of $\sim 3 \times 10^{15}$ cm and a mean expansion velocity of $\sim 9$ km s$^{-1}$. The SiO masers are located within a radius of about $10^{15}$ cm and have expansion velocities $< 6$ km s$^{-1}$ (Lane 1984).

The H$_2$O emission region is as extensive as the main-line OH region and the distribution of these masers is asymmetric. The H$_2$O masers lie approximately east-west whilst the main-line OH masers lie approximately north-south. The 1612 MHz OH emission also shows evidence of asymmetries. The maps at this frequency show shell-like structure which is strongest in the east and west and has gaps in the north and south. The envelope asymmetries may be due to the influence of the stellar magnetic field. The magnetic field in the envelope is determined from Zeeman splitting of the main-line OH spectral lines to be $\sim 5$ mG at a radial distance of $3 \times 10^{15}$ cm. A Zeeman pattern is confirmed by the coincidence in position and velocity of OH masers at 1665 MHz and 1667 MHz.

The velocity field of the circumstellar envelope as traced by SiO, H$_2$O and OH maser lines shows a systematic increase of velocity with radial distance from the star. The observed velocity curve cannot be produced by a $1/R^2$ radiation pressure law and it is suggested that grain material condenses to a radial distance of about 100 stellar radii.


References


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MODELLING EXTENDED CHROMOSPHERES

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1. Evidence for and parameters of extended chromospheres

Since Deutch's (1956) pioneering study of the $\alpha$ Herculis system, ample evidence has accumulated for the existence of extended envelopes and large mass loss rates from late-type giants and supergiants (cf. reviews by Cassinelli 1979 and Linsky 1985a). This evidence includes blue-shifted absorption components in infrared, optical, and ultraviolet lines from single stars and from binary systems containing a hot component (i.e., the $\zeta$ Aurigae and VV Cephei systems), infrared emission from dust, molecular absorption features, and microwave continuum and line emission. Although certain diagnostics like infrared emission from dust indicate cool circumstellar material
at several hundreds of degrees, other diagnostics can only be explained by the presence of warm (5,000–10,000 K) gas and/or large electron densities that imply that hydrogen is significantly ionized in the outer atmospheres of luminous K and M stars. The geometrically extended nature of this warm gas is implied by the He I 10830 emission observed by Zirin (1982) in the K bright giants and supergiants, direct speckle imaging in the Hα line (Beckers 1985), C II and Si III emission features in α Boo (Ayres, Simon and Linsky 1982) and other stars, and microwave free-free emission observed in α Boo, ρ Per, μ Gem, α¹ Her, and β UMi (Drake and Linsky 1985, 1986a). The microwave fluxes, which are proportional to the emission measure, imply emitting regions for the ionized gas at 6 cm twice that of the photosphere, but the particularly important 11 μm spatial heterodyne interferometry measurements of Sutton et al. (1977) (cf. Low 1979) show that the dust shell around α Orionis is not significant until about 12 photospheric radii from this star.

The inevitable conclusion from this very brief summary is that warm, partially ionized plasma, which I call a chromosphere, extends outward at least several photospheric radii from these stars. While the existence of these extended chromospheres is well established, their plasma properties are difficult to measure. For example, Stencel et al. (1981) proposed that the ratio of lines within the C II 2325 Å multiplet is a useful density diagnostic in the range 10⁷ < nₑ < 10⁹ cm⁻³. Carpenter, Brown and Stencel (1985) estimated the geometrical extent of the C II emitting regions using this density diagnostic together with the emission measure in the 2325 Å multiplet and temperature estimates. This work shows that chromospheres are geometrically thin compared to the photospheric radius in the G giants, which are located to the left of the transition region dividing line (Linsky and Haisch 1979, Simon, Linsky and Stencel 1982), but are geometrically extended (several photospheric radii) in the K-M giants and supergiants, which are located to the right and above. However, Judge (1985) has pointed out inadequacies in this diagnostic technique and further work is needed before it can be relied upon for estimates of temperature, density, and geometrical extent. Also, the C II 2325 Å multiplet is almost certainly formed at the base of the chromosphere where nₑ is large and contains little information on the extended chromosphere. No real estimates of plasma properties in the extended chromospheres have as yet been forthcoming from the other diagnostics already mentioned, except one estimate of temperature for the extended chromosphere of the ζ Aur system 32 Cygni made by Che-Böhnstenstengel (1984) using Fe II line ratios, but this estimate depends crucially on the uncertain hydrogen ionization fraction.

A very different approach involves computing an extended atmosphere model directly from the energy and momentum equations with input parameters chosen to achieve reasonable agreement between observed and computed line fluxes and profiles. Hartmann and Avrett (1984) pursued this synthetic approach in deriving a model for α Ori. They assumed Alfvén waves as their source of momentum and heating in a spherically-symmetric steady-state atmosphere, and adjusted the initial Alfvén flux and damping length to produce a mass loss rate of 10⁻⁶ M⊙ yr⁻¹, reasonable terminal velocity, and fluxes and profiles for the Ca II, Mg II, Si II, Hα, and microwave emission similar to observations. In their model, chromospheric temperatures of 5000–8000 K extend out to about 10 photospheric radii (rₑ). This approach should be pursued...
further, but the derived model parameters are dependent upon many assumptions that are not readily verified. In this paper I pursue an alternative strategy.

2. Modelling technique

Optically thick spectral lines formed in an extended atmosphere for which the expansion velocity increases outward contain information about the chromospheric parameters as a function of radial position in their line profiles. Decoding this information requires a physically realistic description of the line formation process, reasonable accuracy for the assumptions of spherical symmetry and monotonic outflow velocity with radial position, and good fortune that the solutions are reasonably unique. The latter two concerns can only be addressed when the first is properly handled. Three spectral features almost certainly formed well out in the extended chromospheres of luminous cool stars are the resonance lines of Mg II (h and k lines at 2802 and 2796 A), O I (1302, 1305, and 1306 A) and H I (Lz at 1216 A). I describe here calculations for the Mg II k line because this ion is relatively simple and the interstellar Mg II absorption feature is often small or can be corrected for. However, the techniques described here can and should be applied to all of these lines together to confirm the results obtained using the k line only.

The dominant excitation process for optically thick resonance lines formed in low density plasmas is resonance scattering, with collisional excitation creating the photons which are then scattered $10^4$–$10^6$ times before they leave the atmosphere or are destroyed by a collisional de-excitation from the upper level. Changes of frequency during each scattering are due to Doppler motions of the scattering ion, collisional changes of the energy of the excited state, radiative damping, and excitations to higher states followed by radiative decay. These processes must be included at least approximately if the calculations are to be relevant to the modelling process. The term partial redistribution (PRD) is used to describe the scattering process in the observer’s frame which is noncoherent (photons are completely redistributed) within the line core due to Doppler motions, and which is nearly coherent in the line wings, because scattering is coherent in the atom’s frame and Doppler velocities are small compared to the velocity separation between the wings and line centre. Linsky (1985b) and Hubeny (1985) have reviewed the development and present status of PRD radiative transfer techniques and calculations.

For a static plane-parallel homogeneous atmosphere, methods for computing emergent profiles including PRD are well in hand; however, there are complexities in the treatment of interlocking transitions to other levels and the collisional redistribution rates that are commonly glossed over in order to make the computations tractable. Relaxing the static and plane-parallel geometry assumptions raises additional complexities and there are few calculations in the literature relevant to cool star extended chromospheres.

In two early papers Vardavas (1974) and Cannon and Vardavas (1974) calculated PRD line profiles for an expanding atmosphere using the $R_l(x',x)$ redistribution function (which includes Doppler motions but unrealistically assumes zero intrinsic line width) in the observer’s frame. [For the definition of the term “redistri-
bution function” and a description of the different types, see Linsky (1985b)]. They found large differences between line profiles computed with CRD (complete redistribution within the absorption line profile) and PRD, which Magnan (1974) argued must be incorrect. Mihalas et al. (1976b) showed that in a moving atmosphere accurate source functions can be computed in the observer’s frame only when the full angular and frequency coupling is taken into account for the frequency redistribution, which is computationally very expensive. However, calculations in the comoving frame of the fluid are greatly simplified because then the scattering atoms see a nearly isotropic radiation field of limited frequency range, so that angle-averaged static redistribution functions are a good approximation. They then formulated the PRD transfer problem in the comoving frame. Vardavas (1976) showed that accurate solutions to the PRD differentially expanding atmosphere problem can be obtained using the comoving frame formulation in an isothermal atmosphere, and Vardavas and Cannon (1976) did the same for a schematic chromosphere model with \( R_{\text{I}}, A (x', x) \) redistribution, which includes Doppler motions and radiation damping for the upper state.

In subsequent theoretical developments, solutions of the radiative transfer equation were generally formulated in the comoving frame for spherically symmetric geometries. These calculations generally assumed CRD or, nearly equivalently, the Sobolev approximation, since the intended applications were for atmospheres of hot stars where the expansion velocities are much larger than the thermal and turbulent motions. To test the validity of the CRD approximation, Mihalas, Kunasz and Hummer (1976a) solved the transfer equation for spherically symmetric flows with an angle-averaged redistribution function in the comoving frame using a variable-Eddington factor iterative scheme. Mihalas (1980) extended this work to include the full angle and frequency dependence of redistribution.

Since winds in late-type giants and supergiants have expansion velocities of only a few times the combined thermal-turbulent half-width, the CRD and Sobolev approximations should not be accurate. To explore the range of velocities and chromospheric geometries that should be representative for the late-type giants, Drake and Linsky (1983) made a series of calculations using the comoving frame PRD code developed by Mihalas et al. (1976b). In their exploration of parameter space they found that with increasing flow speed and geometrical extension, the effect on the Mg II line profiles is to suppress the blue emission peak, shift the central minimum \( (k_2) \) to the blue, and enhance the red emission peak giving the Mg II lines a P Cygni-like character. For all cases in which the maximum flow speed is less than six times the Doppler width, the PRD profiles are very different from the CRD case. They also explored the effects on the line profile of changing the chromospheric temperature gradient and the location of the chromosphere.

3. Calculations of Mg II line profiles for Arcturus

I report here on calculations by Drake (1985) and Drake and Linsky (1986b) for the Mg II k line of Arcturus (\( \alpha \) Boo, K2 III), assuming a spherically-symmetric chromosphere, two-level Mg²⁺ ion, and angle-averaged \( R_{\text{I}}, A (x', x) \) redistribution.
functions. These calculations were based on an original program kindly provided by Paul Kunasz and modified to include partial redistribution of this particular type.

We have adopted a semi-empirical approach to fitting an observed Mg II profile. For the photosphere and temperature minimum region of the atmosphere of α Boo where \( v_{\text{wind}} \ll v_{\text{Dopp}} \), we have adopted the Ayres and Linsky (1975) plane-parallel model atmosphere. This assumption has little effect on the modelling of the Mg II k line emission core. For the outer region of the chromosphere, where the outflow velocity is significant, we choose on a trial and error basis the following quantities: \( T_e(r), \ v_{\text{wind}}(r), \ v_{\text{Dopp}}(r), \) and \( dM/dt \). The electron density is computed from a rudimentary non-LTE calculation of the hydrogen ionization equilibrium and an LTE treatment for the metals. (The density \( \rho(r) \) is implicitly fixed, given \( dM/dt \) and \( v_{\text{wind}}(r) \), by the equation of continuity). Since all of these parameters help determine the resultant line profile, it is difficult to determine the actual atmospheric structure uniquely and more than one valid solution that fits the observed profile might well exist.

Given this input model atmosphere, the radiative transfer in the specified line is carried out in the co-moving frame, and the emergent line profile calculated after a transformation into the observer's frame (Mihalas et al. 1976a). The theoretical line profile of say, the k line, is then compared with the observed line profile (see Figure 1), and the nature of the discrepancies between the two is generally helpful in guessing a new "improved" set of input parameters. This procedure is repeated for many iterations until “optimal” agreement is reached between the two profiles. Because of the number of free parameters and the wide range of parameter space that must be explored, this iterative process requires a large amount of time and the decision as to when the “optimal” or best fit has been reached is clearly subjective. Since this process may not lead to unique atmospheric parameters, we believe that it is crucial to compare the predicted properties of the “optimal” fit model(s) with as many other observational constraints as possible.

In Figure 1, we compare the observed Mg II k profile in α Boo with that predicted by two model atmospheres. The agreement is not perfect: Model A has about the right peak emission but the blue-shifted absorption is weaker than is observed, while model B fits the absorption well but underestimates the peak emission by a factor of 2. In Figure 2, the temperature and velocity structure of Models A and B are shown. The other properties of these particular model atmospheres can be summarized as follows:

(i) The wind velocity and electron temperature climb steeply to their maximum values of \( 35-40 \) km s\(^{-1}\) and \( 8000-8400 \) K, respectively, in a fairly short distance above the photosphere.

(ii) There is a broad high temperature plateau with \( T_e \gg 7 \times 10^3 \) K extending from 1.2 to \( \sim 13 \) \( r_s \).

(iii) There is a region further out, extending from 13 to 50-100 \( r_s \), where the temperature is rather cooler \( (T_e \sim 5 \times 10^3 \) K\).

(iv) The mass loss rate is \( 2 \times 10^{-10} \) M\(_\odot\) yr\(^{-1}\), and the ionization fraction of the outer atmosphere is about 50%.

(v) The maximum microturbulence reached is \( 5 \) km s\(^{-1}\).
The observed profile of Mg II k in α Boo is compared with calculated profiles for Model A and Model B. The ordinate is monochromatic luminosity in ergs s$^{-1}$ Hz$^{-1}$ and the abscissa is radial velocity relative to the stellar photosphere in km s$^{-1}$. The models are discussed in the text.

The major difference between Model B and Model A is that the latter has an even more extensive high temperature plateau region extending as far as 35 r$_{\odot}$, which is required to produce the stronger Mg II peak emission of this model. Plotted in Figure 3 are various parameters for Model B as a function of radial distance in units...
of the photospheric radius ($r_\text{\#}$). Specific predictions that can be made based on Model B are:

(i) The angular size of Arcturus in the Mg II k line is large: compared to a photospheric angular diameter of 0.021 arcsec, the predicted size of $\alpha$ Boo in the integrated Mg II emission is $\sim 0.27$ arcsec.

(ii) The free-free emission from the ionized wind of $\alpha$ Boo should make it a radio continuum source of $\sim 0.4$ mJy at 6 cm and the 6 cm angular diameter (half-power points) should be 1.7 times the photospheric angular diameter.

(iii) The angular size of Arcturus imaged in the blue absorption component of Mg II k is $\sim 1$ arcsec.

The only one of the above predictions that has been tested to date is that $\alpha$ Boo should be a radio source. Drake and Linsky (1985, 1986a) have detected $\alpha$ Boo as a 0.27 mJy source at 6 cm, in good agreement with the predicted value. The remaining predictions should be easily verifiable or disproven by the Faint Object Camera on Space Telescope.

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![Graph](image)

**Figure 2.** The wind velocity and electron temperature for Models A and B are shown as a function of radial distance above the photosphere.
Figure 3. Parameters for Model B as a function of radial distance in units of the photospheric radius \(r_\star\). The quantities plotted are the electron density \(n_e\) and optical depths at the centre of the k line \(\tau_{\text{line}}\), adjacent continuum \(\tau_{\text{cont}}\), and in the microwave continuum at 6 cm \(\tau_{6\text{ cm}}\).
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