LINEAR CLUSTERS OF GALAXIES: A999 AND A1016

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ABSTRACT

We have measured 44 new redshifts in A999 and 40 in A1016: these clusters are both "linear" according to Rood and Sastry (1971) and Struble and Rood (1982, 1984). With 20 cluster members in A999 and 22 in A1016, we can estimate the probability that these clusters are actually drawn from spherically symmetric distributions. By comparing the clusters with Monte Carlo King models, we find that A999 is probably intrinsically spherically symmetric, but A1016 is probably linear. We estimate that 2% of a catalog of spherically symmetric clusters might be erroneously classified as linear. We use the data to estimate the virial masses for these systems. The standard form of the virial-mass estimator assumes isotropy and leads to an underestimate of the uncertainty in the mass-to-light ratio for anisotropic systems. We recast the virial-mass estimator for use in anisotropic cases, and find $\frac{M}{L_B} = 180 \pm 340$ for A999, and $\frac{M}{L_B} = 260 \pm 340$ for A1016. These values are lower than the mean $\frac{M}{L_B}$ of $\sim 340$ for groups in the CfA Redshift Survey (Huchra and Geller 1982; Geller 1984). We reassess the cluster–galaxy alignment analysis of Adams, Strom, and Strom (1980) and examine the relationship between the luminosity and morphological type of the cluster members and the cluster itself. In the convincingly linear system A1016, we find no evidence for alignment. Neither cluster shows evidence for segregation.

I. INTRODUCTION

Detailed studies spanning the entire range of morphologies of rich clusters of galaxies are important for understanding the formation and evolution of these systems. Both optical and x-ray data show that many clusters are anisotropic (Rood and Sastry 1971; Struble and Rood 1982; Forman and Jones 1982). Subclustering in a number of well-observed systems (Forman et al. 1981; Geller and Beers 1982; Forman and Jones 1982) suggests that at least some aspects of the cluster formation process can still be observed (Geller 1984; Cavaliere et al. 1985).

Ten relatively nearby systems appear so flattened that Rood and Sastry (1971) classified them as "linear." Struble and Rood (1982, 1984) have identified many more distant linear clusters. These systems constitute 8% of the Abell catalog to $D<4$ and 16% for $D = 5$. The classification of cluster morphology "by eye" depends on the distribution of a small number of the brightest galaxies. Here we develop a statistic to measure the "linearity" of a system and we apply the statistic to simulated spherically symmetric clusters. These simulations indicate that 2% of clusters with 20 known members will appear linear even if they are intrinsically spherically symmetric.

Adams, Strom, and Strom (1980, hereafter referred to as AdSS) took $R$ band plates of seven of the ten linear clusters classified by Rood and Sastry (1971). We have measured redshifts for galaxies in two of the clusters in the AdSS sample: A999 and A1016. With 44 redshifts in A999 (20 cluster members) and 40 in A1016 (22 cluster members), we can assess the probability that the systems are truly flattened. We conclude that the galaxies in A999 are probably drawn from a spherically symmetric distribution; those in A1016 probably are not.

We use the redshift surveys to examine the dynamics of flattened systems. An early study of A194 by Chincarini and Rood (1977) and our own analysis (Chapman, Geller, and Huchra 1987) indicate that the mass-to-light ratio of this flattened system is lower than that typical of other clusters. Analyses of both A999 and A1016 produce low values ($\frac{M}{L_B}$ for A999 and $\frac{M}{L_B}$ for A1016). We examine the effect of anisotropy on the determination of cluster masses from the virial theorem. We find that if the shortest axes of these clusters are close to the line of sight (and are significantly shorter than the observed axes), the mass-to-light ratios may be underestimated by $\sim 50%$. In this case, the values would come into agreement with those for other systems.

Flattened systems also offer an opportunity to test for cluster properties predicted by the adiabatic or “pancake” picture of cluster formation (Zel’dovich 1972, 1978). Because galaxy formation occurs late in this model, the orientations of galaxies in clusters should be correlated with the cluster axis (see Thompson 1976 for a study of A2197). The models predict that the major axes of elliptical galaxies and the rotation axes of spirals should lie in the plane of the “pancake” (Dekel 1985; Doroshkevich 1970). The major axes of ellipticals (as seen on the sky) should then be aligned parallel to the major axis of the cluster; spirals should be perpendicularly aligned. The model also predicts large-scale alignments of the axes of rich clusters in superclusters (Doroshkevich et al. 1980; Zel’dovich 1978; Binggeli 1982; Gregory, Thomson, and Tiff 1981; Einasto, Jöe veer, and Saar 1980), although neighboring clusters may also be brought into alignment by tidal distortion (Binney and Silk 1979).

In contrast, the hierarchical clustering model (Peebles 1980; Peebles and Dicke 1968) predicts that the orientations of galaxies (which predate clusters in these models) are unrelated to the cluster axis except where recent environmental processes (e.g., cannibalism) are important. With member-
ship based on redshift, we can reassess the AdSS analysis of cluster–galaxy alignments.

Section II describes the data and includes a discussion of cluster membership. Section III contains the statistical tests and models. Section IV is a discussion of cluster dynamics and includes the derivation of an anisotropic form of the virial theorem. Section V examines the relationship between various properties of the cluster members and the cluster itself.

II. OBSERVATIONS

a) Spectroscopic Data

AdSS list all galaxies within 30' of the cluster centers down to a limiting $m_{R} = 17.9$ for A999 (omitting one bright foreground galaxy) and $m_{R} = 17.6$ for A1016. We measured the redshifts of 44 galaxies in the region of A999 and 40 around A1016. These samples are complete to $m_{R} = 16.2$ for A999, and to $m_{R} = 15.5$ for A1016. The sample of A1016 is only two galaxies (1025 + 1117A and 1024 + 1129) short of completeness to $m_{R} = 16.3$. In addition, we found the redshifts of four galaxies near A1016 in the Catalogue of Radial Velocities of Galaxies (Palumbo, Tansella-Nitti, and Vettolani 1983) and use the redshift of NGC 3253 (near A999) from the CfA Redshift Survey. Tables I and II list the heliocentric redshifts and errors (where available).

We use the cross-correlation technique (Tonry and Davis 1979) to extract redshifts from medium-resolution spectra obtained at the Whipple Observatory. Forty of these spectra were obtained with the photon-counting Reticon detector system—the "Z-machine" (Latham 1982)—on the 1.5 m Tillinghast telescope. A 600 line mm$^{-1}$ grating was used in first order, giving a resolution of 5 Å over a wavelength range of $\lambda \lambda$ 4500–7100 Å. The total integration time for these galaxies was typically 20–50 min. The remaining 44 spectra were obtained using a similar detector system at the Multiple Mirror Telescope (Latham 1979) with a 300 line mm$^{-1}$ grating in first order to give a resolution of 7 Å over a range of $\lambda \lambda$ 3400–7000 Å. The typical integration time for these (fainter) galaxies was 10–20 min.

The mean internal error of the Tillinghast data is 37 km s$^{-1}$, comparable with the mean external error of ~35 km s$^{-1}$ in the CfA Redshift Survey (Huchra et al. 1983). The MMT data have a mean internal error of 49 km s$^{-1}$.

b) Positions of Galaxies

Columns 4 and 5 of Tables I and II contain the 1950 epoch coordinates of each galaxy in our sample. The positions of the galaxies observed with the MMT are obtained from the telescope coordinates. For the galaxies observed with the Tillinghast reflector, we used the CfA measuring engine to obtain the galaxy coordinates from the Palomar Observatory Sky Survey print (#238) which covers both clusters. The engine was calibrated with the epoch 1950 coordinates of 15 SAO stars, giving an internal error of 1.5 arcsec. A comparison of the coordinates of ten galaxies obtained at the MMT with their positions from the measuring engine yields a mean external error of 4°. Figures 1 and 2 are plots of the positions of all galaxies within 30' of the centers of A999 (to a limit of $m_{R} = 17.2$) and A1016 (to $m_{R} = 17.3$), respectively.

c) Galaxy Morphologies, Position Angles, and Magnitudes

These data (columns 3, 6, and 7 of Tables I and II) are from AdSS and come from $R$ band (RG 610 + IIIa-F) plates taken with the Mayall 4 m telescope. These plates were scanned with the Kitt Peak National Observatory PDS microphotometer. The instrumental magnitudes were calibrated with photoelectric photometry of four galaxies in each cluster. No errors are quoted by AdSS, but they should be <15%.

AdSS also obtained position angles for individual galaxies from the digitized images. The major-axis position angles at the 25.2 mag arcsec$^{-2}$ isophote, relative to north (such that a galaxy aligned SE–NW has a position angle of 135°) have $\pm 5^\circ$ error (AdSS).

We have examined reproductions of the AdSS plates of A999 and A1016 and generally agree with the galaxy morphologies (while noting that SO's are particularly difficult to distinguish).

d) Cluster Membership

Our data consist of magnitude-limited samples within 30' of the cluster centers, and include any other galaxies with known redshift within 2' of the cluster center. Figure 3(a) shows the velocity histogram of A999 from 0 to 46 000 km s$^{-1}$, binned in 500 km s$^{-1}$ intervals. Figure 3(b) shows the data for A1016; Figs. 4(a) and (b) expand the region between 8100 and 10 600 km s$^{-1}$ (binned in 50 km s$^{-1}$ intervals) for A999 and A1016, respectively. The solid curves in Figs. 3(a) and 3(b) show the distribution expected in these samples for uniformly distributed galaxies. To obtain the expected distribution we use the Schechter (1976) luminosity function with parameters from the CfA Redshift Survey (Davis and Huchra 1982): $\alpha = -1.3$, $M^* = -20.9$, $\phi^* = 0.0143$ Mpc$^{-3}$.

The redshift distributions along the line of sight toward the two clusters are highly correlated. A999 and A1016 probably form a small supercluster: they are 27' (or 3.1 h$^{-1}$ Mpc) apart on the sky, and have the same redshift (within the limits of observational error).

There are four galaxies near A999 and two near A1016 that are between 30' and ~1' of the cluster centers and are also at the appropriate redshift. We include these galaxies in Tables I and II, but exclude them from cluster membership because of our lack of knowledge about the structure of the clusters between 30' and 1' from their cores. The exclusion of these galaxies does not materially affect our analysis.

The four galaxies with redshifts between 7500 and 9000 km s$^{-1}$ are probably not members of A999; the galaxy with the highest redshift of these four is over 400 km s$^{-1}$ from the nearest cluster member. The 20 cluster galaxies [see Fig. 4(a)] have a velocity dispersion of 250 km s$^{-1}$.

A1016 is quite distinct in redshift space and includes all 22 galaxies with redshifts between 9100 and 10 200 km s$^{-1}$ [Fig. 4(b)]. The cluster has a dispersion of 240 km s$^{-1}$, and the nearest nonmember is more than 3700 km s$^{-1}$ from the cluster mean. Cluster members are marked with an asterisk in Tables I and II, and by a dark circle in Figs. 1 and 2.

This intuitive definition of cluster membership gives the same results as the following more rigorous procedure: we initially assume that all galaxies within 30' of a cluster are members, and that the velocity distribution of the cluster is Gaussian. We then use the $\chi^2$ test to reject any galaxy with a redshift far enough from the sample mean to make the red-
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* denotes cluster membership

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Column (8): Heliocentric redshift of galaxy
Column (9): Error in redshift
Column (10): Source of redshift data:
M: MMT unpublished observations
T: Tillinghast 1.5m unpublished observations
Z: CfA Redshift Survey
P: Palumbo, Tansella-Nitti and Vettolani 1983

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Table II. Positions and redshifts of galaxies in Abell 1016.

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* denotes cluster membership
Columns as in Table 1
The classification of linear clusters (Rood and Sastry 1971; Struble and Rood 1982, 1984) relies on selection "by
eye”: if four of the ten brightest members of a cluster lie close to a line or arc, the cluster is “linear.” A significant fraction of linear clusters may in fact be spherically symmetric, only appearing flattened because of a chance alignment of a few bright cluster members. The inclusion of foreground (or background) galaxies may similarly lead to confusion. In order to connect cluster morphology with the physics of cluster formation, we must quantify the departures of linear clusters from spherical symmetry; we need an objective measure of linearity. We can then apply this measure to simulated spherically symmetric clusters to examine the significance of apparent anisotropies in the observed systems.

A subjective definition of cluster morphology is probably influenced by the distribution of fainter galaxies surrounding the ten brightest. Examination of POSS prints suggests that ~20 galaxies might influence the categorization of a linear cluster. We thus estimate the fraction of linear systems that would appear in a catalog of intrinsically spherically symmetric clusters in which ~20 cluster members were responsible for the visual impression of cluster morphology.

a) Test for “Linearity”

To test for linearity, we draw a band across the cluster and count the number of cluster members within it ($N_\parallel$). We determine the maximum number of galaxies ($N_L$) that lie within a band of the same width but perpendicular to the first. We then vary the width, position, and orientation of the

![Fig. 3. (a) Velocity histogram of A999 from 0 to 46,000 km s$^{-1}$, with bins 500 km s$^{-1}$ wide. (b) Velocity histogram of A1016 from 0 to 46,000 km s$^{-1}$, with bins 500 km s$^{-1}$ wide.](image)

![Fig. 4. (a) Velocity histogram of A999 from 8100 to 10,600 km s$^{-1}$, with bins 50 km s$^{-1}$ wide. (b) Velocity histogram of A1016 from 8100 to 10,600 km s$^{-1}$, with bins 50 km s$^{-1}$ wide.](image)
L tends to the axial ratio (a/b) in the limit of bands with zero cluster surface-density distribution is an elliptical function. If the original strip to maximize the linearity $L = N_0^*/N_1$ in the limit of bands with zero width.

We constrain $N_1 > aN$ (where N is the total number of cluster members, and $0 < a < 1$). A low value of a makes the test sensitive to particularly close alignments of a few cluster members, which may not be representative of the overall cluster morphology. On the other hand, an a close to 1 leads to a cluster position angle determined by the outermost cluster members only. The values of $N_1$ and L in rows 5 and 6 of Table IV are the results of the $a = 0.4$ test performed on A999 and A1016. A comparison with simulated spherically symmetric systems further amplifies the difference between the two systems.

**b) Simulated Clusters**

In order (1) to obtain a confidence level for our linearity statistic, and (2) to estimate the fraction of cataloged linear clusters that are likely to be truly flattened systems, we simulate 60 spherically symmetric clusters with 20 members each. The simulated clusters have Hubble density profiles:

$$\sigma(r) = \sigma(0) \left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-1}.$$ (2)

We limit $r < r_{max}$, a cutoff radius, and choose $r_{max}/r_c = 10$, so that the appearance of the simulated clusters is approximately that of A999 and A1016. The linearity statistic is not sensitive to the choice of either $r_c$ or $r_{max}$. Figure 5 shows two of the Monte Carlo clusters: cluster #8 [in Fig. 5(a)] is a typical (nonlinear) example, while cluster #10 [Fig. 5(b)] has the highest linearity. The apparent axial ratios of these clusters are $(a/b) = 1.3 \pm 0.3$ for #8 and $(a/b) = 2.0 \pm 0.3$ for #10. Their linearities are 2.00 and 3.67, respectively.

**IV. Dynamics**

The three factors that enter the calculation of the cluster redshift are the motion of the Sun around the Galaxy, the peculiar motion of the Local Group, and the “Virgo flow”—the distortion from the Hubble flow caused by the Local Supercluster. For A999 and A1016, these corrections are small. We take the solar motion with respect to the centroid of the Local Group to be: $\Pi = 300 \text{ km s}^{-1}$, $\Theta = 79 \text{ km s}^{-1}$, and $Z = 36 \text{ km s}^{-1}$ (Yahil, Tammann, and Sandage 1977). We use the dipole ($\gamma = 2$) model of Aaronson et al. (1982) to estimate the Virgo flow correction, adopting their best-fit values for the Local Group motion $250 \pm 100 \text{ km s}^{-1}$ towards $12^h \ 28^m$, $+12^\circ \ 40'$ (1950) and the “true” Virgo redshift ($1305 \text{ km s}^{-1}$). The corrected redshifts and their uncertainties are in row 2 of Table III. The distance to each
cluster (row 6) is calculated from Mattig's relation (Mattig 1958) with $q_0 = 0.1$ and $H_0 = 100$ km s$^{-1}$ Mpc$^{-1}$.

We next determine the cluster luminosities. The total apparent magnitude of known cluster members is $m_R = 11.2$ for A999, and $m_R = 11.1$ for A1016. As noted in Sec. IIa, the magnitude limits are $m_R < 16.2$ for A999, and $m_R < 16.3$ for A1016. The distance modulus for both clusters is 34.9 mag; with $M_R = 4.31$, the $R$ band luminosities are $1.7 \times 10^{11}$ $h^{-2} L_\odot$ for A999 and A1016, respectively. The values (row 7, Table III) are corrected for redshift, but have not been corrected for Galactic obscuration, $K$ dimming, or for the missing contribution from the faint end of the luminosity function. The uncertainty in mean corrected redshift (2%) is much less than the error in the individual magnitude measurements (~15%).

Using our assessment of cluster membership, we find the differential luminosity functions of the clusters (Figs. 7 and 8). We note that both functions have maxima at $m_R \approx 15$. Despite this, we estimate the fraction of the cluster luminosity from cluster members fainter than the magnitude limit by fitting the observed luminosity function for each cluster to the Schechter (1976) function:

\[ \phi(L) dL = \phi^* \left( \frac{L}{L^*} \right)^{\alpha} e^{-L/L^*} d\left( \frac{L}{L^*} \right) \]

The values of $M^*_R$ (corresponding to $L^*$) (row 8, Table III) are calculated by taking $\alpha = -1.3$ (the mean for the CfA Redshift Survey—Davis and Huchra 1982). We find $M^*_R = -20.9$ for A999, and $M^*_R = -20.4$ for A1016, or $m^*_R = 14.0$ and $m^*_R = 14.5$, respectively. We therefore sample the luminosity functions of both clusters to a limit approximately two magnitudes fainter than $M^*$. The reduced chi-square of the fits are 4.0 and 6.8 for A999 and A1016, respectively. We estimate the uncertainty in $M^*_R$ by treating $\alpha$ as a free parameter. The best fits are then $\alpha = -2.1$, $M^*_R = -20.3$, $\chi^2 = 1.0$ for A999; and $\alpha = -2.0$, $M^*_R = -19.5$, $\chi^2 = 4.6$ for A1016. The uncertainty in $M^*_R$ is thus approximately 1 mag. The mean $M^*_R$ for the CfA Redshift Survey is $M^*_R = -19.4$ (Davis and Huchra 1982) or $M^*_R = -21.2$, consistent with the cluster values. The total luminosity is the sum of the observed luminosity and the integral of Eq. (3) for galaxies fainter than the survey limit. The error in the total luminosity (row 9, Table III) reflects...
the uncertainty in the luminosity-function parameters due to the fact that the Schechter function is not a good description of the luminosity functions of these clusters. The errors should therefore be upper limits on the uncertainty in cluster luminosity.

We next estimate the mass of each system. The "projected mass estimate" (Bahcall and Tremaine 1981) for a group of galaxies is (Heisler, Tremaine, and Bahcall 1985)

\[ M_{\text{pm}} = \frac{10.2}{G(N-1.5)} \sum v_{\parallel}^2 r_{\parallel} \]

where \( G \) is the gravitational constant, \( N \) is the number of galaxies, \( v_{\parallel} \) is the line-of-sight velocity of the \( i \)th cluster member with respect to the mean, and \( r_{\parallel} \) is the projected separation of the \( i \)th galaxy from the mean position of cluster members.
Fig. 8. Differential luminosity function of A1016. The symbols are as in Fig. 3. Note that two galaxies are absent from the sample at $m_R = 15.6$.

For A999, we find $\mathcal{M}_{PM} = 5.8 \pm 0.4 h^{-1} 10^{13} M_\odot$, and for A1016 $\mathcal{M}_{PM} = 3.6 \pm 0.3 h^{-1} 10^{13} M_\odot$, with errors estimated using the statistical jack-knife procedure (Diaconis and Efron 1983; Efron 1982).

We also use the virial theorem to estimate the mass of each cluster. We examine the effects of anisotropy on the application of the virial theorem to these flattened systems. In the absence of rotation and bulk motion, the tensor virial theorem takes on a simple form (Binney 1981):

$$\Pi_{xx} + W_{xx} = 0$$

for each axis of the cluster, where

$$\Pi_{xx} = \int \rho(r)(v_x - \langle v_x \rangle)^2 d^3r$$

and

$$W_{xx} = \int \rho(r)x \frac{\partial \Phi}{\partial x} d^3r$$

or

$$W_{xx} = -\frac{1}{2} G \int \frac{\rho(r)\rho(r')(r-x)^2}{|r-r'|^3} d^3r d^3r'. \quad (8)$$

The volume of integration includes the whole cluster, $\rho(r)$ is the mass density at position $r$, $\Phi$ is the gravitational potential, and $v_x$ is the velocity component along the $x$ axis. If the cluster is not rotating, we are free to choose any orthogonal set of axes. The scalar potential $W$ is the trace of the potential energy tensor:

$$W = \int \frac{\rho(r)\rho(r')(r-x)^2}{|r-r'|^3} d^3r d^3r' \quad (9)$$

and $W_{xx}$ is one of the trace elements of the tensor.

Following the derivation of the conventional virial-mass estimator, we assume that the galaxies trace the mass and velocity distributions with equal "weight." For a system of $N$ discrete masses, Eq. (5) becomes

$$\langle N-1 \rangle \sigma_x^2 - \frac{G \mathcal{M}_{VT}}{2N} \sum_{i \neq j} \frac{(x_i - x_j)^2}{r_{ij}^3} = 0, \quad (10)$$

where $r_{ij}$ is the spatial separation of the $i$th and $j$th cluster members, $\sigma_x$ is the component of the velocity dispersion along the $x$ axis, and $\mathcal{M}_{VT}$ is the anisotropic virial-mass estimate for the cluster. We can choose an $x$ axis parallel to the line of sight, and define $(x^2/r^3)$ as

$$\frac{x^2}{r^3} = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{(x_i - x_j)^2}{r_{ij}^3}. \quad (11)$$

We rewrite Eq. (10):

$$\mathcal{M}_{VT} = \frac{2}{G} \sigma_{cor} \left( \frac{x^2}{r^3} \right)^{-1}, \quad (12)$$

with $\sigma_{cor}$ defined by Eq. (1).

This form of the virial theorem is valid for nonrotating systems. If the velocity and spatial-separation vectors are isotropically distributed, $\langle 1/r \rangle = 3 \langle x^2/r^3 \rangle$ and $\sigma_2 = 3\sigma_{cor}^2$, making Eq. (11) identical to the conventional virial-mass estimator. We make the relationship between these two estimators explicit by defining a form factor $f$:

$$f = \frac{2}{3\pi} \left( \frac{1}{r_p} \right) \left( \frac{x^2}{r^3} \right)^{-1}. \quad (13)$$

Equation (12) now becomes

$$\mathcal{M}_{VT} = \frac{3\pi}{G} f^{cor} \left( \frac{1}{r_p^2} \right)^{-1}. \quad (14)$$

Thus $f$ is the ratio of the anisotropic virial-mass estimator to the isotropic estimate.

To use this form of the virial theorem, we must calculate $f$ for an appropriate model of the cluster density distribution. We show in the Appendix that if the scalar potential is an analytic function that can be rescaled to spherical symmetry, $(x^2/r^3)$ may be obtained by differentiation. For less symmet-
ric models, numerical integration or Monte Carlo simulations may be necessary.

As an example, we consider a constant-density ellipsoid. If the envelope of the ellipsoid is given by \( s_1^2/e_1 + s_2^2/e_2 + s_3^2/e_3 = 1 \), where \( s_i \) are vectors along the principal axes, the potential is (Chandrasekhar 1969, pp. 43, 33–39, 57, and 128)

\[
W = -\frac{3}{10} G m^2 \frac{A_i e_i}{e_i e_k},
\]

or

\[
W = -\frac{3}{10} G m^2 \left( A_1 e_1^2 + A_2 e_2^2 + A_3 e_3^2 \right),
\]

where

\[
A_1 = \left[ F(\psi,k) - E(\psi,k) \right] \frac{g}{k^2},
\]

\[
A_2 = \left[ E(\psi,k) - k^{-2} F(\psi,k) \right] \frac{g}{(kk')^2},
\]

\[
A_3 = \left[ \frac{e_1}{e_3} \sin \psi - E(\psi,k) \right] \frac{g}{k^2},
\]

and

\[
g = 2e_i e_j e_k \sin \psi.
\]

We note that when the distribution is isotropic \((e_i = e_j = e_k = R)\) or \(\psi = 0\) \((x^2/r^2)\) has a constant value \((1/R - 1)\) independent of orientation. For a face-on plane \((\theta = \phi = \pi/2\) and \(e_i = 0\)) and an end-on infinite rod \((\theta = 0\) and \(e_i = \infty\)), we see that \((x^2/r^2)\) is 0, as expected.

We find the mean projected separation \((1/r_p)\) of the distribution by noting that \((1/r) = 2W/Gm^2\). We substitute \(e_i = a, e_j = b, \) and \(e_k = 0\) into Eq. \((16)\):

\[
\frac{1}{r_p} = \frac{1.2}{a} \frac{F(\pi/2)}{\sqrt{1 - b^2/a^2}},
\]

where \(a:b\) is the axial ratio of the projected density distribution. In the isotropic case, \((1/r_p) = \pi R/R\) so that \(f = 1\).

Substituting \((1/r_p)\) and \((x^2/r^2)\) into Eq. \((13)\), we find the form factor \(f\). We expect the greatest deviations from the isotropic virial-mass estimate when either the major or the minor axis lies along the line of sight \((\theta = 0, \) and \(\phi = \pi/2\), respectively). When \(\theta = 0, \) we have \(a = e_1\) and \(b = e_2\); when \(\theta = \phi = \pi/2, \) \(a = e_1\) and \(b = e_2\); Table \(V\) lists the values of \(f\) for a range of observed axial ratios \((a:b)\) and line-of-sight scale lengths \(x\): we assume that \(x \geq b/2\). The entries in Table \(V\) show that \(f\) is much less sensitive to the upper limit on \(x\). In the last row of Table \(V\) we list \(x_o\), the value of \(x\) for which \(f\) has a minimum.

Using a cluster axial ratio of \((b/a) = 0.50\) (Sec. IIIa), we estimate that the uncertainty in the virial mass of A999 introduced by the anisotropy of the cluster is +52% to −3.6%. For A1016, which has \((b/a) = 0.56\), the error image is +47% to −4.0% (Table \(V\)).

Another problem in virial-theorem analyses is the measurement of \((1/r_p)\): background contamination and close pairs of galaxies typically lead to large uncertainties. We use the statistical jack-knife to estimate the error in \((1/r_p)\), and find \((1/r_p)^{-1} = 0.150 \pm 0.007^\circ\) for A999, and \((1/r_p)^{-1} = 0.127 \pm 0.005^\circ\) for A1016. The corresponding values of \((1/r_p)^{-1}\) are in row \(10\) of Table \(III\).

Using Eq. \((14)\) with \(f = 1\), we have an isotropic virial mass for A999 of \(3.4 h^{-1} \times 10^{13} \mathcal{M}_\odot\) and \(2.7 h^{-1} \times 10^{13} \mathcal{M}_\odot\) for A1016. These masses are lower than (but consistent with) the corresponding projected mass estimates, as predicted by Heisler, Tremaine, and Bahcall (1985). The errors quoted in rows \(12\) and \(13\) of Table \(III\) include the uncertainty due to anisotropy.

The virial-mass estimator is robust, in that small departures of \((1/r_p)\) dominate the uncertainties in the projected mass estimates.
tures from isotropy lead to small errors in the estimated mass. Without detailed modeling, it is difficult to say whether the projected mass estimator is more (or less) sensitive to anisotropy.

Dividing the virial mass of each cluster by its total \( R \) band luminosity, we find mass-to-light ratios of 140 and 100 \((h\,M_\odot/L_\odot)\) for A999 and A1016, respectively. If we assume an average galaxy \( B - R \) color of 1.8, and a solar \( B - R \) of 1.17, we have

\[
\frac{\mathcal{M}}{L_B} = 1.79 \frac{\mathcal{M}}{L_B},
\]

which leads to \( \mathcal{M}/L_B \) values of 260 for A999 and 180 for A1016. These results are consistent with the low values found for several poor clusters (Beer et al. 1984) but are only consistent with the \( \mathcal{M}/L_B \approx 340 \) for groups in the CfA Redshift Survey (Huchra and Geller 1982; Geller 1984) at the upper end of the range in the uncertainty in our cluster mass-to-light ratios. In comparison, the Coma cluster has \( \mathcal{M}/L^r = 360 \) (Kent and Gunn 1982), while the Perseus cluster has \( \mathcal{M}/L_B \approx 600 \) (Kent and Sargent 1983). Taking a mean galaxy \( B - V = 1.4 \), and the solar \( B - V = 0.65 \), the Perseus \( \mathcal{M}/L_B \) is 1200, considerably higher than the \( \mathcal{M}/L \) ratios for A999 and A1016.

V. RELATIONSHIP BETWEEN GALAXY AND CLUSTER PROPERTIES

a) Alignments

The alignment of the major axes of the members of a flattened cluster with the cluster major axis (AdSS; Binggeli 1982; Nemiroff, Ftaclas, and Struble 1985) has been cited as evidence that the shape and orientation of both clusters and their members have a common origin. In particular, AdSS and Thompson (1976) claim such alignments for A999 and A2197, respectively.

The analysis of Sec. III shows that the distribution of the 22 galaxies in A999 is consistent with a spherically symmetric distribution. AdSS find that the galaxies in A999 have a net orientation parallel to the cluster axis, which has a position angle of \( 161^\circ \pm 5^\circ \) (AdSS). We note that 1021 + 1253, the fifth-brightest galaxy listed by AdSS as a member of A999, is actually not a cluster member. This galaxy lies on the axis identified by AdSS and is probably crucial to the identification by Rood and Sastry (1971) of A999 as a linear cluster. The 20 observed members of A999 form no real axis: for \( 0.2 < a < 0.7 \), "best-fit axis" varies between 115° and 154°, with no strong preference for any of the solutions. The meaning of the net orientation of galaxy position angles found by AdSS is thus unclear.

In contrast to A999, A1016 has a well-defined axis: for \( 0.2 < a < 0.6 \), we find a position angle of \( 51^\circ \pm 2^\circ \). AdSS find \( 49^\circ \pm 10^\circ \). In Fig. 9 we show the best-fit \( a = 0.4 \) band in A1016, together with the corresponding perpendicular band.

To obtain the orientations of individual galaxies, AdSS scanned their \( R \) band plates with the Kitt Peak National Observatory PDS microphotometer: these data were reduced with the IPPS system at KPNO. AdSS obtained 49 galaxy position angles in the field of A1016, which are accurate to \( \pm 5^\circ \). We give this angle (counterclockwise from north, in degrees) in column 6 of Tables I and II. Without redshift data to determine cluster membership, AdSS rejected galaxies fainter than a given magnitude limit (\( m_B = 16.5 \) for A1016): their position-angle samples then include 43 galaxies in A1016, of which 22 are members according to our redshift data.

AdSS conclude that the galaxies in A1016 have an isotropic distribution of position angles. Limiting the sample to the 22 known cluster members, we reach the same conclusion. We use the \( F \) test and \( \chi^2 \) test to search for differences between the position-angle distributions of spirals and ellipticals in A1016: the data are consistent with no differences.
b) Tests for Segregation in Luminosity or Galaxy Morphology

A correlation between galaxy luminosity or morphology with radial distance from the cluster center could place constraints on cluster evolution or galaxy formation. With linear clusters, we can look for correlations with distance from the cluster axis. We perform these tests for A1016, but find no significant correlations.

We use the Kolmogorov-Smirnov (KS) two-tailed test (Siegel 1956) to look for evidence of cluster rotation. We rotate the coordinates so that X is the distance along the cluster axis, and Y is the distance from the axis. We find the maximum absolute difference (Δ) between the cumulative frequency as a function of X of cluster members whose redshift is greater than the cluster median (9642 km s⁻¹) and those whose redshift is less than the median. Thus the range of Δ is 0 < Δ < 1. We have 22 cluster members in A1016: to have confidence at the 10% level that the distribution of redshifts is nonrandom, we require Δ > 0.520, but find only Δ = 0.273. A similar test listing the galaxies in order of ascending Y gives Δ = 0.455. There is no evidence for rotation in A1016.

We use a similar two-tailed KS test for luminosity segregation as a function of distance from the cluster axis. The median distance of members of A1016 from the cluster axis is 1.6'. We find Δ = 0.273: this test provides no evidence for luminosity segregation in A1016.

We note that AdSS find marginally significant evidence for axial luminosity segregation in A999. Our test would agree with the AdSS result (significant axial luminosity segregation at the 5% level) if the cluster axis was determined by the positions of the brightest cluster members only. We caution that basing the cluster axis on the positions of the brightest members alone is an intrinsically biased procedure for evaluating luminosity segregation in linear systems.

We next look for correlations between the morphology of a cluster member and some of its other properties. Following the classification of AdSS there are six ellipticals, 13 spirals, and one SO galaxy in A999, and four ellipticals, 12 spirals, and six SO galaxies in A1016. We use the KS test to determine whether the elliptical galaxies are significantly more luminous than the spirals. The 10% significance level for n₁ = 6, n₂ = 13 is Δ₀₁ = 0.602: in A999 we find Δ = 0.436, which is not significant. We make the same test for A1016, but here the quantization (in 0.1 mag intervals) of the magnitude data precludes an absolute determination of Δ because some magnitude bins contain both elliptical and spiral galaxies. We find that Δ = 0.455 or Δ = 0.75, with equal probability: thus Δ = 0.542. Comparing SO's and spirals, we find Δ = 0.333: for SO's and ellipticals, we find Δ = 0.3. The significance levels are Δ₀₁ = 0.704 for n₁ = 4, n₂ = 12; Δ₀₁ = 0.610 for n₁ = 6, n₂ = 12; and Δ₀₁ = 0.788 for n₁ = 6, n₂ = 4. There is no evidence of nonrandom luminosity ranking in either cluster.

We also test for a concentration in A1016 of either ellipticals or SO galaxies close to the cluster axis. Ranking elliptical and spiral galaxies in order of distance from the axis, we find Δ = 0.25, which is not significant (Δ₀₁ = 0.704, as above). We find Δ = 0.25 when comparing the ranking of spiral and SO galaxies (Δ₀₁ = 0.610). For SO's and ellipticals, Δ₀₁ = 0.788, but we find only Δ = 0.25. We thus find no evidence of differing spatial distributions relative to the cluster axis of any morphological type.

Finally, we compare the measured velocity dispersions for galaxies of different morphological type in both clusters. We use the F test to determine whether the differences are significant. In A999, the six ellipticals have a velocity dispersion of 242 km s⁻¹ and the 13 spirals have σ₁ = 271 km s⁻¹. The ratio of the variances is F₁₂ = 1.25, with v₁ = 12 and v₂ = 5. We would need F₁₂ > 4.7 to reject at the 5% level the hypothesis that both morphological types belong to the same parent population. Similarly, in A1016, the velocity dispersions are

- Sprial: σ₁ = 277 km s⁻¹ (12 galaxies),
- Elliptical: σ₁ = 177 km s⁻¹ (4 galaxies),
- SO galaxies: σ₁ = 180 km s⁻¹ (6 galaxies).

Comparing either SO's or ellipticals to spirals, we have F₁₂ = 2.4. Even if we combine the elliptical and SO populations, we find F₁₂ = 3.1 at the 5% level (with v₁ = 11 and v₂ = 9).

We thus find no evidence that the members of these clusters are segregated by morphological type in either velocity dispersion or luminosity. In the case of the linear cluster A1016, there is no significant evidence of preferential concentration towards the cluster axis of either the brighter cluster members or of galaxies of any given morphological type.

VI. CONCLUSIONS

The linear clusters classified by Rood and Sastry (1971) and Struble and Rood (1982, 1984) provide candidate systems that might be used to test some predictions of models for galaxy and cluster formation. The use of these systems for evaluation of physical models requires quantitative evaluation of their properties. Our linearity statistic applied to simulations of spherically symmetric clusters, each with 20 members, indicates that at least 2% of clusters in a catalog of spherical systems with 20 easily identifiable members will appear linear. This fraction is to be compared with the ~ 8% of the Abell catalog D < 4 actually found to appear linear by Struble and Rood (1982). Smaller numbers of easily identifiable members imply a larger fraction of apparently linear systems. We suggest that the increase in the fraction of linear Abell clusters with distance class may be caused by this statistical effect.

Two of the three nearby linear systems for which we have acquired extensive redshift data are probably highly flattened. A1016 is more linear than all but one of the simulated spherically symmetric systems. Chapman et al. (1987) show that A194 (with 64 confirmed members) is significantly more linear than 30 simulated spherically symmetric clusters. On the other hand, the distribution of galaxies in A999 is consistent with that in most of the simulated systems. We suggest that a superposed foreground galaxy is responsible for the classification in this case.

Naively calculated mass-to-light ratios for these systems tend to be small. The application of the virial theorem in its usual isotropic form leads to a mass of ~ 3 h⁻¹ 10¹³ M☉ for both A999 and A1016. The mass-to-light ratios (M/L_k) for these systems are 140 and 100, respectively. These values are considerably below the median for groups in the CfA survey and for richer clusters. Using a form of the virial theorem that accounts for anisotropy, we show that if the clusters are filamentary and have their shortest axis nearly along the line of sight, the mass-to-light ratio may be under-
estimated by $\sim 50\%$. In this case the values would come into agreement with those for other systems.

Although the samples are small, galaxies of all morphologies appear to share the same luminosity and velocity distributions in these two systems. We find no significant evidence for alignments of individual cluster members with the cluster axis in the convincingly linear cluster A1016. There is similarly no evidence of segregation by luminosity or morphological type in A1016. The earlier claims by AdSS of alignment and segregation in A999 may be affected by lack of redshift data. Larger samples, of course, provide stronger tests; Chapman et al. (1987) show that there is also no significant alignment in the better sampled system A194.

Although linear clusters are generally poor and hence hard to study, their extreme geometry (if confirmed by study of a larger sample) may make them one of the more reliable laboratories for examining the relationship between the large-scale cluster geometry and the properties of individual cluster members. Because the number of accessible members in a system is limited, a convincing demonstration that these systems depart substantially from spherical symmetry requires examination of a larger number of clusters. The statistics of the observed systems can then be compared with a set of simulations.

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APPENDIX

We provide some expressions that may be useful in calculating the form factor $f$ for ellipsoidal density distributions, and show that $\langle x^2/r^2 \rangle$ may be calculated as a function of the scalar potential $W$ in cases where the density distribution can be rescaled to spherical symmetry:

$$\rho(s_1,s_2,s_3) = \frac{1}{e_i e_j e_k} n(\xi),$$

(A1)

where $\xi = (\xi_1,\xi_2,\xi_3)$, $\xi_1 = s_1/e_1$, $\xi_2 = s_2/e_2$, and $\xi_3 = s_3/e_3$. The coordinates $s_1, s_2, s_3$ refer to the principal axes of the density distribution, and the axial ratios of the distribution are $e_1, e_2, e_3$. We are free to choose $e_1 > e_2 > e_3$. From Eq. (9), we have

$$W = \frac{1}{2} G \int \int \frac{n(\xi)n(\xi')d^3\xi d^3\xi'}{[\xi^2_1(\xi_1 - \xi'_1)^2 + \xi^2_2(\xi_2 - \xi'_2)^2 + \xi^2_3(\xi_3 - \xi'_3)^2]^{1/2}},$$

(A2)

so that

$$\frac{\partial W}{\partial \xi_i} = \frac{e_i}{2} G \int \int \frac{n(\xi)n(\xi')\xi_i^2(\xi_1 - \xi'_1)^2 d^3\xi d^3\xi'}{[\xi^2_1(\xi_1 - \xi'_1)^2 + \xi^2_2(\xi_2 - \xi'_2)^2 + \xi^2_3(\xi_3 - \xi'_3)^2]^{3/2}},$$

(A3)

and

$$\frac{1}{e_i} \frac{\partial W}{\partial \xi_i} = \frac{1}{2} G \int \int \frac{\rho(r)\rho(r')\langle s_1 - s'_1 \rangle^2/\xi_i d^3r d^3\nu'}. \quad \text{(A4)}$$

Thus

$$W_{11} = -e_1 \frac{\partial W}{\partial \xi_1}, \quad \text{(A5)}$$

and similarly for $W_{22}$ and $W_{33}$. From Eqs. (24):

$$\langle x^2/r^2 \rangle = \frac{2}{G d^2} \left[ e_1\cos^2\theta \frac{\partial}{\partial \xi_1} + e_2\sin^2\theta \cos^2\phi \frac{\partial}{\partial \xi_2} + e_3\sin^2\theta \sin^2\phi \frac{\partial}{\partial \xi_3} \right] W, \quad \text{(A6)}$$

with $\theta$ and $\phi$ defined by Eq. (21).

We demonstrate this method of calculating $\langle x^2/r^2 \rangle$ by applying it to a homogeneous ellipsoid. From Eq. (16), we have

$$\frac{2e_1}{G d^2} \frac{\partial W}{\partial \xi_1} = \frac{3}{5} \left[ e_1 [A_1 - e_i (\partial A_1/\partial \xi_i)] - e_j [A_j - e_i (\partial A_j/\partial \xi_i)] - e_k [A_k - e_i (\partial A_k/\partial \xi_i)] \right], \quad \text{(A7)}$$

with $A_1, A_2$, and $A_3$ defined in Eqs. (17)-(19). Now

$$\frac{\partial F}{\partial \psi} = (1 - k^2 \sin^2 \psi)^{-1/2} = e_1 \quad \text{and} \quad \frac{\partial F}{\partial \psi} = (1 - k^2 \sin^2 \psi)^{1/2} \frac{k}{\psi} \quad \text{or} \quad \frac{\partial F}{\partial \psi} = \frac{1}{e_1} \cot \psi, \quad \text{(A12)}$$

and

$$\frac{\partial E}{\partial \phi} = \frac{E - F}{k} \quad \text{and} \quad \frac{\partial E}{\partial \phi} = \frac{\partial F}{\partial k} \frac{k}{\psi} \quad \text{(A13)}$$

Also,

$$\frac{\partial E}{\partial \xi_i} = \frac{1}{e_1} \cot \psi \quad \text{(A14)}$$
\[ \frac{\partial \psi}{\partial \epsilon} = 0, \quad (A15) \]
\[ \frac{\partial \psi}{\partial \epsilon_1} = - (\epsilon_1 \sin \psi)^{-1}, \quad (A16) \]
\[ \frac{\partial k}{\partial \epsilon_1} = \frac{k^2}{\epsilon_1 k \sin^2 \psi}, \quad (A17) \]
\[ \frac{\partial k}{\partial \epsilon_2} = - \epsilon_2, \quad (A18) \]
\[ \frac{\partial k}{\partial \epsilon_3} = \frac{k \cot \psi}{\epsilon_3 \sin \psi}. \quad (A19) \]

We note that
\[ \epsilon_1 k^2 \frac{\partial}{\partial \epsilon_1} \left( \frac{1}{k^2} \right) = - 2 \frac{k^2}{k^2} \csc^2 \psi, \quad (A20) \]
\[ \epsilon_2 k^2 \frac{\partial}{\partial \epsilon_2} \left( \frac{1}{k^2} \right) = 2 \csc^2 \psi (1 - k^2 \sin^2 \psi) \frac{1}{k^2}, \quad (A21) \]
\[ \epsilon_3 k^2 \frac{\partial}{\partial \epsilon_3} \left( \frac{1}{k^2} \right) = - 2 \cot^2 \psi, \quad (A22) \]
\[ \epsilon_4 k^2 \frac{\partial}{\partial \epsilon_4} \left( \frac{1}{k^2} \right) = 2 \csc^2 \psi, \quad (A23) \]
\[ \epsilon_5 k^2 \frac{\partial}{\partial \epsilon_5} \left( \frac{1}{k^2} \right) = - 2 \csc^2 \psi (1 - k^2 \sin^2 \psi) \frac{1}{k^2}, \quad (A24) \]

and
\[ \epsilon_x k^2 \frac{\partial}{\partial \epsilon_x} \left( \frac{1}{k^2} \right) = 2 \frac{k^2}{k^2} \cot^2 \psi. \quad (A25) \]

Also,
\[ \frac{\partial \psi}{\partial \epsilon_1} = - \frac{E - k^2 F}{\epsilon_1 k^2 \sin^2 \psi}, \quad (A26) \]
\[ \frac{\partial \psi}{\partial \epsilon_2} = \frac{E}{\epsilon_2 k^2 \sin^2 \psi}, \quad (A27) \]
\[ \frac{\partial \psi}{\partial \epsilon_3} = \cot \psi \left[ \frac{E}{k^2} - \frac{k^2 \sin^2 \psi}{\epsilon_3 \sin \psi} \right] - \frac{1}{\epsilon_1 \sin \psi}, \quad (A28) \]
\[ \frac{\partial \psi}{\partial \epsilon_4} = \epsilon_4 \cot \psi + \frac{k^2 (E - F)}{\epsilon_1 k^2 \sin^2 \psi}, \quad (A29) \]
\[ \frac{\partial \psi}{\partial \epsilon_5} = \epsilon_5 (E - F), \quad (A30) \]
\[ \frac{\partial \psi}{\partial \epsilon_6} = \epsilon_6 \cot \psi (E - F), \quad (A31) \]
\[ \frac{\partial \psi}{\partial \epsilon_7} = \epsilon_7 \cot \psi, \quad (A32) \]
\[ \frac{\partial \psi}{\partial \epsilon_8} = 1 - \frac{3}{5} \csc^2 \psi. \quad (A33) \]
\[ \frac{\partial \psi}{\partial \epsilon_9} = 1 - \frac{3}{5} \csc^2 \psi. \quad (A34) \]
\[ \frac{\partial \psi}{\partial \epsilon_{10}} = 1 - \frac{3}{5} \csc^2 \psi. \quad (A35) \]
\[ \frac{\partial \psi}{\partial \epsilon_{11}} = 1 - \frac{3}{5} \csc^2 \psi. \quad (A36) \]
\[ \frac{\partial \psi}{\partial \epsilon_{12}} = 1 - \frac{3}{5} \csc^2 \psi. \quad (A37) \]
\[ \frac{\partial \psi}{\partial \epsilon_{13}} = 1 - \frac{3}{5} \csc^2 \psi. \quad (A38) \]
\[ \frac{\partial \psi}{\partial \epsilon_{14}} = 1 - \frac{3}{5} \csc^2 \psi. \quad (A39) \]
\[ \frac{\partial \psi}{\partial \epsilon_{15}} = 1 - \frac{3}{5} \csc^2 \psi. \quad (A40) \]
\[ \frac{\partial \psi}{\partial \epsilon_{16}} = 1 - \frac{3}{5} \csc^2 \psi. \quad (A41) \]
\[ \frac{\partial \psi}{\partial \epsilon_{17}} = 1 - \frac{3}{5} \csc^2 \psi. \quad (A42) \]
\[ \frac{\partial \psi}{\partial \epsilon_{18}} = 1 - \frac{3}{5} \csc^2 \psi. \quad (A43) \]
\[ \frac{\partial \psi}{\partial \epsilon_{19}} = 1 - \frac{3}{5} \csc^2 \psi. \quad (A44) \]
\[ \frac{\partial \psi}{\partial \epsilon_{20}} = 1 - \frac{3}{5} \csc^2 \psi. \quad (A45) \]
\[ \frac{\partial \psi}{\partial \epsilon_{21}} = 1 - \frac{3}{5} \csc^2 \psi. \quad (A46) \]

Substituting Eqs. (A35)-(A43) into Eq. (A7), we have
\[ \frac{\partial \psi}{\partial \epsilon_1} = - \frac{3}{5} \frac{\partial \psi}{\partial \epsilon_3}, \quad (A47) \]

which is the result given in Eq. (25).
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