CURRENT-DRIVEN MAGNETOHYDRODYNAMIC THERMAL INSTABILITIES
IN SHEARED FIELDS

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ABSTRACT

We study thermal instabilities in a magnetized dissipative medium. The equilibrium structure for the magnetic field is assumed to be a planar force-free solution, with uniform temperature and density. We obtain an algebraic dispersion relation that can be solved numerically. Furthermore, we derive approximate analytical solutions to determine necessary conditions for instability. We discuss our results and compare them with previous calculations which neglected the effect of shearing of the magnetic field. The maximum growth rate \( \Re (\tilde{\eta}) \) and the typical scale lengths \( \lambda \) of the instability for a "precoronal" atmosphere and for the lower transition region of the Sun are found to be, respectively, \( \Re (\tilde{\eta}) \approx 10^{-5}-10^{-4} \) s\(^{-1} \); \( \lambda \approx 10^{3}-10^{5} \) cm; and \( \Re (\tilde{\eta}) \approx 1-10^{2} \) s\(^{-1} \), \( \lambda \approx 10^{2}-10^{3} \) cm.

Subject headings: hydromagnetics — instabilities — Sun: corona

I. INTRODUCTION

Magnetic structuring is recognized to have an important influence on the energetics of the solar outer atmosphere. Dissipation processes considered for coronal heating, such as tearing modes (Galeev et al. 1981) or Joule heating with anomalous resistivity (Rosner et al. 1978; Hinata 1982; Benford 1983), require inhomogeneities on small scales; recently Rabin and Moore (1984) have discussed how the energetic balance of the transition region may also be understood by considering the dissipation of currents in small filaments. A viable mechanism for the creation of strong, fibril inhomogeneities in astrophysical plasmas has been proposed by Heyvaerts (1974): a current flowing along a magnetic field becomes unstable, for an appropriate variation of the resistivity with temperature, to modes which concentrate currents in narrow filaments along the magnetic field. This instability has been successively studied by Chiuderi and Van Hoven (1979) for a hot sheared infinite unstratified medium, by Ferrari, Rosner, and Vaiana (1982) for a cool infinite homogeneous medium and by Bodo et al. (1985) and Massaglia et al. (1985) for finite and gravitationally stratified cool structures. These latter authors investigated how a hot corona can be created by starting from a "precoronal" initial state at low temperature and density. These currents channeled in filaments may be directly responsible for heating the lower transition region (Rabin and Moore 1984) and may also favor more efficient heating processes in the coronal region. In addition, such overheating instabilities have been considered as a triggering mechanism for tearing modes in solar flares (Coppi and Friedland 1971; Spicer 1977).

The original approach by Heyvaerts (1974), which was also followed by succeeding authors (Ferrari, Rosner, and Vaiana 1982; Bodo et al. 1985), requires a reexamination from two points of view. First, the assumption of a uniform background magnetic field is, in principle, inconsistent with the presence of a current flowing along it (cf. Chiuderi and Van Hoven 1979). Second, the limitation to very small values of \( x \approx 4\pi J_0/cB_0 \), which is used for simplifying the dispersion relation, must be rediscussed. In this paper we carry out a stability analysis using a self-consistent sheared equilibrium structure for the magnetic field; we show that the shearing introduces an overstable model at small wavenumbers. In addition, we are able to clarify the ordering implicit in the limit considered by Heyvaerts and, therefore, we derive the range of validity of his limit. In particular, we find that the results obtained by Heyvaerts are not valid for large wavenumbers in the direction transverse to the magnetic field, so that the condition for instability has to be modified (Massaglia et al. 1985).

The organization of the paper is as follows: in § II we study approximate solutions of the dispersion relation, together with the conditions for instability, and discuss the range of validity of these approximations; the numerical solutions of the general dispersion relation are discussed in § III. Our results are summarized and discussed in § IV.

II. STABILITY ANALYSIS

We analyze the stability of a uniform, isothermal medium with a planar, force-free magnetic field against current-driven filamentation modes. The magnetic field in the equilibrium state must therefore fulfill the equation

\[
\mathbf{V} \times \mathbf{B}_0 = \alpha \mathbf{B}_0, \tag{1}
\]

whose solution, for \( \alpha = \) constant, is given in Cartesian coordinates \((x, y, z)\) by

\[
\mathbf{B}_0 = (0, B_0 \sin \alpha x, B_0 \cos \alpha x) ;
\]

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\( \alpha^{-1} \) therefore represents the scale length of the magnetic field shear. The condition for energetic equilibrium can be written as:

\[
\frac{J_0^2}{\sigma_0} = E_R - H,
\]

where \( \sigma_0 \) is the classical (Spitzer) electrical conductivity, \( E_R \) represents the optically thin radiative losses per unit volume, assumed proportional to \( \rho^x T^3 \), and \( H \) is a general heating term (viscous dissipation of acoustic waves, radiative heating, etc.; usually \( \eta = 2 \)). We set \( \nabla \cdot F = 0 \), where \( F \) is the conductive flux, because the background plasma is assumed isothermal.

Before going into the details of the stability analysis, it is useful to briefly consider the processes leading to thermal instability under various physical conditions.

1. Isochoric perturbations.—The thermal stability of a medium in the absence of magnetic fields was first studied by Parker (1953). He considered the energy equation only and found instability when radiative losses decrease while the temperature increases, i.e., when \( \delta < 0 \). This is valid for perturbations that do not affect the density (incompressible perturbations).

2. Isobaric perturbations.—If we instead consider the complete set of fluid equations (Field 1965), we can see that, since the radiative time scale is much longer than any of the dynamical time scales, perturbations will be almost isobaric. We must therefore take into account the effect of density variations on radiative losses; thus the criterion for instability becomes \( \delta > \eta \).

3. MHD case.—The inclusion of frozen-in magnetic fields changes this picture considerably because in this case perturbations with \( k \perp B \) will keep the total (magnetic plus gas) pressure constant. Furthermore, because a density change results in a magnetic pressure build-up, density perturbations will be inhibited (see Massaglia et al. 1985), with consequences for thermal stability.

4. Resistive effects.—The stability analysis can be made yet more realistic by including resistive effects, which allow magnetic field diffusion and further contribute to the energetic balance via Ohmic dissipation. For small spatial scales, where magnetic diffusion is important on radiative time scales, instability is favored because density variations are not inhibited by magnetic pressure; perturbations are again isobaric.

5. Current-driven modes.—Joule heating by currents flowing along the magnetic field constitutes a new source of instability (Joule mode; Heyvaerts 1974) if \( \sigma \) increases with temperature. In this case (Bodo et al. 1985), a temperature increase leads to an increase of the electrical conductivity, which enhances the Ohmic heating and leads to a further temperature increase. This effect will be discussed in this paper.

Let us then linearize the standard MHD conservation equations for mass, momentum, and energy, together with Maxwell's equations and Ohm's law:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + \nabla \mathbf{F} = \nabla p,
\]

\[
\frac{\partial p}{\partial t} = \frac{\gamma - 1}{\gamma} \left[ -E_R + \frac{J^2}{\sigma} + H + \nabla \cdot (\hat{k} \cdot \nabla \mathbf{T}) \right],
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \frac{4 \pi J}{c},
\]

\[
\frac{\partial \mathbf{E}}{\partial t} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}.
\]

where \( \hat{k} \) is the thermal conductivity tensor and the remaining quantities have their customary meaning. We consider perturbations varying in the \( x \)-direction only: these are the most unstable because the damping effect due to thermal conduction along the magnetic field is turned off by virtue of assuming an isothermal initial state. In this case, the perturbed magnetic field has zero component along the \( x \) direction; we can therefore write the linearized system in terms of the components of the perturbed field parallel and perpendicular to \( B_0 \). We then obtain a system with constant coefficients that can be Fourier-analyzed in terms of perturbations \( \alpha \exp (\mathbf{i} t - \mathbf{i} \mathbf{k} \cdot \mathbf{r}) \), where \( \mathbf{i} \) is, in general, complex; modes are unstable when \( \text{Re}(\mathbf{i} \mathbf{k} \cdot \mathbf{r}) > 0 \). As far as the energy equation (2) is concerned, we perturb the Ohmic and the radiative terms but consider \( H \) independent of both \( T \) and \( p \).

In the following, we will denote the equilibrium and perturbed quantities by the subscripts "0" and "1," respectively, and the components of the perturbed magnetic field parallel and perpendicular to \( B_0 \) [in the plane \( (y, z) \)] by \( b_1 \) and \( b_2 \), respectively. Furthermore, in order to nondimensionalize our equations, we define:

\[
\beta = \frac{8 \pi \rho_0}{B_0^2}, \quad S = \frac{4 \pi \sigma_0 v_A}{c^2 \alpha}, \quad P_m = \frac{4 \pi \sigma_0 T_0 k_\perp}{p_0 c^2}, \quad \Sigma_0 = \frac{\partial \ln \sigma}{\partial \ln T},
\]

\[
K = \frac{k}{\alpha}, \quad \omega_R = \frac{E_R}{\rho_0}, \quad n = \frac{\tilde{\eta}}{\omega_R}, \quad \omega = \frac{J_0^2}{\sigma_0 p_0}, \quad \chi = \frac{J_0^2}{E_R} \frac{\omega_1}{\omega_R}.
\]

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where \( v_A \) is the Alfvén speed. The parameter \( S \) is the magnetic Reynolds number, which is always extremely large under astrophysical conditions; \( P_m \) is the magnetic Prandtl number, i.e., the ratio between the thermal and magnetic diffusivities; and \( K \) is the ratio of the magnetic field shear scale length to the perturbation wavelength. We note that \( 0 \leq \chi \leq 1 \); in particular, when \( \chi = 1 \) (i.e., \( H = 0 \)) radiative losses are exactly balanced by Ohmic heating.

We thus obtain the following linearized system:

\[
\begin{align*}
(\gamma - 1)(n - \eta) \frac{\rho_1}{\rho_0} + \left[ -n + (\gamma - 1) \left( -\delta - \frac{1}{2} \beta P_m K^2 - 2\chi\Sigma_0 \right) \right] \frac{T_1}{T_0} + 2\chi(\gamma - 1)b_{\parallel} + 2i\chi K(\gamma - 1)b_{\perp} &= 0, \\
-n + \chi\beta P_m K^2 + \frac{1}{2} \chi\beta K^2 b_{\parallel} + \left[ n + \frac{1}{2} \chi\beta K^2 + 1 \right] b_{\parallel} - i2\chi K b_{\perp} &= 0, \\
i \frac{n}{K} \frac{\rho_1}{\rho_0} + \frac{1}{2} \beta K\chi \Sigma_0 \frac{T_1}{T_0} - i2\chi K b_{\parallel} &+ \left[ n + \frac{1}{2} \chi\beta K^2 + 1 \right] b_{\perp} = 0.
\end{align*}
\]

In the limit that the diffusion time of the magnetic field is much smaller than the instability time scale, which corresponds to \( K^2 \beta \gg 1 \), system (4) can be reduced to

\[
\begin{align*}
(8n^2 + 4\chi^2 S^2 \beta^3 - 4\chi^2 S^2 \beta n) \frac{\rho_1}{\rho_0} + \chi^2 S^2 \beta^3 \frac{T_1}{T_0} &= 0, \\
(\gamma - 1)(n - \eta) \frac{\rho_1}{\rho_0} + \left[ -n + (\gamma - 1) \left( -\delta - \frac{1}{2} \beta P_m K^2 + 2\chi\Sigma_0 \right) \right] \frac{T_1}{T_0} &= 0.
\end{align*}
\]

From equation (5b) we can see that if the density perturbation is negligible, we obtain

\[
\text{Re} (n) = (\gamma - 1)(-\delta - \frac{1}{2} \beta P_m K^2 + 2\chi\Sigma_0), \quad \text{Im} (n) = 0
\]

which is the dispersion relation obtained by Heyvaerts (1974). From equation (5a) we see that this last relation is consistent only if \( K^2 \beta \ll 1 \), but because \( K^2 \beta \gg 1 \), we must require \( S^2 \beta^3 \ll 1 \). This latter condition is hardly ever fulfilled under typical astrophysical situations because \( S \) is always extremely large (see Table 1). Obviously, equation (6) is also not valid for \( K \to \infty \), i.e., for very small perturbation wavelengths. We note that the quantity \( K^2 S^2 \beta^3 \) can be rewritten in terms of the force-free field parameter \( \alpha \),

\[
K^2 S^2 \beta^3 = \frac{k^2 v_A^2 c^2}{4n\sigma_o} \alpha^2.
\]

Thus the condition \( \alpha \to 0 \) of Heyvaerts (1974) is necessary, but not sufficient, for the above approximation (6) to be valid. In order to study the dependence of the validity of approximation (6) on the parameters \( S \) and \( \beta \), and on the wavenumber \( K \), we have solved the full dispersion relation (4). A full analysis is presented below in § III; here we only note that for any fixed value of the product \( S^2 \beta^3 \) (but with \( S^2 \beta^3 \ll 1 \)), there exists a wavenumber range in which Heyvaert's approximation, and hence relation (6), is roughly satisfied. For example, in Figure 1 we show the solution to the system (4) for \( S^2 \beta^3 = 10^{-11} \); in this case, \( \text{Re} (n) \) is reasonably well approximated by relation (6) in the range \( 10^3 < K < 10^4 \).

On the other hand, if we introduce the opposite condition \( K^2 S^2 \beta^3 \gg 1 \), and if this is the largest quantity in equation (5a), we find

\[
\frac{n}{K} \frac{\rho_1}{\rho_0} = - \frac{T_1}{T_0},
\]

so that the perturbations are isobaric. This conclusion can also be reached by noting that the instability time is much longer than the Alfvén time, on which pressure equilibrium is restored. In this isobaric approximation, the dispersion relation obtained from

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|c|}
\hline
\textbf{Location} & \textbf{\( \beta^\ast \)} & \textbf{\( S^\beta \)} & \textbf{\( P_m \)} \\
\hline
Corona & \( 10^{-5} - 10^{-1} \) & \( 10^3/\alpha \) & \( 10^{-3} \) \\
Chromospheric network & \( 10^{-5} - 10^{-1} \) & \( 10^3/\alpha \) & \( 10^{-1} \) \\
Photospheric flux tube & \( 0.1 - 11 \) & \( 10^3/\alpha \) & \( 10^{-3} \) \\
Lower transition region & \( 10^{-2} \) & \( 10^3/\alpha \) & \( 10^{-2} \) \\
Precoronal atmosphere & \( 10^{-8} - 10^{-4} \) & \( 10^3/\alpha \) & \( 10^{-5} \) \\
\hline
\end{tabular}
\caption{Typical Plasma Parameters}
\end{table}

\begin{itemize}
\item \( \beta^\ast \approx \beta_{\text{gas}}/P_{\text{mag}} \)
\item Magnetic Reynolds number.
\item Magnetic Prandtl number.
\end{itemize}

\begin{itemize}
\item \( T \approx 10^4 \) K, \( \rho \approx 10^{-14} \) g cm\(^{-3} \).
\end{itemize}
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Fig. 1.—Growth rate Re \((n)\) vs. wavenumber \(K\) for \(S^2\beta^2 = 10^{-10}, P_m = 10^{-6}, \delta = 1,\) and with Joule heating exactly balancing the radiative losses \((\chi = 1)\). The condition \(K^2S^2\beta^2 \ll 1\) is fulfilled in the range \(10^3 \leq K \leq 10^6\). Note that the growth rate is scaled by the radiative cooling time scale, and the wavenumber is scaled by the shear length \(a^{-1}\) of the magnetic field.

The equations (5a)-(5b) read

\[
\text{Re} (n) = \frac{\gamma - 1}{\gamma} \left( \eta - \delta - \frac{1}{2} \chi \beta P_m K^2 + \chi \Sigma_0 \right), \quad \text{Im} (n) = 0. \quad (7)
\]

If we recall from equation (5a) that the isobaric constraints requires \(K^2\beta^2 \gg 1\), then we see from equation (7) that we can find a range of wavenumbers \(K\) for which thermal conduction can be neglected as long as

\[P_m \ll \beta.\]

In this case the growth rate attains the maximum value

\[
\text{Re} (n) = \frac{\gamma - 1}{\gamma} (\eta - \delta + \chi \Sigma_0). \quad (8)
\]

III. NUMERICAL RESULTS

Nontrivial solutions for the general system (4) are obtained when its determinant is zero; this condition leads to a fifth-order algebraic dispersion relation:

\[a_0 n^5 + a_1 n^4 + a_2 n^3 + a_3 n^2 + a_4 n + a_5 = 0, \quad (9)\]

whose coefficients \(a_i\) are given in the Appendix.

Before discussing the full dispersion relation, it is useful to show which typical values the parameters \(S, P_m,\) and \(\beta\) assume in different regions of solar and stellar atmospheres. These values are summarized in Table 1. Figure 2 shows \(\text{Re} (n)\) as a function of wavenumber \(K\) for different values of \(\beta\), with \(\chi = 1\). The general behavior of the instability is similar to that discussed in Bodo et al. (1985): for large \(K\)'s the instability is stabilized by thermal conduction and for small \(K\)'s is affected by the small diffusion rate of the magnetic field. The importance of these two effects can be appreciated by comparing the typical time scales associated with these two effects to the instability time scale, which is of order of \(\omega_j^{-1}\). The ratio of the diffusion rate to \(\omega_j\) is \(\sim \beta P_m K^2\), while the ratio of the conduction rate to \(\omega_j\) is \(\sim \beta P_m K^2\). It is now obvious why the wavenumber domain in which the instability is important shifts to smaller wavenumbers with increasing \(\beta\) in Figure 2. The value of \(K\) for which the growth rate decreases (for both small and large wavenumbers) scales as the inverse square root of \(\beta\).

In Figure 2 we can see that for values of \(K \approx O(1)\), the behavior of the solutions changes abruptly (dashed part). In this region we have “overstable” modes \([\text{Im} (n) \neq 0]\); these modes appear in the wavenumber range where the sheared structure of the field is important, and therefore cannot be found by an analysis that neglects the field shear. In order to understand the origin of these modes, we approximate the dispersion relation (9), in the range of \(K\)'s of interest, by assuming \(S\beta \gg 1\) and \(\beta \ll 1\), and by looking for

\[\frac{\gamma - 1}{\gamma} (\eta - \delta + \chi \Sigma_0). \quad (8)\]
Fig. 2.—Growth rate Re (n) vs. wavenumber K as a function of the plasma β, for $S = 10^{5}$, $P_m = 10^{-5}$, $\delta = 1$, $\gamma = 1$; $\beta = 10^{-1}$ (curve a), $\beta = 10^{-2}$ (curve b), and $\beta = 10^{-3}$ (curve c). The dashed part of the curves at small wavenumbers represents the overstable region. We see that the short wavelength cutoff scales like $\beta^{-1/2}$.

From equation (10) we can easily find the necessary and sufficient condition that there be overstability in some wavelength interval near $K \sim 1$, namely, that $\chi \Sigma_0 + 3\delta \geq \frac{1}{2}$. The physical reality of this mode is open to some question, however, since the background field evolves in this case on the time scale of the instability [i.e., Re (n) $\approx \beta$, so that the inverse growth rate is of order of the diffusion time scale for the background], contrary to the basic time scale assumption inherent in our stability analysis. We therefore do not pursue this point any further here. This mode will be discussed in greater detail by Bodo and Rosner (1986).

Figure 3 shows the growth rate as a function of wavenumber for different values of the magnetic Prandtl number $P_m$, again with $\chi = 1$. Curve a is for $P_m \ll \beta$; we see that the maximum value attained by the growth rate is that given by equation (8). From curves b and c it is evident that for increasing $P_m$, the maximum value of the growth rate decreases, and stabilization occurs for smaller wavenumbers. The points of marginal stability can be obtained from the condition $\delta_{\Sigma} = 0$ (see Appendix) which, in the limit $\beta \ll 1$, reduces to the quadratic equation

$$\chi^2 P_m K^4 + 2\beta(\delta - \chi \Sigma_0)K^2 + 4\eta \Sigma_0 = 0.$$  

The marginal stability points are thus given by

$$K^2 = \frac{1}{\chi P_m \beta} \left\{ (-\delta + \chi \Sigma_0 + \eta) \pm \left[ (-\delta + \chi \Sigma_0 + \eta)^2 - 4\chi \eta \Sigma_0 P_m \right]^{1/2} \right\}.$$  

These solutions are real if

$$-\delta + \chi \Sigma_0 + \eta > 0$$

and if the discriminant of equation (11) is positive definite; these constraints lead to the condition

$$P_m < \frac{(\eta - \delta + \chi \Sigma_0)^2}{4\eta \chi \Sigma_0}.$$  

The last two conditions can be in fact regarded as general conditions for instability (cf. eq. [8]).

The behavior of the growth rate versus wavenumber for different values of the ratio of Joule heating to radiative cooling $\chi$ is shown in Figure 4. We can see that even when Ohmic heating does not balance the radiative losses, we still have instability (but with reduced maximum growth rate).
Fig. 3.—Growth rate $\text{Re}(n)$ vs. wavenumber $K$ as a function of the magnetic Prandtl number $P_m$ for $S = 10^5, \beta = 10^{-3}, \delta = 2, \chi = 1; P_m = 10^{-3}$ (curve a), $P_m = 10^{-4}$ (curve b), and $P_m = 10^{-3}$ (curve c). The maximum growth rate in curve a, for which $P_m \ll \beta$, is given by relation (8).

Fig. 4.—Growth rate $\text{Re}(n)$ vs. wavenumber $K$ as a function of the ratio of Joule heating to radiative cooling, $\chi$, for $S = 10^5, \beta = 10^{-3}, P_m = 10^{-3}, \delta = 2; \chi = 1$ (curve a), $\chi = 0.5$ (curve b), and $\chi = 0.1$ (curve c).
IV. CONCLUSIONS

We have examined the stability of a magnetized medium to current-driven modes, investigating in detail approximate analytical solutions of the dispersion relation and the appropriate ranges of validity. We have also solved numerically the full problem, and analyzed the effects of the various plasma parameters on the instability.

While the general behavior of the instability is not strongly affected by the presence of sheared magnetic fields (see, e.g., Bodo et al. 1985), our analysis has allowed us to consistently examine the stability problem in the small wavenumber regime, where the shear length is comparable to the mode wavelength. Moreover, the use of a self-consistent equilibrium state has clarified the validity of some approximations found in the literature, showing how the dispersion relation and the stability criterion have to be modified. In particular, our stability criterion shows that instability occurs if radiative losses depend strongly on temperature.

Thermal instabilities in the presence of a magnetic field naturally lead to the formation of filamented structures. This filamentation is the consequence of two combined effects: the first is the anisotropy of thermal conduction in the presence of a magnetic field, the second is the lifting of the constant total pressure constraint by magnetic diffusion, which allows the formation of large density perturbations. The latter effect ensures that the modes with large wavenumber are the most unstable, and—together with efficient longitudinal thermal conduction—that they have a filamented structure aligned with the magnetic field, while transverse thermal conduction places an upper bound on the transverse wavenumber of the unstable modes. The domain of instability is thus determined by the competition between thermal and magnetic diffusivities; and it is the ratio between the two, expressed by the Prandtl number $P_m$, that fixes the region of unstable transverse wavenumbers.

Ohmic dissipation of filamentary electric currents, flowing along an ambient magnetic field, has been invoked for heating the solar lower transition region (Rabin and Moore 1984) and for creating coronae in stars starting from cool, low-density "precoronal" atmospheres (see Ferrari, Rosner, and Vaiana 1982). In both cases the creation of a highly structured state is required to favor magnetic-related heating processes, and it can be achieved via current-driven thermal instabilities. We thus apply our results to these two particular cases.

We assume the magnetic field $B_0 < 10^2$ G in both cases; for a "precoronal" atmosphere appropriate values are (Bodo et al. 1985) $T < 10^4$ K, $\rho \approx 10^{-14}$ g cm$^{-3}$ and the current density required to balance the radiative losses results $J_0 \approx 10^4$ esu; for the lower transition region (Rabin and Moore 1984) we have $10^4 < T < 2 \times 10^5$ K, $\rho T \approx 10^{-5}$ g cm$^{-3}$ K and $J_0 \approx 10^2$-10$^3$ esu. The corresponding values of the parameters $\beta$, $P_m$, and $S$ relevant for the model, are listed in Table 1. The values for the solar corona and the chromospheric network and for a photospheric flux tube are given as a comparison. The results obtained from the calculations are shown in Figure 5. For a "precorona" (solid line) the maximum growth rate is $\Re (\alpha) \approx 0.4$, i.e., $\Re (\alpha) \approx 10^2 - 10^3$ s$^{-1}$ in physical units, and it is attained at $10^3 < K < 10^5$, corresponding to wavelengths $10^3 \lesssim \lambda \lesssim 10^4$ cm; for the lower transition region (dashed line) the maximum growth rate is $\Re (\alpha) \approx 0.2$, i.e., $\Re (\alpha) \approx 1 - 10$ s$^{-1}$, and occurs at $K \approx 10^2$, corresponding to $\lambda \approx 10^2 - 10^3$ cm.

The energy balance in the lower transition region between Joule heating and losses, due to both radiation and thermal conduction across the magnetic field, requires individual current filaments of thickness larger than $\sim 1$ cm and smaller than $\sim 1$ km; it is therefore encouraging that the above linear analysis leads to instability in the right range of wavelengths. Obviously, a study of

![Figure 5](image-url)
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The nonlinear evolution of the instability considered here is required in order to follow the onset of these processes; we are therefore planning to carry out this nonlinear analysis in a forthcoming paper.

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APPENDIX

We define

\[ \Delta_0 = (\gamma - 1)(\delta + \frac{1}{2} \chi \beta P_m K^2 + \chi \Sigma_0), \quad \Delta_1 = (\gamma - 1)(\delta + \frac{1}{2} \chi \beta P_m K^2 + \chi \Sigma_0 - \eta). \]

The coefficients of the algebraic dispersion relation (9) are

\[ a_0 = 1, \quad a_1 = \Delta_0 + \chi \beta (K^2 + 1), \]

\[ a_2 = \chi^2 \frac{\beta^2}{4} (K^2 - 1)^2 + \chi \beta (K^2 + 1)[\Delta_0 - (\gamma - 1)\chi \Sigma_0] + \chi^2 K^2 S^2 \frac{\beta^2}{4} \left( \frac{\gamma}{2} + 1 \right), \]

\[ a_3 = \chi^2 \frac{\beta^3}{4} (K^2 - 1)^2 [\Delta_0 - 2(\gamma - 1)\chi \Sigma_0] + \chi^2 K^2 S^2 \frac{\beta^3}{8} \left[ \Delta_1 + \gamma \chi \beta (K^2 + 1) + (\gamma - 1)\chi (4 + \Sigma_0) + \chi (K^2 - 1) + \frac{2\Delta_0}{\beta} \right], \]

\[ a_4 = -\chi^2 K^2 S^2 \frac{\beta^4}{4} \left[ -\frac{\beta}{2} (K^2 + 1)\Delta_1 - \gamma \chi \frac{\beta^2}{8} (K^2 - 1)^2 \right. \]

\[ - \frac{1}{2} (K^2 - 1)\Delta_0 + \frac{\beta}{4} \chi \Sigma_0 (\gamma - 1)(3K^2 + 1) + \frac{\eta}{2} \Sigma_0 (\gamma - 1) + \chi \Sigma_0 (\gamma - 1)(K^2 - 1) \left], \right. \]

\[ a_5 = \chi^2 K^2 S^2 \frac{\beta^5}{8} (K^2 - 1) \left( \frac{\beta}{4} (K^2 - 1)[\Delta_1 - 2(\gamma - 1)\chi \Sigma_0] + (\gamma - 1) \frac{\eta}{2} \Sigma_0 \right). \]

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