MODELS AND THEORETICAL SPECTRA OF ACCRETION DISCS IN DWARF NOVAE

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МОДЕЛИ И ТЕОРЕТИЧЕСКИЕ СПЕКТРЫ ДИСКОВ АККРЕЦИИ В КАРЛИКОВЫХ НОВЫХ

Предложен новый метод моделирования дисков аккреции в карликовых новых, позволяющий согласовано определить структуру диска вместе с полем излучения. Цилиндрически симметричный диск разделен в ряд концентрических колец. О каждом из них предполагается, что оно ведет себя независимо как плоскопараллельный излучающий слой. Вертикальная структура каждого кольца определена совместным решением уравнений гидростатического равновесия, энергетического равновесия и переноса излучения. Соответствующие граничные условия зависят от радиального расстояния от центральной звезды и соответствуют канонической модели стационарного диска аккреции. Результатирующая система нелинейных уравнений решается модифицированным методом полной линеаризации.

Численные вычисления были выполнены для стационарного диска с центральной звездой массой 1 M⊙ и радиуса 5 × 10⁸ см. Поток массы через диск принимался 10⁻⁷, 10⁻¹⁰ и 10⁻¹¹ массы Солнца в год. Теоретическое распределение потока излучения сравнивалось с наблюдаемым распределением карликовой новой WX Hui. Результаты сравнения указывают на то, что спокойному состоянию WX Hui соответствует поток массы через диск 10⁻¹¹ а вспышка по-видимому вызвана повышением потока массы на 1.5 порядка.

We present a new method of modelling accretion discs in dwarf novae devised to determine self-consistently the structure of a disc together with the radiation field. The basic idea is to divide a (cylindrically symmetric) disc into a set of concentric rings, each of them behaving like an independent plane parallel radiating slab. The vertical structure of each ring is determined by solving simultaneously the hydrostatic equilibrium, the energy balance, and the radiative transfer equations. The appropriate boundary conditions, which depend on the radial distance from the central star, correspond to the canonical model of stationary accretion discs. The resulting set of non-linear, highly coupled equations is solved by a suitable modification of the complete linearization technique.

The numerical calculations are carried out for a stationary disc model with a white dwarf central star of mass 1 M⊙ and radius 5 × 10⁸ cm; the mass flux through the disc is taken to be 10⁻⁷, 10⁻¹⁰, and 10⁻¹¹ M⊙/year. A comparison of the theoretical radiative flux distribution with the observed distribution for WX Hui indicates that the quiescent state of WX Hui corresponds to a mass flux of about 10⁻¹¹ M⊙/year, while its superoutburst is probably caused by an increase of the mass flux by 1.5 dex.

Key words: stars: binaries: c’ose, accretion, accretion discs, atmospheres of—novae

1. Introduction

There is a number of papers devoted to modelling of stationary accretion discs and computing the corresponding theoretical spectra. However, neither the approximate law \( F \propto v^{1/3} \) (or \( F \propto \lambda^{-2.33} \)), deduced by Lynden-Bell (1969; see also Bath et al., 1980) for very extended discs radiating as a superposition of blackbodies, nor subsequent, more involved calculations using the blackbody approximation, have matched the observed spectra satisfactorily. Likewise, recent sophisticated studies using classical model atmosphere fluxes instead of blackbody fluxes


All the models mentioned above were computed assuming that the disc is optically thick everywhere for continuum radiation. Yet, dwarf novae exhibit strong emission lines (Warner, 1976), which indicates that the quiescent discs may be at least partly transparent for continuum radiation. Indeed, the spectra taken during quiescence mostly differ from the theoretical spectra (see, e.g., Hassall et al., 1983), Tylenda (1981) and Williams and Ferguson (1982) elaborated a simple
method for modelling optically thin discs, yet their approach reduces to the blackbody approximation for the optically thick parts of a disc. Also the empirical treatment of optically thin parts of discs used by Meyer and Meyer-Hofmeister (1983) is only coarse. Generally speaking, the existing approaches do not treat the interaction of radiation with disc matter and consequent radiative transfer phenomena properly.

On the other hand, contemporary detailed spectrophotometric observations of dwarf novae in quiescence as well as in outbursts (see, e.g., Hassall et al., 1983; Wargau et al., 1983; Verbunt et al., 1984; Schwarzenberg-Czerny et al., 1985) merit an interpretation by means of appropriate theoretical spectra.

Therefore, we propose a new self-consistent method which avoids some simplifications used in the previous studies, namely

(i) the disc is not considered as a semi-infinite atmosphere but rather as a finite slab of a gas;
(ii) its optical thickness is a parameter that directly follows from the model;
(iii) the total radiative flux is not constant as in the classical atmospheres, but increases upwards (i.e. from the central plane of the disc to its surface); its value being determined through the energy balance equation;
(iv) analogously, the gravity acceleration is not constant and depends on the distance from the central plane;
(v) the vertical structure of a disc (temperature, density, radiation field, etc.) follows from the appropriate model calculations.

Meanwhile we assume the LTE approximation to be valid.

2. Adopted Physical Picture of a Disc

We shall adopt the principles of the canonical picture of accretion discs introduced by Shakura and Sunyaev (1973) and Lynden-Bell and Pringle (1974; hereinafter referred to as LBP). Specifically, we consider a cylindrically symmetric, Keplerian disc. In this case, the rotation velocity, the angular velocity, and the specific angular momentum are respectively given by

\[
\begin{align*}
\varv_\varphi &= \left( \frac{Gm_\ast R}{R^2} \right)^{1/2}, & \Omega &= \left( \frac{Gm_\ast}{R^3} \right)^{1/2}, \\
\frac{h}{R^2} &= \left( \frac{Gm_\ast}{R^3} \right)^{1/2}.
\end{align*}
\]

Here \(G\) is the gravitational constant, \(m_\ast\) the mass of the central star, and \(R\) distance from the axis of symmetry going through the centre of the star and perpendicular to the orbital plane of the disc.

It is assumed that the thickness of the disc is much less than \(R\).

According to LBP, the couple between the material inside a cylinder of radius \(R\) and the material outside it due to the viscous stress is

\[
g_\varphi = -2\pi R^3 \vartheta \Sigma \frac{\partial \Omega}{\partial R},
\]

\(\Sigma\) being the surface density defined as

\[
\Sigma = \int_{-\infty}^{\infty} \vartheta \, dz,
\]

and \(z\) is the distance from the central plane of the disc; \(\vartheta\) is the viscosity per unit surface density (we use index \(v\) because \(v\) is reserved for frequency); \(\vartheta\) being the density.

For a stationary disc with the zero viscous couple between the disc and the central star, LBP deduced that the couple

\[
g_\varphi = \dot{m}_\ast \left[ (Gm_\ast R)^{1/2} - (Gm_\ast R_\ast)^{1/2} \right],
\]

the surface density

\[
\Sigma = \frac{\dot{m}_\ast}{3\pi v_\varphi} \left[ 1 - \left( \frac{R_\ast}{R} \right)^{1/2} \right],
\]

and the frictional dissipation rate per unit surface area

\[
D = \frac{3Gm_\ast v_\varphi}{4\pi R^3} \left[ 1 - \left( \frac{R_\ast}{R} \right)^{1/2} \right],
\]

where \(R_\ast\) is the radius of the central star, and \(\dot{m}_\ast\) the increase of the stellar mass corresponding to the mass flux through the disc.

The crucial problem of all calculations concerning the structure of accretion disc is the knowledge of the nature and the properties of the viscosity. Even the usually used \(\alpha\)-approach (see Shakura and Sunyaev, 1973) is not exactly justified, for it is based on a more or less arbitrary scaling factor. To avoid difficulties connected with the temperature-dependent viscosity in \(\alpha\)-discs, we express the viscosity through the mediation of the Reynolds number \(Re\), i.e.

\[
\frac{\vartheta}{R^2} = \frac{\dot{m}_\ast}{\dot{m}_\ast} = \frac{h}{Re}.
\]

According to LBP, the effective Reynolds number of the accretion flow should be equal to the critical Reynolds number for the onset of turbulence. Thus, we assume that \(Re\) is constant throughout the disc, and treat it as an empirical model parameter. Such a simplified procedure has been already used by Kříž (1982) and Williams and Ferguson (1982). We have
adopted $Re = 5000$ in our calculations; further discussion is presented in the Appendix.

Equations (2.5) and (2.6) only provide a global information, i.e. they describe some properties of the disc integrated over the coordinate $z$. Consequently, we can use $\Sigma$ and $D$ as boundary conditions, but we need some additional information in order to be able to specify relevant equations that determine the disc stratification along the $z$-direction. To this end, we have to consider the momentum and the energy balance along the $z$-direction.

(i) We assume that the hydrostatic equilibrium along the $z$-direction holds, i.e. that the gradient of the total pressure (a sum of the gas, radiation, and turbulent pressures — see also Sect. 3) is balanced by the $z$-component of the gravity acceleration, $g$ (notice that the $R$-component is balanced by the centrifugal force). Since $z \ll R$, the gravitational potential is simply given by

$$U = -\frac{Gm_\ast}{R} \left(1 - \frac{z^2}{2R^2}\right),$$

and thus

$$g = Q \cdot z, \quad Q = \frac{Gm_\ast}{R^3}.$$

(ii) As an energy conservation constraint, we assume that the net radiation loss per unit volume, $D_{rad}$, is balanced by the mechanical energy dissipated per unit volume, $D_{mech}$. The latter quantity may be calculated using the formulae given by LBP (p. 610; $D_1$ in their notation). Neglecting the radial component of velocity, we obtain after some arithmetic

$$D_{mech} = \frac{9}{4} \frac{Gm_\ast}{R^3} \bar{v}_e \rho,$$

where $v_e$ is the ($z$-dependent) viscosity per unit mass. We emphasize that $\bar{v}_e$, entering Eqs (2.5) and (2.7) has the meaning of the $z$-averaged value of $v_e$. We assume that the viscosity is a decreasing function of $z$, parametrized by a simple power law

$$v_e = \bar{v}_e(m/M)^\zeta,$$

where $m$ is the Lagrangian mass coordinate defined by

$$dm = -\rho \, dz, \quad m = \int_x^{\infty} \rho \, dz,$$

$M$ being the Lagrangian mass at the central plane, i.e. $M = \Sigma/2$, $v_e$, is the viscosity at the central plane, and $\zeta$ free parameter. The averaged viscosity $\bar{v}_e$ and the local viscosity $v_e$ should yield the same dissipation per unit surface area, i.e.

$$\frac{D}{2} = \int_0^\infty \frac{9}{4} \frac{Gm_\ast}{R^3} \bar{v}_e \rho \, dz = \int_0^\infty \frac{9}{4} \frac{Gm_\ast}{R^3} \bar{v}_e(m/M)^\zeta \, dz.$$

Therefore,

$$v_e = \bar{v}_e(\zeta + 1).$$

It remains to specify the radiative cooling $D_{rad}$. However, this is a well-known quantity from the theory of stellar atmospheres (see, e.g., Mihalas, 1978), and is given by

$$D_{rad} = 4\pi \int_0^\infty \left[\eta(v, z) - \chi(v, z) J(v, z)\right] \, dv,$$

where $\chi(v, z)$ and $\eta(v, z)$ are the absorption and emission coefficients, respectively, $J(v, z)$ is the mean intensity of radiation at frequency $v$ (we suppress explicit indication of the $R$-dependence of the above quantities). Notice also that the usual form of the energy conservation equation applied in the case of stellar atmospheres consists in assuming the so-called radiative equilibrium, $D_{rad} = 0$.

Since the overall energy balance is quite sensitive to the radiation field, we have to determine the radiation intensity self-consistently with other structural parameters. In other words, we have to solve the radiative transfer equation together with the hydrostatic equilibrium and the energy balance equations. However, this is a difficult task, because the absorption and emission coefficients depend on the atomic level populations of atoms and/or ions which are the dominant opacity sources. The level populations in turn depend on the radiation field and other structural parameters (temperature, density, etc.).

Thus, in view of a rather limited knowledge about $D_{mech}$, we feel it is reasonable to simplify the complex problem of the interaction between radiation and matter by assuming local thermodynamic equilibrium (LTE). Specifically, the atomic level populations are assumed to obey the Boltzmann-Saha law, and the velocity distributions of all kinds of particles are assumed Maxwellian. In practice, this means that the equations of statistical equilibrium need not be solved and that the thermal part of the source function is equal to the Planck function. Yet, the radiative transfer equation has still to be solved together with the constraints of hydrostatic equilibrium and energy balance.

The basically three-dimensional problem of constructing models of accretion disc can be reduced to a set of one-dimensional problems assuming that the physical properties of a disc vary much more slowly with $R$ than with $z$ and, consequently, that a sufficiently small part of the disc behaves like an
independent plane parallel radiating slab. Computing a model of a disc thus consists in calculating a set of submodels for an appropriately chosen set of rings bounded by radial distances \( R_i - \Delta R_i \) and \( R_i + \Delta R_i \). Each submodel describes a vertical structure of a ring. The emergent radiation from a disc may then be calculated by an appropriate summation over the contributions from the individual rings.

3. Basic Equations for the Vertical Structure

The set of basic equations, which we present below, closely resembles that used for a description of classical stellar atmospheres (CSA) — see, e.g., Mihalas, (1978). This will enable us to adopt the highly developed approaches used in constructing CSA models. Therefore, we shall specifically concentrate on stressing differences between the present equations and those met in the CSA theory.

For sake of brevity, we shall also use a terminology borrowed from the CSA theory. Thus, each individual ring is referred to as an “atmosphere”; the “lower boundary condition” means an appropriate condition at the central plane of the disc, \( z = 0 \); and the “upper boundary condition” means that at \( z = z_{\text{max}} \).

(i) Radiative Transfer Equation

This equation has the usual CSA form, viz.

\[
\frac{dI_{\nu}}{d\nu} = \frac{\chi_{\nu}}{\varrho} (I_{\nu} - S_{\nu}),
\]

where \( I_{\nu}, S_{\nu}, \) and \( \chi_{\nu} \) are the specific intensity of radiation, the source function, and the (volume) absorption coefficient, respectively, at frequency \( \nu \); \( \varrho \) is the cosine of polar angle (for more detail, refer, e.g., to Mihalas, 1978). The upper boundary condition is also the same as in CSA, viz.

\[
I_{\nu}(m = 0, \mu < 0) = 0.
\]

Actually, the numerical integration start from some very small mass-coordinate \( m_1 \). We assume that Eq. (3.2) holds even for \( m = m_1 \).

However, the lower boundary condition is different from the CSA case, namely

\[
I_{\nu}(m = M, \mu) = I_{\nu}(m = M, -\mu),
\]

since the disc is assumed symmetric with respect to the central plane.

(ii) Hydrostatic Equilibrium Equation

In contrast to CSA, the gravitational acceleration is no more an input, depth-independent parameter. Instead, it is given by Eq. (2.9). The hydrostatic equilibrium equation thus reads

\[
\frac{d}{dm}(p + p_r + p_t) = Qz,
\]

\( p, p_r, \) and \( p_t \) being the gas, radiative, and turbulent pressure, respectively; and are given by

\[
p = NkT,
\]

\[
p_r = (4\pi/\varrho) \int_0^\infty K_{\nu} \, dv,
\]

\[
p_t = \frac{1}{2} v_t^2.
\]

Here, \( N \) is the total particle number density, \( T \) the temperature, \( K_{\nu} = \int_0^1 I_{\nu}(\mu) \mu^2 \, d\mu \) is the second moment of the specific intensity of radiation, and \( v_t \) the turbulent velocity.

The upper boundary condition also differs from the CSA form and reads (we write \( p_1 = p(m_1) \), etc.)

\[
\frac{p_1}{m_1} \left( 1 - \frac{p_1}{Q_0 z_1^2} \right) = Qz_1.
\]

Equation (3.8) can be derived by integrating Eq. (3.4) from \( m = 0 \) to \( m = m_1 \), assuming \( T(m < m_1) = T(m_1) \), and neglecting \( p_r \) and \( p_t \) for \( m < m_1 \). Writing \( P = \gamma \varrho \), where \( \gamma \) is a constant, Eq. (3.4) yields

\[
\varrho = \varrho_1 \exp \left[ -Q(x^2 - z_1^2)/2\gamma \right]
\]

and then

\[
m_1 = \varrho_1 \exp (Qz_1^2/2\gamma) \left( \frac{\pi \gamma}{2Q} \right)^{1/2} \text{erfc} \left[ \sqrt{\left( \frac{Q}{2\gamma} \right) z_1} \right].
\]

Expressing \( p_1 = \gamma \varrho_1 \), and using the approximation \( \pi^{1/2} \times \exp(x^2) \text{erfc}(x) \approx 1 - 1/(2x^2) \) we obtain, after some arithmetic, Eq. (3.8).

(iii) Energy Balance Equation

Taking into account Eqs (2.10)–(2.15), the condition \( D_{\text{rad}} = D_{\text{mech}} \) may be written as

\[
\int_0^\infty \chi_{\nu} S_{\nu} \, dv = \int_0^\infty \chi_{\nu} I_{\nu} \, dv + E(\zeta + 1) (m/M)^3 \varrho,
\]

where

\[
E = \frac{9}{16\pi} \frac{Gm_*}{R^3} v_p.
\]

Using the first moment of the transfer equation (3.1), one may derive an alternative form of Eq. (3.11), viz.

\[
\frac{dH}{dm} = -E(\zeta + 1) (m/M)^3,
\]
or

\[ H(m) = E[M - m(M)^2] , \]

where \( H = \int_0^\infty \int_{-1}^1 d\nu d\mu I(\nu, \mu) \mu \) is the frequency-integrated first moment of the specific intensity, which is proportional to the total radiative flux. Thus, in contrast to CSA, the total radiative flux is not conserved within an atmosphere.

(iv) Definition of the Lagrangian Mass Coordinate

On the contrary to CSA, the present case involves an explicit dependence of the gravitational acceleration on the geometrical coordinate \( z \). The basic set of equations has thus to contain an explicit relation between \( z \) and \( m \), viz.

\[ \frac{dz}{dm} = \frac{1}{q} . \]

The lower boundary condition is obvious

\[ z(m = M) = 0 . \]

(v) Statistical Equilibrium Equations

Since we assume LTE, there is, in fact, no need to consider the equations of statistical equilibrium explicitly. Yet from the point of view of the adopted numerical technique and also in view of a future extension of this work to non-LTE situations, we consider atomic level populations explicitly, namely

\[ n_{i} = n_{e} n_{\ast} \Phi_{i}(T) , \]

where \( n_{e} \) is the electron number density, \( n_{\ast} \) the population of the ground level of the next higher ion, and \( \Phi_{i}(T) \) is the Saha-Boltzmann factor (see Mihalas, 1978),\( T \) being the temperature. The set of statistical equilibrium equations is complemented by the particle conservation equation and the charge conservation equation.

Finally, we shall briefly discuss two important auxiliary points, namely a) calculation of the emergent flux, b) the fundamental model parameters.

a) Calculation of the Emergent Flux

In the case CSA, the most suitable quantity which is to be compared with observations is the theoretical physical flux at the star’s surface, \( \mathcal{F}_\lambda = 4 \pi H_\lambda \lambda \) being the wavelength, and \( H_\lambda \) the first moment of the specific intensity defined above. The monochromatic flux measured at the distance \( D \) from a star is then given by \( f_\lambda = \mathcal{F}_\lambda (R_\ast/D)^2 \).

It is advantageous to introduce a quantity analogous to \( \mathcal{F}_\lambda \) also for accretion discs. It is straightforward to show that the observed flux can be given by the same formula even for the accretion discs, viz.,

\[ f_\lambda = \mathcal{F}_\lambda \{R_\ast/D\}^2 , \]

if we define the equivalent flux as

\[ \mathcal{F}_\lambda = 2\pi \cos i \int_{r_1}^{r_2} I_\lambda (r, i) \, dr . \]

Here, \( I_\lambda \) is the specific intensity of radiation at the disc surface; \( r \) the relative radius, \( r = R/R_\ast \); \( r_1 \) and \( r_2 \) the inner and the outer radii of the disc, respectively; and \( i \) is the inclination (the angle between the disc symmetry axis and the direction towards observer; \( i = 0 \) for a disc seen pole-on). The integral in (3.19) may easily be evaluated numerically by the trapezoidal rule.

b) Fundamental Model Parameters

As follows from Sect. 2, the fundamental parameters of an overall disc model are the mass and the radius of the central star, \( m_\ast \) and \( R_\ast \), and the mass flux \( \dot{m}_\ast \). Obviously, the next fundamental parameter is, as in the case of CSA, the chemical composition. Further, one has to specify the empirical scaling factors \( \text{Re} \) and \( \zeta \) that describe the viscosity.

The fundamental parameters of the individual rings (i.e. pseudo-atmospheres), \( E, M, Q \) (or \( T_{\text{eff}}, M, Q \) — see below) are all given as function of the fundamental disc parameters \( m_\ast, R_\ast, \dot{m}_\ast \) (together with \( \text{Re} \) and \( \zeta \)), and the radial distance from the center of symmetry, \( R \).

Pursuing an analogy with CSA further, one may define an “effective temperature” for each individual ring (pseudo-atmosphere) as

\[ \frac{\sigma}{4\pi} T_{\text{eff}}^4 = E \cdot M \quad \text{i.e.} \quad T_{\text{eff}} = \]

\[ = (4\pi EM/\sigma)^{1/4} = (D/2\sigma)^{1/4} , \]

where \( E \) and \( M = \Sigma/2 \) are given by Eqs (3.12) and (2.5), respectively, and \( \sigma \) is the Stefan-Boltzmann constant. The last equality in Eq. (3.20) follows from Eqs (2.6), (2.5), and (3.12). The physical meaning of \( T_{\text{eff}} \) readily follows from Eqs (3.14) and (2.6): the effective temperature measures the total radiation flux at \( m = 0 \), i.e. the total rate of energy dissipated per unit surface. Since the total radiation flux in a CSA is equal to \( \sigma T_{\text{eff}}^4 \), the above effective temperature is the most appropriate quantity when comparing models of individual rings with corresponding models of CSA’s. On the other hand, an actual frequency distribution of emergent flux is obviously different for a CSA and a pseudo-atmosphere with the same \( T_{\text{eff}} \). This is
a straightforward consequence of differences in basic equations as well as of the limited optical thickness of a disc.

From Eqs (2.6) and (3.20) follows that the maximum value of effective temperature occurs at \( R/R_\ast = (7/6)^2 = 1.3611 \), and is given by

\[
T_{\text{eff}}^{\text{max}} = \left( \frac{3Gm_\ast\dot{m}_\ast}{56\pi\sigma R_\ast^3} \right)^{1/4}
\]

(3.21)

4. Computational Method

The physical state of a given plane-parallel layer is described by the set of vectors \( \psi_d \) at every discretized depth point \( m_d \). In our case the vector \( \psi_d \) has the following components

\[
\psi_d \equiv \begin{pmatrix} J_{1,1} & \ldots & J_{N_J,1} & N_d & T_d & n_{e,d} & z_d & n_{1,d} & \ldots & n_{N_d,d} \end{pmatrix},
\]

where \( J_{1,d}, \ldots, J_{N_J,d} \) are the mean intensities of radiation in the discrete set of frequency points \( v_i, i = 1, \ldots, N_J \); \( N_d \) is the total particle number density; \( n_{e,d} \) is the electron number density; \( T_d \) is the temperature; \( z_d \) is the geometrical distance from the central disc plane corresponding to the Lagrangian mass column \( m_d \); and \( n_{1,d}, \ldots, n_{N_d,d} \) are the populations of energy levels included explicitly.

Equations (3.1)–(3.17) form a highly coupled, non-linear set of equations. Therefore, we used the method of complete linearization (Auer and Mihalas, 1969) which is ideally suited for such a class of problems. The linearization of our equations represents a straightforward generalization of the procedure used for CSA (for a detailed description, see Mihalas et al., 1975). Numerical results presented below have been obtained by the appropriately modified computer code TLUSTY (Hubeny, 1983) that was originally devised for calculating non-LTE model stellar atmospheres. This was, in fact, the reason for considering atomic level populations as independent variables, in spite of the fact that we could, in principle, employ more economical numerical techniques better suited for LTE situations (Mihalas, 1978). On the other hand, our code is quite capable of handling more general non-LTE models of accretion disc without substantial modifications. Notice that this feature is also present in the computer code of Mihalas et al. (1975).

In addition to obvious modifications of the traditional complete linearization scheme that follow from the differences between Eqs (3.4), (3.8) (3.11), (3.16), (3.17) and those for CSA, we have also found some modifications in the computational strategy to be necessary for our purposes. First, the starting solution has to be obtained by a completely different procedure than in the case of CSA. We shall describe our procedure in the next section.

In the complete-linearization solution of LTE models, we have found that the differential form of the energy conservation equation (3.13) is numerically much more stable than the integral form (3.11). In fact, this approach is also used in CSA model calculations, following the suggestion of Gustafsson (1971) and Frandsen (1974). Strictly speaking, CSA modelling uses the differential form in deep layers of an atmosphere, while at the surface the integral form has to be used. Interestingly enough, we have found that in our case the differential form has to be used everywhere. Nevertheless, this is not surprising because our case involves a stronger coupling between the temperature (determined from the energy equation) and other structural parameters, up to the most superficial layers of an atmosphere, than in the case of CSA. The appropriate upper boundary condition of the energy conservation equation reads

\[
\sum_{i=1}^{N_J} w_i f_{HI} J_{i,1} = E_0 \left[ M - \left( \frac{m_1}{M} \right)^5 m_1 \right],
\]

where \( w_i \) are the quadrature weights for a frequency integration, and \( f_{HI} \) is the variable Eddington factor (Auer and Mihalas, 1970).

Our code typically yields a converged model with all the relative \( \delta T, \delta n_\nu \) of the order of \( 10^{-3} \) or less at all depths after 4–5 iterations. We also tested the overall consistency of models by considering relative errors in satisfying the energy balance equation: our final models always yield the relative differences between the r.h.s. and the l.h.s. of Eq. (3.11) less than 0.1%.

In some cases, we have encountered numerical instabilities in atmospheric regions with \( m < 10^{-5} \). Nevertheless, those regions have a negligible effect on the emergent continuum radiation, so that we have calculated all models with \( m_1 \approx 10^{-5} \).

5. Starting Model

a) Formulation of the LTE-gray Approximation

According to the well-established methodology of the CSA model calculations, the so-called LTE-gray model serves as an excellent starting approximation for the subsequent complete-linearization calculations. The LTE-gray approximation consists in assuming a frequency-independent opacity, \( \chi = \chi \). Conse-
sequently, the energy balance equation for the CSA has a very simple form \( S = J \) (\( S, J \), etc., without subscripts denote the corresponding frequency-integrated quantities), and the radiative transfer equation thus yields a general relation between the local temperature and the mean optical depth, independently on the equation of hydrostatic equilibrium. The latter equation basically establishes a relation between the mean optical depth and a geometrical depth coordinate (and thus the gas pressure and related quantities).

However, applying the above procedure to the case of pseudo-atmospheres representing the individual rings of an accretion disc yields considerable complications. The gray approximation of the energy balance equation now reads

\[
S = J + (\sigma/\chi) E(\zeta + 1) (m/M)^2.
\]

Due to the explicit dependence of \( S \) on the Lagrangian mass \( m \), the solution of the radiative transfer equation can no longer be separated from the solution of other equations.

Thus, we have to solve the radiative transfer equation simultaneously with the energy balance equation. To this end, let us consider the first moment of the transfer equation (3.1), viz.,

\[
\frac{dK}{dm} = \frac{\chi}{\sigma} H,
\]

where \( K \) is the usual second moment of specific intensity, \( K = \frac{1}{2} \int_0^\infty dv \int_1^\infty d\mu I_v(\mu) \mu^2 \). Next we introduce the Eddington factors \( f \) and \( f_H \) such as

\[
K = fJ \quad H(m = 0) = f_H = f(m = 0).
\]

Since the gray model only serves as a starting approximation for subsequent detailed calculations, we are satisfied with the Eddington approximation, \( f = 1/3, f_H = 1/3 \).

The frequency-integrated source function is written as

\[
S = \frac{\chi}{\chi} B + \frac{n_e \sigma_e}{\chi} J,
\]

where \( \chi = \chi_t + n_e \sigma_s, \chi_t \) being the total opacity (including scattering), \( \chi_t \) has the meaning of the absorption coefficient for pure absorption (thermal part); \( B = \sigma T^4 \pi \) is the integrated Planck function, and \( \sigma_e \) the cross-section for electron scattering. We consider a more general Eq. (5.4) instead of taking simply \( S = B \) because there may be a considerable portion of electron scattering in the outer parts of a disc. As in the case of CSA, the most suitable mean opacity \( \chi \) is the Rosseland mean opacity (see, e.g., Mihalas, 1978).

Using Eqs (3.14), (5.1), (5.3), and (5.4), we can eliminate the radiation moments \( H \) and \( K \) from Eq. (5.2) and write

\[
\frac{d}{dm} \left\{ \frac{\sigma T^4}{\pi} \frac{\chi}{\chi_t} E(\zeta + 1) (m/M)^2 \right\} f = \frac{\chi}{\sigma} E[M - m(m/M)^2].
\]

Analogously, assuming that the second Eq. (5.3) is valid for \( m = m_1 \), writing \( H_1 = H(m_1) \), etc., and using the fact that \( m_1 \ll M \) and then \( H_1 = E \cdot M \) (see Eq. (3.14)), we obtain the upper boundary condition

\[
T_1 = \left\{ \frac{\pi E}{\sigma} \left[ \frac{M}{f_H} + (\zeta + 1) (m/M)^2 \right] \frac{\theta_1}{(\chi_1)^2} \right\}^{1/4}
\]

Equations (5.5), (3.4) and (3.15), supplemented by the boundary conditions (5.6), (3.8), and (3.16), form the set of basic equations describing the LTE-gray approximation for pseudo-atmospheres. Obviously, this set has to comprise also the auxiliary equations, such as the Saha-Boltzmann equations, the particle and charge conservation equations, and the definition of the Rosseland mean opacity.

In contrast to the CSA, the introduction of the mean optical depth \( \chi = \chi_t + n_e \sigma_s \) does not allow to solve for the relation \( T = T(\zeta) \) analytically. Therefore, we again apply the method of complete linearization. The basic vector of model parameters, \( \psi_d \), now has only three elements,

\[
\psi_d = \{ T_d, q_d, z_d \}
\]

The linearization of Eqs (5.5), (5.6), (3.4), (3.15), (3.8), and (3.16) is straightforward.

b) Initial Estimate

Since even the LTE gray model is calculated iteratively, the knowledge of some initial estimate \( \psi_d = \{ T_d^0, q_d^0, z_d^0 \} \) is now required. Numerical experience shows that the following zero-order estimate is satisfactory: The temperature is taken constant throughout the atmosphere and equal to the effective temperature — see Eq. (3.20).

\[
T^0 = T_{eff} = (4\pi E M/\sigma)^{1/4}
\]

The equation of state can now be written in the form

\[
p = \frac{k}{\mu m_H} T_{eff} q,
\]
where \( k \) is the Boltzmann constant, \( m_H \) the mass of hydrogen atom and \( \mu \) the effective molecular weight (should not be confused with the designation of the polar angle used in Sect. 3). Although \( \mu \) generally depends on ionization degree of the individual chemical species present in an atmosphere, we do not evaluate the ionization balance in detail, but rather assume some suitably chosen fixed values of \( \mu \) (generally different for individual pseudo-atmospheres).

The hydrostatic equilibrium equation (3.4) may then be solved, using Eqs (5.8) and (5.9), and neglecting \( p_r \) and \( p_t \), to yield

\[
V^0 = Q_c \exp \left[ - (\gamma z)^2 \right],
\]

where \( Q_c \) is the density at the central disc plane,

\[
\rho_c = (4/\pi)^{1/2} \gamma M,
\]

and

\[
\gamma = \left( \frac{G m_H \mu m_H}{2R^2 k T_{eff}} \right)^{1/2}.
\]

Finally, the relation between \( m \) and \( z \) now becomes

\[
m(z) = M \operatorname{erfc}(\gamma z).
\]

c) Numerical Procedure

At the beginning of calculations we choose \( N_D \) discretized depth points \( m_d \) \((d = 1, \ldots, N_D)\) equidistantly spaced in log \( m \) between \( m_1 \) and \( m_{N_D} = M \). The first depth point \( m_1 \) approximately corresponds to a prechosen minimum Rosseland optical depth \( \tau_1 \), and can easily be estimated by assuming that the dominant source of opacity is the electron scattering.

Then we solve Eq. (5.13) numerically in order to obtain \( z^0(m_d) \), and calculate \( q^0(m_d) \) from Eq. (5.10); \( T^0(m_d) = T^0 \) being given by Eq. (5.8). Having obtained the initial guess \( \psi^0 \), we apply the complete-linearization method to obtain the LTE-gray solution as described in Sect. 5a. The Rosseland mean opacity is evaluated by a direct frequency quadrature using the preselected frequency grid \( \nu_i \) \((i = 1, \ldots, NJ)\); the same grid is also used in calculating the complete LTE model.

6. Calculated Models and Comparison with Observations

The models presented below are calculated for the following values of the fundamental disc parameters. The mass of the central white dwarf is taken to be \( m_* = 1 \, m_\odot \), because (i) it is a typical value of the white dwarf mass in cataclysmic binaries; (ii) this value is often used in theoretical model calculations; and (iii) we shall compare our model predictions with the observed spectra of the dwarf nova WX Hyi for which the existing mass estimates yield \( m_* = 0.9 \, m_\odot \) (Scholmb and Vogt, 1981) and \( m_* = 1.1 \, m_\odot \) (Hassall et al., 1983). The corresponding radius of a white dwarf is \( R_* = 5 \times 10^8 \) cm (Kippenhahn and Thomas, 1965). We have chosen three values of the mass flux through disc, \( \dot{m}_* = 10^{-8}, 10^{-10}, \) and \( 10^{-11} \, m_\odot/\text{year} \) (Henceforth, mass flux will always be expressed in units of \( m_\odot/\text{year} \)). The corresponding values of \( T_{\text{eff}}^{\max} \) are then 94 500, 29 900, and 16 800 K, respectively.

The abundances (by number) of hydrogen and helium are taken to be \( X = 0.9, Y = 0.1 \), respectively;

<table>
<thead>
<tr>
<th>( \dot{m}_* )</th>
<th>Parameters of calculated models</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{-11} )</td>
<td>( 10^{-10} )</td>
</tr>
<tr>
<td>( R/R_* )</td>
<td>( T_{\text{eff}} )</td>
</tr>
<tr>
<td>1.05</td>
<td>13 000</td>
</tr>
<tr>
<td>1.2</td>
<td>16 300</td>
</tr>
<tr>
<td>1.3611</td>
<td>16 800</td>
</tr>
<tr>
<td>1.6</td>
<td>16 400</td>
</tr>
<tr>
<td>2.0</td>
<td>15 100</td>
</tr>
<tr>
<td>3.0</td>
<td>12 200</td>
</tr>
<tr>
<td>5.5</td>
<td>8 30C</td>
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<tr>
<td>8</td>
<td>6 500</td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td></td>
</tr>
</tbody>
</table>

Notes: \( T_{\text{eff}}, M, g, z \) in CGS units.
\( \log g \) and \( z \) are given at the depth where \( m = M/10 \).
the contribution of heavy elements is neglected. The following opacity sources are considered: HI, H\(^-\), He I, He II bound-free and free-free transitions, and electron scattering.

The effective Reynolds number for the accretion flows has been fixed to Re = 5000. The parameter \( \zeta \) has been arbitrarily taken to be \( \zeta = 0.667 \). In fact, numerical experience showed that the models with the (physically unrealistic) value \( \zeta = 0 \) exhibit a spurious temperature rise near the outer boundary and, moreover, the iteration procedure often fails to converge. The turbulent pressure corresponding to sonic turbulence at the relevant effective temperature, i.e., calculated by Eq. (3.7) with \( v_\ast = (kT_{\text{eff}}/\mu m_\ast)^{1/2} \) is considered in the equation of hydrostatic equilibrium.

The fundamental parameters for the individual submodels (pseudo-atmospheres) of the three global disc models are summarized in Tab. 1. The outer radial boundary of a disc is somewhat arbitrarily taken at the last \( R_t \) for which \( T_{\text{eff}} \geq 6000 \) K. Such a cutoff is induced above all by the practical limitations of our computational procedure. At lower effective temperatures, there is a considerable contribution of metal as well as molecular opacities and, moreover, deeper parts of the optically thick models may become unstable to convection. Consequently, such models would require more sophisticated computational approaches. On the other hand, the cool region beyond our adopted outer radial boundary of a disc has likely a minor effect on the total emergent ultraviolet and visible flux.

Figure 1 shows the temperature and density structure of the calculated disc models; Figure 2 displays the dependence of the monochromatic optical thickness of the disc on the wavelength.

It is readily seen that the whole disc with \( \dot{m}_\ast = 10^{-8} \) is optically thick, while a large part of the disc with \( \dot{m}_\ast = 10^{-11} \) is semitransparent. The intermediate model with \( \dot{m}_\ast = 10^{-10} \) (not displayed here) exhibits a behavior intermediate between both extreme models; qualitatively, its outer parts are semitransparent as well.

Notice that for the optically thick parts of discs (cf. Fig. 2), the isotherms have roughly the same shape as isodensities. In the optically thin parts of discs, the dependence of \( T \) on \( R \) dominates over the dependence on \( z \).

The most interesting feature is the presence of a very hot core below the surface for the model with
Fig. 2. Optical thickness of the calculated disc models as a function of wavelength for various distances from the centre of symmetry. Specifically, $\tau$ denotes the monochromatic optical thickness between the central disc plane and the disc surface measured along the normal to the central plane. The upper and the lower system of curves correspond to the model with $\dot{m}_* = 10^{-8}$ and $10^{-11} M_\odot/\text{year}$, respectively. The individual curves correspond to the following distances from the centre of symmetry, $r = R/R_*$: A for $r = 1.05$; B for $r = 1.3611$ (maximum $T_{\text{eff}}$); C for $r = 3.0$ (maximum $\tau$). Further, for model with $\dot{m}_* = 10^{-8}$; D for $r = 20.0$; E for $r = 45.0$; and for model with $\dot{m}_* = 10^{-11}$; D for $r = 5.5$; E for $r = 8.0$.

$\dot{m}_* = 10^{-8}$, the local temperature reaches values about three times higher than $T_{\text{eff}}^{\text{max}}$. On the contrary, the model with $\dot{m}_* = 10^{-11}$ does not possess such a hot core; the maximum local temperature throughout the disc only slightly exceeds $T_{\text{eff}}^{\text{max}}$.

Figure 1 also illustrates the consistency of our assumption that the thickness of the disc is much smaller than its radius. Indeed, it is readily seen that the disc radius is always about 1.5 dex larger than its thickness.

The theoretical emergent flux for the calculated disc models is plotted in Fig. 3. For comparison, the approximate law $F_\lambda \propto \lambda^{-2.33}$ (Lynden-Bell, 1969) that corresponds to the flux from an infinite stationary disc composed of a superposition of black bodies radiating energy at the local rate of dissipation is also plotted. While the agreement between the $\lambda^{-2.33}$ law and the calculated flux for $\dot{m}_* = 10^{-10}$ is quite good, both extreme models show significant departures; the calculated flux for $\dot{m}_* = 10^{-8}$ is considerably steeper, that for $\dot{m}_* = 10^{-11}$ is flatter. This is obviously caused by departures of the calculated flux from the individual rings (pseudospheres) from the black-body distribution. Nevertheless, the steepness of the computed flux distribution for $\dot{m}_* = 10^{-8}$ may partly be explained as an artifact of our choice of the outer boundary, $R/R_* = 45$, which still has a relatively high $T_{\text{eff}} = 10^7$ K. Consequently, an increase of the disc radius would yield an increase of long-wavelength radiation.

A very interesting feature is the significant level of the soft X-ray radiation predicted for the model with $\dot{m}_* = 10^{-8}$. However, it should be stressed that this phenomenon may in fact be spurious due to the insufficient treatment of the relevant opacity sources in this wavelength region.

In order to compare our theoretical calculations with observations, we plot in Fig. 3 the observed energy distribution for WX Hydra by Hassall et al. (1983). The theoretical flux for $\dot{m}_* = 10^{-11}, i = 30^\circ$ matches the observed flux in the quiescent phase satisfactorily. Moreover, a semitransparency of this model in the ultraviolet and visible continua is consistent with the presence of intense emission lines.
observed in the quiet phase of WX Hya (Hassall et al., 1983).

On the other hand, the spectra taken during the super-outburst phase are best fitted with the model for $\dot{m}_{} = 3 \times 10^{-10}$, $i = 30^0$, obtained by interpolating from the calculated models. While the agreement is very good in the visible region, the observed ultraviolet flux distribution is somewhat flatter than predicted. This is probably caused by an insufficient treatment of the far-UV opacity sources and/or by non-stationary phenomena occurring during the outburst, which are not accounted for by means of our modelling. Anyway, we may conclude that the superoutburst of WX Hya is probably caused by an increase of the mass flux through the accretion disc by a factor of $30-40$. The corresponding model is optically thick, which qualitatively agrees with the absence of strong emission lines in the outburst spectrum.

The spectrum taken during the rise from quiescence to outburst cannot be satisfactorily matched by any calculated flux distribution. A common characteristic feature of spectra observed in the rise is their flatness as compared to both quiescent and outburst spectra. This phenomenon is most probably connected with a non-stationarity of the disc at the beginning of the outburst.

The most straightforward possibility of explaining the flatness of the energy distribution appears to be an increase of the temperature in cool, distant parts of the disc with respect to the temperature following from the stationary canonical model. Consequently, the rise is probably accompanied by an increase of the mass flux in the outer parts of a disc.

In order to test this suggestion quantitatively, we plot in Fig. 4, besides the flux from the whole disc, the theoretical flux calculated for the outer half of the disc,
i.e. calculated by putting $r_1 = 0.5 \ r_2$ in Eq. (3.19). As expected, the flux in the latter case is considerably flatter, particularly for the models with $\dot{m}_a = 10^{-10}$ and $10^{-11}$. The observed flux for VW Hydri after Hassall et al. (1983) is plotted in the lower part of the figure. We did not attempt to fit directly the observed energy distribution of VW Hydri by our models because this system has rather different parameters than those adopted in our models (e.g., the white dwarf mass is much lower than 1 $M_\odot$ — see Hassall et al., 1983; Schoembs and Vogt, 1981). Nevertheless, the similarity of the (even much flatter than for WX Hydri) observed flux in the period of rise with the calculated flux for the outer part of the model with $\dot{m}_a = 10^{-10}$ gives a support to the idea that rise may indeed be accompanied by an increase of the mass flux through the outer parts of a disc. However, we stress again that this conclusion should be viewed as tentative due to an inadequacy of our approach to treat non-stationary phenomena.

7. Conclusions

We have demonstrated that a more consistent treatment of the interaction between radiation and matter plays an important role in constructing models of accretion discs of dwarf novae. Consequently, the predicted flux distribution based either on a superposition of black bodies (like, e.g., the $\lambda^{-2.33}$ law) or on a superposition of classical model atmosphere fluxes should be viewed with caution.

We summarize our basic result below:

(i) For moderate and low values of the mass flux ($\dot{m}_a \sim 10^{-10} \sim 10^{-11} \ M_\odot$/year), large parts of the disc are optically thin in the visible and UV continua.

(ii) The vertical structure of the disc may significantly differ from the classical stellar atmospheric structure.

(iii) For large values of the mass flux ($\dot{m}_a \sim 10^{-9}$), our models predict a very hot core beneath the disc surface ($T \sim 3 \times 10^5$ K) which may emit a significant X-ray radiation.

(iv) A comparison of our model predictions with the observed spectra of WX Hydri shows that the model with $\dot{m}_a = 10^{-11} \ M_\odot$/year fits the observed quiescent spectrum well; the outburst is relatively well explained by a model with the mass flux enhanced by a factor of 30–40.

(v) A preliminary analysis reveals that a strange-looking shape of the energy distribution observed during the rise from quiescence to outburst (e.g. for VW Hydri) is probably caused by an enhanced mass flux in the outer parts of the disc.

The primary aim of this paper was to outline a more exact solution of the radiative transfer problem in the accretion discs. However, our approach is still insufficient for constructing entirely reliable models. The following improvements may easily be worked out: inclusion of further opacity sources, calculation of non-LTE effects, allowance for the convective energy transport in convectively unstable regions, calculation of line profiles. More fundamentally, from the dynamical point of view, a more sophisticated law for viscosity should be applied, and non-stationary effects should be taken into account. The papers by Bath and Pringle (1981), Faulkner et al. (1983), Meyer

Fig. 4. Rough estimate of the effect of an enhanced mass flux in the outer parts of the disc. The three systems of lines correspond (from top to bottom) to $\dot{m}_a = 10^{-8}, 10^{-10}$ (dashed lines), and $10^{-11} \ M_\odot$/year. In each system, the upper curves always correspond to the radiation from the whole disc (i.e. calculated as in Fig. 3 for the complete model); the lower curves correspond to the radiation from the outer half of the disc. All curves correspond to inclination $i = 30^\circ$. The observed energy distribution $f$ for VW Hydri after Hassall et al. (1983) is plotted in the lower part of the figure. In order to avoid overlapping of spectra taken at different phases, the ordinate scale for the quiescent spectrum is shifted with respect to those for the rise and the outburst.
and Meyer-Hofmeister (1984), Smak (1984), and references cited therein, show a possible way of improvement in this direction. Completion of time-dependent models with properly calculated emergent spectra could help to interpret observations and reveal the nature of the dwarf nova outbursts, superoutbursts, and related phenomena.

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Appendix

Relation between the Reynolds number Re and the parameter $\alpha$.

It can be argued that we use too large Reynolds number and that Re corresponding to $\alpha \sim 0.1$ should be much smaller. Also Pringle (1981) in his excellent review paper writes “For dwarf nova outbursts, value of Re in the range $10^2 - 10^3$, or alternatively values of $\alpha$ in the range $0.1 - 1$ provide reasonable fits to the data”. We shall thus discuss this question in more detail.

As $\alpha$ is not exactly defined in the original paper by Shakura and Sunyaev ([1973] – $z_0$ is specified as the thickness of the disc on page 343 and as half-thickness on page 344), we use the unambiguous definition by Pringle (1981):

$$\alpha = \frac{R \nu}{t_\phi}$$

(A 2)

where $c_s$ is the sound speed and $t_\phi$ is the geometrical thickness of the disc. The Reynolds number $Re = \frac{R \nu}{t_\phi}$ and, therefore,

$$\alpha = \frac{R \nu}{c_s t_\phi}$$

(A 2)

Using a very approximate expression for the disc thickness

$$t_\phi = \frac{R c_s^2}{\nu} \quad \text{(Pringle, 1981)}$$

we have

$$\alpha = Re^{-1} \left( \frac{\nu}{c_s} \right)^2$$

or

$$\alpha = Re^{-1} (R/H)^2$$


To express Eq. (A.4) in terms of the fundamental disc parameters, we approximate $c_s$ by the isothermal speed of sound in the ionized hydrogen, i.e. $c_s^2 = \frac{2kT}{m_H}$, and assume that $T$ corresponds to the effective temperature given by Eqs. (3.18) and (2.6).

Then

$$\alpha = \frac{1}{Re} \frac{m_H}{2k} \left( \frac{8\pi \sigma}{3} \right)^{1/4} \frac{G m_*}{R_*^{1/4}} q(r)$$

(A 5)

where

$$q(r) = \left[ r(1 - 1/\sqrt{r}) \right]^{-1/4}, \quad r = R/R_*$$

As $q(r)$ varies in the range 2.5–0.3 for $r$ in the range 1.05–100, we put $q(r) = r$ for rough estimates.

Adopting $Re = 5000$, $m_* = 1 m_\odot$ and $R_* = 5 \times 10^8$ cm, we have $\alpha = 9.4$, 5-2 and 1.7 for $m_* = 10^{-11}$, $10^{-10}$ and $10^{-8} m_\odot/year$, respectively.

Consequently, it seems that $Re = 5000$ is not too high and that even higher $Re$ should be chosen to obtain $\alpha \approx 0.1 - 1$.

Equation (A 5) is independent of our model calculations. However, it overestimates $\alpha$ as the effective temperature is usually much lower than the temperature in the interior of the disc. Moreover, Eq. (A 3) is inaccurate. Therefore, we have also calculated $\alpha$ directly from our models using the original Eq. (A 2). We have put $c_s = \sqrt{(p_e/q_e)}$ and $t_\phi = 2\pi(q = q_e/10)$, i.e. the disc thickness has been tentatively defined by the points where the density is of one order of magnitude lower than the density at the central plane of the disc. $p_e$ and $q_e$ are pressure and density at the central plane.
The resulting $\alpha$ is displayed in Fig. 5. It is evident that our models correspond to reasonable values of $\alpha$. Only the model with $\dot{m}_* = 10^{-11} M_\odot$/year yields a slightly large $\alpha$ and the Reynolds number $Re$ should be increased further.

REFERENCES


TIDAL CONTRIBUTION OF PLANETS TO REMOVING ANGULAR MOMENTUM FROM THE SUN

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ПРИЛИВНЫЙ ВКЛАД ПЛАНЕТ В ПЕРЕНОС МОМЕНТА КОЛИЧЕСТВА ДВИЖЕНИЯ СОЛНЦА

Вычислен угол между плоскостью экватора Солнца и неизменяемой плоскостью Лапласа солнечной системы (5.84°). Это явление вряд ли следует считать случайным. Напрашивается известная идея о быстром вращающемся прото-Солнце, большая часть момента количества движения которого была перенесена на окружающий его диск, из которого образовались планеты. Дается оценка приливного переноса момента количества движения Солнца, вызванного Юпитером, Венерой и Меркурием. Отмечается, что для объяснения современного малого момента количества движения Солнца необходимо было бы предполагать, что орбиты планет были в процессе их образования существенно ближе Солнцу.

The angle between the solar equatorial plane and the Laplace invariant plane of the Solar System has been computed (5.84°). This phenomenon is to be considered as not accidental. The well known idea arises about a fast-spinning early Sun, the angular momentum of which was mostly shared by the surrounding close disc which gave rise to the planets. The tidal removing of the angular momentum from the Sun by Jupiter, Venus and Mercury has been estimated. It has been concluded that the explanation of the present small angular momentum of the Sun would require the planetary orbits to have been much closer to the Sun during their past evolution.

Key words: solar system: invariable plane, angular momentum