RESONANCE ABSORPTION OF MAGNETOHYDRODYNAMIC SURFACE WAVES:
PHYSICAL DISCUSSION

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ABSTRACT

We show how the phenomenon of incompressible magnetohydrodynamic surface wave resonance absorption can be described in simple terms, both physically and mathematically, by applying the "thin flux tube equations" to the finite-thickness transition layer which supports the surface wave. The thin flux tubes support slow-mode waves which are driven by fluctuations in the total pressure, \( \delta p_{\text{tot}} \), which exist in virtue of the presence of the surface wave. We show how the equations for the slow-mode waves can be reduced to a simple equation, equivalent to a driven harmonic oscillator. Certain field lines within the transition layer are equivalent to a harmonic oscillator driven at resonance. Neighboring field lines are effectively driven at resonance as long as the driven oscillator and the corresponding normal mode do not get out of phase during the lifetime of the wave. There thus develops a layer (the "energy-containing layer") which secularly extracts energy from the surface wave. The energy-containing layer is much thinner than the transition layer. We have thus succeeded in showing how resonance absorption can be reduced to a simple problem in classical mechanics. One important virtue of this approach is that it becomes possible to treat more general problems than in the purely mathematical analyses which have been heretofore available. By more clearly revealing the physics of resonance absorption, our analysis indicates that nonlinear effects may destroy the resonance which is crucial to the whole effect. Numerical estimates indicate that nonlinear effects may be important in the corona, but a detailed nonlinear treatment of compressible surface waves appears to be called for.

Subject heading: hydromagnetics

I. INTRODUCTION

The possibility that magnetohydrodynamic (MHD) surface waves may decay through a process called "resonance absorption" was first discussed by Ionson (1978) in the context of the solar atmosphere. He pointed out that resonance absorption can occur when the surface waves are supported not by a truly discontinuous surface but rather by a thin "transition layer" with a finite spatial structure. Ionson noted that the relevant linearized MHD equations possess a singularity at some "resonant field line" within the transition layer, and by studying some special cases he found that one of the consequences of the singularity is the appearance of a pole in the complex frequency plane. The pole lies off the real frequency axis, and its sign implies a decay of the surface wave. Ionson interpreted the pole in terms of the decay rate of a normal mode, thus implying wave dissipation and plasma heating. Resonance absorption of surface waves was thus advanced as a viable candidate for solar coronal heating.

Lee (1980), and others, recognized that there can be no true dissipation in the system, since Ionson's analysis used the ideal (i.e., dissipationless) MHD equations *ab initio*. Lee showed that the conflict arises because the system really possesses no normal modes (see also Hasegawa and Uberoi 1982), so that Ionson's interpretation strictly speaking does not apply.

The actual flow of energy in the dissipationless system was recently studied by Lee and Roberts (1986). They studied in detail a special case. They considered an initial-value problem which basically starts with a standing MHD surface wave. The medium was taken to be incompressible (a poor assumption for the solar corona) with uniform density throughout. The equations were linearized, and \( B_0^2 \) was assumed to vary linearly (see Fig. 1a) across the transition layer (\( B_0 \) is the strength of the background magnetic field, which was assumed to be unidirectional and in the direction of wave propagation). The resonance is then exactly in the middle of the transition layer. As usual, the transition layer was assumed thin, such that \( mA \ll 1 \) (\( m \) is the wavenumber of the surface wave and \( 2A \) is the thickness of the transition layer). Lee and Roberts (1986) showed that energy does indeed flow out of the surface wave, and at long time appears in a thin layer surrounding the resonant field line; the thickness of this "energy-containing layer" was found to be of order \( mA^2 \), i.e., the initial surface wave energy becomes deposited in a small fraction of the transition layer. However, the rate at which the surface wave energy decays and becomes replaced by the energy in the thin layer is essentially the rate implied by the aforementioned pole in the complex frequency plane.

These previous studies of the resonance absorption phenomenon have been highly mathematical and limited to special cases. In this paper we present a less mathematically exact analysis of the resonance absorption phenomenon, but one which plainly reveals its physical basis. Besides being physically based, our analysis has other virtues: we will provide a simple explanation of why the thickness of the energy-containing layer is of order \( mA^2 \) in Lee and Roberts (1986); we can immediately generalize the results of Lee and Roberts (1986) to cases where the mass density \( \rho \) is not uniform (but we shall still assume the fluid to be incompressible, so that \( \mathbf{V} \cdot \mathbf{V} = 0 \), where \( \mathbf{V} \) is the fluid velocity) and where \( v_\lambda^2 = B_0^2/(4\pi \rho) \) has an arbitrary profile across the thin transition layer (there can be several resonant locations where \( v_\lambda \) equals some critical value, denoted \( v_{\lambda*} \), in Fig. 1b); and we shall be able to offer some speculations that under coronal conditions nonlinear effects may destroy or inhibit the resonance absorption...
before the larger part of the energy has been lost from the surface wave, thus making resonance absorption a less viable candidate for coronal heating; but a proper nonlinear analysis remains to be carried out.

The essence of our approach is as follows: We first consider a wave on a discontinuous surface, and note that the total pressure \( p_{\text{tot}} \) (i.e., fluid pressure \( p \) plus magnetic pressure \( B^2/(8\pi) \)) varies along the surface but is continuous across the surface. We then allow the surface to have a small but finite thickness (the transition layer). We take the transition layer to be so thin that the "external" surface wave is essentially unaltered, and that \( p_{\text{tot}} \) is still essentially continuous across the transition layer. We then show in a simple way (§ II) that the continuity of \( p_{\text{tot}} \) requires the introduction of time dependence, even in the frame moving with the surface wave. For detailed analysis, we then study the case of a standing surface wave, in order to make contact with the work of Lee and Roberts (1986). The essence of our analytical approach is to treat the thin transition layer as a collection of thin flux tubes (actually thin flux "slabs" in our two-dimensional configuration) which obey the so-called thin flux tube equations. In a crude initial approach to the problem (§ III), we assume that the external surface wave is completely unaltered by the presence of the thin transition layer, and we consider only the dynamics of the thin energy-containing layer. The temporal variations of \( p_{\text{tot}} \) associated with the surface wave, resonantly drive slow-mode waves in the energy-containing layer. The energy in the slow-mode waves grows secularly with time. This energy must ultimately come from the surface wave. The analysis is thus not self-consistent, but it does allow us to estimate the energy decay rate of the surface wave and the thickness of the energy-containing layer. These estimates agree with Lee and Roberts (1986), but are easily generalized as mentioned above. For a more detailed analysis (§ IV), we try to take the surface wave decay into account by assuming that its amplitude decays as \( \exp(-|a| t) \). This, too, is not self-consistent, since the surface wave decays only approximately, but not exactly, as \( \exp(-|a| t) \) (Lee and Roberts 1986). We also consider the behavior of the entire transition layer by assuming that it consists of an infinite collection of infinitesimally thin flux tubes, each obeying the thin flux tube equations. By requiring that the initial energy in the surface wave equal the final energy in the entire transition layer, we obtain an expression for \(|a| \) which is identical with that found by Lee and Roberts (1986). The analysis also yields a more precise description of the motions in the transition layers, and it is easily generalized to cases not treated by Lee and Roberts (1986). The simplicity of our analysis also allows us to offer some speculations (§ V) about the importance of nonlinearity.

### II. THE SURFACE WAVE: SIMPLE VIEW

Consider a wave on a free planar surface (a tangential discontinuity) in an incompressible fluid. The wave is assumed to propagate along the magnetic field direction. The fields on both sides of the surface are parallel but of unequal magnitudes. We work in the frame moving with the wave. In this frame there is a plasma flow coming in from infinity at the phase speed, and \( \partial/\partial t = 0 \). (See Fig. 2.) The magnetic field \( B \) and velocity \( V \) are then everywhere parallel if the electrical resistivity is zero. In that case, Bernoulli's equation becomes

\[
p_{\text{tot}} + \rho^* V^2/2 = \text{constant along streamlines},
\]

where

\[
\rho^* \equiv \rho - B^2/(4\pi V^2).
\]

Both \( \rho \) and \( \rho^* \) are constants along streamlines (Boyd and Sanderson 1969; see their § 5-6). Now suppose that the surface wave consists of a small bump, as in Figure 2. The flow on top of the bump is accelerated, while the flow below is decelerated (as indicated by \( \delta V \) in the figure). In linearized theory, the acceleration and deceleration are equal in magnitude. If the total pressure variations \( \text{along} \) the surface are to balance \( \text{across} \) the surface, we must have

\[
\rho^* = -\rho^*.
\]

This yields the dispersion relation and the phase speed, \( v_{ph} \):

\[
v_{ph}^2 = \frac{B_{01}^2 + B_{02}^2}{4\pi (\rho_1 + \rho_2)}.
\]

It appears that a magnetic field component normal to the plane of Figure 2 could be added without changing the result. This field component would be frozen into the flow. Since \( \rho \) does not change, the normal magnetic field component would not change, and thus would not affect the pressure balance.

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**Fig. 1.—(a) The spatial variation of the background magnetic field strength across the transition layer used by Lee and Roberts (1986). (b) The analysis of this paper considers a more general situation where the Alfvén speed, \( v_A \), can have a arbitrary variation across the transition layer. The analysis allows for more than one resonant field line, where \( v_A = v_{Ac} \).**

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**Fig. 2.—A free surface wave as viewed in the frame moving with the wave. The flow accelerates (decelerates) over a crest (trough). The total pressures must balance across the surface.**
The reader can readily verify that the magnetic and velocity fields below the surface \((y < 0)\) are

\[
\frac{\delta B_{x_2}}{B_{02}} = -\gamma m \sin (\omega t) \sin (mx)e^{\nu y},
\]

\[
\frac{\delta B_{x_2}}{B_{02}} = \gamma m \sin (\omega t) \cos (mx)e^{\nu y},
\]

\[
\delta V_{x_2} = \gamma \cos (\omega t) \cos (mx)e^{\nu y},
\]

\[
\delta V_{x_2} = \gamma \cos (\omega t) \sin (mx)e^{\nu y},
\]

where \(B_0\) contains the algebraic sign of the \(x\)-component.

Above the surface \((y > 0)\), the velocity and magnetic field fluctuations can be obtained from equations (7)-(10) by changing the signs of \(\gamma\) and \(m\), and by replacing subscript 2 with subscript 1.

The total pressure variations, \(\delta p_{\text{tot}}\), can be readily evaluated by inserting equations (7)-(10) in the ideal MHD momentum equation. We quote, however, only the behavior of \(\delta p_{\text{tot}}\) along the surface; in linearized theory the behavior along the surface can be obtained by evaluating \(\delta p_{\text{tot}}\) at \(y = 0\). We obtain

\[
\delta p_{\text{tot, surf}} = \gamma m \rho_1 \rho_2 \left(v_{A1}^2 - v_{A2}^2\right)(\rho_1 + \rho_2)^{-1} \sin (\omega t) \sin (mx).
\]

We will also need the total energy (kinetic plus magnetic) in the surface wave. This can be obtained from equations (7)-(10) by integrating over \(y\) and averaging over \(x\). Denoting this total energy by \(E\), we obtain

\[
E = \gamma^2 (\rho_1 + \rho_2) c^2 \pi^2/(4m).
\]

Equipartition between kinetic and magnetic energies is true globally, but not locally.

We now introduce the thin flux tube equations, which have been discussed in detail by Roberts and Webb (1979). (Recall that we are really dealing with thin slabs, but we shall continue to use the standard nomenclature.) In essence, these equations consider the motion along a thin collection of magnetic field lines. The region in question is taken to be thin enough so that one can in effect consider only averages across the tube cross section; this is precisely analogous to "one-dimensional channel flows" in ordinary fluids, except that here one allows the slab width, \(z\), of the flux tube to be a function of time as well as of distance along the tube, which we denote by \(s\). Flows transverse to \(s\) are implicitly included in terms such as \(\delta z/\delta t\).

The equation for mass conservation is

\[
\delta \delta V_s = -\frac{1}{\rho_0} \frac{\delta \delta z}{\delta t},
\]

after linearizing. By the frozen-in theorem, \(B_s z = \text{constant}\), and thus

\[
1 = \frac{\delta \delta B_s}{B_{0s} \delta t} = \frac{\delta \delta z}{\rho_0 \delta t},
\]

after linearizing. Combining equations (13) and (14) yields

\[
\frac{\delta \delta B_s}{\delta t} = B_{0s} \frac{\delta \delta V_s}{\delta s}.
\]

Linearized momentum balance along the tube requires

\[
\rho \frac{\delta \delta V_s}{\delta t} = \frac{\delta \delta p}{\delta s},
\]
where $p$ is fluid pressure. Finally, the total pressure inside the tube is $p_{\text{tot}} = p + B^2/8\pi$, and

$$\delta p = \delta p_{\text{tot}} - B_0 \delta B(4\pi)^{-1}. \quad (17)$$

These equations can be combined into

$$\left(\frac{\partial^2}{\partial t^2} - v_A^2 \frac{\partial^2}{\partial s^2}\right) \delta V_s = -\rho^{-1} \frac{\partial \delta p_{\text{tot}}}{\partial s} \delta t, \quad (18)$$

where $v_A^2 = B_0^2/(4\pi \rho)$ and $\rho$ is the density inside the thin tube. Equation (18) is a wave equation whose homogeneous solution corresponds to slow-mode incompressible MHD waves propagating along the tube. These waves can be driven by $\delta p_{\text{tot}}$.

Now the essential point is that the presence of a surface wave implies that $\delta p_{\text{tot}} \neq 0$ in the transition layer. Thus slow-mode waves are driven on the thin flux tubes which make up the transition layer. If we take $\delta p_{\text{tot}} = \delta p_{\text{tot, surf}}$ (as given by eq. [11]), equation (18) becomes

$$\left(\frac{\partial^2}{\partial t^2} + m^2v_A^2\right) \delta V_s = m^2\omega^2 v_A^2 \rho^{-1}(\rho_1 + \rho_2)^{-1}(v_{A2}^2 - v_{A1}^2) \cos(\omega t) \cos(mx), \quad (19)$$

where $\delta V_s = \delta P_s \cos(mx)$ and we have taken $s = x$, which is valid for the small-amplitude waves considered here. Equation (19) describes a harmonic oscillator with natural frequency $mv_A^2$, harmonically driven at the surface wave frequency $\omega$. If $\omega/m = v_A$ (corresponding to $p^* = 0$ in § II), then the harmonic oscillator is resonantly driven and its energy grows secularly with time. If we consider an initial-value problem with $\delta V_s = \delta B_s = 0$ at $t = 0$, the solution to equation (19) is then

$$\delta V_s = \gamma \Gamma \sin(\omega t) \cos(mx), \quad (20)$$

where

$$\Gamma \equiv (m^2/2)\rho_1 \rho_2 \rho^{-1}(\rho_1 + \rho_2)^{-1}(v_{A2}^2 - v_{A1}^2). \quad (21)$$

Now equation (20) applies only to a single field line which is strictly in resonance with the surface wave. However, we shall show below that a layer of finite thickness $x_0$ can be regarded as being effectively in resonance. The energy $\epsilon$ in this energy-containing layer is

$$\epsilon = \rho(\gamma \Gamma)^2 x_0 t^2/4. \quad (22)$$

In obtaining equation (22) we have averaged over $x$ and over the (assumed) rapid fluctuations at $\omega t$, and we have allowed for equipartition between magnetic and kinetic energies in the energy-containing layer. The inconsistency in the analysis is now manifest. The energy in the energy-containing layer grows without limit. But this energy is driven by the surface wave (via $\delta p_{\text{tot}}$), which contains only a finite amount of energy at $t = 0$, given by equation (12). So equation (22) must eventually fail. To estimate the time $t_\sigma$ at which equation (22) fails, we take

$$\epsilon = E. \quad (23)$$

Roughly speaking, at $t = t_\sigma$ all the energy initially in the surface wave has reappeared in the energy-containing layer; thus $t_\sigma$ is an estimate of the damping rate of the surface wave. We find

$$t_\sigma = \frac{\omega^2(\rho_1 + \rho_2)}{|m|\Gamma^2 x_0}. \quad (24)$$

It remains still to define the thickness of the energy-containing layer, $x_0$. A field line with natural frequency $mv_A$ will effectively be in resonance with the driver at frequency $\omega$ as long as $mv_A t$ and $\omega t$ are approximately in phase up to $t = t_\sigma$. At later times the surface wave has decayed away, and it does not matter if things get out of phase. We require

$$|mv_A - \omega|t_\sigma \leq \pi/2. \quad (25)$$

The criterion $\pi/2$ is somewhat arbitrary, but it happens to yield the exact result obtained by Lee and Roberts (1986) for the special case of Figure 1a, and the more exact result of the analysis in the following section. We expand $v_A$ in a Taylor series about the resonant field line, i.e.,

$$mv_A \approx \omega + m \frac{dv_A}{dy} \Delta y, \quad (26)$$

where $\Delta y$ is distance from the resonant field line. Combining equations (25) and (26) yields

$$x_0 = \pi(m_2 v_A)^{-1}. \quad (27)$$

where $v_A = |dv_A/dy|$ evaluated at the resonant layer. Combining equations (24) and (27) yields the following expression for the surface-wave damping rate:

$$t_\sigma^{-1} = \frac{\rho \Gamma^2 \pi}{\omega^2 v_A^2 (\rho_1 + \rho_2)}. \quad (28)$$

Using definition (21), and recalling that $\omega^2 \propto m^2$, we have

$$t_\sigma^{-1} \propto m^2(v_{A2}^2 - v_{A1}^2)/v_A^2. \quad (29)$$

The thickness of the energy-containing layer can be found from equation (27). We will not give the explicit result, but merely note that

$$x_0 \propto m^2(v_{A2}^2 - v_{A1}^2)/v_A^4. \quad (30)$$

For the specific case considered by Lee and Roberts (1986), we have $\rho_1 = \rho_2 = \rho$, and

$$v_A = m|v_{A2}^2 - v_{A1}^2|/(4A_\omega^2), \quad (31)$$

where $2A$ is the thickness of the transition layer. Also,

$$\Gamma = m^2(v_{A2}^2 - v_{A1}^2)/4 \quad (32)$$

for this case. Thus the damping rate is

$$t_\sigma^{-1} = \pi A^2 m^2(v_{A2}^2 - v_{A1}^2)(8\omega)^{-1}, \quad (33)$$

which, except for notation, is identical with the equation at the bottom of page 433 of Lee and Roberts (1986). Also, we obtain

$$x_0 = (\pi^2/4) m A^2. \quad (34)$$

The result that the thickness of the energy-containing layer is proportional to $mA^2$ agrees with the result Lee and Roberts (1986); the physics of this result is embodied in equation (25).

Note that the results of this section can be readily generalized to cases where there are several resonant field lines, as in Figure 1b. In that case one can treat each such field line separately, and sum over the individual energy-containing layers. This procedure fails, however, if the energy-containing layers overlap.

IV. THE TRANSITION LAYER

The previous section revealed the essential physics of resonance absorption, and the mathematical results of Lee and Roberts (1986). It suffers two significant defects, however: (1) the driving term, i.e., the surface wave, in equations (18) and
dependence of \( \nu_0 \). At long times, the phase is a very rapidly varying function of \( \Delta y \). Thus, as \( t \) increases, \( \delta P \) develops increasingly strong cross-field gradients; this result was also found by Lee and Roberts (1986). The reason, of course, is that each field line is now oscillating at its natural frequency \( m_B \nu_0 \), and neighboring field lines get out of phase; this is commonly referred to as “phase mixing.” The energy, however, is contained within an envelope of constant width, given roughly by equation (27) or equation (39). Thus phase mixing has nothing to do with the buildup of energy in the energy-containing layer.

The kinetic energy in the energy-containing layer can be found by multiplying \( \rho/2 \) by the square of equation (41), and then integrating over the \( y \)-coordinate. We also average over the \( x \)-coordinate, and over the rapid time variations at \( m_B \nu_0 \).

Since the energy-containing layer is thin, we use equation (26), take \( \rho \) = constant, and extend the \( y \)-integration to \( \pm \infty \). If we include an equal amount of magnetic energy, the energy works out to

\[
\epsilon = \frac{\pi \rho \nu_0 \Gamma^2}{4m_B} |a| \ .
\]

To obtain \( |a| \), we require that the energy in the energy-containing layer at long times be equal to the initial energy in the surface wave, given by equation (12) with \( \gamma = \gamma_0 \). The result is

\[
|a| = t_{\epsilon}^{-1} \ ,
\]

where \( t_{\epsilon}^{-1} \) is given by equation (28).

As in the previous section, this result is easily generalized to cases where there are several resonant field lines, as long as the energy-containing layers do not overlap.

V. NONLINEARITY: SPECULATIONS

We have shown that the resonance absorption phenomenon can be regarded as a driven harmonic oscillator problem; see equation (19). The resonant frequency of the oscillator is \( m_B \nu_0 = m_B (4\pi \nu_0)^{-1/2} \). Note that only \( B_0 \), a constant, appears in the resonant frequency. This is a consequence of the linearization. In reality, as the slow-mode oscillations build up in the energy-containing layer, the magnetic field there will depart from \( B_0 \). In addition, parallel flows build up in the energy-containing layer; the effects of these flows also do not appear in equation (19), in virtue of the linearization. It seems clear that once the magnetic and velocity fluctuations become large enough, the behavior of the energy-containing layer will not be well approximated as a harmonic oscillator with constant resonant frequency; in a general sense, the resonant frequency will then be a nonlinear function of the field and flows in the layer. We suggest that when this occurs, the resonance will be destroyed, and the resonant buildup of energy in the energy-containing layer will either slow down or cease altogether, and so too will the decay of the energy in the surface wave.

To estimate the time \( t_{\nu_0} \), at which this occurs, we ask when the energy in the energy-containing layer has built up to the level of the magnetic energy originally in the layer, i.e., we ask when

\[
\epsilon = \nu_0 B_0^2 / 8\pi \ ,
\]

where \( \epsilon \) is given by equation (22). We find

\[
t_{\nu_0} = 2^{1/2} \nu_0 |\nu_0 \Gamma|^{-1} \ .
\]
The linearized theory of resonance absorption will apply if $t_{NL} \gg t_d$, but will fail if the inequality is not satisfied.

For a specific example, we consider the case studied by Lee and Roberts (1986). We then obtain

$$t_{NL} = 2^{-1/2} \pi A \gamma^{-1},$$

(46)

where equations (32) and (33) have been used, and we have taken $\gamma \omega = \omega$ in the energy-containing layer. Roughly speaking, equation (46) says that nonlinear effects become important when the amplitude of the surface wave displacement, $\gamma$, becomes equal to the thickness, $2A$, of the transition layer.

For a numerical estimate, we note that $\gamma \omega$ is the amplitude of $\delta V_2$; see equation (10). Then equation (46) is

$$t_{NL} = (2A)^{2}(2^{1/2} \frac{1}{t_d})^{-1},$$

(47)

where $T = 2\pi/\omega$. For the solar corona we might have $|\delta V_2| = 50 \text{ km s}^{-1}$, $T = 300 \text{ s}$, and $2A = 10^5 \text{ km}$. Then $t_{NL}/t_d = 0.47$, indicating that the resonance absorption could be inhibited by nonlinear effects before the surface wave has decayed away. Of course, it is really inappropriate to apply our incompressible analysis to the solar corona, but we recall that Ions's cold plasma analysis (which is a good approximation for the corona) yielded nonlinear flows, of the order of $v_A$, in the energy-containing layer, again suggesting the possibility that nonlinearity could inhibit the resonance absorption. In fact, the lack of observational evidence for such large flows in the corona can be interpreted as an indication that resonance absorption breaks down before such large flows can build up. But the fact that $t_{NL}/t_d = O(1)$ suggests that a proper nonlinear analysis should be carried out. We intend this to be the subject of a future paper.

VI. CONCLUSIONS

We have shown how the phenomenon of MHD surface wave resonance absorption can be described in simple terms, both physically and mathematically, by applying the "thin flux tube equations" to the finite-thickness transition layer which supports the surface wave. The thin flux tubes support incompressible slow-mode waves which are driven by fluctuations in the total pressure, $\delta p_{tot}$, which exist in virtue of the presence of the surface wave. We show the equations for the slow-mode waves can be reduced to a simple equation, equivalent to a driven harmonic oscillator. Certain field lines within the transition layer are equivalent to a harmonic oscillator driven at resonance. Neighboring field lines are effectively driven at resonance as long as condition (25) is satisfied. There thus develops a layer (the "energy-containing layer") which secularly extracts energy from the surface wave. The energy-containing layer is much thinner than the transition layer; the physics governing its thickness is embodied in condition (25). We have thus succeeded in showing how resonance absorption can be reduced to a simple problem in classical mechanics. One important virtue of this approach is that it becomes possible to treat more general problems than in the purely mathematical analyses which have been heretofore available.

By more clearly revealing the physics of resonance absorption, our analysis indicates that nonlinear effects may destroy the resonance which is crucial to the whole effect. Numerical estimates indicate that nonlinear effects may be important in the corona, but a detailed nonlinear treatment of compressible surface waves appears to be called for. We plan to investigate this problem in a future paper.

Before closing, we refer the reader to two other papers on resonance absorption. Steinolfson (1984) studied the problem numerically, but he did not address the essential physics as we have tried to do here. Mok and Einaudi (1985) analytically addressed a problem similar to the one considered here and by Lee and Roberts (1986), viz., a standing surface wave in an incompressible fluid. They correctly recognized that there is no normal mode in a purely dissipationless case, but they instead considered a system with dissipation, in which case there are normal modes. They found parameters for which there is a damped mode, but the damping rate is independent of the dissipation rate. This corresponds to the damping of the surface wave by resonance absorption. But their damping rate disagrees with ours and with the result of Lee and Roberts (1986). We believe that this represents an error in their work. In particular, their expression for the damping rate contains the fluid pressure $p$ explicitly; this indicates an error, since $p$ is really a free parameter in incompressible fluids.

Finally, we refer the reader to a recent paper by Buti (1986) which considers a number of other interesting aspects of surface waves in different contexts.

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