, \omega_o=7.0, \Theta_o=1.6, \) and \( E_o=15.0 \). The Weber–Davis (1967) model then gives \( \beta=1.4 \) and \( E_o=-0.46 \), and we also find \( f_1=0.48 \) and \( f_2=-0.24 \). The centrifugal force in this model is half of the Keplerian rotation, that is, \( f_2=0.5 \). Negative \( f_2 \) means that the centrifugal effect is significant. We would have \( f_2 \sim -1.5 \) for purely centrifugal winds. If \( f_2 \sim 1 \) like in our example, \( f_1 \) is approximated by \( (C_{s*}/V_{K*})(V_{A*}/V_{K*})^{-1} \), where \( V_{K*}^2=\varphi_{*} \) is the Keplerian rotational speed squared. Therefore, \( f_2=0.48 \) means that the effect of thermal pressure in our example is smaller but significant compared to the gravitational and centrifugal forces.

Figure 3a shows the poloidal field lines near the disk. The three curves labeled as s, A, and f are the slow-, Alfvén-, and fast-mode critical surfaces, respectively. Figure 3b covers a wider area, showing the tendency of poloidal field lines to bend toward the \( z \)-axis. The original potential field lines, which are radial, are shown by dashed lines in the same figure.
Fig. 3. (a) Solution for the magnetic field in the poloidal plane. Three curves labeled as $s$, $A$, and $f$ represent the location of slow-, Alfvén-, and fast-mode critical surfaces, respectively. (b) The solution plotted on a larger scale. The dashed lines are the original radial field lines in the absence of rotation. The bending of the field lines toward the $z$-axis is evident.

Figure 4 shows the distribution of $v_\varphi$ (solid contours) and $B_\varphi$ (dashed contours). Roughly speaking, $v_\varphi$ increases up to the Alfvén surface, because the magnetic field enforces the corotation of the plasma. Beyond the Alfvén surface $v_\varphi$ declines, by conserving the angular momentum. Therefore, the rotation velocity has a maximum near the Alfvén surface along each field line. The toroidal magnetic field $B_\varphi$ decreases monotonically along the wind flow.

Figure 5 shows the distribution of density (contours) and of poloidal velocity (arrows). The enhancement in density near the rotation axis is due to the collimation of the flow. The speed of the wind is smaller near the side boundary, because the
Fig. 4. The distribution of $v_\phi$ (solid) and $B_\phi$ (dashed) in the poloidal plane. Contour intervals are 0.2 on a logarithmic scale. The toroidal velocity $v_\phi$ has a local maximum near the Alfvén critical surface.

Fig. 5. The density distribution in contours, together with velocity fields represented by arrows, in the poloidal plane. The contours are equally spaced in log $y$, ranging from $y=100$ near the origin to $y=10^{-3}$. The lengths of arrows are proportional to the magnitudes of dimensionless velocity. The enhancement in the density near the rotation axis is due to the collimation of the wind.

rotation of the underlying disk is slowest there. The wind speed increases toward the rotation axis due to the centrifugal acceleration. In the very vicinity of the axis, however, the wind is thermally driven and the wind velocity there can be made larger or smaller as one likes by suitably adjusting the parameter $\Theta_e$.

Figure 6 depicts a three-dimensional view of the helical field lines. This figure and figure 4 imply that the collimation of the wind is due to the toroidal pinching effect. This proves to be the case by evaluating the magnitudes of various forces.
Fig. 6. A three-dimensional view of the field lines, which show a helically twisted structure. A thick horizontal bar represents the disk. A part of the field lines are not drawn in order to avoid overlapping.

The electric currents flowing in the negative $z$-direction along the dense collimated flow are responsible for this pinching.

The detailed features in the solution will change depending on the values of the dimensionless parameters, although the general character of the solution does not change. By reducing the thermal parameter $\Theta$, the thermal driving force decreases and the slow-mode critical surface is shifted outward (i.e., it takes more time for the wind to be accelerated to a supersonic velocity.) When $\omega$ is increased, the three critical surfaces become more and more separated from each other ($r_s \ll r_A \ll r_t$). The decrease in $r_s$ is due to the centrifugal acceleration of the wind. A large separation between $r_A$ and $r_t$ is the result of the tightly wound magnetic field lines in the wind beyond the Alfvén surface. As the magnetic field becomes more and more toroidal, there arises a large difference between the poloidal Alfvén speed $V_{Ap}=B_p/\sqrt{(4\pi\rho)}$ and the fast-mode speed $V_{fp}$, which is roughly $B/\sqrt{(4\pi\rho)}$. Therefore the wind, after passing through the Alfvén surface, has to travel a longer distance to reach the fast-mode critical surface.

In addition, the increase in $\omega$ (and hence the build up of the toroidal magnetic field) leads to the collimation of the flow toward the $z$-axis. The opening of the flow channel around the $z$-axis is narrowed consequently, and this results in the reduced efficiency in the acceleration. The opposite situation takes place in the region far from the rotation axis. The enhanced toroidal magnetic pressure has the effect of widening the flow channel, so that the wind is accelerated more efficiently in such a region compared to the undisturbed, radial magnetic field configuration. This feedback is nonlocal in nature. For example, if the value of $\omega$ is increased only near the edge of the disk, the degree of collimation is enhanced as a whole and the wind velocity near the $z$-axis will be reduced.
5. Conclusion

We showed that the wind from a rotating magnetized disk will take the form of a collimated jet along the direction of the rotation axis. The basic principle is very simple (paper I). The poloidal and toroidal magnetic field components will decay as $B_p \sim r^{-2}$ and $B_t \sim r^{-1}$ for sufficiently large $r$, because of the magnetic flux conservation. Therefore, the field becomes increasingly toroidal far from the disk. On the other hand $B_p = 0$ on the $z$-axis. Consequently the magnetic pressure is directed toward the rotation axis. This is the ultimate driver of the collimation. A naive expectation that the field lines will be blown away from the rotation axis due to the centrifugal force does not hold. Such effects may be present near the disk, but outside the Alfvén radius the centrifugal force is no longer a dominant driver.

As was discussed in section 2.1, the similarity solution of Blandford and Payne (1982) shows less collimation than in the corresponding current-free configuration. The reason is as follows. The asymptotic behavior of their solution at large distances is such that

$$B_p \sim \frac{1}{\omega^{1-1/(4(1-\alpha_l))} z^{2\alpha_l/4(1-\alpha_l)}} ,$$

(21)

where $\alpha_l$ is a positive constant less than 1 [equation (2.30) of Blandford and Payne (1982)]. Namely $B_p$ decreases faster than $\omega^{-1}$ in the $\omega$-direction, so that the outward magnetic pressure gradient $-(1/8\pi) \partial B_p / \partial \omega$ is greater than the inward tension $B_t / (4\pi \omega)$. Therefore the toroidal magnetic force is directed away from the $z$-axis. On the contrary, $B_p$ in our model does not have a singularity on the $z$-axis. Therefore both the pressure and the tension of the toroidal magnetic field are directed inward, promoting the collimation of the flow.

The author thanks Drs. H. C. Spruit and R. Pudritz for discussions. This work was supported in part by the Scientific Research Fund of the Ministry of Education, Science, and Culture under Grant Nos. 60540150 and 61740137.

Appendix. Current-Free Magnetic Fields Threading a Disk

We will look for axisymmetric, current-free magnetic fields. The magnetic field $B$ can be written as

$$B_\omega = -\frac{\partial A}{\partial z} , \quad B_z = \frac{1}{\omega} \frac{\partial}{\partial \omega}(\omega A) . \quad (A1)$$

From $\nabla \times \mathbf{B} = 0$ $A$ is found to satisfy

$$0 = \frac{\partial B_z}{\partial \omega} - \frac{\partial B_\omega}{\partial z} = \frac{\partial^2 A}{\partial \omega^2} + \frac{1}{\omega} \frac{\partial A}{\partial \omega} - \frac{A}{\omega^2} + \frac{\partial^2 A}{\partial z^2} . \quad (A2)$$

Solutions to this equation will in general be written as

$$A = \int_0^\infty A_k J_1(k \omega) \exp(-kz) dk \quad (A3)$$

© Astronomical Society of Japan • Provided by the NASA Astrophysics Data System
Fig. A1. Potential field lines with a power-law distribution of the magnetic flux on the disk. The magnetic field strength on the disk varies as $\bar{\omega}^{-\mu}$. Since the shape of the field lines is self-similar in this model, only the field lines that start from $(\bar{\omega}=1, z=0)$ are shown. The dotted curve is the self-similar MHD solution by Blandford and Payne (1982), which also has a power-law flux distribution with $\mu=5/4$.

and the corresponding magnetic field components are

$$B_\bar{\omega} = \int_0^\infty k A_\bar{\omega} J_1(k\bar{\omega}) \exp(-kz)dk,$$  

(A4)

$$B_z = \int_0^\infty k A_\bar{\omega} J_1(k\bar{\omega}) \exp(-kz)dk.$$  

(A5)

In particular when $k A_\bar{\omega} = C \bar{\omega}^{-\mu}$ ($\mu>0$), we find

$$B_\bar{\omega} = \begin{cases} 
C \bar{\omega} \Gamma\left(\frac{\mu+1}{2}\right) F\left(\frac{\mu+1}{2}, \frac{\mu+2}{2}, 2, -\frac{\bar{\omega}^2}{z^\mu}\right), & z>0, \\
C \frac{2^{\mu-1}}{\bar{\omega}^\mu} \Gamma\left(\frac{\mu+1}{2}\right) / \Gamma\left(1-\frac{\mu-1}{2}\right), & z=0,
\end{cases}$$

(A6)

$$B_z = \begin{cases} 
C \frac{\Gamma\left(\frac{\mu}{2}\right)}{\bar{\omega}^\mu} F\left(\frac{\mu}{2}, \frac{\mu+1}{2}, 1, -\frac{\bar{\omega}^2}{z^\mu}\right), & z>0, \\
C \frac{2^{\mu-1}}{\bar{\omega}^\mu} \Gamma\left(\frac{\mu}{2}\right) / \Gamma\left(1-\frac{\mu}{2}\right), & z=0,
\end{cases}$$

(A7)

where $F$ is the hypergeometric function and $\Gamma'$ is the gamma function (Watson 1922). Field lines starting from the point $\bar{\omega}=1$ and $z=0$ are shown in figure A1. Solutions are self-similar so that the scale is arbitrary. Generally the field lines bend toward the $z$-axis, because the models have an infinite amount of magnetic flux towards $\bar{\omega} \rightarrow \infty$. Magnetic sources far from the $z$-axis therefore produce the inward-directed mag-
netic field component, which deflects the field lines inward. This bending of the field lines has nothing to do with the effect of toroidal magnetic fields. The case of $\mu=5/4$ has the same magnetic flux distribution as the model by Blandford and Payne (1982), and their solution is also shown in figure A1. Their MHD self-similar solution turns out to be less collimated compared to the current-free, untwisted, magnetic field.

References