As we have intimated, a cluster’s integrated luminosity depends largely on how many AGB stars it contains, while its infrared color indices will depend on the number of member carbon stars. It therefore is important to see how the number of these stars depend on the cluster parameters. Table II gives our computed numbers of oxygen stars (M) and carbon stars (C) in the helium shell thermal-flash stage for different ages t, heavy-element abundances Z (by mass), and mass-loss laws (the coefficient α). These values are strongly age-dependent, increasing toward younger clusters—an effect easily understood once we realize that the duration of the thermal-flash AGB stage is insensitive to stellar mass; hence the situation is the same as for 1–2 M⊙ stars passing through the early AGB phase (see the second paragraph above). Both M and C stars will rarely occur in the flash AGB stage in clusters ≥10−10 yr old. We have, then, a natural explanation for the absence of carbon stars from old clusters (Iben13 considers that either the material in the members of these clusters has remained unmixed, or the stars have lost their envelopes prior to the thermal-flash stage).

Our results of Table II also imply that a cluster will have fewer AGB stars if we were to assume a more intensive mass loss during this phase. If the mass-loss rate should increase abruptly when a star reaches a luminosity log (L/L⊙) = 4.1, the number of AGB stars would be sharply reduced in clusters ≥0.2–10−9 yr old, but in younger clusters the number of AGB stars would rise again. Finally, as Z increases from 0.002 to 0.006 the number of C stars drops steeply, because a greater amount of carbon has to be brought up from the interior into the envelope in order to make the abundance ratio of C and O atoms greater than unity.

This investigation was carried out at the Astronomical Council, USSR Academy of Sciences, in Moscow.

### TABLE I. Doppler Measurements of 160-min Solar Oscillations

<table>
<thead>
<tr>
<th>No.</th>
<th>Observatory</th>
<th>Type of observation</th>
<th>Mean amplitude m/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>Crimean Astrophysical[1] (1976)</td>
<td>Differential: center zone ((r &lt; 0.06)) - limb zone ((0.06 &lt; r &lt; 1))</td>
<td>0.5 ± 0.1</td>
</tr>
<tr>
<td>1b</td>
<td>Crimean Astrophysical[3]</td>
<td>Differential as in No. 1a, with equatorial belt (</td>
<td>\phi</td>
</tr>
<tr>
<td>2</td>
<td>Stanford Univ.</td>
<td>Differential: center zone ((r &lt; 0.5)) - outer zone ((0.5 &lt; r &lt; 0.8))</td>
<td>0.2 ± 0.05</td>
</tr>
<tr>
<td>3</td>
<td>Birmingham Univ.</td>
<td>Radial velocity averaged over entire disk</td>
<td>1.1</td>
</tr>
<tr>
<td>4</td>
<td>South Pole[5]</td>
<td>Radial velocity averaged over entire disk</td>
<td>0.3 ± 0.1</td>
</tr>
</tbody>
</table>

Table I summarizes the available Doppler-shift data on the 160-min solar oscillations and gives the average amplitudes measured (here and below the radius of the solar disk is taken to be \(r = 1\)). The systematic error in the velocity calibration[8] may reach 30%. These data constitute the basic observational results.

No consensus presently exists as to how the 160-min oscillations should be interpreted. Table II lists the theoretical hypotheses of which we are aware. Their diversity reflects the inadequacy of the empirical data and above all the lack of information on the way that the oscillation amplitude is distributed over the solar surface, for if one of the normal gravity modes (g-modes) is observable, then the surface distribution of the amplitude will yield the order of the corresponding spherical harmonic. Thus, to develop a reliable theory, one should first ascertain from the observations whether the 160-min pulsation does represent a nonradial mode of natural solar oscillation, and then establish the order of its spherical harmonic.

### TABLE II. Theoretical Interpretations of 160-min Oscillation

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Normal g-mode</td>
<td></td>
</tr>
<tr>
<td>Excitation mechanism:</td>
<td></td>
</tr>
<tr>
<td>1. Resonance interaction g modes</td>
<td>Christensen-Dalsgaard &amp; Gough[11]</td>
</tr>
<tr>
<td>2. Resonance interaction of 5-min oscillations</td>
<td>Kotov et al.[9], Guenther &amp; Demarque[12]</td>
</tr>
<tr>
<td>3. Breakup instability</td>
<td>Bismovtii-Kogan[22]</td>
</tr>
<tr>
<td>4. Nuclear mechanism</td>
<td>Soli[13], Vandavrov[14]</td>
</tr>
<tr>
<td>5. Internal magnetic field</td>
<td>Wolff[15], Zatsepin et al.[16]</td>
</tr>
<tr>
<td>6. Convection motions</td>
<td>Drimmelowski et al.[17]</td>
</tr>
<tr>
<td>7. Anisotropic turbulent pressure</td>
<td>Dolginov &amp; Musilov[18]</td>
</tr>
<tr>
<td>8. Low-mass star passing near the sun</td>
<td>Drobyshavski[19]</td>
</tr>
<tr>
<td>9. Gravitational waves encoutersing the sun</td>
<td>Kosovichev &amp; Severnyi[20]</td>
</tr>
<tr>
<td>Delache et al. (see Walgata[21]), Kotov[22]</td>
<td></td>
</tr>
</tbody>
</table>

| II. Other hypotheses |
| 1. Nonlinear anharmonic oscillation | Dappen & Perdang[23] |
| 2. Soliton in core of sun | Childress & Perdang[3] |
| 4. Universal period of gravitational interaction | Kotov[26] |
where R, M are the radius and mass of the sun and 
\( \omega \) is the oscillation frequency.

We have calculated the normal g-modes of oscillations with periods of about 180 min and we find that the value (2) is accurate to within 10% or better. From Eqs. (1), (2) one can calculate the average velocity \( v_0 \) of the oscillations along the line of sight as a function of the numbers \( \ell, m \) for each type of observation described in Table 1:

\[
v_0 = V \cdot S_{\ell m},
\]

in which the \( S_{\ell m} \) values determined for observing programs Nos. 1-4. We have here used the expressions given by Christensen-Dalsgaard and Gough, somewhat modified for case 1b. Similar calculations (except for 1b) have been carried out by Christensen-Dalsgaard and Dziembowski et al., but in the case of the Crimean observations these authors have erroneously taken the difference in mean radial velocity between the entire solar disk and the limb zone, whereas the quantity actually observed was the difference between the center zone (\( r \leq 0.66 \)) and the remaining annulus. Furthermore, in our calculations the symmetry axis of the normal modes is aligned with the sun’s rotation axis, so that the effects of rotation can be accommodated. In other respects our results are in accord with the earlier calculations (Refs. 29, 17).

Since all the radial-velocity measurements were symmetric about the solar equator, modes with an odd sum \( \ell + m \) will have \( 4^4 S_{\ell m} = 0 \); hence these observations could have recorded only those normal modes for which \( \ell + m \) is even. One should also bear in mind that, as Christensen-Dalsgaard et al. have shown (Ref. 31), the 160-min normal mode cannot be radial (that is, with \( \ell = 0, m = 0 \)).

From the values quoted in Tables I and III together with Eq. (3), we can draw several conclusions:

1. The 160-min pulsation cannot be a normal quadrupole mode (\( \ell = 2 \)); if it was, then as the difference between the corresponding spatial filters indicates, it would not have been observed in the same phase at Stanford and in the Crimea.

2. Nor can the oscillation be an \( \ell \geq 4 \) mode: radial-velocity observations averaged over the whole disk are insensitive to oscillations of high orders \( \ell \) (because of the mutual averaging), and the amplitude of such a mode measured for the full disk would have been lower than in the differential observations by an order of magnitude.

3. If the oscillation were an \( \ell = 1 \) normal mode, then according to our calculations the measurements in the Antarctic and by the Birmingham group would have yielded an amplitude nearly 20 times that recorded at Stanford and in the Crimean, again contravening Table I.

4. On the other hand, the data of Tables I and III are compatible with the possibility that the 160-min oscillation is an \( \ell = 3 \) mode. In this event the computed and measured amplitudes would not be in conflict.

Thus an intercomparison of the phase and amplitude data obtained at the different observatories suggest that the 160-min pulsation might be an \( \ell = 3 \) mode.

Hill et al. have recently sought to ascertain what mode the 160-min oscillation is by comparing their solar-diameter measurements against the Crimean and Stanford Doppler data. In contrast to our practice, however, they calculate the spatial filters on the basis of an apparent-velocity hypothesis, whereby the measured Doppler line shifts would represent not physical displacements of material on the solar surface but perturbations due to temperature fluctuations conjoined with solar rotation effects. Accordingly, Hill et al. conclude that the 160-min oscillation can be classified as an \( \ell = 2, m = 2 \) mode — a hypothesis which they introduce to account for the 2-dex disparity between the oscillation amplitudes in the Doppler-velocity and the solar-diameter measurements.

In support of this finding, Hill and Logan have calculated the linear nonadiabatic oscillations in the solar envelope and have determined that the relative temperature perturbations amount to \( \approx 10^5 \) times the relative changes which such oscillations would produce in the radius of the sun due to nonlocal radiation effects in the solar atmosphere. But calculations by other authors (see Christensen-Dalsgaard’s summary) fail to indicate so strong an influence of these effects upon the sun’s natural oscillations, and thus do not confirm the \( \ell = 2, m = 2 \) suggestion of Hill et al. Clearly before it is accepted further observations and theoretical analysis will be needed: Meanwhile in our own interpretation we shall adhere to the point of view adopted earlier: the Doppler line shifts observed do represent oscillatory motions of material on the solar surface.

Now let us consider the rotational-splitting effect. The oscillation eigenfrequencies form multiplet
TABLE IV. Rotational-Splitting Parameters of 160-min Mode

<table>
<thead>
<tr>
<th>( i )</th>
<th>( c_{kl} )</th>
<th>( \Delta \nu / 2\pi ), ( \mu )</th>
<th>( P_{n,k,\mu} ), min</th>
<th>( (m = m_{\pm} \pm 3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.503</td>
<td>0.290</td>
<td>158.44</td>
<td>160.61</td>
</tr>
<tr>
<td>2</td>
<td>0.157</td>
<td>0.705</td>
<td>150.83</td>
<td>161.10</td>
</tr>
<tr>
<td>3</td>
<td>0.075</td>
<td>0.975</td>
<td>150.82</td>
<td>161.22</td>
</tr>
<tr>
<td>4</td>
<td>0.045</td>
<td>0.809</td>
<td>158.78</td>
<td>161.26</td>
</tr>
</tbody>
</table>

structures:  

\[ \omega_{n,m} = \omega_{ad} + m (1 - c_{ad}) \Omega_{z}, \tag{4} \]

where \( \Omega_{z} \) is the sun’s mean angular rotational velocity and the \( c_{n,k} \) are numerical constants that depend on the oscillation mode and the model for the internal structure of the sun. Equation (4) expresses the eigenfrequencies in an inertial coordinate system.

For comparison with the observations one must correct for the earth’s orbital motion about the sun at angular velocity \( \Omega_{e} \). An observer on earth will then record solar oscillations with eigenfrequencies

\[ \omega_{n,m} = \omega_{ad} + m (1 - c_{ad}) \Omega_{z} - m \Omega_{e}. \]

Suppose that the 160-min oscillation is a mode described by parameters \( n_{1}, \xi_{1}, m_{2} \). Since oscillation modes with the same \( n_{1}, \xi_{1} \) values but differing \( m \) will have similar physical parameters, it is entirely possible that along with the 160-min mode oscillations having \( n = n_{1}, \xi = \xi_{1}, m \neq m_{2} \) will be excited. A search can be made for such modes in the empirical power spectrum, although one should keep in mind that these oscillations might not be observed if the 160-min pulsation were to be excited by a mechanism that operates only for certain \( m \) (such as mechanisms 1, 1, 3, 5 of Table II). As we have said, the observations are sensitive to the internal modes with even \( \pm m \) values, so in addition to the 160-min mode (\( \xi = \xi_{1}, m = m_{2} \)) the power spectrum might exhibit peaks corresponding to oscillation modes with \( \xi = \xi_{2} \) and with \( m = m_{1} \pm 2, m = m_{2} \pm 4, \ldots \)

The closest modes, those with \( m = m_{2} \) and \( m = m_{1} \pm 2 \), will have a frequency difference

\[ \Delta \omega_{\xi} = \omega_{n_{1},\xi_{1},m_{2}} - \omega_{n_{1},\xi_{2},m_{2}} = \pm \left[ 2 (1 - c_{ad}) \Omega_{z} - 2 \Omega_{e} \right]. \tag{5} \]

Adopting the standard model for the sun’s internal structure we have computed for several \( \xi \) the \( c_{n,k} \) values for g-modes having periods near 160 min. The results are given in Table IV (similar values are obtained for other solar models), along with values of \( \Delta \omega_{\xi} / 2\pi \) computed from Eq. (5) for \( \Omega_{z} = 2\pi/(25^{*}30) \) and \( \Omega_{e} = 2\pi/(365^{*}20) \), as well as the periods of the \( m = m_{2} \pm 2 \) modes (\( P_{n_{1},\xi_{2},m_{2}} = 160.01 \) min).

On comparing these computed periods with those observed by our group, we find agreement in just one case: the observed 159.3-min period corresponds to the tabular \( \xi = 1 \) value, 158.4 min. Hence if the 160.61- and 159.3-min oscillations represent components of the same rotational multiplet, then both would have a spherical-harmonic order \( \xi = 1 \), with \( m = -1 \) for the first and \( m = 1 \) for the second. It is noteworthy that a superposition of these two oscillations would yield the effect detected in 1983: an amplitude modulation of the 160-min oscillation with a period close to the sun’s 27th rotation period [1/(159.3 min) - 1/(160.0 min) = 1/253], so that one maximum in the oscillation amplitude would in fact be observed in each solar rotation cycle.

We see, then, that when the observed amplitude phases are analyzed by calculating spatial filters and by considering the rotational splitting of the g-mode periods, contradictory results are obtained. In the first case we have found that the 160-min oscillation most likely represents the g-mode with \( \xi = 3 \), while in the second case it turns out to be a dipole mode (\( \xi = 1 \)). If the former result is right, then we have erred in supposing that the observed 159.3-min fluctuation is caused by rotational splitting of the modes; presumably it would have an order \( \xi \) different from that of the 160-min oscillation. In structure, we should conclude that the amplitude of the 160-min oscillation has been incorrectly determined; it would differ from the actual value by nearly an order of magnitude, either for the Crivine and Stanford observations or for the whole disk measurements. For the time being the available data are inadequate to warrant a definite choice.

In acquiring further information it would certainly be best to observe the sun with different spatial filters but one and the same telescope. When using the differential technique one may select filters such that oscillations with a particular \( \xi \) value would not be observed; that is, the Doppler-shift difference between the central and annular zones of the solar disk would be zero. Table V gives the spatial filters calculated so as to exclude any \( \xi = 2, 3, 4 \) oscillations from the Crivine observations. In particular, to tell whether the 160-min oscillation is an \( \xi = 3 \) mode one would have to measure the Doppler-signal differential between a central zone of 0.35 radius and the ring 0.35 \( \leq r \leq 0.76 \). If no oscillation is detected, one would have evidence that the 160-min effect is indeed an \( \xi = 3 \) normal mode.

The second alternative, \( \xi = 1 \), clearly cannot be tested in this way, but a different approach can be taken here. Since our second possibility presupposes that rotational mode splitting is being observed, we can check it by attempting to discriminate a third component of the rotational multiplet (\( \xi = 1, m = 0 \)) which cannot be observed with a
symmetrical filter. This oscillation ought to have a 159.7-min period. It could be recorded by covering the images of the sun's northern and southern hemispheres with opaque screens. One should then observe all three $\ell = 1$ components: $m = -1, 0, 1$. Since the differential center-limb observing technique is insensitive to the 160-min $\ell = 1$ mode (Table III), a more effective method could be applied: measuring the difference in radial velocity between the sun's entire northern and southern hemispheres. Our calculations indicate that in this event only the $\ell = 1, m = 0$ mode ought to be observed, with a $\approx 159.7$-min period and an amplitude about 10 times that recorded in center-limb observations.

Our results do demonstrate that the possibilities of interpreting the 160-min pulsation as a nonradial normal mode of the sun have by no means been exhausted. In our opinion too little information is currently available for there to be any need to resort to radical speculations.

We should like to thank Dr. Henry A. Hill for communicating the results of his research in advance, as well as Professor W. A. Dziembowski for a valuable discussion.