ON THE THERMAL INSTABILITY OF GALACTIC AND CLUSTER HALOS

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ABSTRACT

We present a detailed study of thermal instabilities in cooling flows associated with galaxies and clusters of galaxies. In the case of purely radiation-driven accretion onto a central object such as the cD galaxy M87, we find that the gas is largely subject to overstability, rather than to monotonic instability. If thermal conductivity is taken into account, the flow is stabilized on scales of several kiloparsecs, even if the conductivity is appreciably reduced (e.g., \( \sim 1\% \)) with respect to the Spitzer value. In no case do we find global perturbations (i.e., perturbations with comparable radial and azimuthal dimensions) to be monotonically unstable. We present numerical solutions of the local dispersion relation for the cooling flow in M87 and discuss the possible consequences of our results for a correct understanding of cooling flows.

Subject headings: galaxies: clustering — galaxies: structure — instabilities

I. INTRODUCTION

Studies of hot gas observed in a number of galaxies and clusters of galaxies by, for example, the Einstein Observatory, have shown that radiative cooling can be important throughout much of the volume of this gas. In the simplest equilibrium model, in which heat sources and thermal conduction are neglected, such cooling may lead to significant subsonic inflow of gas toward the center of the system (see Silk 1976; Cowie and Binney 1977; Fabian and Nulsen 1977; Fabian, Nulsen, and Canizares 1984, hereafter FNC; and review by Sarazin 1986), with mass inflow rates ranging from a few \( M_\odot \, yr^{-1} \) to 400 \( M_\odot \, yr^{-1} \) (Stewart et al. 1984b), up to an exceptional case of 1000 \( M_\odot \, yr^{-1} \) (Fabian et al. 1985). In several cases, such models lead to the prediction that the gas must “drop out” of the flow over a wide range of radii (Stewart et al. 1984a, hereafter SCFN; Thomas et al. 1986), at a rate large enough to possibly form a large galaxy in less than a Hubble time (Fabian 1986).

The obvious question is how it is possible for some fraction of the inflowing gas to settle out at various radii. One suggestion (Nulsen 1986; FNC) is that thermal instability in the cooling flow is responsible for removing the gas from the flow itself. That is, in order to account for the fact that there is no direct observational evidence for the final fate of this gas throughout most of the volume of the halo (Fabian, Nulsen, and Canizares 1982; Fabian, Nulsen, and Arnaud 1986), it has been argued that a large number of very low mass stars may form in the particular environment of cooling flows (Sarazin and O’Connell 1983), thus also providing an explanation for the “dark matter halos” found in some galaxies such as M87 (Fabricant and Gorenstein 1983, hereafter FG). However, consideration of heating processes (see, for example, the review by Sarazin 1986) and of thermal conduction from the outer intergalactic medium (Rosner, Tucker and Najita 1987, hereafter RTN; Bertschinger and Meiksin 1986; Volkov 1985) show that the mass inflow required to fit the data can be significantly reduced. For example, if thermal conduction is not strongly suppressed in the presence of weak, tangled magnetic fields (RTN), it can play a significant role in stabilizing the cooling flow.

It is thus evident that a detailed study of the thermal stability of a cooling flow is in order. Several authors (Mathews and Bregman 1978; Cowie, Fabian, and Nulsen 1980; SCFN; Nulsen 1986) have discussed approximate stability criteria, which are based on a simple “bubble” picture of the fluid, and have pointed out the constraints that instability imposes on cooling flow models. In this paper, we address this problem from a more rigorous perspective by linearizing the full set of hydrodynamic equations for a gravitationally stratified fluid, deriving the appropriate local dispersion relation, and considering the nature of the unstable modes in the present context. White and Sarazin (1986) have recently derived a similar dispersion relation, but their study focused only on growth rates and did not consider the nature of the instabilities (which turns out to be a crucial point in this problem). We consider a number of cases which have appeared in the literature, including cooling flows with no heating and thermal conduction, and cooling flows with the Spitzer and a reduced thermal conductivity. As a specific example, we evaluate the solutions to this dispersion relation numerically for the well-studied cooling flow in M87 (FG; SCFN). Finally, we discuss the consequences of our results for cooling-flow model of hot gaseous halos in galaxies and in clusters of galaxies.

II. STABILITY ANALYSIS

We consider a standard spherically symmetric cooling-flow model, described by the time-independent hydrodynamic equations, including thermal conduction and radiative cooling. In the high-temperature region of interest, the flow is highly subsonic (e.g. \( v \ll c_s \equiv \text{sound speed} \); see Sarazin 1986), so that we shall consider the simplified set of single-fluid hydrodynamic equations for mass, momentum, and energy conservation:

\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 v \right) = \frac{1}{r^2} \frac{d}{dr} \left( \frac{\dot{m}}{4\pi} \right), \quad (1a)
\]

\[
\frac{dp}{dr} = -\frac{\rho}{T} \left( \frac{dT}{dr} + \frac{\mu d\phi}{R} \right), \quad (1b)
\]

\[
\frac{dF_\varepsilon}{dr} = -\frac{2F_\varepsilon}{r} - \left( \frac{\rho}{\mu n} \right)^2 \Lambda(T) + \frac{\dot{m}}{4\pi r^2} \left( \frac{2.5}{\mu} \frac{d\rho}{dr} + \frac{d\phi}{dr} \right). \quad (1c)
\]
where the thermal conductive heat flux $F_c$ is given by

$$F_c = -\kappa T^{5/2} \frac{dT}{dr},$$ (1d)

and where $\dot{m}$ is the mass accretion rate, $R$ is the gas constant, $\mu$ is the mean molecular weight, $\Lambda(T)$ represents the radiative cooling function $[\Lambda(T) \propto T^{1/2}$ for thermal bremsstrahlung], and $\phi$ is the gravitational potential of the galaxy onto which the gas is accreting. In our numerical example, we will consider the potential due to the mass distribution of M87, as given by model C in SCFN. The thermal conduction coefficient $\kappa$ will be written in the form $\kappa = \alpha \times 10^{-7}$, where $\alpha \leq 1$; the parameter $\alpha \leq 1$ accounts for effects which might reduce the conductivity with respect to the Spitzer formula (Spitzer 1962). This system of ordinary differential equations is easily integrated numerically, given appropriate boundary conditions (see RTN); note that if $\alpha$ is set to zero (no thermal conduction), then we have simply that $F_c = 0$, so that equation (1c) just gives the temperature gradient. The results. The roots of equation (4) are

$$\hat{n}^2 \pm \hat{n} \left[ \frac{\gamma - 1}{\gamma} (\omega_f + \kappa T^{5/2} \frac{dT}{dr}) + \frac{k^2}{k^2} \omega_{BV}^2 \right] = 0; \hspace{1cm} (4)$$

this simpler dispersion relation is essentially equivalent to that given by Defouw (1970) in an entirely different context. The roots of equation (4) are

$$\hat{n}_+ = -\frac{1}{2} \left( \frac{\gamma - 1}{\gamma} (\omega_f + \kappa T^{5/2} \frac{dT}{dr}) + \frac{k^2}{k^2} \omega_{BV}^2 \right)^{1/2}, \hspace{1cm} (5)$$

where $\omega_f \equiv \omega_f + \kappa T^{5/2} \frac{dT}{dr}$. In the absence of thermal conduction ($\omega_f = 0$), we thus immediately recover Field’s criterion for thermal instability (Field 1965), but for the more general situation of a stratified medium,

$$\omega_f + \kappa T^{5/2} \frac{dT}{dr} - \omega_{BV} < 0.$$ (5a)

In addition, the presence of a restoring force due to buoyancy can change the character of the instability from monotonically to overstable (Defouw 1970); this occurs if

$$\omega_{BV} > \frac{4}{\gamma} [(\gamma - 1)/\gamma]^2 (k^2/k^2) (\omega_f + \kappa T^{5/2} \frac{dT}{dr} - \omega_{BV})^2.$$ (5b)

If this latter inequality holds, the perturbed fluid will oscillate with exponentially growing amplitude about an equilibrium point which is itself comoving with the fluid. Neglect of this inequality can thus lead to an entirely erroneous interpretation of the physical nature of this instability. In fact, the instability in question is rather well known: it is the result of thermal instability of a strongly subadiabatic, stratified atmosphere (Defouw 1970). As shown in Figure 1 (and as just mentioned), the growth rate for this mode is somewhat smaller than the local buoyancy frequency; hence, the buoyancy-restoring force dominates the motion of a local disturbance. That is, if we adiabatically displace a “test bubble” radially outward, it will
become more dense than its immediate surroundings and be antibuoyant; hence, it will drop back down and oscillate about its initial starting point (since the background atmosphere is highly subadiabatic). If the bubble is instead thermally unstable (as is the case here), then the bubble will become slightly more dense than it would have been had it been adiabatic because the instability leads to cooling at constant pressure; hence, it will be yet more antibuoyant and, when it returns to its equilibrium position, will overshoot with more kinetic energy than it had to start with—overstability.

In the presence of thermal conduction, the instability criterion becomes

\[ \omega_c < \omega_p - \omega_T , \]  

(6b)

we see that the thermal conduction stabilizes perturbations whose wavelengths are smaller than the critical wavelength

\[ \lambda_{cr} = \left[ \frac{1}{\lambda_c} \left( \frac{1}{\lambda_p} - 1 \right) \right]^{1/2} , \]  

(7)

where

\[ \lambda_{p,T} \equiv 2 \pi c_s / \omega_{p,T} , \quad \lambda_c \equiv 2 \pi c T_0^{1/2} / c_s p_0 . \]

Numerically,

\[ \lambda_{cr} \approx 24.4 \left( \frac{T}{2 \times 10^7 \text{ K}} \right)^{3/2} \left( \frac{N}{10^{-3} \text{ cm}^{-3}} \right)^{-1} \left( \frac{\alpha}{10^{-2}} \right)^{1/2} \text{ kpc} . \]  

(8)

III. RESULTS AND DISCUSSION

We have applied the above results to the hot gas surrounding M87, which is probably the best-observed cooling flow to date. We adopted the model for M87's mass distribution given as model C by SCFN, and integrated equations (1) with appropriate boundary conditions to fit the observational data of SCFN in the region 4-100 kpc. We focused attention on two examples: model A, with no thermal conduction and with mass inflow \( m = 3(r/10 \text{ kpc})^{3/2} M_\odot \text{ yr}^{-1} \) (SCFN), and model B, with \( \alpha = 0.01 \) and constant \( m = 0.1 M_\odot \text{ yr}^{-1} \) (we note that in presence of thermal conduction, the final results are fairly insensitive to the accretion rate).

The first point to make is that for both models, thermal instabilities are of the overstabil form throughout the flow; this is seen in Figure 1, where we plot the Brunt–Väisälä frequency and the thermal instability growth rate \( \omega_{\text{Th}} = \omega_T - \omega_p \) versus radius for model A; for model B we obtain essentially the same curves. Thus, the picture of the thermal instability suggested by Nulsen (1986) and others, in which thermally unstable “bubbles” (with \( k_t \) of order \( k_r \)) separate from the flow and fall inward, is incorrect. Instead, if we ignore thermal conduction, then “bubble”-like perturbations will oscillate about their starting points and will be advected inward, with an amplitude whose growth time scale is comparable to the advection time scale to the center of the halo gas. However, as White and Sarazin (1986) points out, pancake-like radial modes (with \( k_t \ll k_r \)) have slightly larger growth rates (by a factor of ~2), and from Figure 1, we see that for sufficiently small \( k_t / k_r \), these modes will be monotonic. Since these modes also roughly comove with the fluid, their advection time scale is also of order of their growth time scale; furthermore, to the extent that such pancake-like perturbations do move relative to the fluid (i.e., radially inward and hence perpendicular to the plane of the “pancake”), one would expect these “pancakes” to fragment, leading again to “bubbles” with \( k_t \approx k_r \).

The solutions to dispersion relation (3) are given in Figure 2; we plot the real and imaginary parts of the unstable (isobaric) solutions (the other solutions correspond to damped sound waves and are not of interest) versus the total wavelength \( \lambda_{\text{tot}} \).
[where \( k_r = k_t = 2\pi(2)^{1/2}/\lambda_{ct} \)]; the values of the required physical quantities are obtained from model B above at a radius of 4 kpc. Note that perturbations whose wavelength is smaller than \( \lambda_{ct} \approx 3 \) kpc (at \( r = 4 \) kpc) are damped: perturbations whose wavelengths are smaller than this value have negative real part and are hence stable, whereas perturbations whose wavelength is larger than this value have positive real part, so that overstable oscillations set in. Since \( \lambda_{ct} \) is comparable to \( r \) at \( r = 4 \) kpc, we in fact have the result that all perturbations are damped on wavelengths small enough to be consistent with the local dispersion relation (3). Since \( \lambda_{ct} \) is a rather rapidly increasing function of radius (see eq. [8]), we can conclude that this is true for all radii \( r \geq 4 \) kpc (see, e.g., Fig. 3).

In summary, we have found that as long as thermal conduction is neglected, isobaric perturbations are unstable, and that since the buoyancy frequency is much larger than the growth rate of the perturbations themselves, the instability is in fact "overstable" for most values of \( k_t/k_r \). This result does not accord with the calculations of Nulsen (1986), in which "blobs" fragment after becoming thermally unstable (thereby limiting the spatial scale of possible thermal instabilities); neither do our results conform with those of Cowie, Fabian, and Nulsen (1980), who considered the evolution of thermally unstable "bubbles" with finite amplitude initial density contrasts. That is, even if a fully nonlinear treatment verifies the validity of, for example, the Cowie et al. results, our calculations show that once the initial generation of finite-amplitude density perturbations has "condensed out" from the flow, it is essentially impossible to produce any further such "bubbles." Instead, thermally unstable blobs (with \( k_t \approx k_r \)) oscillate with initially exponentially growing amplitude, while their equilibrium position sinks into the galaxy or cluster potential well in concert with the mean flow. The dominant process for nonlinear saturation of this instability remains unclear; in other contexts, the instability typically saturates because the increasing gradients in, for example, temperature, eventually inhibit further growth. Fragmentation does not seem to be a likely saturation process in this case because it does not directly affect the basic competition between thermal instability and the buoyancy restoring force, which determines the evolution of the instability. In the case of M87, the growth rate of the perturbations is comparable to the time for the equilibrium position to reach the center, at least in the linear regime we are considering; hence we tentatively conclude that thermal instabilities are not effective in removing the gas from the flow, except possibly close to the center of M87, even if one entirely ignores thermal conduction. Finally, we note that when heat conduction is taken into account in modeling cooling flows,
the formation of cold fragments becomes yet more problematic. In particular, thermal conduction severely restricts the spatial scales of possibly thermally unstable gas “blobs.”

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