THE CHEMICAL COMPOSITION OF CARBON STARS. I. CARBON, NITROGEN, AND OXYGEN IN 30 COOL CARBON STARS IN THE GALACTIC DISK

David L. Lambert
Department of Astronomy, University of Texas, Austin

Bengt Gustafsson
Stockholm Observatory and Astronomical Observatory, Uppsala

Kjell Eriksson
Astronomical Observatory, Uppsala

AND

Kenneth H. Hinkle
Kitt Peak National Observatory

Received 1985 November 11; accepted 1986 March 7

ABSTRACT

The chemical compositions of 30 Galactic carbon stars are determined from high-resolution infrared (1.5–2.5 \mu m) spectra and new model atmospheres.

Elemental C, N, and O abundances (with respect to hydrogen) are obtained from lines of the C2 Phillips system's \Delta v = -2 sequence, the CN red system's \Delta v = -2 sequence, and the CO X 1Σ + vibration-rotation second-overtone bands. The \(^{12}\)C/\(^{13}\)C ratios are extracted from the CN \Delta v = -2 sequence and, in two semi-independent ways, from the CO first- and second-overtone bands. Metal abundances are determined from a handful of atomic lines.

Spectra at 3–4 \mu m for a few stars provide additional lines which are used as checks. These include the HCN 3 \mu m bands, the CN red system's \Delta v = -3 sequence, and the NH X 3Σ - and the CH X 2Π fundamental vibration-rotation bands. The H\(_2\) 1–0 S(0) line near 2.22 \mu m is also available as a check.

Model atmospheres accounting for the fine blanketing by the polyatomics HCN and C2H2 were computed for a grid of effective temperatures and surface gravities. Selected models for differing opacities and assumptions (e.g., surface heating) were computed. Effective temperatures are on the scale established by lunar occultation angular diameters. The surface gravity is based on a mean stellar mass and absolute luminosity.

The principal results are as follows:

1. The dispersion in C, N, and O abundances for the 30 program stars is small.
2. The stars show only modest carbon enrichments; the most carbon-rich star has C/O ~ 1.6, and about 50% of the stars in the sample have C/O < 1.1.
3. The \(^{13}\)C abundance is, in general, low, with 30 < \(^{12}\)C/\(^{13}\)C < 70 encompassing the larger part of our sample, and the very \(^{13}\)C-rich (\(^{12}\)C/\(^{13}\)C ~ 4) stars are the exceptions, not the rule.
4. The oxygen abundances are, in general, slightly subsolar and in good agreement with results for M and MS giants (progenitors) and carbon-rich planetary nebulae (descendants).
5. The nitrogen abundances are subsolar and below the values measured for M and MS giants and planetary nebulae. This difference may reflect an error in the CN dissociation energy or possibly in the effective temperature scale.
6. The metal abundances are approximately solar.
7. The weak H\(_2\) lines are not evidence for hydrogen-deficient atmospheres.

It is shown that the observed CH line strengths are less than the predicted strengths. This discrepancy is discussed but not resolved. Possible explanations include non–LTE effects in the excitation and/or formation of CH, strengthened by thermal inhomogeneities, and a significant underestimate of the effective temperature scale.

For the typical star with a low \(^{13}\)C content, conversion to a carbon star was achieved by addition of pure \(^{12}\)C, a product of helium burning, to the envelope of a typical G, K, or M giant. The \(^{13}\)C-rich stars are not nitrogen-rich and, therefore, may not have undergone severe exposures to the hydrogen-burning CNO cycles.

It is shown that the abundances may be used to calibrate published spectrophotometric indices of C2, CO, and CN bands. Such a calibration exposes fainter carbon stars to quantitative analysis.

Subject headings: stars: abundances — stars: carbon

\(^{1}\)Visiting Observer, Kitt Peak National Observatory, National Optical Astronomy Observatories, operated by the Association of Universities for Research in Astronomy, Inc., under contract with the National Science Foundation.
I. INTRODUCTION

The visual spectrum of a cool carbon star is dominated by strong bands of the C$_2$ Swan and CN red and violet systems. The prominence of these carbon-containing molecules and the complete absence of the bands of metal oxides (especially TiO) so striking in the spectra of the M giants indicates that carbon must be more abundant than oxygen in the atmosphere of a carbon star, a reversal of the relative abundances in the M giants and the unevolved stars. The C enhancement in the carbon stars has long been attributed to the addition of freshly nucleosynthesized material to the atmosphere.

While inspection of the visual spectra suffices to classify unambiguously the coolest red giants as C-rich or O-rich, the fierce spectral congestion has often deterred spectroscopists interested in quantitative abundance analyses. A careful analysis is needed to show whether carbon stars are the result of an addition of C or a depletion of O in the atmosphere. If freshly synthesized $^{12}$C from the $\alpha$ process is added to the atmosphere of an O-rich giant, a carbon star is produced with possibly no appreciable change in the N and O abundances; some O may be produced with the C in the He-burning shell. On the other hand, production of a carbon star by addition of CNO tri-cycle products must lead to large changes in the N and O abundances; the C enhancement in the carbon stars has long been attributed to the addition of freshly nucleosynthesized material to the atmosphere.

A determination of the O abundance is possible only from the infrared vibration-rotation $(V-R)$ lines of CO. Since the first-overtone bands near 2.3 $\mu$m include many saturated lines, the abundance determination should be based on the weaker second-overtone 1.6 $\mu$m lines. Thompson's (1977) inspection and synthesis of low-resolution spectra of the semiregular variable carbon stars RS Cyg and RR Her led him to conclude that the O abundance was inconsistent with a CNO tri-cycle composition in which both C and O are reduced to achieve C/O $> 1$. His CO spectra were satisfactorily explained with a normal (solar) O abundance.

Although the changes in the C, N, and O abundances result in dramatic differences in the spectra of M- and N-type red giants, the total amount of processed material added to the atmospheres is expected to be small, and so hydrogen and helium should remain the principal constituents. The $H_2$ fundamental quadrupole vibration-rotation lines provide a tracer of the hydrogen content. Goorvitch, Goebel, and Augason (1980) discovered that the 1–0 $S(1)$ line of $H_2$ was absent from the spectrum of UU Aur, even though synthetic spectra for model C-rich atmospheres predicted the line to be extremely strong. Johnson et al. (1983) examined spectra of 16 carbon stars and reported the $H_2$ line to be present only in the coolest stars, and then at an intensity far below the predicted values. Possible resolutions of the large discrepancy between the observed and predicted $H_2$ line intensities were aired by Johnson et al.: (1) the model atmospheres are defective (non-LTE; polyatomic molecules may be major contributors of opacity; a chromosphere may replace the cool extended boundary layer where the $H_2$ lines are formed) or (2) the atmosphere of a carbon star is hydrogen-deficient. Any abundance analysis for C, N, and O which fails to address the $H_2$ lines must be regarded with some suspicion.

Identification of the $\alpha$ process as the origin of the C enrichment coincided with the birth of a vigorous theoretical effort to explain how $\alpha$-processed material is mixed out into the atmosphere of an evolved star. On the asymptotic giant branch (AGB), thermal instabilities (Schwarzschild and H"arm 1967) are predicted to occur thanks to the close proximity of hydrogen- and helium-burning shells. While these instabilities occur in all stars, a dredge-up of $\alpha$-processed material to the surface was predicted to occur only in intermediate-mass stars $3 \leq M/M_{\odot} \leq 8$; see Iben and Renzini (1983) for a review. However, there is considerable observational evidence suggesting that the dredge-up occurs in lower mass stars. Indeed, Richer (1981), from an analysis of an H-R diagram compiled for the Magellanic Clouds, concludes that all stars with $1.2 M_{\odot} \leq M \leq 2 M_{\odot}$ evolve to carbon stars. Recent theoretical work (Iben and Renzini 1982) suggests that mixing is possible in low-mass AGB stars.

In this paper we present CNO elemental and isotopic abundances, based upon high-resolution infrared spectra, for 30 cool carbon stars. Our inventory of questions included the following: What is the distribution function for the $^{13}$C/$^{12}$C ratio? Do $^{13}$C-rich stars such as WZ Cas and Y CVn have elemental abundances strikingly different from those of carbon stars with less $^{13}$C? Are the CNO abundances compatible with the addition of pure $\alpha$-processed material to a previously convectively mixed oxygen-rich giant? While our principal objective was to answer such questions and to pursue the

---

When there is little chance of confusion, we shall let the chemical symbol refer to the number density of that species.
implications for mixing in AGB stars, we envisaged several applications of our results. A thorough quantitative analysis of these Galactic carbon stars provides a basis for the calibration of low-dispersion spectra and narrow-band photometry for studies of larger samples of fainter stars, e.g., the carbon stars in external galaxies such as the Magellanic Clouds.

II. OBSERVATIONS

a) Spectra

The observing program consisted of all 32 carbon stars with $m_K \leq 1.00$ and a declination north of $-32^\circ$. The $K$-magnitudes were taken from Neugebauer and Leighton (1969). W CMa ($m_K = 2.02$) was included because it may be a massive star: $M > 10 \ M_\odot$ (Herbst, Racine, and Richer 1977; Scalo and Miller 1979).

The spectra were acquired with the Kitt Peak 4 m Mayall reflector and the Fourier transform spectrometer (Hall et al. 1978). The region from 4000 to 6600 cm$^{-1}$ (2.5–1.5 µm) was sampled at 0.07 cm$^{-1}$ ($\sim$ 4 km s$^{-1}$) resolution. Table 1 summarizes the spectra. All spectra were apodized by Norton and Beer (1976) apodizing function F2, and the signal-to-noise ratio and resolution presented in Table 1 reflect the increase in signal-to-noise ratio and decrease in resolution resulting from this operation. The stars were observed principally during morning daylight hours over the course of a year. It was not practical in such an observing program to observe a hot standard star during each observing session, and, hence, no effort was made to remove the telluric lines from the spectra. This is not a serious limitation, because many stellar lines are uncontaminated by telluric features.

Initial inspection showed the V Cyg spectrum to be dominated by circumstellar emission as spectrophotometry has suggested (Forrest, Gillett, and Stein 1975). In a plot of $K - 4 \mu m$ versus $4 \mu m - 11 \mu m$ color, V Cyg falls between the stars with nearly opaque circumstellar shells (e.g., IRC +10216 and CIT 6) and the bulk of our sample. For R Lep, S Cep,

<table>
<thead>
<tr>
<th>Name</th>
<th>Spectral Type</th>
<th>Variable Star Class</th>
<th>K-Magnitude</th>
<th>Date Observed</th>
<th>Integration Time (minutes)</th>
<th>S/Na</th>
</tr>
</thead>
<tbody>
<tr>
<td>AQ Sgr</td>
<td>N3</td>
<td>SRb</td>
<td>0.78</td>
<td>1980 Aug 26</td>
<td>146</td>
<td>113</td>
</tr>
<tr>
<td>BL Ori</td>
<td>N0</td>
<td>Lb</td>
<td>0.75</td>
<td>1980 Jan 4</td>
<td>154</td>
<td>61</td>
</tr>
<tr>
<td>R Lep</td>
<td>N6, C7, 4+</td>
<td>Mira</td>
<td>0.06</td>
<td>1980 Feb 27</td>
<td>121</td>
<td>128</td>
</tr>
<tr>
<td>R Sco</td>
<td>Np, C6, 5</td>
<td>SRa</td>
<td>0.03</td>
<td>1979 Jul 7</td>
<td>81</td>
<td>82</td>
</tr>
<tr>
<td>RT Cap</td>
<td>N</td>
<td>SRb</td>
<td>0.55</td>
<td>1980 Oct 30</td>
<td>146</td>
<td>72</td>
</tr>
<tr>
<td>RV Cyg</td>
<td>N5, C6, 4</td>
<td>SRb</td>
<td>0.36</td>
<td>1980 Mar 30</td>
<td>129</td>
<td>100</td>
</tr>
<tr>
<td>RY Dra</td>
<td>N4, C4, 4</td>
<td>SRb</td>
<td>0.27</td>
<td>1979 Oct 28</td>
<td>89</td>
<td>50</td>
</tr>
<tr>
<td>S Cep</td>
<td>Ne, C7, 4</td>
<td>Mira</td>
<td>-0.15</td>
<td>1980 Jan 25</td>
<td>81</td>
<td>92</td>
</tr>
<tr>
<td>S Scy</td>
<td>N3, C5, 4</td>
<td>SR</td>
<td>0.56</td>
<td>1980 Apr 28</td>
<td>81</td>
<td>46</td>
</tr>
<tr>
<td>SS Vir</td>
<td>Ne, C6, 3</td>
<td>Mira</td>
<td>0.73</td>
<td>1979 Dec 6</td>
<td>146</td>
<td>56</td>
</tr>
<tr>
<td>ST Cam</td>
<td>N5, C6, 4</td>
<td>SRb</td>
<td>0.42</td>
<td>1979 Jul 7</td>
<td>113</td>
<td>88</td>
</tr>
<tr>
<td>T Lyr</td>
<td>Np, C6, 5; R6</td>
<td>Lb</td>
<td>0.45</td>
<td>1980 Jan 24</td>
<td>154</td>
<td>82</td>
</tr>
<tr>
<td>TU Gem</td>
<td>N3, C4, 6</td>
<td>SRb</td>
<td>1.00</td>
<td>1979 Nov 2</td>
<td>129</td>
<td>31</td>
</tr>
<tr>
<td>TW Oph</td>
<td>Nb, C6, 5</td>
<td>SRb</td>
<td>0.26</td>
<td>1980 Mar 29</td>
<td>113</td>
<td>114</td>
</tr>
<tr>
<td>TX Psc</td>
<td>N0, C6, 2</td>
<td>Lb</td>
<td>-0.76</td>
<td>1979 Jul 7</td>
<td>65</td>
<td>146</td>
</tr>
<tr>
<td>U Cam</td>
<td>N5, C6, 4E</td>
<td>SRb</td>
<td>0.38</td>
<td>1979 Jul 6</td>
<td>113</td>
<td>81</td>
</tr>
<tr>
<td>U Hya</td>
<td>N2, C7, 3</td>
<td>SRb</td>
<td>-0.67</td>
<td>1979 Nov 2</td>
<td>41</td>
<td>71</td>
</tr>
<tr>
<td>UU Aur</td>
<td>N3, C5, 3</td>
<td>SRb</td>
<td>-0.71</td>
<td>1979 Sep 29</td>
<td>57</td>
<td>87</td>
</tr>
<tr>
<td>UX Dra</td>
<td>N0, C7, 3</td>
<td>SRa</td>
<td>0.18</td>
<td>1980 Jan 25</td>
<td>113</td>
<td>59</td>
</tr>
<tr>
<td>V Aql</td>
<td>N6, C6, 4</td>
<td>SRb</td>
<td>-0.14</td>
<td>1980 Apr 28</td>
<td>81</td>
<td>103</td>
</tr>
<tr>
<td>V Cyg</td>
<td>Ne</td>
<td>Mira</td>
<td>0.82</td>
<td>1980 Feb 7</td>
<td>210</td>
<td>84</td>
</tr>
<tr>
<td>V460 Cyg</td>
<td>N1, C6, 3; R</td>
<td>Lb</td>
<td>0.23</td>
<td>1980 Apr 26</td>
<td>170</td>
<td>92</td>
</tr>
<tr>
<td>V Hya</td>
<td>N; C7, 5</td>
<td>SRa</td>
<td>-0.40</td>
<td>1979 Nov 27</td>
<td>121</td>
<td>154</td>
</tr>
<tr>
<td>VX And</td>
<td>N7</td>
<td>SRa</td>
<td>0.91</td>
<td>1979 Jul 8</td>
<td>129</td>
<td>69</td>
</tr>
<tr>
<td>VY UMa</td>
<td>N0, C6, 3; R8</td>
<td>Lb</td>
<td>0.56</td>
<td>1979 Nov 27</td>
<td>170</td>
<td>78</td>
</tr>
<tr>
<td>W CMa</td>
<td>N; R8</td>
<td>Lb</td>
<td>1.02</td>
<td>1980 Jan 4</td>
<td>121</td>
<td>43</td>
</tr>
<tr>
<td>W Ori</td>
<td>N5, C5, 3</td>
<td>SRb</td>
<td>-0.33</td>
<td>1979 Sep 29</td>
<td>65</td>
<td>77</td>
</tr>
<tr>
<td>W Cas</td>
<td>Np, C9, 1</td>
<td>SRa</td>
<td>0.49</td>
<td>1979 Jul 6</td>
<td>146</td>
<td>103</td>
</tr>
<tr>
<td>X Cnc</td>
<td>N3, C5, 4</td>
<td>SRb</td>
<td>0.30</td>
<td>1979 Oct 28</td>
<td>194</td>
<td>61</td>
</tr>
<tr>
<td>Y Hya</td>
<td>N3, C5, 4</td>
<td>SRb</td>
<td>0.54</td>
<td>1980 Oct 30</td>
<td>129</td>
<td>115</td>
</tr>
<tr>
<td>Y CVn</td>
<td>N3, C5, 4; R</td>
<td>SRb</td>
<td>-0.77</td>
<td>1979 Nov 27</td>
<td>49</td>
<td>105</td>
</tr>
<tr>
<td>Y Tau</td>
<td>N2; C7, 4e</td>
<td>SRa</td>
<td>0.39</td>
<td>1979 Nov 2</td>
<td>97</td>
<td>69</td>
</tr>
<tr>
<td>Z Psc</td>
<td>N0; C7, 3</td>
<td>SRb</td>
<td>0.80</td>
<td>1979 Jul 8</td>
<td>129</td>
<td>65</td>
</tr>
</tbody>
</table>

a One standard deviation at peak signal. All spectra apodized by function F2 of Norton and Beer 1976.
and V Hya, which are displaced slightly from the remainder of our stars, dust emission may contribute slightly at 2 μm. Our spectra show the lines of the Mira variables S Cep and SS Vir to be certainly double and probably double, respectively. V Cyg, S Cep, and SS Vir were excluded from the abundance analysis. We assume that spectra of the remaining 30 stars record the photospheric spectrum free from significant contamination by a circumstellar dust shell.

Spectra from 2380 to 2800 cm\(^{-1}\) (4.2–3.6 μm) were available from archival tapes for five carbon stars: TX Psc, UU Aur, U Hya, Y CVn, and RY Dra. The spectra were observed with the KPNO prototype Fourier transform spectrometer (FTS). The resolution is less, 0.08 cm\(^{-1}\) (~9 km s\(^{-1}\)), than that in the 4000–6600 cm\(^{-1}\) region. In order to remove the telluric lines, the spectra were ratioed to a spectrum of IRC +10216 obtained on the same night. IRC +10216 is featureless in this spectral region (Barnes et al. 1977). The spectrum of TX Psc is displayed by Ridgway et al. (1984). An additional spectrum of TX Psc from 2400 to 3050 cm\(^{-1}\) was obtained in 1981 June.

In the following subsections, we discuss the principal molecular and atomic lines contributing to the abundance analysis. Basic data for the lines are presented in Appendix A.

b) Molecular Hydrogen

Discussion to date of the H\(_2\) quadrupole lines in spectra of carbon stars has focused on the 1–0 \(S(1)\) transition at 4712.9 cm\(^{-1}\) (Goorvitch, Goebel, and Augason 1980; Johnson et al. 1983). Unfortunately, this line is sandwiched between two CN lines, and a thorough spectrum synthesis is required to define H\(_2\)'s contribution. The first member of the \(S\)-branch—\(S(0)\) at 4497.84 cm\(^{-1}\)—is present on our spectra and, while neighboring lines intrude upon the H\(_2\) line profile, the line's central depth may be measured and an equivalent width accurate to about ±20% estimated for most of our stars. Sample spectra in Figure 1 show the H\(_2\) line in four stars. Johnson et al. detected the \(S(1)\) line in only the coolest stars of their sample, including T Lyr, RV Cyg, and WZ Cas from Table 1. Our detection of the \(S(0)\) line in almost all of the stars confirms Johnson et al.'s suggestion that the H\(_2\) equivalent width \(W_e\) increases with decreasing effective temperature (Fig. 2; our estimates of the effective temperatures are discussed later), but striking exceptions are apparent (e.g., WZ Cas).

The identification of the \(S(0)\) line appears secure. The observed stellar line matches the predicted frequency of the \(S(0)\) line. The intensities of the \(S(0)\) and \(S(1)\) lines are well correlated. A search of the C\(_2\) and CN line lists revealed no competing identifications. More important, we note key exceptions to the general correlation between the \(S(0)\) line's \(W_e\) and the intensity of C\(_2\) and CN lines; for example, RY Dra and T Lyr with strong C\(_2\) lines both show a strong H\(_2\) line, but so does WZ Cas with very weak C\(_2\) lines, and Y CVn with strong \(^{12}\)CN and \(^{13}\)CN shows only a weak H\(_2\) line.

c) Diatomic Carbon C\(_2\)

The interval 4360–5100 cm\(^{-1}\), bounded by a strong H\(_2\)O telluric band and the CO 2–0 bandhead, was searched for

![Fig. 1.—H\(_2\) 1–0 \(S(0)\) line in four cool carbon stars. The broken line in this and subsequent figures denotes the continuum, which was based on a considerably broader spectral interval than that reproduced here.](image_url)
CHEMICAL COMPOSITION OF CARBON STARS. I.

Teff [K]

Fig. 2.—Equivalent width $W_a$ of the H$_2$ 1--0 S(0) line vs. effective temperature, $T_{\text{eff}}$, of the cool carbon star. The inset gives the variable class (M = Mira). Filled symbols are stars for which Tsuji (1981a) gives $T_{\text{eff}}$. The $T_{\text{eff}}$ values for the other stars (open symbols) are from Table 3.

$^{12}\text{C}_2$ lines from both the singlet Phillips ($\Delta v = -2$) and Ballik-Ramsay ($\Delta v = 0$) systems. Several $^{12}\text{C}^{13}\text{C}$ Phillips lines are detectable in spectra of $^{13}\text{C}$-rich stars. No search was conducted for $^{12}\text{C}^{13}\text{C}$ Ballik-Ramsay lines. The Ballik-Ramsay 0--0 bandhead at 5650 cm$^{-1}$ was rejected because it is generally very saturated and blended with many telluric lines. No serious search was made for lines of the Phillips $\Delta v = -1$ bands which occur with CO and CN lines within the 5600--6600 cm$^{-1}$ window. Figure 3 shows representative $^{12}\text{C}_2$ lines in sample spectra. For a typical star, we measured about 10 Phillips and five Ballik-Ramsay lines. In the few cases in which all $^{12}\text{C}_2$ lines were very strong, weak $^{12}\text{C}^{13}\text{C}$ lines and an estimate of the $^{12}\text{C}/^{13}\text{C}$ ratio replace the $^{12}\text{C}_2$ lines in the analysis.

d) The CN Red System

The 4360--5100 cm$^{-1}$ interval provides an adequate number of weak $^{12}\text{CN}$ and $^{13}\text{CN}$ lines; Figure 4 shows typical CN lines. Our primary line list for $^{12}\text{CN}$ consisted of about 140 lines that were effectively unblended. The $^{13}\text{CN}$ list with slightly less than 100 entries includes single $^{12}\text{CN}$ lines as well as blends of $^{13}\text{CN}$ lines. For a typical star, we measured about 80--100 $^{12}\text{CN}$ and 30--50 $^{13}\text{CN}$ lines.

In the warmer stars, where the CN lines are weakest, the continuum is defined by numerous points whose envelope matches the convolution of the instrumental sensitivity with the expected stellar flux distribution. Unless the entire window is depressed rather uniformly by a blanket of molecular lines, location of the continuum is not a significant contributor to the $W_a$ uncertainties for $^{12}\text{C}_2$ and CN. Both the number and the width of the continuum points are reduced in the cooler stars.

Spectra at 4$\mu$m were searched for unblended $^{12}\text{CN}$ $\Delta v = -3$ lines. Selected lines from the 0--3, 1--4, and 2--5 bands were measured in TX Psc, UU Aur, U Hya, Y CVn, and RY Dra. A similar number of $^{13}\text{CN}$ lines from the 1--4 and 2--5 bands was measured in Y CVn and RY Dra. At 4$\mu$m, continuum windows are wider and more numerous than at 2$\mu$m, and the

© American Astronomical Society • Provided by the NASA Astrophysics Data System
were sufficiently complete and the unidentified lines so rare were identified by hand, and their equivalent width was 0.

Fines and outputs a plot showing the central depth r against fines suitable for an abundance analysis. A semiautomatic bad point corresponding to the initial members of these bands. In retrospect, these exceptions were to be expected because such lines are identified by extrapolating the characteristic curves, and the inevitable presence of blends biases the extrapolation. Second, if the glaring exceptions are excluded, the individual second-overtone bands give remarkably similar results; for example, the CO abundance (relative to the selected model) for UU Aur is the same within the statistical errors for all bands of the \( \Delta v = 3 \) sequence. As a result of our scrutiny of the six stars, we rejected the lowest and highest rotational lines. Then our tests show the search by computer to provide an unbiased list of CO lines.

First-overtone bands beginning with the 2–0 R-branch bandhead at 4363 cm\(^{-1}\) provide \(^{13}\)C\(^{16}\)O lines that are too strong for an abundance analysis. We retained the lines with central depths of 50% or less of the continuum. The deeper lines carry little information on the oxygen abundance and should ultimately provide novel information on the structure of the outer atmosphere. An interesting discrepancy occurs between the abundances obtained from the strongest lines of our sample of \( \Delta v = 2 \) lines [log(\( W_\lambda / \sigma \)) \( \approx \) -4.8] and the abundance provided by the \( \Delta v = 3 \) lines. While the few weaker \( \Delta v = 2 \) lines agree with the \( \Delta v = 3 \) lines, the stronger \( \Delta v = 2 \) lines often give systematically lower abundances in the cooler stars. This tendency does not occur for the hotter stars (\( T_{\text{eff}} \approx 2900 \) K). In the coolest stars the discrepancy amounts to about 0.8 dex and is a remarkable 2.0 dex for V Hya. The strongest \(^{13}\)C\(^{16}\)O lines in the \( \Delta v = 2 \) bands show a similar but not an identical abundance–line depth relation. We suggest that overlying bands of HCN, \(^2\)C\(_2\)H\(_2\), and \(^2\)C\(_2\)H (Curl, Carrick, and Merer 1985) provide a quasi-continuous opacity, which, in conjunction with the increasing density of telluric lines toward the low-wavelength end of the interval, affects the continuum location; an error in the adopted continuum level would lead to much greater effects on the abundance extracted from saturated lines than from the weak lines actually used in the analysis. Revisions to the outermost layers of the models and a consideration of non-LTE effects may be required to account for the differing behaviors of strong \(^{13}\)CO and \(^{13}\)CO fines. For some stars, e.g., V Hya, emission from circumstellar dust grains may dilute the photospheric spectrum and, hence, result in spuriously low oxygen abundances. V Hya is the sole member of our sample with a pronounced excess of flux at long wavelengths (see below).

It is noteworthy that the phenomenon just discussed does not occur for the measured CN \( \Delta v = -2 \) in the interval 4360 < \( \sigma < 5000 \) cm\(^{-1}\), i.e., the microturbulence parameter needed to force the stronger CN lines to give the same abundance as the weaker lines is consistent with that derived from the CO \( \Delta v = 3 \) lines (see below).

The weakest CO \( \Delta v = 2 \) lines provide abundances consistent with that given by the \( \Delta v = 3 \) lines. Hence, the \( \Delta v = 2 \) bands do provide useful lines from the least abundant species \(^{13}\)C\(^{16}\)O, \(^{13}\)C\(^{16}\)O, and \(^{13}\)C\(^{16}\)O. Extraction of the \(^{13}\)C/\(^{12}\)C ratio from CO lines is discussed below. Oxygen isotopic abundances are discussed elsewhere (Harris et al. 1986).

The strongest \( \Delta v = 2 \) lines are 2–0 lines with \( J'' > 40 \). These low-excitation lines are known to be formed in a region kinematically distinct from the photosphere in oxygen-rich Mira variables (Hinkle, Hall, and Ridgway 1982). Examina-
Fig. 6.—Logarithmic CO abundance provided by the weaker \( \Delta v = 3 \) lines \([\log(\frac{W_v}{\sigma}) \leq -4.8]\) in the spectrum of UU Aur. It is seen that the individual bands give very consistent abundance results. Note, however, the systematically higher \( \delta(\text{CO}) \) values provided by the highest excitation lines of an individual band.

f) The CH Vibrational-Rotation Lines

The fundamental vibration-rotation bands of the CH \(^2\Pi\) ground state cross the interval recorded on the 4 \( \mu \)m spectra. The \( P \) and \( R \)-branches with the initial \( Q \) lines of both the 1–0 and the 2–1 bands are readily traced in the five stars for which 4 \( \mu \)m spectra are available. Undoubtedly, the 3–2 band, and possibly others, contribute weak lines, but term values for \( v \geq 3 \) are uncertain. Unfortunately, the first-overtone band 2–0 is in the 1.8 \( \mu \)m atmospheric absorption band. Our identifications represent the first detection of the CH spectra with 200 or more CO lines measured (BL Ori, W CMa, U Hya, and UX Dra) a statistical trend of \( \Delta v = 3 \) velocity with excitation potential is apparent. The lowest excitation lines have velocities +0.7 km s\(^{-1}\) (0.01 cm\(^{-1}\) at 6000 cm\(^{-1}\)) relative to the highest excitation lines. A similar trend of velocity with excitation potential was noted in the MS-type Mira \( \chi \) Cygni by Hinkle, Hall, and Ridgway.
Fig. 7.—The CH 1–0 P3 quartet in the spectrum of (a) TX Psc, (b) UU Aur, (c) Y CVn, and (d) U Hya.

V–R bands in any extraterrestrial source; Ridgway et al. (1984) independently identified CH lines in TX Psc. Figure 7 shows several CH lines.

g) The NH Vibration-Rotation Lines

A thorough search of the 4 μm spectra of TX Psc was conducted for NH vibration-rotation lines from the 1–0 and 2–1 P-branches. For each NH line, CN and CH lines within about ±10 cm⁻¹ were identified, so that the CN or CH contribution to the putative NH line could be assessed. Our result is that NH lines are present in TX Psc's spectrum; the 1–0 P5, P7, and P9 lines are present with minor contributions from CH, P6 and P8 are present but blended, P10 and P11 are blended beyond recognition, and higher lines correspond to tentative identifications or to upper limits that are consistent with predictions based on the identified stronger lines. Figure 8 illustrates these novel identifications for spectra of carbon stars. The 2–1 branch is present, with P2, P6, and P9 identified tentatively and other lines providing consistent upper limits. Confidence in the new identifications is heightened by the fact that few lines remain unidentified after CN, CH, HCl, and now NH lines are accounted for in the interval 3050 > σ > 2585 cm⁻¹; CS first-overtone bands contribute many lines for σ < 2585 cm⁻¹. At this low frequency for unidentified lines, the NH identification are most unlikely to be chance coincidences.

Fig. 8.—(a) NH 1–0 P5 and (b) NH 1–0 P7 triplets in the spectrum of TX Psc. Two CH quartets and the H³Cl 1–0 R2 line are identified. Selected ¹⁴CN lines are marked. The breaks in the spectrum occur where strong telluric lines cause this ratioed spectrum (TX Psc/Sun) to be unreliable.

Spectra at 2380–2800 cm⁻¹ are available also for Y CVn, U Hya, UU Aur, T Lyr, and RY Dra. Tentative identifications of NH are provided for 1–0 P12 and 2–1 P6 and P8. The most convincing identifications for TX Psc lie outside this spectral interval.

h) The HCN 3 μm Lines

Archival spectra of the broad 3 μm band were inspected. Ridgway, Carbon, and Hall (1978) identified HCN and C₂H₂ bands as the source of the stellar 3 μm band in these spectra of TX Psc, UU Aur, Y Tau, and T Lyr. The band is quite weak in TX Psc, and several individual lines of the leading HCN bands are sufficiently well resolved to permit a measurement of their Wₚ values. These measurements of HCN lines were intended to provide a consistency check on the abundance analysis; C₂H₂ lines are not obviously present in TX Psc (Ridgway, Carbon, and Hall 1978). The 3 μm band in the other stars is so strong that a detailed synthesis is demanded and lies outside the scope of this paper.

i) Atomic Lines

A search for atomic lines was carried out for the 4360–5100 cm⁻¹ interval. Species with two or more unblended or slightly blended lines include Ca i, Sc i, Ti i, and Fe i. The weaker lines that are at most lightly blended with a CN line include Ca i 4413 and 5045 cm⁻¹ and Fe i 4466, 4491.1 and 4491.7 cm⁻¹. A fine at 4685 cm⁻¹ was tentatively identified as a Na i line.

III. MODEL ATMOSPHERES

a) The Grid

Our analysis makes use of a new grid of model atmospheres whose salient features are sketched in this section. Outstanding characteristics of the models include (1) their sensitivity to
the assumed composition and (2) the opacity contributed by polyatomic molecules (notably HCN and C$_2$H$_2$) with concomitant influence on the atmospheric structure. Our analysis responds to these characteristics through (1) the use of a dense grid of model atmospheres spanning a range of $T_{\text{eff}}$, $g$, and CNO abundances and (2) the adoption of the best available opacities for HCN and C$_2$H$_2$ as well as for abundant diatomic molecules.

The standard model atmospheres were based on the usual assumptions of plane-parallel stratification, hydrostatic equilibrium, conservation of radiative flux, and LTE. The effects of spherical extension were investigated by calculating a number of spherically symmetric models, using the method of Nordlund (1984), and were found to be unimportant for models in hydrostatic equilibrium. This situation, which is in accordance with the results of Scholz and Tsuji (1984), is different from that for M giant models with comparable temperatures and gravities (Scholz and Wehrse 1982 and references cited therein). In addition to the radiative flux, the convective flux in the mixing-length approximation was taken into account in some models, and introduced negligible effects on the calculated spectra. This is also true of the convective fluxes calculated on the assumption that the convective velocities are constant through the convectively unstable layers—a reasonable assumption in these low-density atmospheres. Some models were calculated with a turbulent pressure added to the gas and radiation pressures. It was found that this led to temperature-pressure structures very close to those of corresponding models calculated with a smaller surface gravity, $g'$, where

$$g' = g/(1 + 160 \beta c_s^2/T_{\text{eff}});$$

where $g$ is the true gravity, $T_{\text{eff}}$ is the effective temperature in Kelvins, and $\beta$ is dependent on the angular velocity distribution but is thought to be in the interval $1/2 \leq \beta \leq 1$, and $c_s$ is the characteristic turbulent velocity in kilometers per second. This equation is easy to derive (cf. Gustafsson et al. 1975) from an assumption of hydrostatic equilibrium.

The molecular-line opacity was handled with the opacity distribution function (ODF) technique. In practice, individual ODFs were calculated for the molecules C$_2$, CN, CO, HCN, and C$_2$H$_2$ for a number of temperatures and pressures. During the model atmosphere calculations, interpolations to the proper temperature/pressure combinations were made in those tables, and then the ODFs were folded together by means of the procedure described by Saxner and Gustafsson (1984).

The basic assumption of the ODF technique is, roughly speaking, that the relative strength of the opacity at different wavelengths within each ODF wavelength interval at a certain depth correlates well with that at other depths (Carbon 1984). This assumption may not be true, especially not when the absorptions of different molecules are important at different depths. This is certainly the case for the cooler carbon stars, where the absorption from the polyatomic molecules dominates in the upper layers, while the diatomic molecules are most important deeper in the atmospheres. In fact, detailed opacity-sampling models with several times $10^4$ frequency points have proved that the ODF approximation for models with $T_{\text{eff}} \leq 2900$ K may lead to significant errors in the atmospheric structure (Ekberg, Eriksson, and Gustafsson 1985). We repeated the abundance analysis using some of these more detailed ODF models, but found the errors in the derived abundances to be small.

The basic molecular data used for the model atmospheres were essentially those described by Eriksson et al. (1984). Since then, a new set of detailed quantum-mechanical ab initio calculations for HCN have been performed by Jørgensen et al. (1985) in order to improve the opacity data for the numerous vibration-rotation bands of HCN. Although these data have not been found to change the absorption noticeably in many spectral regions, the model atmospheres (and the resultant abundances) calculated with the new data for HCN are not drastically different. The C$_2$H$_2$ opacities are still very uncertain, but considerable systematic changes (by 1 order of magnitude in the absorption coefficient) result in minor alterations to the abundances.

The present models are successful in predicting the intensity of the quadrupole H$_2$ lines (see below) and Na D lines. Typical observed D-line widths (H. B. Richer 1985, unpublished observations) are 20 Å (BL Ori), 25 Å (Z Psc and VY UMa), and 30 Å (X Cnc). At the adopted effective temperatures (see below), these values agree well with the predictions of synthetic spectra for the “polyatomic” models, while models constructed without the polyatomic opacity predict these lines to be stronger by a factor of 2–8. In order to fit the “nonpolyatomic” model predictions, the effective temperatures would have to be raised by several hundred degrees in order to decrease the pressures sufficiently to reduce the damping widths of the D lines to the observed values.

However, the predicted fluxes from our (polyatomic) models show stronger bands (e.g., the 3 μm band) from polyatomic molecules than those observed. This problem is further discussed by Ekberg (1986) and is illustrated in Figure 9. The discrepancies are not diminished significantly when the new ab initio calculations are used for HCN or when the C$_2$H$_2$ opacity is totally excluded.

Although our models predict the total HCN band strengths, including that of the “3 μm band” observed at low resolution, to be far too strong, the individual lines of the fundamental bands identified in the spectrum of TX Psc are predicted to have roughly the observed strengths (see below, § IV/). The individual HCN lines identified at high resolution are fairly saturated and thus rather weakly dependent on the individual oscillator strengths or the HCN abundances. Nevertheless, the agreement for the individual lines suggests that the HCN abundances in TX Psc are approximately correct and that either the calculation of the oscillator strengths of the enormous number of higher excitation HCN lines, severely affecting the low-resolution scan, is in error, or the observed strengths of these lines are strongly reduced by non-LTE effects. The former suggestion could possibly be relevant because of certain problems in the treatment of the highly excited states in the ab initio calculations (the handling of degeneracy, the identification of vibronic quantum numbers, and the distribution of opacity remaining when all “explicitly treated lines” have been considered). The possibility that departures from LTE would affect the equivalent widths of the hot-band lines sufficiently to account for the discrepancy

© American Astronomical Society • Provided by the NASA Astrophysics Data System
Fig. 9.—Observed and predicted flux distributions for TX Psc. The observed distribution was taken from Johnson et al. (1985) and is shown in each panel by the thick line with gaps corresponding to the opaque telluric bands. The predicted fluxes for an appropriate model from the standard grid are shown in the bottom panels. Prediction for a H-deficient model [log e(H) = 10 rather than the standard log e(H) = 12] are shown in the middle panel. The top panel shows that observed and predicted fluxes agree better when the opacity contributed by HCN and C$_2$H$_2$ is reduced by a factor of 10.
may not seem very attractive. However, the HCN lines are formed and block radiation in a rather thin layer on top of an atmosphere that is, at certain wavelengths, relatively transparent. Thus, the mean intensity, determined by the conditions in the deeper layers, is considerably greater than the local Planck function, in particular for wavelengths shorter than 3 μm. Therefore, the source function may deviate very significantly from the Planck function, especially for weak lines and for lines formed in scattering processes. Preliminary calculations with a simple two-level model for the molecule have indicated that very severe effects of this nature may occur for lines weaker than 50 mA and at wavelengths shorter than 2.5 μm; such lines may be reduced in strength by typically a factor of 2. More detailed studies with more adequate model molecules and with a realistic treatment of the background opacity from the wealth of other HCN lines will be undertaken. It should be noted that if the HCN lines are formed in scattering, the line opacity decouples from the temperature structure and the effects on the model atmospheres are much less.

The measurement of individual 3 μm HCN lines has not been possible in cooler stars, where the discrepancy between predicted and observed strengths of the polyatomic bands is more severe than for TX Psc. For these stars it is not improbable that the abundance of the polyatomic species is reduced relative to the models by non–LTE effects or by heating of the outer atmospheric layers. An extensive heating of the surface layers would significantly reduce the column density of polyatomic molecules. The heating needed to reduce the calculated polyatomic bands enough to reproduce the scans must start between $T_{\text{Ross}} = 0.1$ and $T_{\text{Ross}} = 0.01$, and be chosen such that the photosphere outside this depth is roughly isothermal. Such a heating would result from a transport of mechanical energy through the deeper photosphere and deposited in the upper layers in a suitable manner; the fraction of the total stellar flux that has to be transported in this way is, however, very great, on the order of several percent of the total flux. Deep chromospheres of this type were discussed by Johnson et al. (1983) as a possible explanation for the weak H$_2$ quadrupole lines. Another possible mechanism for producing such a flat temperature structure in the upper photosphere would be the heating effects of dust, formed in the outer layers of the photospheres. Ekberg (1986) has found that a condensation temperature significantly above 2000 K is required for this mechanism to produce the observed flux distribution. However, the heated models predict the individual HCN fundamental lines to be about a factor of 2 too weak in the spectrum of TX Psc.

Another possible explanation or the discrepancy between the predicted and observed polyatomic band strengths may be that the adopted effective temperatures are too low by about 100 K for the hotter stars and by 300 K for the cooler stars. This possibility will be further discussed in §§ IV/(vi).

Whatever the cause for the discrepancy, it seems quite possible that the effects of molecular opacities of HCN and C$_2$H$_2$ were severely overestimated in the present models. From comparisons between calculated and observed scans and colors (cf. Ekberg 1986), it is found that a depletion of the polyatomic opacity ranging from a factor of 1/20 to a factor of 1/5000, when $T_{\text{eff}}$ varies from 3000 to 2700 K, is needed in order to fit the scanner observations. Models with such opacity modifications produce significantly stronger H$_2$ lines than observed, provided that the surface layers are not heated, since a significant increase of the pressure, and thus of the H$_2$ abundance, occurs in the upper layers of these models. Consequently, in order to reproduce the observed H$_2$ line strengths with such models, the hydrogen content must be reduced significantly. Such reductions are, however, found to lead to conflicts with other observations (see below, § IVe). Alternatively, the pressures in the outer layers have to be reduced by means of opacity sources other than HCN and C$_2$H$_2$, such as absorption by vibration-rotation or electronic transitions from other polyatomic molecules (C$_3$, SiC$_2$, and so on) or by dust absorption.

In view of the uncertainties discussed above, we have constructed extensive grids of additional models, with various degrees of reductions of polyatomic opacities, as well as models with no absorption from HCN and C$_2$H$_2$ at all, models with reduced hydrogen abundances, and models with "artificially" flattened temperature gradients in their outer layers. These models were then used in the abundance analysis for studying the effects of the model uncertainties. The results of this study are presented in § IIIe.

The present grid of models was calculated for $^{12}$C/$^{13}$C = 10 and for a microturbulence $\xi = 2$ km s$^{-1}$. Some models were constructed for $^{12}$C/$^{13}$C = 3 and 99, respectively, and for $\xi = 3$ km s$^{-1}$. The subsequent changes in the models, and, thence, in the derived abundance results, are minor. The blanketing from metal lines using the method of Gustafsson et al. (1975) was found to be of no importance for the present models and was therefore neglected.

The standard grid of models is summarized in Table 2. The grid will be described in detail in a forthcoming publication. The response of the temperature-pressure profile to changes in various parameters is shown in Figure 10. Variations arising from alterations to the polyatomic opacity, the H content, and heating of the outer layers are shown in Figure 11. The choice of fundamental parameters $T_{\text{eff}}$ and $g$ for the models is discussed in the following sections.

b) Effective Temperature

Determination of effective temperatures for cool carbon stars is now possible by the two methods that we label the "method of lunar occultations" and the "infrared flux method." Temperatures provided by these methods are the
foundation for our adopted $T_{\text{eff}}$ values. These methods provide results for just one-half of our sample. For the remaining stars, we employ empirical relationships between $T_{\text{eff}}$ and an observational quantity such as a color index.

A lunar occultation provides an estimate of the star's angular diameter that, in combination with the apparent bolometric magnitude, yields an estimate of $T_{\text{eff}}$. Results for six carbon stars are discussed by Ridgway et al. (1980, 1981); five of the stars are on our program: Y Tau, X Cnc, TW Oph, AQ Sgr, and TX Psc. Published error estimates range from ±70 to ±150 K; the uncertain reddening of TW Oph introduces a larger systematic error.

The ratio of a monochromatic flux to the bolometric flux is a unique and calculable function of $T_{\text{eff}}$ for a blackbody, and
CHEMICAL COMPOSITION OF CARBON STARS. I.

Fig. 11.—Temperature-pressure profiles for nonstandard models for an effective temperature $T_{\text{eff}} = 2800$ K and a gravity $\log g = 0.0$. (a) The influence of the polyatomic molecules is shown by the three profiles: the open squares describe a standard model with the polyatomics' opacity included; the open circles refer to the corresponding model without the polyatomics' opacity. An intermediate case (plus signs) with their opacity reduced by a factor of 1000 is also shown. (b) Changes introduced as the H content is reduced. Normal abundance of H[$\log e(\text{H}) = 12$] provides the profile defined by the open squares. Reductions to $\log e(\text{H}) = 11$ and 10 provide the profiles identified by plus signs and open circles, respectively. (c) Standard model (open squares) and one of a series in which the temperature profile was raised in the outer layers to simulate deep heating (plus signs). Nonstandard models of these types were used to compute the flux distributions shown in Fig. 9.

As noted above, our fluxes agree well with observed fluxes and colors except for wavelength intervals affected by (predicted) polyatomic absorption. Since such discrepancies have yet to be resolved, we postponed a final recalibration of the infrared flux method using the present grid and elected to rely on the overall consistency of Tsuji's predicted fluxes with ours and of his $T_{\text{eff}}$ scale with that based on angular diameters. The $T_{\text{eff}}$ scale obtained from our model fluxes after empirical corrections for the polyatomic absorption in the $L$ band, following Noguchi et al. (1977), agrees well with Tsuji's $T_{\text{eff}}$ scale.

The infrared flux method supplies $T_{\text{eff}}$ for half of our stars. To obtain $T_{\text{eff}}$ for the remainder of our sample, we looked at several potential thermometers. Four are discussed briefly: (1) the [1.25]–[2.2] color index (Frogel and Hyland 1972); (2) the ($I$ – $L$) color index with observations taken from the extensive list by Noguchi et al. (1981) (this photometry is not on the Johnson system); (3) the ($H$ – $K$) color index with independent calibrations constructed from photometry by Noguchi et al. and Catchpole et al. (1979); and (4) the [8.4]–[3.5] color index (Gillett, Merrill, and Stein 1971).

Published photometry shows that the Mira and some of the SRa variables cannot be calibrated using 1, 2, and 3 above because these indices for these types of stars show large variations. For example, Catchpole et al. (1979) report ($H$ – $K$) for R Lep (Mira) to vary from 0.57 to 1.12, but Tsuji’s (1981a) calibration stars show a range of just 0.25 mag as $T_{\text{eff}}$ runs from 3000 to 2500 K. Fortunately, the [8.4]–[3.5] color changes only slightly during a Mira variable’s pulsation cycle.

Fig. 12.—Variation of the [8.4]–[3.5] color with effective temperature. Colors are taken from Gillett, Merrill, and Stein (1971; filled circles) except for the measurement of UU Aur by Dyck et al. (1971; open circle). Effective temperatures are provided by the infrared flux method (Tsuji 1981a).
Forrest, Gillett, and Stein (1975) find a $\pm 0.1$ mag change across R Lep's pulsation cycle, an amplitude only slightly larger than that reported for the SRb variable UU Aur; a $\pm 0.1$ mag change in the color index translates to $\pm 130$ K in $T_{\text{eff}}$. With Tsuji's stars showing a scatter of only $\pm 60$ K around the mean relation in the range $2500 \leq T_{\text{eff}} \leq 2900$ K (Fig. 12), the [8.4]–[3.5] color may be the most accurate of our interpolating procedures. In view of the small number of stars, except for the calibration stars, for which a [8.4]–[3.5] colors is available, this index has not played a major role in the temperature calibration. One may expect circumstellar dust emission to influence the color and to introduce scatter in a color-$T_{\text{eff}}$ plot.

Temperature estimates derived from the four different color indices are given in Table 3. The scatter of the calibration stars about the mean color-$T_{\text{eff}}$ relations is not obviously correlated with the C, N, and O abundances. A molecular band’s intensity is generally more sensitive to temperature than to chemical composition and, hence, the scatter attributable to composition differences may be of minor importance.

The strength of the 3 $\mu$m band composed of HCN and C$_2$H$_2$ lines increases sharply with decreasing effective temper-

<table>
<thead>
<tr>
<th>STAR</th>
<th>TYPE</th>
<th>IR Flux$^a$</th>
<th>Occultations$^b$</th>
<th>3 $\mu$m Band$^c$</th>
<th>(I - L)$^d$</th>
<th>(H - K)$^e$</th>
<th>[1.25]–[2.2]$^f$</th>
<th>[8.4]–[3.5]$^g$</th>
<th>Adopted</th>
</tr>
</thead>
<tbody>
<tr>
<td>AQ Sgr</td>
<td>SBa</td>
<td>2630</td>
<td>2680</td>
<td>2800</td>
<td>2970</td>
<td>2930</td>
<td>2960</td>
<td>2960</td>
<td>2960</td>
</tr>
<tr>
<td>BL Ori</td>
<td>Lb</td>
<td>3420</td>
<td>3100</td>
<td>2730</td>
<td>2510</td>
<td>2820</td>
<td>2510</td>
<td>2630</td>
<td>2630</td>
</tr>
<tr>
<td>R Lep</td>
<td>M</td>
<td>2500</td>
<td>2895</td>
<td>2760</td>
<td>2830</td>
<td>2860</td>
<td>2740</td>
<td>2800</td>
<td>2800</td>
</tr>
<tr>
<td>R Sco</td>
<td>SRa</td>
<td>2620</td>
<td>2390</td>
<td>2540</td>
<td>2780</td>
<td>2960</td>
<td>2500</td>
<td>2950</td>
<td>2950</td>
</tr>
<tr>
<td>RT Cap</td>
<td>SRb</td>
<td>2730</td>
<td>2510</td>
<td>2820</td>
<td>2510</td>
<td>2630</td>
<td>2630</td>
<td>2630</td>
<td>2630</td>
</tr>
<tr>
<td>RV Cyg</td>
<td>SRb</td>
<td>2730</td>
<td>2510</td>
<td>2820</td>
<td>2510</td>
<td>2630</td>
<td>2630</td>
<td>2630</td>
<td>2630</td>
</tr>
<tr>
<td>RY Dra</td>
<td>SRb</td>
<td>2500</td>
<td>2895</td>
<td>2760</td>
<td>2830</td>
<td>2860</td>
<td>2740</td>
<td>2800</td>
<td>2800</td>
</tr>
<tr>
<td>S Sco</td>
<td>SRa</td>
<td>2895</td>
<td>2760</td>
<td>2830</td>
<td>2860</td>
<td>2740</td>
<td>2800</td>
<td>2800</td>
<td>2800</td>
</tr>
<tr>
<td>ST Cam</td>
<td>SRa</td>
<td>2380</td>
<td>2810</td>
<td>2830</td>
<td>2820</td>
<td>2600</td>
<td>2770</td>
<td>2770</td>
<td>2770</td>
</tr>
<tr>
<td>T Lyr</td>
<td>Lb</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
</tr>
<tr>
<td>TU Gem</td>
<td>SRb</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
</tr>
<tr>
<td>TW Oph</td>
<td>SBa</td>
<td>2450</td>
<td>3010</td>
<td>2630</td>
<td>2530</td>
<td>2520</td>
<td>2430</td>
<td>2500</td>
<td>2500</td>
</tr>
<tr>
<td>TX Psc</td>
<td>Lb</td>
<td>2670</td>
<td>3100</td>
<td>2670</td>
<td>2670</td>
<td>2670</td>
<td>2670</td>
<td>2670</td>
<td>2670</td>
</tr>
<tr>
<td>U Cam</td>
<td>SRb</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
</tr>
<tr>
<td>U Hya</td>
<td>SRb</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
</tr>
<tr>
<td>UX Dra</td>
<td>SRa</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
<td>2825</td>
</tr>
<tr>
<td>V Aql</td>
<td>SBa</td>
<td>2610</td>
<td>3010</td>
<td>2610</td>
<td>2610</td>
<td>2610</td>
<td>2610</td>
<td>2610</td>
<td>2610</td>
</tr>
<tr>
<td>V Hya</td>
<td>SRa</td>
<td>2650</td>
<td>2770</td>
<td>2650</td>
<td>2920</td>
<td>2920</td>
<td>2920</td>
<td>2920</td>
<td>2920</td>
</tr>
<tr>
<td>VX And</td>
<td>SRa</td>
<td>2720</td>
<td>2720</td>
<td>2720</td>
<td>2720</td>
<td>2720</td>
<td>2720</td>
<td>2720</td>
<td>2720</td>
</tr>
<tr>
<td>VY UMa</td>
<td>Lb</td>
<td>2855</td>
<td>2855</td>
<td>2855</td>
<td>2855</td>
<td>2855</td>
<td>2855</td>
<td>2855</td>
<td>2855</td>
</tr>
<tr>
<td>V460 Cyg</td>
<td>Lb</td>
<td>2845</td>
<td>2770</td>
<td>2845</td>
<td>2990</td>
<td>2990</td>
<td>2990</td>
<td>2990</td>
<td>2990</td>
</tr>
<tr>
<td>W CMa</td>
<td>Lb</td>
<td>2888</td>
<td>2888</td>
<td>2888</td>
<td>2888</td>
<td>2888</td>
<td>2888</td>
<td>2888</td>
<td>2888</td>
</tr>
<tr>
<td>W Ori</td>
<td>SRa</td>
<td>2680</td>
<td>2680</td>
<td>2680</td>
<td>2680</td>
<td>2680</td>
<td>2680</td>
<td>2680</td>
<td>2680</td>
</tr>
<tr>
<td>WZ Cas</td>
<td>SRa</td>
<td>2480</td>
<td>2480</td>
<td>2480</td>
<td>2480</td>
<td>2480</td>
<td>2480</td>
<td>2480</td>
<td>2480</td>
</tr>
<tr>
<td>X Cnc</td>
<td>SBa</td>
<td>2670</td>
<td>2670</td>
<td>2670</td>
<td>2670</td>
<td>2670</td>
<td>2670</td>
<td>2670</td>
<td>2670</td>
</tr>
<tr>
<td>X CVn</td>
<td>SBa</td>
<td>2730</td>
<td>2730</td>
<td>2730</td>
<td>2730</td>
<td>2730</td>
<td>2730</td>
<td>2730</td>
<td>2730</td>
</tr>
<tr>
<td>Y Hya</td>
<td>SRb</td>
<td>2520</td>
<td>2520</td>
<td>2520</td>
<td>2520</td>
<td>2520</td>
<td>2520</td>
<td>2520</td>
<td>2520</td>
</tr>
<tr>
<td>Y Tau</td>
<td>SRa</td>
<td>2520</td>
<td>2520</td>
<td>2520</td>
<td>2520</td>
<td>2520</td>
<td>2520</td>
<td>2520</td>
<td>2520</td>
</tr>
<tr>
<td>Z Psc</td>
<td>SBa</td>
<td>2790</td>
<td>2790</td>
<td>2790</td>
<td>2790</td>
<td>2790</td>
<td>2790</td>
<td>2790</td>
<td>2790</td>
</tr>
</tbody>
</table>

$^a$Tsujii 1981a, Table 2 (Table 3 for Y Tau).
$^b$Ridgway et al. 1980. For TW Oph (Ridgway et al. 1981) we assume $E_{B-V} = 0.6$. The estimate of Ridgway et al. 1980 of $T_{\text{eff}} = 2385$ K for X Cnc is increased to 2560 K to reflect the smaller angular diameter reported by Ridgway et al. 1982.
$^c$The optical depth $\tau_{3 \mu m}$ adopted for VX And, Y Hya, and WZ Cas was derived from the mean relation between the 3 $\mu$m index and $T_{\text{eff}}$ (Tsuji 1981a, Table 2). The estimate of Noguchi et al. 1981 rather than their original estimate $\tau_{3 \mu m} = 0.91$ measured by Noguchi et al. 1981 was adopted for VX And, Y Hya, and WZ Cas.
$^d$All photometry from Noguchi et al. 1981.
$^e$Estimates for all stars except Z Psc are based solely on Noguchi et al.’s photometry. For Z Psc, we average $T_{\text{eff}} = 2910$ and 3070 K from calibrations of photometry by Noguchi et al. 1981 and Catchpole et al. 1979, respectively.
$^f$Photometry by Frogel and Hyland 1972.
$^g$Photometry by Gillett, Merrill, and Stein (1971), but with $[8.4]–[3.5] = -0.53$ (Dyck et al. 1971; Forrest, Gillett, and Stein 1975) for UU Aur rather than $-1.12$ (Gillett, Merrill, and Stein, 1971), which must be in error. The Y Tau estimate from photometry by Forrest et al. involves an extrapolation of the calibration.
$^h$BL Ori with $T_{\text{eff}} = 3420$ K departs markedly from all of the calibrations, and, therefore, we discarded it as a calibration star.
Fig. 13.—Variation of the optical depth ($\tau_{3 \mu m}$) of the 3 $\mu$m band with effective temperature. The adopted $\tau_{3 \mu m} - T_{\text{eff}}$ calibration is shown by the solid line. The three $^{13}$C-rich stars T Lyr, RY Dra, and Y CVn may have systematically smaller $\tau_{3 \mu m}$. The discrepant point at $T = 2520$ K and $\tau_{3 \mu m} = 0.35$ corresponds to Y Tau.

ature (Tsuji 1981b). We adopt the band’s optical depth ($\tau_{3 \mu m}$; see Noguchi et al. 1977, 1981) as a measure of its intensity. The calibration is shown in Figure 13. There is a hint that the $^{13}$C-rich stars are displaced slightly ($\Delta T \sim 70$ K) from the general relation defined for $2600 < T_{\text{eff}} < 2900$ K. Unless circumstellar emission contributes at 3 $\mu$m, $\tau_{3 \mu m}$ could be a reliable thermometer even for Mira variables. Observations are presently too sparse to test this assertion. Temperatures based on $\tau_{3 \mu m}$ are listed in Table 3.

Baumert (1972) obtained a color temperature ($T_c$) for a large sample of carbon stars with narrow-band near-infrared photometry. Ridgway et al. (1980, 1981) proposed a relation between the $T_{\text{eff}}$ values obtained from lunar occultations and $T_c$ as corrected for line blocking by CN and C$_2$ lines. This empirical correction used band intensities measured by Gow (1977). When the $T_{\text{eff}}/T_c$ relation is applied here, some odd $T_{\text{eff}}$ values are obtained; for example, $T_{\text{eff}} = 1900$ K for U Cam, for which $T_{\text{eff}} \sim 2500$ K is indicated by other methods. The values of $T_{\text{eff}}$ provided by the infrared flux method and the values of $T_c$ are uncorrelated. Inspection of a $T_{\text{eff}} - T_c$ plot shows no obvious trends with the CN, C$_2$, and CO band intensities. We surmise that detailed syntheses of the narrow-band photometry will be needed to calibrate $T_c$ as a (poor) thermometer.

In principle, the excitation equilibrium of molecules may serve as thermometers. Both CO and CN are richly represented by lines spanning a range of 1–2 eV in excitation potential. Unfortunately, the line selection is less than ideal. Figure 14 shows the distribution of $^{12}$CN 2 $\mu$m lines for TX Psc. Although the lines span 1 eV in excitation potential (x"), the overlap at a fixed equivalent width is extremely restricted. With a run of $W_\sigma$ versus x" that is narrow, the lines serve primarily to fix the microturbulent velocity ($\xi_\tau$). The lines at $W_\sigma \sim 0.05$ cm$^{-1}$ do span a 1 eV range in x" that, after a thorough examination of the lines for blends, should provide an estimate of $T_{\text{eff}}$ with little dependence on the adopted $\xi_\tau$.

The distribution of $^{12}$CO $\Delta v = 3$ lines is shown in Figure 15 for UU Aur. The expected trend of $\log (W_\sigma/\sigma)$ with x" is apparent. A total range in x" of about 2.3 eV is covered, but, at a fixed intensity, the range is cut to about 1 eV. In practice, the lines displayed in Figure 15 carry as much information about the microturbulent velocity as the excitation/effective temperature. Examination of the CO abundances line by line

FIG. 14.—Run of equivalent width $W_\sigma$ with excitation potential x" for $^{12}$CN lines at 2 $\mu$m from TX Psc. The figure shows that the range in x" at a fixed $W_\sigma$ is small except for a thinly populated strip at $W_\sigma \sim 0.05$ cm$^{-1}$ spanning 1 eV.

FIG. 15.—$^{12}$CO lines at 1.6 $\mu$m from UU Aur. The run of $W_\sigma$ with x" shows that the range in x" at a fixed $W_\sigma$ is less than the total range of about 2.4 eV. Different bands, but not the individual P- and R-branches, are distinguished.
and band by band (see Fig. 6) shows no strong trend with excitation potential; a spread of less than ±0.05 dex over a range of 1 eV suggests an uncertainty \( \Delta T_{\text{eff}} \lesssim 200 \text{ K} \), i.e., the adopted \( T_{\text{eff}} \) scale is confirmed by the analysis of the CO lines. As with CN, a thorough analysis of critical low- and high-excitation lines might convert CO to a precision thermometer.

c) Surface Gravity

The stellar surface gravity (\( g \)) is usually derived from neutral and ionized lines of one or more elements with the condition that both species provide an identical abundance. This method, and even its generalization to the dissociation equilibrium between atom and molecule in practice cannot be applied to cool carbon stars; a lack of ionized atomic lines and suitable atom/molecule pairs are crucial factors. Pressure-broadened atomic or molecular lines are of potential use for estimating the surface gravity, but the lack of suitable lines in our windows and the uncertain continuum location at shorter wavelengths seriously limit this approach. Hence, we must calculate \( g \) from an estimate of the stellar mass (\( M \)) and the radius provided from estimates of the bolometric magnitude (\( M_{\text{bol}} \)) and effective temperature. The standard relation is

\[
\log g = \log \left( \frac{M}{M_{\odot}} \right) + 4 \log T_{\text{eff}} + 0.4 M_{\text{bol}} - 12.47.
\]

A kinematical analysis (Dean 1976) suggests that N-type carbon stars have \( M \sim 1.4 M_{\odot} \). Catchpole and Feast (1985) suggest that the motions of Galactic SC stars, the supposed immediate progenitors of carbon stars, indicate a mass \( M \sim 2.0 M_{\odot} \). In the galactic clusters NGC 2660 and NGC 2477 containing a carbon star, the main-sequence turn-off is at \( M \sim 1.8 M_{\odot} \). Wallerstein (1973) notes that \( M \sim 2 \) to 5 \( M_{\odot} \) is suggested by those carbon stars belonging to binary systems. Although proposed memberships of W CMA in the OB1/CMA R1 complex (Herbst, Racine, and Richer 1977) and BL and W Ori in the Pleiades group (Eggen 1979) would require considerably higher masses (e.g., \( M \sim 20 M_{\odot} \) for W CMA), the arguments for membership are weak and insufficient to overturn our conclusion that the typical cool Galactic carbon star has \( M \sim 2 M_{\odot} \) (±50%).

Extensive studies of carbon stars in the Magellanic Clouds have provided new results on luminosities and masses. In particular, Aaronson and Mould (1985) claim that (1) for stars with \( M \gtrsim 3 M_{\odot} \), the thermal pulsing is unable to convert the star to a carbon star before severe mass loss ensues as the AGB; (2) for stars with \( M \approx 2 M_{\odot} \), thermal pulsing increases the carbon abundance so that the M giant is converted first to a S star and then to a carbon star with the transition of oxygen to carbon star occurring at about \( M_{\text{bol}} \sim -4.5 \). These claims are quite consistent with our adopted mass \( M \approx 2 M_{\odot} \). Methods of obtaining mass estimates refer, in general, to the initial or main-sequence mass. We assume that little mass is lost before the AGB.

Estimates of \( M_{\text{bol}} \) for Galactic carbon stars are subject to large uncertainties. The few stars in Galactic clusters indicate \( M_{\text{bol}} \sim -4.4 \) to –5.2, a range supported by the studies of the Magellanic Clouds (Bessell, Wood, and Lloyd Evans 1983). The lower bound is precisely the value obtained by Catchpole and Feast (1985). Magnitudes for Galactic carbon stars from statistical parallaxes, membership in binaries, and other traditional methods span a wide range but are generally consistent with \( M_{\text{bol}} \sim -5 \pm 1 \) (Wallenstein 1973; Scalo 1976). For the Large Magellanic Cloud, the mean is close to \( M_{\text{bol}} \sim -5 \) (Richer 1981; Cohen et al. 1981). More luminous stars may exist (e.g., \( M_{\text{bol}} \sim -7.2 \) for W CMa; see Herbst, Racine, and Richer 1977), but the evidence is far from convincing. We adopt \( M_{\text{bol}} = -4.8 \pm 0.5 \) as a mean value. The star-to-star scatter in \( M_{\text{bol}} \) may be larger than ±0.5 mag. Since our sample is effectively an apparent-magnitude–limited one, the Malmquist bias makes the mean \( M_{\text{bol}} \) of the sample brighter than that of a complete sample within a given volume. If the true scatter were 0.5 mag in \( M_{\text{bol}} \), the bias amounts to about 0.3 mag and has been neglected here.

The set of average parameters—\( M = 2 M_{\odot}, M_{\text{bol}} = -4.8 \), and \( T_{\text{eff}} = 2700 \text{ K} \)—corresponds to \( g = -0.4 \). If each of the parameters is pushed to limits of the quoted ranges, we obtain \(-1.1 < \log g < 0.3 \). If \( M \sim 20 M_{\odot} \) and \( M_{\text{bol}} \sim -7.2 \) for W CMa, the gravity is close to this mean, i.e., \( \log g \sim -0.2 \).

IV. THE ABUNDANCE ANALYSIS

a) Trends and Variations

Several of our major results on the chemical composition are illustrated effectively by mean equivalent widths (\( \langle W_a \rangle \)) for key species. The values of \( \langle W_a \rangle \) were based on selected lines. When a particular line was unavailable, its \( W_a \) was interpolated from the other measured lines of that species. The reader is warned that lines contributing to the \( \langle W_a \rangle \) values are not always unsaturated and, hence, \( \langle W_a \rangle \) differences and ratios underestimate abundance differences. The full abundance analysis is based on weaker lines than the contributors to the \( \langle W_a \rangle \) values which were assembled prior to the detailed analysis.

The utility of the values of \( \langle W_a \rangle \) may be highlighted at once. Controversy has long surrounded the \( 12^C/13^C \) ratios for cool carbon stars. Early work suggested that the \( 12^C/13^C \) ratio was, in general, low. Recently this claim has been disputed and the suggestion advanced that the \( 12^C/13^C \) ratio for a typical carbon star is higher than that provided by even slight exposures to the CN cycle (Fujita and Tsuji 1977; Dominy et al. 1978). Published results on the \( 12^C/13^C \) ratio are based almost exclusively on analysis of the near-infrared bands of the CN red system, where weak lines are scarce and the continuum is elusive and, hence, systematic errors may affect a \( 12^C/13^C \) ratio.

In Figure 16 we compare the \( \langle W_a \rangle \) values for \( 12^C \) and \( 13^C \) lines. Note that the means are compiled from differing selections of lines: the \( 12^C \) include satellites with small \( f \)-values, whereas the \( 13^C \) lines are \( P, Q, \) or \( R \) lines with much larger \( f \)-values. One important conclusion may be drawn from Figure 16: the majority of the sample stars are \( 13^C \)-poor, with \( 30 \lesssim 12^C/13^C \lesssim 70 \) as a rough estimate based upon the ratio of the \( f \)-values and the observation that \( W_{12}^P \sim W_{13}^Q \) for the selected lines. The spread in the \( 12^C/13^C \) ratios is intrinsic and not a reflection of \( W_a \) measurement errors; note the tighter correlation of \( \langle W_a \rangle \) values for the \( C_2 \) Phillips and Ballik-Ramsay systems (Fig. 17). Another conclusion is that a
handful of the sample stars are indeed $^{13}$C-rich. The familiar stars in this group, WZ Cas, Y CVn, T Lyr, and RY Dra, have long been touted as rich in $^{13}$C. (The precise $^{12}$C/$^{13}$C ratio is not obtainable directly from Fig. 16 because the $^{13}$CN (W$_a$) corresponds to a saturated line.)

Several additional characteristics of the compositions of the carbon stars may be illustrated by means of the mean equivalent widths ($\langle W_a \rangle$) and predictions for representative model atmospheres. Molecular equilibrium of C, N, and O in carbon stars is dominated by the two molecules, CO and N$_2$, into which O and N, respectively, are almost fully associated, and, as a consequence, the partial pressure of C$_2$, CN, and CO and the elemental abundances $e_C$, $e_N$, and $e_O$ [where $e_X = n(X)/n(H)$] are described by the relations

$$p(C_2) \propto (e_C - e_O)^2 P_T(H)^2,$$
$$p(CN) \propto (e_C - e_O)^{1/2} P_T(H)^{3/2},$$
$$p(CO) \propto e_O P_T(H),$$

where $P_T(H)$ is the pressure of H were H$_2$ dissociated fully.

As our guide to the expected trend of line intensity with effective temperature, we adopt a set of models with $e_C/e_O$ (here, C/O) = 1.02 and 1.35; solar abundances of nitrogen, oxygen, and metals; a gravity of log $g = -1$; and a preliminary estimate of the HCN and C$_3$H$_2$ opacities.

Our primary indicators of the C, N, and O abundances are the C$_2$ Phillips, CN red system 2 $\mu$m, and CO 1.6 $\mu$m lines. Figure 18 compares the observed and predicted variation of equivalent width for these indicators. Recall that the contributions to $\langle W_a \rangle$ include partially saturated lines, and, therefore, the observed and predicted molecular column densities show a greater relative variation than $\langle W_a \rangle$.

The C$_2$ lines (Fig. 18a), which strengthen with decreasing $T_{\text{eff}}$, are predicted satisfactorily by the models. Predictions are rather insensitive to surface gravity and to whether and how the opacity contributed by HCN and C$_3$H$_2$ is included. The principal result is that the C/O ratio in cool carbon stars is close to unity. Carbon stars severely enriched in carbon C/O $> 1.6$ are apparently rare. Our detailed analyses confirm these results.

Throughout the photosphere, oxygen is fully associated into CO, so that CO is a direct monitor of the O abundance, with C and N exerting only a weak influence via their contributions to the total line blanketing. Comparison of observation and prediction in Figure 18c reveals a persistent result of our analyses: oxygen is mildly and rather uniformly underabundant. The predicted $W_a$ values are sensitive to the adopted surface gravity. Predictions for log $g = 0$ included in Figure 18c show that the O underabundance is reduced but persists, and would be eliminated only for log $g = -1$, a gravity beyond the estimated upper limit. Our detailed analyses confirm the O deficiency, but the equivalent widths of the atomic lines suggest that the stars may also be slightly metal-poor, so that the oxygen-to-metal ratio is approximately normal.

The C, N, and O abundances each influence the intensity of a CN line. Observed and predicted equivalent widths of CN lines are compared in Figure 18b, which hints at another
persistent feature of our analyses: the N abundance is at or below the solar value. In particular, note that the observed \( \langle W_a \rangle \) values for the cooler stars fall consistently below the line labeled \( C/O = 1.35 \), a ratio suggested by the \( C_2 \) observations. More direct information on the N abundance is provided by the ratio

\[
\langle W_a (CN) \rangle / \langle W_a (C_2) \rangle,
\]

which for weak lines is expected to be proportional to the N abundance; see Figure 19, which underscores the hint that nitrogen is underabundant \( \{ x_N < x_N (\odot) \} \). Figures 18b and 19 show also that the N abundances are surprisingly uniform across the sample. WZ Cas would appear to be the most N-rich star and, perhaps remarkably, the sole member of the \( ^{13}C \)-rich quartet to be distinguished by a relatively high N abundance.

Our final preliminary remarks concern the \( H_2 1-0 S(0) \) line. Observed and predicted values of \( W_a \) are compared in Figure 20. The disparity in Figure 20 between the observations and those predictions for model atmospheres constructed without opacity contributed by the polyatomic molecules HCN and \( C_2H_2 \) illustrates published remarks by Goorvitch, Goebel, and Augason (1980) and by Johnson et al. (1983). The key point is that when the opacity contributions of HCN and \( C_2H_2 \) are recognized, the predicted \( H_2 W_a \) decreases sharply and is dependent on the adopted C/O ratio. For the models selected for Figure 20, the observed \( H_2 W_a \) values suggest C/O ratios within the range provided more directly by Figure 18. This preliminary survey does not confirm published suggestions that the \( H_2 \) lines are anomalously weak and that the atmospheres are possibly deficient in H.

In summary, this reconnaissance provides these tentative results:

1. The dispersion in C, N, and O abundances for the 30 program stars is small.
2. The stars show only modest C enrichments: C/O \( \leq 1.6 \).
3. The \( ^{13}C \) abundance is, in general, low, with 30 < \( ^{13}C/^{12}C \) < 70 encompassing the majority of our sample stars, and the very \( ^{13}C \)-rich (\( ^{13}C/^{12}C \sim 4 \)) stars are the exceptions, not the rule.
4. The weak \( H_2 \) lines are not evidence for H-deficient atmospheres.

Our detailed analysis extending these conclusions is discussed next.

\( b ) \) Standard CNO Analysis

Tables representing curves of growth for representative lines of each molecule were computed for the model atmospheres. Arguments in these tables were \( \log \eta, \lambda, \) and \( \xi, \) where \( \log \eta = \log (N_{gf} - 5040 \chi / T'), \chi \) denotes the excitation energy of the lower state of the transition, \( T' \) is the temperature of the model at \( \log T_{\text{eff}} = -1.4, \lambda \) is the wavelength, and \( \xi \) is the microturbulent velocity. Through interpolation in the stored tables, the correction to the molecular abundance was estimated from a comparison of observed and predicted values of \( W_a \) for each line; the logarithmic correction for molecule AB is denoted by \( \delta(AB) \). For each molecule, a mean estimate of \( \delta(AB) \) was constructed from the available weak lines for a grid of models spanning a range in \( T_{\text{eff}} \) and chemical composition \( \{ \epsilon(O), \epsilon(N), \) and \( \log (C/O) \} \) at a fixed surface gravity \( \log g = 0 \) or \(+1\), metal abundance \( (\text{M/H}) = 0 \) or \(-0.3\) and the standard He/H (0.1) ratio. As the final step in the CNO analysis, an interpolation routine locates the chemical composition providing \( \delta(AB) = 0 \) for each molecule at the adopted \( T_{\text{eff}} \) for the star. This approach ensures that the derived CNO abundances match those adopted for the construction of the “interpolated” model. Tests with special models calculated for the final abundances verify that the scheme gives self-consistent results. The CNO abundance analysis was completed using the measured \( C_2, \) CN, and CO lines from the 1.5–2.5 \( \mu m \) spectra. On completion of this...
Curves of growth for infrared lines flatten out at smaller values of $W_e$ than do curves for shorter wavelengths. Consequently, although our spectra are of high quality, few weak unsaturated lines on the weak-line portion of the curve of growth appear in the line lists. Of course, the high density of lines also reduces the number of measurable weak lines. Weak lines of $C_2$ are especially rare because the list of potential lines is far shorter than for CN and CO. For these reasons, the microturbulent velocity ($\xi_t$) was derived from CO and CN lines prior to the CNO analysis. The usual method was employed of adjusting $\xi_t$ until $\delta(\text{AB})$ was independent of $W_e$. Independent estimates were obtained from the $^{12}$CN and the $^{12}$CO $\Delta v = 3$ lines, with checks provided by $^{13}$CN and $^{13}$CO lines when adequate lists for the latter species were available. Lines with $\log (W_e/\sigma) \geq -4.6$ were rejected because, when $\xi_t$ was being determined, they were sensitive to the uncertain damping constants and the structure of the outer atmosphere, and, in some cases, they required extrapolations of the tabulated curves of growth. Too few weak $^{12}$CO and $^{13}$CO $\Delta v = 2$ lines were measurable to permit these bands to be included.

Estimates of $\xi_t$ from CN and CO are pleasingly concordant; the mean difference $|\xi_{\text{CN}} - \xi_{\text{CO}}| = 0.2 \text{ km s}^{-1}$, with identical mean values for the full sample of stars from the two molecules. The maximum difference of $\pm 0.6 \text{ km s}^{-1}$ occurs for two stars. This level of consistency is compatible with the estimated precision of $\pm 0.3 \text{ km s}^{-1}$ for a single determination. We adopt the average of the two estimates as the microturbulence to be employed in the abundance analyses. Table 4 includes the adopted $\xi_t$, and the range across the sample is shown by a histogram (Fig. 21). The grand mean of the sample is $\langle \xi_t \rangle = 2.2 \text{ km s}^{-1}$. No correlations between $\xi_t$ and atmospheric parameters or composition are apparent. The three smallest values of $\xi_t$ are provided by stars with distinctive features: R Lep, the Mira variable in the sample; R Scl, for which the $\delta(\text{CO})$ values are a strong function of wavelength; and V Hya, for which the strong $\Delta v = 2$ CO lines provide markedly lower $\delta(\text{CO})$ values. If this trio is excluded, there is a hint that $\xi_t$ is larger in the cooler stars. It is of interest to note that these values of $\xi_t$ are subsonic and, also,
In the abundance analysis we based the mean $\langle AB \rangle$ for each molecule on weak lines. For the CO $\Delta v = 3$ bands in most stars and for CN in most stars, the criterion was 

$$\log (W_c/\sigma) \leq -5.0.$$ 

For CN in eight stars and $C_1$ in all stars, selection was based on $\log (W_c/\sigma) \leq -4.8$ in order to enlarge the line sample. In eight stars the great strength of the $C_2$ lines forced us to extend the limit of acceptance to $\log (W_c/\sigma) \leq -4.7$. In the case of the $^{13}C$-rich stars (including VX And), $\delta(^{13}C)$ was obtained from $\delta(^{13}C)$ provided 

$$e(X) = X/H$$ 

and $\log e(C) = 8.67$, $\log e(N) = 7.99$, and $\log e(O) = 8.92$ 

(Lambert 1978).

**Table 4**

<table>
<thead>
<tr>
<th>Star</th>
<th>$T_{\text{eff}}$ (K)</th>
<th>$\xi$ (km s$^{-1}$)</th>
<th>$^{12}C/^{13}C$</th>
<th>$^{13}C/^{13}C$/$^{12}O$</th>
<th>$[^{13}C/^{12}C]/[H]$</th>
<th>$[^{12}C/^{13}C]/[H]$</th>
<th>$[^{16}N]/[H]$</th>
<th>$[^{16}O]/[H]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AQ Sgr</td>
<td>2650</td>
<td>2.3</td>
<td>52</td>
<td>1.033</td>
<td>0.12</td>
<td>-0.23</td>
<td>-0.14</td>
<td></td>
</tr>
<tr>
<td>BL Ori</td>
<td>2960</td>
<td>2.0</td>
<td>57</td>
<td>1.039</td>
<td>-0.02</td>
<td>0.05</td>
<td>-0.29</td>
<td></td>
</tr>
<tr>
<td>R Lep</td>
<td>2390</td>
<td>1.6</td>
<td>62</td>
<td>1.030</td>
<td>0.24</td>
<td>-0.16</td>
<td>-0.02</td>
<td></td>
</tr>
<tr>
<td>R Sco</td>
<td>2550</td>
<td>1.9</td>
<td>19</td>
<td>1.34</td>
<td>-0.21</td>
<td>1.13</td>
<td>-0.59</td>
<td></td>
</tr>
<tr>
<td>RT Cap</td>
<td>2680</td>
<td>2.3</td>
<td>59</td>
<td>1.10</td>
<td>0.10</td>
<td>-0.42</td>
<td>-0.19</td>
<td></td>
</tr>
<tr>
<td>RV Cyg</td>
<td>2600</td>
<td>2.3</td>
<td>74</td>
<td>1.20</td>
<td>0.17</td>
<td>-0.42</td>
<td>-0.16</td>
<td></td>
</tr>
<tr>
<td>RY Dra</td>
<td>2500</td>
<td>2.4</td>
<td>3.6</td>
<td>1.18</td>
<td>-0.06</td>
<td>0.05</td>
<td>-0.38</td>
<td></td>
</tr>
<tr>
<td>S Sco</td>
<td>2895</td>
<td>2.2</td>
<td>45</td>
<td>1.069</td>
<td>0.17</td>
<td>-0.24</td>
<td>-0.11</td>
<td></td>
</tr>
<tr>
<td>ST Cam</td>
<td>2800</td>
<td>2.3</td>
<td>61</td>
<td>1.14</td>
<td>0.08</td>
<td>0.00</td>
<td>-0.23</td>
<td></td>
</tr>
<tr>
<td>T Lyr</td>
<td>2380</td>
<td>2.6</td>
<td>3.2</td>
<td>1.29</td>
<td>-0.14</td>
<td>-0.83</td>
<td>-0.50</td>
<td></td>
</tr>
<tr>
<td>TU Gem</td>
<td>2770</td>
<td>2.2</td>
<td>59</td>
<td>1.12</td>
<td>-0.03</td>
<td>0.08</td>
<td>-0.36</td>
<td></td>
</tr>
<tr>
<td>TW Oph</td>
<td>2450</td>
<td>2.2</td>
<td>65</td>
<td>1.20</td>
<td>0.07</td>
<td>-0.57</td>
<td>-0.27</td>
<td></td>
</tr>
<tr>
<td>TX Psc</td>
<td>3030</td>
<td>2.2</td>
<td>43</td>
<td>1.027</td>
<td>0.16</td>
<td>-0.27</td>
<td>-0.10</td>
<td></td>
</tr>
<tr>
<td>U Cam</td>
<td>2530</td>
<td>2.5</td>
<td>97</td>
<td>1.30</td>
<td>-0.06</td>
<td>-0.42</td>
<td>-0.42</td>
<td></td>
</tr>
<tr>
<td>U Hya</td>
<td>2825</td>
<td>1.8</td>
<td>32</td>
<td>1.043</td>
<td>0.10</td>
<td>-0.37</td>
<td>-0.17</td>
<td></td>
</tr>
<tr>
<td>UU Aur</td>
<td>2825</td>
<td>2.3</td>
<td>52</td>
<td>1.063</td>
<td>0.10</td>
<td>0.15</td>
<td>-0.18</td>
<td></td>
</tr>
<tr>
<td>UX Dra</td>
<td>2900</td>
<td>2.2</td>
<td>32</td>
<td>1.046</td>
<td>0.06</td>
<td>-0.12</td>
<td>-0.21</td>
<td></td>
</tr>
<tr>
<td>V Aql</td>
<td>2610</td>
<td>2.0</td>
<td>82</td>
<td>1.25</td>
<td>0.15</td>
<td>-0.65</td>
<td>-0.20</td>
<td></td>
</tr>
<tr>
<td>V Hya</td>
<td>2650</td>
<td>1.5</td>
<td>69</td>
<td>1.050</td>
<td>0.22</td>
<td>-0.16</td>
<td>-0.05</td>
<td></td>
</tr>
<tr>
<td>VX And</td>
<td>2700</td>
<td>2.2</td>
<td>13</td>
<td>1.76</td>
<td>-0.18</td>
<td>-0.54</td>
<td>-0.68</td>
<td></td>
</tr>
<tr>
<td>VY UMa</td>
<td>2855</td>
<td>2.2</td>
<td>44</td>
<td>1.060</td>
<td>-0.01</td>
<td>-0.31</td>
<td>-0.29</td>
<td></td>
</tr>
<tr>
<td>V660 Cyg</td>
<td>2845</td>
<td>2.2</td>
<td>61</td>
<td>1.062</td>
<td>-0.04</td>
<td>-0.06</td>
<td>-0.32</td>
<td></td>
</tr>
<tr>
<td>W CMa</td>
<td>2880</td>
<td>2.3</td>
<td>53</td>
<td>1.046</td>
<td>0.07</td>
<td>-0.01</td>
<td>-0.20</td>
<td></td>
</tr>
<tr>
<td>W Ori</td>
<td>2680</td>
<td>2.4</td>
<td>79</td>
<td>1.16</td>
<td>-0.04</td>
<td>-0.17</td>
<td>-0.35</td>
<td></td>
</tr>
<tr>
<td>WZ Cas</td>
<td>2850</td>
<td>2.2</td>
<td>45</td>
<td>1.010</td>
<td>0.32</td>
<td>0.01</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>X Cnc</td>
<td>2620</td>
<td>2.1</td>
<td>52</td>
<td>1.14</td>
<td>0.06</td>
<td>-0.56</td>
<td>-0.25</td>
<td></td>
</tr>
<tr>
<td>Y CVn</td>
<td>2730</td>
<td>2.4</td>
<td>3.5</td>
<td>1.087</td>
<td>-0.11</td>
<td>-0.12</td>
<td>-0.40</td>
<td></td>
</tr>
<tr>
<td>Y Hya</td>
<td>2770</td>
<td>2.1</td>
<td>82</td>
<td>1.52</td>
<td>0.21</td>
<td>-0.32</td>
<td>-0.22</td>
<td></td>
</tr>
<tr>
<td>Y Tau</td>
<td>2600</td>
<td>2.3</td>
<td>58</td>
<td>1.040</td>
<td>0.08</td>
<td>-0.17</td>
<td>-0.19</td>
<td></td>
</tr>
<tr>
<td>Z Psc</td>
<td>2870</td>
<td>2.8</td>
<td>55</td>
<td>1.014</td>
<td>0.03</td>
<td>-0.39</td>
<td>-0.23</td>
<td></td>
</tr>
</tbody>
</table>

\(*\) Solar values used for cols. (6), (7), and (8) are $\log e(C) = 8.67$, $\log e(N) = 7.99$, and $\log e(O) = 8.92$ (Lambert 1978).

**Fig. 21.** — Histogram of the microturbulent velocities derived from the CN and CO $\Delta v = 3$ lines.
Table 4 are preserved. For example, substitution of log $g = -1$
for log $g = 0$ decreases $[X/H]$ for C, N, and O by 0.3 dex, or, if a metal abundance $[M/H] = -0.3$ is preferred, $[X/H]$ would be decreased by about 0.13 dex for C, N, and O.

c) Metal Abundances

Our attempt to extract the metal abundance is based on the five lines discussed in Appendix A. In many stars, one or more lines were unmeasurable, due to blending with a telluric line or, as occurs primarily in the $^{13}$C-rich stars, a stellar line of unusual strength. Abundances for Na, Ca, and Fe are given in Table 5. Our analyses of the metal lines used the standard model most closely approximating the adopted assumed $log g$, and derived CNO abundances, and the final microturbulent velocity was adopted. The (more or less) systematic errors in the metal abundances could well be 0.3 dex. The formal error in $[Fe/H]$ derived from the scatter in the results for individual Fe I lines is typically 0.15 dex.

The weak Na I line suggests that these stars are metal-poor by about a factor of 2 relative to the Sun, i.e., $[M/H] < 0.3$.

<table>
<thead>
<tr>
<th>Star</th>
<th>$[Na/H]$</th>
<th>$[Ca/H]$</th>
<th>$[Fe/H]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AQ Sgr</td>
<td>-0.4</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>BL Ori</td>
<td>-0.2</td>
<td>-0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>R Lep</td>
<td>&lt; -0.7</td>
<td>-0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>R ScI</td>
<td>&lt; -0.1</td>
<td>0.4</td>
<td>...</td>
</tr>
<tr>
<td>RT Cap</td>
<td>...</td>
<td>-0.2</td>
<td>...</td>
</tr>
<tr>
<td>RV Cyg</td>
<td>-0.2</td>
<td>...</td>
<td>0.3</td>
</tr>
<tr>
<td>S Sct</td>
<td>-0.3</td>
<td>0.0</td>
<td>-0.3</td>
</tr>
<tr>
<td>ST Cam</td>
<td>&lt; -0.3</td>
<td>-0.4</td>
<td>...</td>
</tr>
<tr>
<td>T Lyr</td>
<td>0.1</td>
<td>...</td>
<td>0.3</td>
</tr>
<tr>
<td>TU Gem</td>
<td>-0.2</td>
<td>-0.5</td>
<td>...</td>
</tr>
<tr>
<td>TW Oph</td>
<td>-0.3</td>
<td>0.7</td>
<td>-0.1</td>
</tr>
<tr>
<td>TX Psc</td>
<td>...</td>
<td>-0.3</td>
<td>-0.3</td>
</tr>
<tr>
<td>U Cam</td>
<td>&lt; -0.1</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>U Hya</td>
<td>&lt; -0.5</td>
<td>-0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>UU Aur</td>
<td>0.2</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>UX Dra</td>
<td>-0.1</td>
<td>-0.3</td>
<td>-0.2</td>
</tr>
<tr>
<td>V Aql</td>
<td>-0.4</td>
<td>0.3</td>
<td>-0.1</td>
</tr>
<tr>
<td>V Hya</td>
<td>...</td>
<td>0.6</td>
<td>-0.4</td>
</tr>
<tr>
<td>VX And</td>
<td>-0.1</td>
<td>0.2</td>
<td>...</td>
</tr>
<tr>
<td>VY Uma</td>
<td>...</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>V460 Cyg</td>
<td>-0.1</td>
<td>...</td>
<td>0.1</td>
</tr>
<tr>
<td>W CMa</td>
<td>-0.3</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>W Orl</td>
<td>-0.3</td>
<td>0.2</td>
<td>-0.2</td>
</tr>
<tr>
<td>WZ Cas</td>
<td>-0.2</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>X Cnc</td>
<td>...</td>
<td>-0.3</td>
<td>...</td>
</tr>
<tr>
<td>Y Cen</td>
<td>-0.2</td>
<td>0.6</td>
<td>-0.2</td>
</tr>
<tr>
<td>Y Hya</td>
<td>...</td>
<td>-0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Y Tau</td>
<td>-0.3</td>
<td>-0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>Z Psc</td>
<td>...</td>
<td>-0.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The weak Na I line suggests that these stars are metal-poor by about a factor of 2 relative to the Sun, i.e., $[M/H] < 0.3$.

This result from a line of, perhaps, less than conclusive identification is confirmed by the Ca I line. (Of course, if the line attributed to Na I is a blend, the Na abundance is less than is indicated here.) However, the evidence from the Fe I lines is not so strongly in support of a metal deficiency. Moreover, differences between predicted and observed $W_A$ values may reflect factors other than metal abundance, e.g., errors in the $gf$-values, misidentifications, non–LTE effects in the excitation and/or ionization equilibrium. An apparent pointer to non-LTE effects within the excitation of Ca I is offered by a Ca I line at 4045 cm$^{-1}$. Although the $gf$-value is uncertain and, hence, an absolute abundance is unobtainable, the line is significant because its observed $W_A$ decreases with decreasing $T_{eff}$, in contrast to the predicted strengthening of the line in the cooler stars; the abundance discrepancy at $T_{eff} \approx 2500$ K is a factor of 2 when the $gf$-value is adjusted to give the same abundance as the Ca I line at 4413 cm$^{-1}$ for $T_{eff} = 2900$ K. Our line at 4413 cm$^{-1}$ does vary with $T_{eff}$ in the expected way. Finally, we should note that an overall metal deficiency by, say, 0.3 dex results in $W_A$ decline for weak lines of Na I and Ca I by only about 0.15 dex because

$^a$ Colons denote uncertain values.
the continuous opacity also declines with decreasing metal abundance. Hence, a modest metal deficiency, \([M/H] \sim -0.3\), cannot be ruled out by the analysis of the atomic lines, including those of Fe I. Then, our conclusion is that the metal abundance of the program stars is in the range \(-0.3 < [M/H] < 0.0\).

\[ \text{d) The } ^{12}\text{C}/^{13}\text{C} \]  

Three molecules, \(C_2\), \(CN\), and \(CO\) present in the 1.5–2.5 \(\mu m\) spectra are potential monitors of the \(^{12}\text{C}/^{13}\text{C}\) ratio. The line selection for the two \(C_2\) systems does not favor their use for this purpose, but the relative intensities of the \(^{12}\text{C}_2\) and \(^{12}\text{C}^{13}\text{C}\) Phillips system lines can be shown to be consistent with the \(^{12}\text{C}/^{13}\text{C}\) ratio provided by the other two molecules. In this section we discuss the extraction of the \(^{12}\text{C}/^{13}\text{C}\) ratio from the CN \(\Delta v = -2\) red system bands and from the CO \(\Delta v = 2\) and \(\Delta v = 3\) vibration-rotation bands.

Thanks to the existence of weak satellite lines of \(^{13}\text{CN}\), the \(^{12}\text{C}/^{13}\text{C}\) ratio is obtainable from the CN red system bands through a comparison of \(^{12}\text{CN}\) and \(^{13}\text{CN}\) lines of similar and overlapping equivalent width and lower excitation potential (see Fig. 4). The key point as stressed by Fujita and Tsuji (1977) in an application of CN red system lines at 8000 \(\AA\) is that the derived ratio ought to be rather insensitive to the atmospheric model. A key advantage of our \(\Delta v = -2\) bands over the several sequences in the near-infrared and visible regions is that the continuum level is more securely defined and the CN lines are weaker, so that an adequate number of suitable lines is available. For the standard model most closely approximating each star, the CN lines were used to obtain the abundance corrections \(\delta(^{12}\text{CN})\) and \(\delta(^{13}\text{CN})\). Features identified as blends of two or more \(^{13}\text{CN}\) lines were analyzed with a special routine that recognized the line separations and relative strengths. The lines were then grouped by strength \([\log (W_s/\sigma)]\) into bins of width 0.05 or 0.1 dex and the mean abundance corrections computed. A very few obviously discrepant lines were rejected from the samples. Almost without exception, a discarded line gave an exceptionally large \(\delta(CN)\) value, suggesting an unidentified contaminant. The lists of discarded lines for individual stars contained, of course, many common lines.

The \(^{12}\text{C}/^{13}\text{C}\) ratio is provided directly by differencing the mean \(\delta(^{12}\text{CN})\) and \(\delta(^{13}\text{CN})\) values over the common interval in \(\log (W_s/\sigma)\). In a few cases, principally those with the highest \(^{12}\text{C}/^{13}\text{C}\) ratios, the selection of \(^{13}\text{CN}\) lines was extended to include lines from the bin just above that populated by the strongest \(^{13}\text{CN}\) lines. Thanks to the earlier extraction of the microturbulent velocity, the abundance corrections are independent of \(\log (W_s/\sigma)\) for all bins populated by \(^{13}\text{CN}\) lines and for several of the adjacent higher bins. Then, the mean \(\delta(^{13}\text{CN})\) and the mean \(\delta(^{13}\text{CN})\) values were obtained and differenced to give the \(^{12}\text{C}/^{13}\text{C}\) ratio.

One estimate of the uncertainty in \(^{12}\text{C}/^{13}\text{C}\) is provided by the line-to-line and bin-to-bin scatter of the [CN] values. This scatter declines with increasing sample size in the manner expected from a random \(W_s\) measurement error and its amplification by saturation. The final estimated uncertainty includes a small additional contribution arising from uncertainty over the choice of the model atmosphere. Analysis with alternative models shows that differences of less than \(\pm 0.02\) dex (often \(\pm 0.01\) dex) in \(\log ^{12}\text{C}/^{13}\text{C}\) are found for changes of \(T_{\text{eff}}\) by \(\pm 500\) K, of \(\log g\) by \(\pm 1\), of \(\xi_t\) by \(\pm 1\) \(\text{km s}^{-1}\), and of the \(\text{C/O}\) ratio over the complete interval spanned by our stars. Systematic errors are likely to be small; for example, the distributions of \(^{12}\text{CN}\) and \(^{13}\text{CN}\) lines across the 4400–5000 \(\text{cm}^{-1}\) interval are quite similar, so that an overlooked depression of the continuum by a blanket of weak lines from polyatomic molecules may introduce systematic errors into the [CN] values, but the \(\delta(^{12}\text{CN})\) and \(\delta(^{13}\text{CN})\) estimates for lines of the same \(W_s\) may be expected to give the correct \(^{12}\text{C}/^{13}\text{C}\) ratio. The demonstrated insensitivity to the model parameters reflects the inherent strength of the method, which combines a \(^{13}\text{CN}\) sample containing several satellite lines and a \(^{13}\text{CN}\) sample composed of similar main branch lines. Therefore, unanticipated errors in the atmospheric structure are also likely to have a small effect on the \(^{12}\text{C}/^{13}\text{C}\) ratio. The final results for the \(^{12}\text{C}/^{13}\text{C}\) ratio and the number of \(^{12}\text{CN}\) and \(^{13}\text{CN}\) lines are given in Table 6. The fraction of the \(^{12}\text{CN}\) lines with \(W_s\) in the range spanned by the \(^{13}\text{CN}\) lines ranges from about \(\frac{1}{3}\) for the \(^{13}\text{C}\)-poor stars to nearly unity for the \(^{13}\text{C}\)-rich stars.

The CO vibration-rotation transitions provide many weak \(^{13}\text{CO}\) lines from the second-overtone \((\Delta v = 3)\) bands and a sufficient number of weak \(^{12}\text{CO}\) lines from the stronger first-overtone \((\Delta v = 2)\) bands to permit two almost independent measurements of the \(^{12}\text{C}/^{13}\text{C}\) ratio. The \(\Delta v = 2\) bands provide many strong \(^{13}\text{CO}\) lines, but, as noted above, the strongest \(^{12}\text{CO}\) and \(^{12}\text{CO}\) line of the \(\Delta v = 2\) bands are anomalously weak relative to the predictions based on the weakest lines. Our analysis of the \(^{12}\text{CO}\) and \(^{13}\text{CO}\) \(\Delta v = 3\) lines followed the scheme applied to the CN lines. One clear difference must be noted between the two analyses. With the exceptions of the \(^{12}\text{C}\)-rich stars, the \(^{13}\text{CO}\) lines are weaker than essentially all of the \(^{12}\text{CO}\) lines; i.e., there is no equivalent among the CO transitions of the CN satellite transitions that provide weak \(^{13}\text{CN}\) lines for comparison with weak \(^{12}\text{CN}\) lines belonging to one of the main branches. Of course, weak \(^{12}\text{CO}\) lines are present as the high rotational members of the \(P\) and \(R\) branches. Unfortunately, the total number of such lines available is small, and, of this total, only a small fraction is unblended. Hence, the distributions of \(W_s\) of \(^{12}\text{CO}\) and \(^{13}\text{CO}\) lines for the typical star show effectively no overlap. A consequence must be recognized: the derived \(^{12}\text{C}/^{13}\text{C}\) ratio is quite sensitive to the adopted atmospheric structure \((T_{\text{eff}}, \xi_t, \text{C/O ratio, and so on})\) because the weakest \(^{13}\text{CO}\) lines are of higher average excitation potential and are formed at different depths than the typical and weaker \(^{12}\text{CO}\) line.

With the standard model most closely approximating the star, the CO \(\Delta v = 3\) lines provide the \(^{12}\text{C}/^{13}\text{C}\) ratios given in Table 6. In general, the estimated uncertainties are similar to those quoted for CN. However, systematic errors are likely to be larger for CO than for CN for several obvious reasons. First, the \(^{13}\text{CO}\) line list may be contaminated with unrecognized blends that introduce a bias into the mean \(\delta(^{13}\text{CO})\) value. The initial selection of \(^{13}\text{CO}\) lines was examined, and, as described earlier, the lines of lowest and highest rotational quantum number were rejected as potentially blended. Then the remaining sample generally provided a set of \(\delta(^{13}\text{CO})\) values with a scatter consistent with the measurement error.
and the scattered by the $^{13}$CO lines. In a few cases, the $\delta_{(13)CO}$ values were rather widely scattered as though blending were a problem. In these instances we based the $^{12}$C/$^{13}$C ratio on the 10 lowest $\delta_{(13)CO}$ values. Second, since the $^{12}$CO lines are weaker than the vast majority of the $^{13}$CO lines, the CO and $^{12}$C/$^{13}$C ratios on the 10 lowest $\delta_{(13)CO}$ values. Second, since the $^{12}$CO lines for W CMa, a typical star, have central depths on our spectra of 7%-15%. (Spectrum synthesis of the $^{13}$CO R-branch bandheads ought to alleviate the difficulties associated with the weakness of the $^{13}$CO lines.) Third, as noted above, the $^{12}$C/$^{13}$C ratio from CO $\Delta v = 3$ is inherently more dependent on the assumed atmospheric structure. Changes in $T_{eff}$, $g$, and $\xi$, have predictable consequences for the $^{12}$C/$^{13}$C analysis. If the adjustments to $^{12}$C/$^{13}$C contributed by the estimated uncertainties of the physical parameters and the composition are added in quadrature, they amount to about $\pm 0.1$ dex. With a typical measurement having an uncertainty also of $\pm 0.1$ dex arising from the line-to-line scatter, the final uncertainty for a typical star is about $\pm 0.14$ dex. The uncertainty may be expected to be less than this for the $^{13}$C-rich stars. The uncertainty quoted in Table 6 does not include the model atmosphere-dependent terms. Measurements are not reported for RV Cyg and Y Hya, owing to an insufficiency of measured lines.

Within the $\Delta v = 2$ bands, strong $^{13}$CO lines abound, thanks to the substantial increase of $g$-values over the $\Delta v = 3$ bands. Our standard method of extracting the $^{12}$C/$^{13}$C ratio used the weaker $^{13}$CO $\Delta v = 2$ lines to establish the $\delta_{(13)CO}$ value and the $^{13}$CO $\Delta v = 3$ lines, to set the $\delta_{(13)CO}$ value. This pairing, apart from the obvious use of two different wavelength intervals, is the CO analog of the combination of main $^{13}$CN and satellite $^{13}$CN lines. As noted earlier, the stronger $^{12}$CO and $^{13}$CO $\Delta v = 2$ lines appear to be anomalously weak in the cooler stars. Even attempts to extract the $^{12}$C/$^{13}$C ratio from these lines via empirical curves of growth were thwarted by...
the fact that the $W_e - \delta$(CO) relations are not identical for $^{12}$CO and $^{13}$CO. Fortunately, there were generally a sufficient number of the weaker $^{13}$CO lines to establish the $\delta$(CO) value. In a few cases (e.g., the $^{13}$C-rich stars), it was necessary to derive the $^{12}$C/$^{13}$C ratio from within the $\Delta v = 2$ bands by differencing the $\delta$(CO) and $\delta$(CO) values provided by a set of weak to medium-strong lines. Use of the $\delta$(CO) value obtained from $\Delta v = 3$ lines makes the resultant $^{12}$C/$^{13}$C ratio insensitive to the atmospheric structure. Estimated uncertainties for $T_{\text{eff}}$, g, $\xi$, and chemical composition contribute negligibly to the total uncertainty, which is dominated by the line-to-line scatter. Table 6 gives the $^{12}$C/$^{13}$C value and the total number of lines measured in the $\Delta v = 2$ bands.

With very few exceptions, the three measurements of the $^{12}$C/$^{13}$C ratio are in good agreement. In Figure 22 the CN and CO $\Delta v = 2$ based results are compared. To within the estimated uncertainties, these results are delightfully concordant. The CN and CO $\Delta v = 3$ comparison (Fig. 23) shows more scatter. This is not at all surprising because the $^{12}$C/$^{13}$C from these CO lines is more sensitive to the adopted atmospheric structure, and these $^{13}$CO lines may be contaminated with unidentified lines; then $^{12}$C/$^{13}$C < $^{12}$C/$^{13}$C, as is observed in Figure 23. The general agreement between the $^{12}$C/$^{13}$C measurements confirms the absence (or a cancellation) of systematic errors in the setting of the continuum location across the 1.5–2.4 $\mu$m windows, the calculation of continuous opacity, the absence (or uniformity) of continuous emission by dust grains, and the generally close approach of the molecular excitation to LTE. Our final mean estimate of the $^{12}$C/$^{13}$C ratio is a weighted average of the three measurements with weights of 4, 2, and 1 for the CN, CO $\Delta v = 2$, and CO $\Delta v = 3$ measurements, respectively. For U Cam, V Aql, and VX And, the lower $^{12}$C/$^{13}$C ratio from the CO $\Delta v = 3$ lines was excluded from the computation of the weighted mean. A histogram of the final $^{12}$C/$^{13}$C ratios given in Table 4 is presented in Figure 24.

e) An Error Analysis for the CNO Abundances

For the present discussion it is natural to divide the stars into two groups, one with less crowded spectra (in practice, relatively small C/O ratios) to which most of the stars belong, group I, and one with richer spectra (greater C/O), group II. The stars in this latter group are R ScI, RY Dra, T Lyr, TW Oph, U Cam, V Aql, and V Hya, with ST Cam, TU Gem, V Hya, and W Ori being intermediate cases. We shall see that the abundances of the group II stars are more uncertain, because the blending problems are more severe and the continuum location is more difficult. Also, the great strength of the lines in these stars has forced us to extend the limit of acceptance slightly beyond $\log (W_e/\sigma) = -4.8$ for C. Moreover, these stars tend to be cooler, and hence, the uncertainties due to effects of polyatomic molecules and dust are more severe.

In the following discussion we follow the path of the analysis, starting with errors in the equivalent widths, next turn to errors in the model atmosphere of various sorts, and finally discuss errors in the line calculations.

i) Errors in Continuum Location and Small Equivalent Widths

We estimate that the uncertainty in the location of the continuum should be less than 5% for the group I stars, but...
CHEMICAL COMPOSITION OF CARBON STARS. I.

Fig. 23.—A comparison of the $^{12}$C/$^{13}$C ratios provided from the CN $\Delta v = -2$ and the CO $\Delta v = 3$ lines. See caption to Fig. 22 for additional details.

Fig. 24.—Histogram of the $^{12}$C/$^{13}$C ratios

may well be of that order of magnitude for the group II stars. We have found that such an error, which ought to be systematic such that the level of the continuum is underestimated, leads to effects of typically $\Delta = (\Delta [O/H], \Delta [N/H], \Delta \log (C/O)) = (+0.1, +0.15, 0.00)$ for the group I stars, assuming that the level of the continuum is underestimated by 5% in the 1.6 and 2.2 $\mu$m regions. If the error occurred only in the 2.2 $\mu$m region, the changes would be $\Delta = (0.00, +0.15, 0.00)$ instead. For group II stars the typical corresponding numbers are $(+0.2, +0.2, -0.01, \text{both wavelength regions})$ and $(0.00, +0.2, +0.04, \text{only the 2.2 } \mu\text{m region})$, respectively.

Systematic errors related to errors in continuum location may in particular affect the equivalent widths of weak lines (so important in any abundance analysis). In an attempt to investigate this further, we have compared the observed relation between line depth and equivalent width with corresponding predicted ones. It is clear from Figure 25 that there is a tendency for the weakest lines to be weaker than predicted by typically 20%; this may show that the contributions from the wings of these lines are underestimated, but may, however, also be the result of systematically smaller macro-turbulence parameters in the deeper layers where these lines are formed. In any case, possible errors in the $W_0$ values should not lead to errors in the abundances greater than the numbers given above. We note in passing that fairly consistent macroturbulence parameters result from Figure 25 (cf. also Fig. 26); all stars but one (V Hya) have a parameter in the range $4 \text{ km s}^{-1} < \xi_{\text{macro}} < 7 \text{ km s}^{-1}$.

ii) Effects of Errors in Basic Assumptions and Structures of the Model Atmospheres

The effects of departures from plane-parallel stratification and from standard mixing-length convection, and the effects of turbulent pressures were commented on earlier.

Departures from local thermodynamic equilibrium may be essential in several respects. In particular, the ionization of the important electron donors in the outer layers of late-type stars (Na, K, Ca, Al) may be out of equilibrium (Auman and Woodrow 1975) and thus increase the $^1\text{H}$ opacity. Photodetachment of $^1\text{H}$ may induce departures from the LTE abundance of $^1\text{H}$ and of $^2\text{H}$ with the associative detachments $^1\text{H} + ^1\text{H} \leftrightarrow ^2\text{H}_2 + e^-$ linking $^1\text{H}^-$ to $^2\text{H}_2$. The dissociation equilibria of the important opacity-contributing carbon molecules may be similarly affected by hot radiation from the deeper atmospheric layers, and the excitation equilibria of molecules may be out of the Boltzmann equilibrium, thus affecting the line blanketing. Also, the line formation process for the lines contributing the blanketing may depart systematically from
Fig. 25.—Predicted relations between observed line depth and equivalent width $W_\sigma$ for CN lines at 2.2 μm. Predictions are shown for lines of different excitation energy and for different macroturbulence parameters and effective temperatures. In all but one case, the microturbulence was chosen to be 2 km s$^{-1}$. Predicted relations for “flat” models are also shown (plus signs and crosses). The calculations were made for model atmospheres with a reduced oxygen abundance $[O/H] = -0.3$, log C/O = 0.04, and $[N/H] = 0.0$, but the composition influences the predictions only slightly. Most of the empirical calibrations (see Fig. 26) fall within the shaded area.

Fig. 26.—Empirical calibrations of $W_\sigma$ vs. line depth at 2.2 μm. For all stars except those denoted explicitly on the figure, the calibrations fall within the shaded area.
CHEMICAL COMPOSITION OF CARBON STARS. I.

We have investigated the maximum effects of photodetachment of $\text{H}^-$ by calculating self-consistent models with the number density of $\text{H}^-$ reduced by the factor $R$:

$$R(\tau) = \int B_0 \sigma_0 \, d\lambda \int J_0 \sigma \, d\lambda,$$

where $\sigma$ is the photodetachment cross section of $\text{H}^-$ (Doughty, Fraser, and McEachran 1966), $B_0$ is the Planck function, and $J_0$ is the mean intensity. The resulting reduction in the number density of $\text{H}^-$ is not very dependent on effective temperature and carbon abundance, as is seen in Figure 27. The resulting surface heating, however, is considerably greater for carbon-poor and cooler models, as is also shown in Figure 27. Nevertheless, the total structural effects are much smaller than those to be discussed below for the artificially heated models, and the effects on the abundances are correspondingly smaller. The effects on the calculated spectral lines, caused by the depleted $\text{H}^-$ abundance, will be discussed below.

The assumption that all the molecular lines were formed in scattering processes leads to more severe effects for the model atmospheres, since it decouples the thermal structure from the radiation field. Thus, the temperature gradient becomes much smaller in the outer layers, with typically 600 K higher temperatures around $\tau_{\text{Ross}} = 0.01$, and the pressures are increased in these layers by 0.5–1.0 dex. These changes, which probably greatly exaggerate the non-LTE effects, lead to changes in the abundances by typically $\Delta = (-0.03, -0.1, 0.00)$.

The possibility that the upper layers of the real stars are significantly hotter than the calculated models, as a result of heating by a substantial mechanical flux, was discussed above as a possible explanation for the polyatomic bands being predicted to be too strong. We have investigated the effects of such flat temperature structures [with $T(\tau_{\text{Ross}} < 0.1) =$ constant] and found changes of $\Delta = (-0.02, -0.2, -0.01)$ for group I stars. The consistency (see below) between the CN lines and the observed NH $V - R$ lines [whose predicted strength is little affected by this change in $T(\tau_{\text{Ross}})$] becomes less satisfactory with these lowered [N/H] abundances; the $\text{H}_3$ lines are predicted to be significantly weaker for the flatter structures, but this is compensated for by the lower C/O ratios. The individual HCN lines for TX Psc are predicted to be weaker by 0.3 dex, and, thus, the fair agreement in Figure 31 is destroyed. The most severe objections, however, to these flat structures are astrophysical: large fluxes need to be deposited in just the right layers of the atmospheres, and the resulting lower nitrogen abundances are not easily accounted for in terms of stellar evolution.

In LTE, the relation between line depth and equivalent width is sensitive to the temperature structure of the surface layers (cf. Fig. 25). Thus, one might argue that the seven deviating stars of Figure 26 are affected by surface heating and that the rest of the stars are less affected and, perhaps, not affected at all. However, the LTE assumption and other uncertainties preclude firm conclusions in this direction.

Another source of error in the models may be erroneous atomic and molecular data. Errors (cf. Appendix A) in the dissociation energies of $\text{C}_2$ and CN have been found to cause marginal abundance errors through the model atmospheres, but are of great significance in the line calculations (see below). The errors in the calculated C$_2$H$_2$ opacity may be very great, but they have been found to be of rather small importance for the abundances, since a large part of the effects of the C$_2$H$_2$ molecules is taken over by the HCN molecules if the C$_2$H$_2$ opacity has been overestimated; an underestimate is less probable, since the predicted polyatomic bands are found to be too strong, as compared with low-resolution scans.
Errors in the HCN opacity ought to be much smaller, in view of the ab initio calculations of Jørgensen et al. (1985). If, however, the opacity of the polyatomic molecules were totally disregarded, the resulting changes $\Delta$ in the abundances would be typically $(+0.15, -0.2, 0.00)$. These numbers are astonishingly small in view of the very great structural changes that are caused by the neglect of the polyatomic opacities (cf. Eriksson et al. 1984). The effective $\Delta$'s are even smaller if the polyatomics' opacity is decreased sufficiently to match the infrared spectrophotometry; e.g., the $\Delta$'s for UU Aur are less than half of those first cited if the polyatomics' opacity is reduced by a factor of 1000.

### iii) Dust Emission

A source of error of another character would be the emission from high-temperature dust in the outermost layers of the atmospheres, which could significantly contribute to the stellar flux around 2 $\mu$m and thus weaken the spectral lines used in the analysis. In fact, we have already mentioned that the star V Cyg was excluded from the program, since its spectrum showed clear evidence for dust emission to be important in this respect. This may well be the case, to a smaller but still significant degree, for many of the program stars.

However, from a comparison between model fluxes and available low-resolution scans and photometry, we estimate that a flux fraction greater than about 15% from dust emission is improbable for the program stars at this wavelength. This number is confirmed by comparing the observed line depths of the CO and CN lines with the predicted depths (allowing for apodization and a macroturbulence parameter of 5 km s$^{-1}$). As is seen in Figure 26, only seven stars deviate significantly from the predicted relations between line depth and equivalent width with $E_{\text{macr}} = 5$ km s$^{-1}$; and in most cases, it is natural to explain the deviation as a result of a larger macroturbulence. However, if the deviation is due to dust emission, it corresponds to less than 10% extra emission from dust except in the case of V Hya, where about 20% of extra emission would be needed. An assumption of 15% dust emission at both 1.6 and 2.2 $\mu$m would lead to changes of typically $(+0.1, +0.1, -0.00)$ for group I stars and $(+0.15, +0.3, -0.00)$ for group II stars. If the dust emission is assumed to be negligible at 1.6 $\mu$m but still 15% at 2.2 $\mu$m (not an impossible assumption in view of the decreasing photometric flux across this wavelength interval and the necessarily relatively cooler dust temperature), we instead find changes in the abundances of $\Delta = (0.00, +0.1, +0.01)$ and $(0.00, +0.3, +0.03)$, respectively. Evidently, the possible existence of considerable amounts of dust in the outer layers of the stars produces one of the more serious sources of error in the present analysis. It should be noted that the temperature in the outer layers of the cooler models ($T_{\text{eff}} \leq 2800$ K) is below 1800 K, which seems to make it possible for SiC or amorphous carbon grains to form (Rowan-Robinson and Harris 1983). However, improvements in the theory of grain formation and of the dynamics of the outer atmospheric layers are necessary in order to predict accurately the number density and the size distribution of the grains.

### iv) Effects of Errors in the Fundamental Parameters of the Models

We estimate the probable errors in $T_{\text{eff}}$ for the stars to amount to $\pm 100$ K. Such revision of the effective temperatures adopted lead to typical changes in the abundances of $\Delta = (+0.01, +0.05, +0.00)$ for the stars of group I, while for the stars of group II the change in [N/H] is less and that in $\log(C/O)$ may amount to about $\pm 0.02$. We note in passing that very considerable errors in effective temperature are necessary before the temperature uncertainty contributes significantly to the total error in the abundance results. Although improbable, such large corrections to the temperature scale lead to interesting consequences; see § IV/(vi) below.

The errors in the value of $\log g = 0.0$ adopted in the analysis should be less than $\pm 0.5$ dex in most cases. A change to $\log g = -0.5$ leads to $\Delta = (-0.15, -0.15, -0.01)$. The macroturbulence is uncertain by less than 0.4 km s$^{-1}$; a reduction by this amount leads to typical changes $\Delta = (+0.05, +0.05, +0.00)$ for group I stars and $(+0.05, +0.10, +0.03)$ for group II stars. In these figures both the direct effects on the line strengths and the indirect effects (less important) through the model atmospheres are included.

Errors in the adopted composition for the metals (i.e., solar or $[\text{M/H}] = 0.0$) may presumably amount to 0.3 dex, with a tendency for the metal contents to be slightly lower than that of the Sun, according to our results. A change to $-0.3$ dex for all elements heavier than oxygen leads to typical changes of $\Delta = (-0.12, -0.15, 0.00)$.

All standard model atmospheres used in the analysis were calculated with a $^{12}$C/$^{13}$C ratio of 10. For a number of stars, we have investigated the effects of adopting models with $^{12}$C/$^{13}$C = 3 and $^{12}$C/$^{13}$C = 99 and found negligible effects for stars in group I, while for stars in group II the effects may amount to $\Delta = (+0.1, +0.15, +0.02)$ in the worst cases (T Lyr, RY Dra) for a decrease of $^{12}$C/$^{13}$C to 3. We have made no further efforts to obtain self-consistency in the analysis in this respect.

### v) Effects of Reduced Hydrogen Abundances

Low H abundances have recently been suggested for the N-type stars by Johnson et al. (1983), Goebel and Johnson (1984), and Johnson et al. (1985) as an explanation for the weakness of the observed H$_2$ 1–0 S(1) line, as compared with predictions from model atmospheres of the same group. For our models, with HCN and C$_2$H$_2$ included in the opacity calculations, the predicted strengths of the H$_2$ 1–0 lines agree reasonably well with the observations (see below). However, two puzzles remain: (1) the models predict the bands of the polyatomics to be stronger than observed and (2) the CH $V - R$ lines around 3.8 $\mu$m are predicted to be too strong (see below). One might wonder whether these puzzles could be resolved by adopting a lower hydrogen abundance for the stars and, if so, what the consequences would be for the CNO abundance determinations.

An explanation of problem 1 in terms of a reduced hydrogen abundance (and a correspondingly increased He abundance; it is assumed that hydrogen is converted into helium) requires a very considerable reduction on the order of a factor of 10$^{3}$. This, however, leads to negligible equivalent widths of the H$_2$ and NH lines, as is demonstrated in Figure 34. Thus,
we conclude that a hydrogen deficiency alone cannot resolve problem 1 but has to be combined with or replaced by some other explanation, such as heated outer atmospheric layers. Problem 2 cannot be resolved as the result of only a hydrogen depletion (a factor of 10^2 is needed for UU Aur), since in this case the calculated H_2 and NH lines become far too weak. Note that we do not find the agreement between observed and calculated fluxes around 1.6 μm, where the H^- opacity is at a minimum, to be substantially improved if the hydrogen abundance is reduced, contrary to the statements by Goebel and Johnson (1984).

We conclude that reductions of the hydrogen abundance are not the explanations for the problems mentioned, but may still be of importance in their effects on the results of the CNO analysis. Such reductions would lead to reductions in the CNO abundances, measured relative to the original hydrogen abundance or to the metals, as a consequence of the decreased continuum opacity. A decrease of the hydrogen abundance by a factor of 10 leads to decreased CNO abundances of typically Δ = (−0.4, −0.7, 0.00); [O/H] and [N/H] are here normalized relative to the initial hydrogen abundances. In order to keep a reasonable consistency with the observed H_2 and NH widths, however, hydrogen deficiencies greater than 0.3 dex are unacceptable, corresponding to Δ = (−0.1, −0.2, 0.00). With a reduction of the H abundance and, hence, of the continuous opacity, sharply reduced metal abundances are demanded in order to account for the observed W_a values of the atomic lines. The fact that the metal abundances derived on the assumption of a normal H abundance fall within the range expected for young disk stars, the kinematic identity of carbon stars (Dean 1976), and the observed metal abundance of two Galactic clusters containing carbon stars (Bessell, Wood, and Lloyd Evans 1983) demand a near-normal H abundance.

One might wonder whether a solution of problems 1 and 2 might not be found if one assumes that, for some reason, the abundances of HCN and C_2H_2, or their opacities, are severely overestimated. As noted earlier, an error of a factor of 20–5000 then has to be advocated; with such a lowering of the polyatomic opacity, one also has to decrease the hydrogen abundance by a factor of 10–1000 (dependent on the effective temperature of the star) in order to obtain H_2 lines of the right strength. However, such a change would lead to NH lines that were too weak and reductions of the CNO abundances of the same order of magnitude as those discussed above for the hydrogen-poor models.

Departures from LTE also may be important for the line equivalent widths, although it is not clear that this is the case for the lines used in the present investigation. Hinkle and Lambert (1975) suggest that the molecular lines in the electronic systems may form by processes that are closer to scattering than to true absorption. If we assume all the lines used in our abundance determinations to be formed in scattering, we obtain changes in the abundance results of typically Δ = (−0.1, −0.15, +0.01) for group I stars and Δ = (−0.2, −0.3, +0.01) for group II stars. If just the CO V − R lines are assumed to be formed in absorption, we obtain only minor changes in the oxygen abundances, while the other corrections are unchanged. If only the C_2 lines are formed in scattering, we obtain Δ = (0.0, −0.3, 0.00) for group I and Δ = (0.0, +0.15, −0.02) for group II, and, correspondingly, if the CN lines form in scattering, we obtain Δ = (0.0, −0.1, 0.00) for group I and Δ = (0.0, −0.4, −0.01) for group II. These are typical corrections; individual stars deviate considerably. The consistency between CO lines of different excitation energy, between the C_2 lines of two different electronic systems, and between the CN lines of different sequences, as well as the consistency with the NH bands and the consistent estimates of the ^13C/^12C ratio (Table 6), seem to indicate that the departures from LTE are of minor importance.

Interesting effects on the abundances of certain molecules may occur as the result of the non–LTE effects in the photodetachment of H^−, discussed above. The H_2 molecules are thought to be formed through the channel H + H^− → H_2 + e^- (see Lambert and Pagel 1968), and, thus, a decrease of the H^- number density may lead to a corresponding decrease of the H_2 density (electrons are mainly provided by the metals). By introducing the same reduction factor R for H_2 as that calculated for H^−, we find that the photodetachment of H^- would lower the equivalent width of the H_2 1–0 S(1) line by 0.2 dex, almost independently of effective temperature and carbon abundance. For V − R lines from polyatomic molecules like HCN, the corresponding reduction would lead to reductions of equivalent widths of single unsaturated lines by about 0.4 dex and less for saturated lines [0.1 dex for log (W_a/σ) ~ −4.7]. Obviously, if the H_2 abundance and thus the H^- density were determining the abundance of HCN, the photodetachment of H^- would be a natural explanation for the steep slope in the relation between observed and calculated HCN widths plotted in Figure 31 (see below). However, the effect is far from great enough to explain the discrepancy between observed and calculated low-resolution scans.

The effects on the lines of diatomic CNO molecules that result from the photodetachment of H^- are much smaller than for H_2, because the lines are formed at much greater depths, where R is closer to 1.0 (cf. Fig. 27) and because there are probably other channels, not affected by the H_2 molecule, which contribute to the formation of these molecules.

vi) Errors in the Line Calculations

The effects of changing the dissociation energy of C_2 to 6.21 eV (cf. Appendix A) are changes in the abundances of typically Δ = (−0.01, +0.15, 0.00) for group I, while for group II log (C/O) changes by typically −0.03. In fact, log (C/O – 1) decreases by about −0.15 for all stars. Similarly, changes in D_0^CN(1) of ±0.1 eV correspond to abundance changes of Δ = (±0.01, ±0.04, ±0.01). Characteristic changes in log (C/O – 1) are ±0.05. Unfortunately, the error in D_0^CN can be greater than 0.1 eV. This is obviously a major source of error in the N abundance determination.

© American Astronomical Society • Provided by the NASA Astrophysics Data System
this, and obtain errors of $\epsilon([O/H]) = \pm 0.3$, $\epsilon([N/H]) = (+0.5, -0.6)$, and $\epsilon(\log(C/O)) = \pm 0.01$ for the group I stars; for group II the corresponding numbers are $\epsilon([O/H]) = \pm 0.3$, $\epsilon([N/H]) = \pm 0.7$, and $\epsilon(\log(C/O)) = \pm 0.04$. Although these numbers were derived from the square sums of the errors in the table, they should be regarded not as mean errors but rather as "probable maximum errors" because the numbers in Table 7 are maximum errors in most cases. It should also be noted that most of these errors would affect all the stars in the sense that the uncertainties in the differential abundances are significantly smaller.

It is of interest to consider what improvements in the analysis would substantially decrease the errors in the abundance determinations. From Table 7 we see that the dominating errors in the oxygen abundances are errors in continuum location and in polyatomic opacities, and errors caused by possible dust emission, as well as errors in the adopted surface gravity and metal abundance. These same sources also contribute to the error in the nitrogen abundance and are joined by uncertainties in atmospheric structure and by the uncertain dissociation energies of $^{12}$C and CN. For the C/O ratio, one should also add the uncertainties in the microturbulence parameter to the list.

Obviously, improvements along several different lines of investigation are necessary in order to increase the accuracy of the analysis significantly. However, two stand out as particularly important. The first improvement would be to establish the reason for the lack of agreement between the predicted and observed polyatomic bands. This would reduce the uncertainties due to opacities and possible surface heating. The second improvement would be to measure or calculate the dissociation energies of $^{12}$C and CN with greater accuracy. To this list one could add that our confidence in the results of the analysis would increase if one could find a natural quantitative explanation for the lack of agreement between the observed and calculated CH $V$ - $R$ lines (see below).

The effects of the various systematic errors discussed above on the $^{12}$C/$^{13}$C determinations have been found to be small. For $\log(^{12}$C/$^{13}$C), as derived from the CO lines, they amount to $\pm 0.1$, where the greatest contribution is caused by the effective-temperature uncertainty. For CN determinations the resulting errors are less than $\pm 0.03$, and most of these are caused by the uncertainty in the microturbulence parameter. These errors should be compared with the formal statistical errors in $\log(^{12}$C/$^{13}$C) from CO or CN due to the scatter in the determinations from the individual lines, which typically amounts to 0.06-0.12. This is reasonably consistent with the scatter in the difference $\log(^{12}$C/$^{13}$C)$_{CO} - \log(^{12}$C/$^{13}$C)$_{CN}$, which amounts to 0.15 (Figs. 22 and 23).

### f) Other Monitors of the CNO Abundances

The CNO abundance analysis combines the $^{12}$C, $^{13}$C, and CO lines from the 1.5-2.5 $\mu$m spectra. Here we investigate whether other molecular lines are consistent with the derived abundances. The lines considered are the Ballik-Ramsay $\Delta v = 0$ sequence of $^{12}$C$_2$, some $\Delta v = 3$ and $\Delta v = 4$ Phillips lines of $^{12}$C$_2$ at 0.8 $\mu$m, the $\Delta v = -3$ sequence of the CN red system, the vibration-rotation transitions of the CH, NH, and HCN ground electronic states, and the $H_2$ 1-0 $S(0)$ line. This mixed bag is used only in a supporting role, for a variety of reasons: the basic molecular data may be less reliable than for similar standard transitions ($^{12}$C$_2$ Ballik-Ramsay); the available observations cover very few of the program stars (the CH, NH, and HCN transitions), or the line is an indirect indicator of the CNO abundances [$H_2$ 1-0 $S(0)$]. In most cases, the checks on the CNO abundances are made by calculating the $W_0$ values of the lines for models with parameters equal to or

<table>
<thead>
<tr>
<th>SOURCE OF ERROR</th>
<th>GROUP I</th>
<th>GROUP II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuum location + 5%</td>
<td>$\Delta [O/H]$ = +0.1</td>
<td>$\Delta [O/H]$ = +0.2</td>
</tr>
<tr>
<td>Overionization</td>
<td>$\Delta [N/H]$ = +0.1</td>
<td>$\Delta [N/H]$ = +0.2</td>
</tr>
<tr>
<td>Scattering line opacity</td>
<td>$\Delta \log (C/O)$ = 0.0</td>
<td>$\Delta \log (C/O)$ = 0.0</td>
</tr>
<tr>
<td>Surface heating</td>
<td>$\Delta [O/H]$ = &lt;0.03</td>
<td>$\Delta [O/H]$ = &lt;0.03</td>
</tr>
<tr>
<td>Polyatomic opacity neglected</td>
<td>$\Delta [N/H]$ = &lt;0.03</td>
<td>$\Delta [N/H]$ = &lt;0.03</td>
</tr>
<tr>
<td>Dust emission only at 2.2 $\mu$m</td>
<td>$\Delta \log (C/O)$ = &lt;0.03</td>
<td>$\Delta \log (C/O)$ = &lt;0.03</td>
</tr>
<tr>
<td>$T_{eff} = \pm 100$ K</td>
<td>$\Delta [O/H]$ = &lt;0.03</td>
<td>$\Delta [O/H]$ = &lt;0.03</td>
</tr>
<tr>
<td>$\log g = \pm 0.5$</td>
<td>$\Delta [N/H]$ = &lt;0.03</td>
<td>$\Delta [N/H]$ = &lt;0.03</td>
</tr>
<tr>
<td>$\xi = \pm 0.4$ km s$^{-1}$</td>
<td>$\Delta \log (C/O)$ = &lt;0.03</td>
<td>$\Delta \log (C/O)$ = &lt;0.03</td>
</tr>
<tr>
<td>$[Fe/H] = -0.3$</td>
<td>$\Delta [O/H]$ = &lt;0.03</td>
<td>$\Delta [O/H]$ = &lt;0.03</td>
</tr>
<tr>
<td>$^{12}$C/$^{13}$C</td>
<td>$\Delta [N/H]$ = &lt;0.03</td>
<td>$\Delta [N/H]$ = &lt;0.03</td>
</tr>
<tr>
<td>H abundance: $-0.3$ dex</td>
<td>$\Delta \log (C/O)$ = &lt;0.03</td>
<td>$\Delta \log (C/O)$ = &lt;0.03</td>
</tr>
<tr>
<td>$D^0_2(C_2) = 0.1$ eV</td>
<td>$\Delta [O/H]$ = &lt;0.03</td>
<td>$\Delta [O/H]$ = &lt;0.03</td>
</tr>
<tr>
<td>$D^0_2(CN) = 0.1$ eV</td>
<td>$\Delta [N/H]$ = &lt;0.03</td>
<td>$\Delta [N/H]$ = &lt;0.03</td>
</tr>
<tr>
<td>Non-LTE (scattering)</td>
<td>$\Delta \log (C/O)$ = &lt;0.03</td>
<td>$\Delta \log (C/O)$ = &lt;0.03</td>
</tr>
</tbody>
</table>
Chemical Composition of Carbon Stars. I.

Although a few lines from the 0–0 and 1–1 bands of the C\(_2\) Ballik-Ramsay system were measured for almost all stars, they were not used directly in the analysis because their \(g_f\)-values were judged to be more uncertain than for the Phillips system. However, these lines provide an interesting consistency check because, since they belong to an electronic transition between two triplet states, whereas the Phillips system occurs between singlet states, an inconsistency in the \(\delta(C_2)\) values from the two systems could indicate departures from LTE in the populations of the C\(_2\) states. Also, the excitation energy of the lower states of the Ballik-Ramsay lines is higher by typically 0.6 eV, so that errors in the effective temperature and atmospheric structure should be reflected in the \(\delta(C_2)\) values.

The C\(_2\) abundances from the Ballik-Ramsay lines do not differ significantly from those provided by the Phillips lines: the mean differences of the \(\Delta C_2 = \delta(C_2)_{BR} - \delta(C_2)_{Ph}\) is 0.04 dex for all the stars in our sample and is smaller than might be expected from the uncertainties afflicting the \(g_f\)-values (see Appendix A). The standard deviation for \(\Delta\) is 0.16 dex, which is accounted for totally by the scatter among the different lines and the uncertainties in the microturbulence. Values of \(\Delta C_2\) for individual stars are not correlated with effective temperature. These values of \(\Delta C_2\) correspond to errors in \(\log(C/O)\) of less than 0.01 dex. We conclude that the Ballik-Ramsay lines confirm abundances drawn from the Phillips lines.

ii) Phillips System C\(_2\) Lines at 8000 Å

Equivalent widths for lines at 0.8 \(\mu\)m from the \(\Delta \nu = 3\) and \(\Delta \nu = 4\) sequences of the Phillips system were published by Querci and Querci (1970) for UU Aur. We have compared these widths with those predicted by a standard model with abundances close to those attributed to UU Aur in Table 4. We find very satisfactory agreement for \(W_\lambda \lesssim 200\) mA [i.e., \(\log(W_\lambda/\sigma) \lesssim -4.6\)], as is shown in Figure 28; the stronger lines are sensitive to the microturbulence in and the structure of the upper atmosphere. Our test of the 0.8 \(\mu\)m C\(_2\) lines was undertaken initially to investigate the H abundance. Since the continuous absorption around 0.8 \(\mu\)m is provided by the bound-free H\(^-\) transitions and the absorption around 2.2 \(\mu\)m is due to free-free H\(^-\), H\(_2\), and He\(^-\), a reduction of the H abundance with a matching enhancement of He results in a change of the ratio of the continuous absorption at 0.8 and 2.2 \(\mu\)m and, hence, a change in the predicted \(W_\lambda\) of the C\(_2\) 0.8 \(\mu\)m lines. In Figure 27 we show predicted values of \(W_\lambda\) for a model with hydrogen reduced by a factor of 100 and the CNO abundances also reduced to maintain the fit to the observed CN, CO, and 2 \(\mu\)m C\(_2\) lines. The expected tendency for the predicted 0.8 \(\mu\)m C\(_2\) lines to become stronger is found, but the effect is small and cannot be used to confirm a hydrogen deficiency for carbon stars unless it exceeds about a factor of 100.

![Fig. 28. Predicted and observed equivalent widths of \(\Delta \nu = 3\) and \(\Delta \nu = 4\) Phillips lines in UU Aur. The effect of decreasing the hydrogen abundance by a factor of 100 and a corresponding increase of the helium abundance is shown by the arrows. The H-poor model also has reduced CNO abundances to an extent such that the observed infrared C\(_2\), CN, and CO values of \(W_\lambda\) are matched.](image-url)
iii) The 4 μm Lines of CN and NH

Spectra at 4 μm with CN, NH, and CH lines were available for RY Dra, TX Psc, U Hya, UU Aur, and Y CVn. The 4 μm Δν = −3 CN lines should provide a simple check on the 2 μm CN lines with both sets belonging to the red system; gf-values for both sequences are taken from Larsson, Siegbahn, and Agren (1983). Predicted and observed values of Wν (see Fig. 29) are in reasonable agreement. The predicted lines for two stars—U Hya and Y CVn—tend to be too strong, with a discrepancy of about 0.1 dex in log (Wν/σ); the controlling influence of N2 on the pressure of free N increases the discrepancy to about 0.2 dex when it is expressed in terms of [N/H]. We suspect that the discrepancy is attributable to a general infrared excess diluting the 4 μm photospheric spectrum. A comparison of the observed (Gezari, Schmitz, and Mead 1984) and predicted infrared fluxes verifies the existence of infrared excesses which may be ascribed to emission from circumstellar dust grains.

Vibration-rotation multiplets of NH were identified readily in TX Psc. Spectral coverage of RY Dra, U Hya, UU Aur, and Y CVn was less extensive, and fewer NH multiplets were identified; in particular, the least blended multiplets seen in TX Psc were outside the interval available for the other four stars. Observed and predicted line intensities are compared in Figure 30. There is a tendency for the predictions for TX Psc to be too weak by an amount equivalent to an abundance correction [N/H] ~ 0.3 dex. Since the CN Δν = −3 lines are predicted very satisfactorily (see Fig. 29), an infrared excess at 4 μm cannot be invoked to account for the small [0.15 dex in log (Wν/σ)] discrepancy in NH. A possible explanation is a small error in the adopted D0(CN) [or D0(C2)]. If D0(CN) were reduced from 7.60 to 7.53 eV, CN and NH would be consistent for TX Psc. As Appendix A notes, available data do not preclude this small revision of D0(CN). Analysis of CN and NH lines in M, MS, and S stars gives consistent N abundances for D0(CN) = 7.60 eV (Smith and Lambert 1986).

iv) The 3 μm Lines of HCN

A selection of HCN lines was measured from a spectrum of TX Psc. Predicted and observed intensities are compared in Figure 31. Oscillator strengths for individual lines were taken from the work of Jørgensen et al. (1985). Since the measured lines are almost certainly superposed upon a quasi-continuous distribution of weak lines from hot bands, our measurements relative to a local continuum most probably underestimate the values of Wν (see Ridgway, Carbon, and Hall’s 1978 band profile based upon the points defining the local continuum). In Figure 31 we show that, with a plausible raising of the continuum by 10%, predicted and observed intensities of the weaker [log (Wν/σ) < −4.7] lines are approximately equal. A

3For NH, we adopt D0(NH) = 3.46 ± 0.03 eV (see Lambert et al. 1984). A recent review of the experimental and theoretical results concludes that D0 is in the range 3.29–3.47 eV (Hofzumahaus and Stuhl 1985). If D0(NH) = 3.29 eV, the stellar NH lines would demand a higher N abundance and a lower D0(CN) in order for the CN lines to give the same abundance.
small mismatch for the stronger lines could be removed by a minor adjustment of the adopted microturbulence in the upper atmospheric layers where the HCN lines are formed.

In fitting predictions from our models to the observed flux curve of TX Psc (see Fig. 9), we find it necessary to reduce the HCN (and C$_2$H$_2$) opacity by a factor of 20. Such a factor, if applied to the HCN abundance or to the $f$-values, destroys completely the good agreement in Figure 31, where the broken lines show the effect on the predicted intensities of a factor of 10 decrease in the HCN $f$-values. This inconsistency suggests that, in the calculations of the predicted flux curves, it is the absorption by the many weak lines from the hot bands, and not the partial pressures of HCN, that is overestimated. As a working hypothesis, we suggest that the polyatomic lines whose region of formation is the upper photosphere may be formed by scattering rather than pure absorption (cf. § IIIa).

v) The H$_2$ 1–0 S(0) Line

Attention was drawn to the H$_2$ abundance in carbon stars by Goorvitch, Goebel, and Augason (1980), who pointed out that the 1–0 S(1) line was absent from the spectra but was predicted to be very strong. As we noted earlier, this remarkable discrepancy vanishes when model atmospheres incorporating the opacity of polyatomic molecules are adopted.

With the improved models, the H$_2$ lines are predicted to be a sensitive monitor of the C/O ratio (see Fig. 20). In Figure 32 we compare predicted and observed line intensities. Within the scatter, a clear pattern is evident: the predicted H$_2$ lines for the stars with C/O = 1 tend to be too great (i.e., the pressures in these models are apparently too high), while the converse is true for those stars with highest C/O ratios. Predictions match the observations quite well for intermediate C/O ratios. In other words, the expansion of the compact, high-pressure models produced by the polyatomic opacity seems to be too dependent on the carbon abundance. Alternatively, the temperatures of the surface layers of the most carbon-rich stars are overestimated, while the converse would be the case for the stars with C = O. Of course, the same result is provided by Figure 20, where the H$_2$ $W_0$ values suggest that the C/O ratio has a similar value for almost all of the sample. Note, however, that the strong H$_2$ line for WZ Cas corresponds to a prediction that the C/O ratio is very low in this star, a result fully confirmed by the abundance analysis: C/O = 1.005 from H$_2$ and 1.01 from the C$_2$ and CO lines. In view of the H$_2$ line's especial sensitivity, in...
vi) The CH Vibration-Rotation Lines

Quartets of the CH vibration-rotation lines from the $X^2\Pi$ state are striking contributors to the 4 $\mu$m spectra and would appear to offer a valuable independent check on the (C–O) abundance difference; the molecule's dissociation energy is accurately determined, and ab initio calculations ought to provide reliable $f$-values. To our dismay, the CH lines are predicted to be stronger than the observed lines. This is demonstrated in Figure 33 for sample 1–0 lines in TX Psc, UU Aur, U Hya, and RY Dra; Y CVn gives very similar results. The 2–1 lines give similar results. As a measure of the discrepancy, we find (see Fig. 33) that predicted and observed $W^\prime$ values may be reconciled by a reduction of the $gf$-value (or the CH number density) by factors of 5 (TX Psc), 15 (UU Aur and U Hya), and 50 (RY Dra). The required large reduction is thus temperature-dependent, ranging from a factor of 5 at $T_{\text{eff}} \approx 3000$ K, 15 at $T_{\text{eff}} \approx 2800$ K, and 50 at $T_{\text{eff}} \approx 2500$ K.

The reason for this discrepancy is as yet unknown but is nontrivial. The reasonable consistency for the NH and CN lines in the same wavelength region excludes the effects of dust emission or an unidentified opacity as explanations for the discrepancy. Also, it may be shown that the ratio of the CH absorption coefficient, relative to that of NH, is only slightly dependent on atmospheric structure, formation of dust, and polyatomic carbon molecules, as well as on errors in the CNO abundances for given CO, C$_2$, and CN line strengths. The ratio is also very little dependent on errors in the adopted hydrogen abundance (cf. Hammarbäck, Eriksson, and Gustafsson 1985). The $f$-values for the CH lines are based on the ab initio calculation of the electric dipole moment function by Lie, Hinze, and Liu (1973), which is similar to that of Meyer and Rosmus (1975). Moreover, the function is essentially linear over the relevant range of the internuclear separations, so that the $gf$-values are proportional to the square of the function's slope. We estimate that the $gf$-values are correct to within 20%. Gross errors are excluded because the calculations predict correctly the permanent electric dipole moment of the ground state.

An error in the dissociation equilibrium of CH is an obvious candidate as an explanation. Uncertainties in the dissociation energy of the molecule are very small and unimportant, even at the low prevailing temperatures. Errors in the C$_2$ dissociation may lead to an erroneous carbon abundance; an underestimate of $D_0^*/(C_1)$ by 0.1 eV would, however, correspond to a change of only about 0.1 dex in the CH number density. Errors in the dissociation energy of CS, a molecule of great importance for the molecular equilibria of the carbon molecules, as well as errors in the adopted (solar) sulfur abundance, would affect both the C$_2$ and the CH equilibria, but these errors would be compensated for in the carbon abundance determination, i.e., the strength of the C$_2$ lines is a measure of the free carbon atoms, which also
determine (together with the atomic hydrogen density) the number density of CH. Likewise, errors in the equilibria of other carbon molecules cannot explain the CH discrepancy.

Suspicion turns inevitably to the H abundance. However, the predicted CH intensities decline only slightly as the H content is decreased; the increase of the gas pressure and the opacity decrease compensate for the H deficiency. Tests show that the H abundance must be cut by a factor of 1000 to bring predicted and observed H lines into agreement. Such a cooling does not affect the abundance analyses significantly, since the lines used in the analyses are formed at much greater depths. However, the NH lines from the surface-cooled models are predicted to be too weak. Other major problems with this explanation of the CH discrepancy would be the conflict with the occultation angular diameter temperature scale and the fact that the predicted colors of the models would be significantly too blue. The first problem might be resolved by invoking extended inhomogeneous atmospheres or scattering in outer dust shells. The second problem may be a consequence of selective dust absorption. Another problem is that the agreement between the effective temperature scale and the excitation equilibrium of CO would be partially impaired. This is less severe, however, in view of the uncertainties involved in the latter method of establishing the effective temperature with the present data. Although this explanation for the CH discrepancy contains several ad hoc assumptions and must be regarded as improbable, it cannot be totally ruled out. We note in passing that an increase of the effective temperature by 300 K for the cooler stars, and less for the hotter ones, would seem more believable. Although this would not resolve the CH problem, these hotter models could reproduce the observed polyatomic band strengths and lead to greater nitrogen abundances. Further attempts should be made to establish an accurate temperature scale based on the excitation equilibrium of photospheric lines.

A structural difference between CH and the other molecules led us to investigate the possibility of departures from LTE in the dissociation equilibrium. The difference is that CH may be dissociated by photons in the near-ultraviolet, where the photon flux is nonnegligible and the ratio of the mean intensity (J) to the Planck function (B_v) is large in the atmosphere’s outer layers. Dissociation is effected by predissociations in the B-X and C-X transitions. The NH molecule with a similar dissociation energy is immune to predissociations from the electronic ground state. The trio C_2, CN, and CO have much higher dissociation energies, so that photodissociation demand shorter wavelength photons that are extremely scarce (unless, perhaps, the star possesses an extensive chromosphere). However, the CH lines are predicted to be formed at depths, 0.01 < r < 0.5, where the ultraviolet radiation field is almost thermalized, J / B_v ~ 1. In the
Fig. 35.—Effects on CNO abundances and predicted equivalent widths for characteristic lines that result from a significant increase of the effective temperature for TX Psc, UU Aur, and RY Dra. Standard choices of Teff are indicated by plus signs; observed line strengths, by arrows.

limit that photodissociation controls the molecule's destruction, the non–LTE prediction for the \( W_\nu \) of a typical CH line is a factor of 1.3 smaller at \( T_{\text{eff}} = 2800 \) K than the LTE prediction, but one is seeking a factor of 15; the predicted lines are saturated, which leads to rather small effects on the \( W_\nu \) values. Also note that the dissociation equilibrium of CH in carbon star atmospheres should be dominated by collision processes, such as \( \text{C} + \text{H}_2 \leftrightarrow \text{CH} + \text{H} \), the rates of which, as adopted from Iglesias (1977) and Iglesias and Silk (1978), seem to outnumber the radiative processes by several orders of magnitude. Radiative dissociation would, however, still be of importance in the present context if the collisions mainly produce CH molecules in excited states which do not immediately decay to the ground state.

Another potential non–LTE effect may be introduced by the radiative excitation of CH to the \( B^2 \Sigma^- \) state followed by a de-excitation to a different vibrational level. This possibility is more likely for the \( B \rightarrow X \) than for the \( A \rightarrow X \) transition, because in the latter the \( \Delta v = 0 \) sequence is so dominant. Since \( J_x / B_x > 1 \) for the \( B \rightarrow X \) transition, this radiatively driven cycle in the absence of collisions with atoms and electrons would result in an overpopulation of the excited vibrational levels and the \( W_\nu \) of the vibration-rotation transitions would be reduced by the extra emission. Detailed statistical equilibrium calculations (Hammarbäck, Eriksson, and Gustafsson 1985) show that this effect is not very important, irrespective of the collision cross sections adopted, because the radiation fields in the electronic transitions thermalize \( (J_x \sim B_x) \) at rather shallow depths.

If non–LTE dissociation or excitation equilibrium is to account for the weak CH lines, thermal inhomogeneities (stellar granulation) would seem to be necessary at the depths at which the CH lines are predicted to be formed; temperature differences of 500 K and granular dimensions equivalent to a horizontal optical depth of about unity at 3500 Å may suffice to reduce the predicted CH densities. The non–LTE effects on the other molecules used in our analysis would appear to be much smaller. Systematic effects will be present.
CHEMICAL COMPOSITION OF CARBON STARS. I.

V. DISCUSSION

a) Internal Checks and Comparisons with Other Studies of Carbon Stars

Before comparing our derived abundances with theoretical predictions for carbon stars produced by thermal pulses, we provide a series of checks, both internal and external. First, we check for significant trends with effective temperature. In Figure 36 we plot $[X/H]$ versus $T_{\text{eff}}$ for $X = C$, N, and O. On these and subsequent plots, we name and underline the four $^{13}\text{C}$-rich stars and name without underlining the two additional stars with $^{13}\text{C}/^{12}\text{C} < 20$. When the $^{13}\text{C}$-rich stars are set apart, $[X/H]$ is independent of $T_{\text{eff}}$; $[\text{N}/H]$ may, however, possibly decline with decreasing $T_{\text{eff}}$. This weak trend would vanish if the effective temperature of the cooler stars were systematically raised by a few hundred K as was discussed above. The apparently larger scatter in the $[\text{N}/H]$ plot is probably a reflection of the fact that the CN lines, on which the N abundances are based, measure the partial pressure of free N, but the N abundance is proportional to the square of this pressure.

In the plots involving $[\text{C}/H]$ and $[\text{O}/H]$, the $^{13}\text{C}$-rich stars appear as the extremists: WZ Cas is the most C- and O-rich member of our entire sample, and the other five $^{13}\text{C}$-rich stars are among the least C- and O-rich members. This hint that WZ Cas may belong to a second class of $^{13}\text{C}$-rich stars is supported by two observations. First, WZ Cas alone has a very high Li abundance. Second, the $^{13}\text{C}$-rich cool carbon (often referred to as $J$-type) stars do not seem to show the strong $s$-process enhancements characteristic of carbon stars (Utsumi 1985), but WZ Cas is an exception with $s$-process enhancements approaching those of the typical carbon star (Dominy 1985; Dominy provides extensive references to papers discussing lines of the heavy elements in cool carbon stars). There is evidence that Li enrichment on a more modest scale occurs in most carbon stars; for example, Wallerstein and Conti (1969) give a mean abundance over a sample of seven stars which is about an order of magnitude larger than that seen in K giants.

Presentations involving $[\text{C}/H]$ and $[\text{N}/H]$ may hide systematic errors because neither quantity is related directly to observed equivalent widths. Recall that $[\text{C}/H]$ is obtained from $^{12}\text{C}$ lines whose intensity is proportional to $(^{12}\text{C}—^{12}\text{O})^2$ and the $^{12}\text{C}^{16}\text{O}$ lines, which provide the number of $^{12}\text{C}$ atoms tied up in CO molecules. This oxygen abundance ($^{16}\text{O}$) is a partial measure of the total $^{12}\text{O}$ abundance, which must include those $^{16}\text{O}$ atoms tied up in $^{12}\text{C}^{16}\text{O}$ molecules. With the CN lines as our primary indicator of the N abundance, the N abundance is inversely proportional to $(^{12}\text{C}—^{12}\text{O})^2$, because of the controlling influence of $N_2$ in the molecular equilibrium of nitrogen. Figure 37 shows that $(^{12}\text{C}—^{12}\text{O})$ tends to be higher for the cooler stars, a trend that is the opposite of that provided by $[\text{N}/H]$.

A direct comparison of $[\text{N}/H]$ and log $(^{12}\text{C}—^{12}\text{O})$ is provided in Figure 38. For the coolest stars, with $T_{\text{eff}} \leq 2600$ K, $[\text{N}/H]$ appears to decline with increasing log $(^{12}\text{C}—^{12}\text{O})$. Warmer stars, $2600 < T_{\text{eff}} < 2800$ K, may show a similar, but weaker trend. However, the slope of the relation between $[\text{N}/H]$ and log $(^{12}\text{C}—^{12}\text{O})$ is not obviously attributable to a systematic error. Suppose that the measurement and analysis of the $C_2$ lines result in a value $(^{12}\text{C}—^{12}\text{O})$ that is in error by a factor $F$; the N abundance is then subject to an error of a factor $F^{-2}$. This relation is a poor fit to the points in Figure 38. Location of the continuum in the coolest stars is a potential source of systematic errors. Since the $C_2$ and CN lines occupy the same spectral interval, their $W_a$ values should be subject to the same error. For a factor $f$ correction to the measured values...
Two external checks are discussed; one with published analyses of carbon stars, and the second with analyses of related objects. Our introduction covered the more important papers providing CNO analyses for carbon stars. In spite of a number of severe uncertainties in these less comprehensive analyses, their results appear to be consistent with ours.

In view of these uncertainties, we restrict our detailed comparison to the $^{13}$C/$^{12}$C ratios, a key product of our analysis and least sensitive to the detailed atmospheric structure and to the uncertainties in the basic molecular data. We restate our result that the $^{13}$C/$^{12}$C ratios provided by the CN 2 $\mu$m lines, the CO 1.6 $\mu$m lines, and the combination of the $^{13}$CO 1.6 $\mu$m and $^{13}$CO 2.3 $\mu$m lines are consistent to within the expected uncertainties (Figs. 22 and 23). With the exception of a few stars rich in $^{13}$C, the $^{13}$C/$^{12}$C ratios for cool carbon stars are in the range 30–100, with $^{12}$C/$^{13}$C $\sim$ 55 as a most common value. This range confirms results provided by Fujita and his colleagues (e.g., Fujita and Tsuji 1977), who advocated the exploitation of the satellite $^{13}$CN lines from the red system bands near 8000 Å. Our $^{13}$C/$^{12}$C ratios are fairly consistently higher than other published analyses involving the CN near-infrared bands, for example, the following results have been reported recently for V460 Cyg, for which we find $^{13}$C/$^{12}$C $= 61$; $^{13}$C/$^{12}$C $= 30 \pm 13$ (Dominy et al. 1978), 24 (Johnson, O’Brien, and Climenhaga 1982), and 22–29 (Little-Maremin and Little 1984). Fujita and Tsuji (1977) obtained $^{13}$C/$^{12}$C $= 100$; their results for two other stars—Z Psc and U Hya—are also similar to ours. Our infrared spectra provide an adequate supply of weak lines that are extremely rare in the near-infrared. In addition to the consistent results from CN and CO, we point to results obtained recently from observations of circumstellar CO pure rotation lines for carbon stars showing extensive mass loss: Knapp and Chang (1985) state that “for the carbon stars, the range of $^{13}$C/$^{12}$C is 30–100,” a range compatible with our histogram (Fig. 24). In particular, their result for the well-studied object IRC $+$10216 is $^{13}$C/$^{12}$C $= 48 \pm 8$, which is close to the peak of the histogram. With improvements in sensitivity, it is to be hoped that circumstellar $^{13}$CO (and $^{12}$CO) lines may be detected for several of our program stars and so provide a direct check on the two techniques.

b) Carbon Stars, Planetary Nebulae, and M–S Stars

Planetary nebulae (PNs) are direct descendants of AGB stars with the C-rich nebulae evolving from the cool carbon stars. The nebulae represent recently ejected material (age $\leq 10^4$ yr) and contain a significant fraction of the original convective envelope (mass of nebula $\sim 0.1 M_\odot$). With the qualification that the pre–PN stars may have experienced additional dredge-up of carbon and/or conversion of carbon (and, perhaps, oxygen) to nitrogen at the base of a hot convective envelope, we expect to find similar compositions for cool carbon stars and PNs. This is an important test because the analysis of the emission-line spectrum of a PN draws on a different set of atomic data and modeling techniques.

Dredge-up of carbon is expected to convert a M giant to a carbon star through the sequence of spectral types M $\rightarrow$ MS $\rightarrow$ S $\rightarrow$ SC $\rightarrow$ CS $\rightarrow$ C. If the dredge-up contaminates the envelope with pure $^{13}$C (and s-process elements), the composi-
Fig. 39.—Histograms of [X/H] for X = C, N, and O for three samples: C-rich planetary nebulae, the carbon stars, and M, MS, and S stars. For the carbon stars a hatched box denotes a star with $^{12}\text{C}/^{13}\text{C} < 5$ and a stippled box a star with $5 < ^{12}\text{C}/^{13}\text{C} < 20$.

The CNO analysis is structured differently for O-rich and C-rich stars. Furthermore, the atmospheric structures of the warmer O-rich M and MS giants differ from those of the cooler C-rich stars.

For the PNs, we draw on the extensive program undertaken by Aller and Czyzak (1983).4 The PNs may be used to dispose of the idea that cool carbon stars are hydrogen-poor. Aller and Czyzak report a mean He abundance for C-rich PNs of log $\epsilon$(He) = 11.03 or He/H = 0.11; the mean for their entire sample, including O-rich PNs, was log $\epsilon$(He) = 11.04. A few He-rich PNs are known with He/H $\sim$ 0.18 (see, for example, Kaler 1983). For comparison, we cite the solar abundance log $\epsilon$(He) = 10.90 ± 0.10 (Grevesse 1984), the ratio He/H = 0.100 ± 0.005 (Peimbert and Torres-Peimbert 1977; see also Dufour, Shields, and Talbot 1982) reported for the Orion Nebula, a typical local H II region, and He/H = 0.10 ± 0.02 (Nissen 1983) as provided for young stars from photospheric He i lines. Clearly, the He enrichment of PNs through dredge-up prior to ejection of the nebula is very slight.

For our comparisons of CNO in PNs, carbon stars, M, MS, and S stars, we construct histograms for [C/H], [N/H], and [O/H] (Fig. 39). We assign weights of 2, 2, 1, and 0 to abundances given a quality rating of A, B, C, and D, respectively, by Aller and Czyzak (1983). Our sample of M, MS, and S stars is that analyzed from near-infrared and infrared spectra by Smith and Lambert (1985, 1986) using the molecules CO for the C abundance, OH for the O−C abundance, and CN for the N abundance.5

Since the oxygen abundance is predicted to be almost unaffected by the dredge-up phases, we begin by comparing the histograms for [O/H]. Inspection shows [O/H] to have essentially identical mean values and dispersions in the three samples. One notes also that similar results are found for samples of G and K giants: the mean [O/H] is −0.01 (Lambert and Ries 1981) or −0.12 (Kjaergaard et al. 1982) where the difference is due almost entirely to the higher $T_{\text{eff}}$ (and log g) values adopted by Lambert and Ries. This level of agreement between the histograms is consistent to within the estimated uncertainties, with the assumption that oxygen is unaffected by dredge-up. Oxygen is a special case because it is derived from the CO lines, for which the $g_f$-values are known quite accurately, and the CO molecule is insensitive to the details of the molecular equilibrium because oxygen is essentially fully associated with CO.

Knapp and Chang (1985) obtain $40 < ^{12}\text{C}/^{13}\text{C} < 130$ for the PN NGC 7027, which is classified as C-rich according to its infrared spectrum (Aitken et al. 1979). This range in $^{12}\text{C}/^{13}\text{C}$ certainly encompasses the values reported here for typical carbon stars (see Fig. 24).

We defer comment on the isotopic ratios to the following section.
The three histograms for \([C/H]\) show the expected trend; the mean value for \([C/H]\) increases from about \(-0.2\) in the M stars to about \(0.0\) in the carbon stars to about \(0.2\) in the PNs. The value for the M stars is confirmed by the independent analyses of larger samples of G and K giants, where the \(C_2\) Swan system (with the \([O\, i]\) lines) is the carrier of information on the carbon abundance: the mean \([C/H]\) is \(-0.34\) (Lambert and Ries 1981) or \(-0.32\) (Kjaergaard et al. 1982), and, since \([C/H]\) is insensitive to the adopted \(T_{\text{eff}}\), one may infer that \([C/H] \approx -0.3\) is representative of G and K giants. A simple interpretation of the higher \([C/H]\) seen in some PNs is that our carbon stars are not in the terminal phase of thermal pulsing and dredge-up. One difficulty with this scenario is that the more C-rich stars close to the termination of evolution up the AGB ought to be more luminous and, hence, members of our magnitude-limited sample.

A qualification must not be overlooked in a discussion of the \([C/H]\) histogram for the carbon stars. Since \(C/O\) is close to unity, the \([C/H]\) and \([O/H]\) histograms must be similar, and large errors in the analysis of the \(C_2\) lines, which measure the difference \(C-O\), are required to push \([C/H]\) to values as high as the value \(0.4\) dex found for a few PNs. On the other hand, the \([C/H]\) (and \([O/H]\)) histogram for PNs must be broadened by measurement errors, and the intrinsic histogram must be more sharply peaked, so that the differences at the high \([C/H]\) end between the PN and carbon star histograms should not be overemphasized.

Histograms for \([N/H]\) in the three samples show interesting differences. First, we note that \(N\) is considerably more enhanced in the M and MS giants than in the carbon stars.\(^6\) Confidence in the M and MS giants as a benchmark is increased by the observation that G and K giants provide an enhancement of \(N\): mean values of \([N/H] = +0.39\) (Lambert and Ries 1981) and \(+0.01\) (Kjaergaard et al. 1982) are obtained. (The difference may be ascribed largely to differences in the adopted \(T_{\text{eff}}\) and log \(g\) scales.) The histograms for the PNs and the carbon stars differ apparently in that only the former includes a significant fraction of N-rich \((|N/H| > 0)\) examples with a maximum enrichment close to that seen in the average G, K, and M giant. We postpone to the following section any discussion of how these differences might or might not be related to evolution of these AGB stars. Here we comment on the possibility that the lack of \(N\) enrichment in carbon stars may be the result of a systematic error.

In the carbon stars, the \(N_2\) molecule controls the partial pressure of free \(N\) and, hence, this pressure and that of molecules such as CN and NH vary as the square root of the \(N\) abundance. Conversely, the derived \(N\) abundance is proportional to the square of the CN column density. Moreover, as we use \(CN\) in the standard analysis, random and systematic errors in the \(C_2\) analysis, our source of the partial pressure of \(C\), also affect the \(N\) abundance. For these reasons, we are reluctant to attribute to the greater width of the carbon stars’ \([N/H]\) histogram any astrophysical interpretation.

There is a simple way in which to boost the stellar \(N\) abundances, namely, adjustments to the adopted \(D_{C}(C_2)\) and \(D_{C}(CN)\). As is noted in Appendix A, the former could be raised \(0.1\) eV without introducing a serious conflict with available experimental results. The latter is notoriously uncertain. Although we have adopted a value near the lower end of the wide range of experimental results, a reduction by \(0.10\) eV cannot be excluded. A simultaneous application of these revisions would raise our \([N/H]\) values by \(0.5\) to \(0.6\) dex (see Table 7) and remove the significant difference between our stellar and the PN histograms. Analysis of the NH lines for TX Psc, the only star for which reliable NH identifications are presently available, provide a higher \([N/H]\) value by \(0.3\) dex. If the NH dissociation energy were reduced slightly (see footnote 3), NH lines would require an even higher abundance.

In suggesting this recipe for \(N\) enrichment, we should not overlook the fact that the CN molecule is also the source of the \(N\) abundances providing the histogram for the M, MS, and S (and the G and K) stars, so that adjustment to \(D_{C}(CN)\) also changes their \(N\) abundances in the same sense. Since these other stars are warmer and \(N_2\) has a weaker influence, the adjustments are smaller. Smith and Lambert (1985) note that \(\Delta D_{C} = -0.1\) eV raises the \(N\) abundance in M–MS stars by about \(0.15\) dex rather than the \(0.4\) dex required for the carbon stars. The net result is that \(\Delta D_{C} = +0.1\) eV and \(\Delta D_{C}(CN) = -0.1\) eV would approximately superpose the \([N/H]\) histogram for carbon stars onto that for the M–S (and G and K) stars. This is yet another illustration of the need for precise (say, \(\pm 0.02\) eV or better) measurements of these dissociation energies.

Just possibly, the explanation for the lower \(N\) abundance in the carbon stars could be a drastic error in the temperature scale, as sketched above in § IV/(vi).

c) Comparison with Theory

Theoretical studies of the third dredge-up in intermediate-mass stars are well developed (Iben and Renzini 1983). The cool carbon stars with masses \(M \sim 2\, M_\odot\) fall below the mass range spanned by intermediate-mass stars. A third dredge-up in the lower mass stars was only recently produced by the theoretical studies (Iben and Renzini 1982). Specific predictions for the changes in composition are not yet available. We may compare our derived abundances with prediction of a simple working hypothesis.

The simplest working hypothesis is that the dredge-up adds \(^{12}\)C but not \(^{13}\)C and \(^{14}\)O to the convective envelope and, hence, to the atmosphere. We further assume that the base of the envelope is cool so that hydrogen burning through the CN cycle does not convert \(^{12}\)C to \(^{13}\)C and \(^{14}\)N. On this hypothesis, the \(^{12}\)C/\(^{10}\)O and \(^{12}\)C/\(^{13}\)C ratios are predictable given the envelope’s composition prior to the dredge-up of \(^{12}\)C.

For this composition we adopt the range displayed by the G, K, and M giants, i.e., \(^{12}\)C/\(^{10}\)O = \(-0.25\) and \(^{12}\)C/\(^{13}\)C = \(-10\)–\(20\). Observations and predictions are compared in Figure 40; a few stars are carbon-rich, \(^{12}\)C/\(^{10}\)O > 1, but have \(^{12}\)C/\(^{13}\)C < 1. If the working hypothesis were valid, the carbon

---

\(^6\)Dominy, Wallerstein, and Suntzeff (1986) analyze 2 \(\mu\)m spectra of SC stars to obtain \(N\) enhancements similar to those in M and MS stars. Note, however, that these authors adopt \(D_{C}(C_2) = 6.16\) eV and \(D_{C}(CN) = 7.52\) eV—a choice that would also increase our nitrogen abundances to similar values. Oxygen deficiencies for the SC stars may be more severe \((|O/H| \sim -0.5)\) than for the typical carbon star.
FIG. 40.—The $^{12}\text{C}/^{16}\text{O}$ and $^{12}\text{C}/^{13}\text{C}$ ratios are shown for the cool carbon stars and two simple models of the dredge-up (see text) in which pure $^{13}\text{C}$ is added to initial envelope compositions representative of a red giant prior to the third dredge-up. Adopted initial compositions of these red giants are indicated on the figure. For the key to the symbols see Fig. 2.

stars would fall between the two full lines in the figure. With the exception of the $^{13}\text{C}$-rich stars, the stars do populate the predicted area. An apparent failure of the hypothesis may be indicated by the distribution of the $^{12}\text{C}/^{13}\text{C}$ ratios, in that a significant fraction of the G and K giants have $^{12}\text{C}/^{13}\text{C} < 10$, but very few carbon stars appear to have evolved from such $^{13}\text{C}$-rich giants. Scalo (1977) predicted the frequency distribution of the $^{12}\text{C}/^{13}\text{C}$ ratio in carbon stars for assumptions essentially equivalent to our hypothesis and obtained a similar result; his distribution with a maximum at $^{12}\text{C}/^{13}\text{C} \approx 30$ lies below our observed peak. This apparent discrepancy may have a simple explanation. There is evidence that the $^{13}\text{C}$-rich G and K giants are lower mass and mildly metal-poor giants (see Lambert and Ries 1981) and, hence, the predicted and observed C/O ratios are lower than for the giants with $^{12}\text{C}/^{13}\text{C} \approx 20$. This change is a reflection of the fact that $[\text{O}/\text{Fe}] > 0$ in metal-poor stars. Such giants require additional $^{13}\text{C}$ to convert them to carbon stars (see broken line in Fig. 40). With this change, the carbon stars populate the predicted region in Figure 40. Of course, the $^{13}\text{C}$-rich stars are not covered by the hypothesis. The lower N abundances in carbon stars are probably not expected by this hypothesis, but, as noted above, a likely explanation lies in minor and plausible adjustments to molecular dissociation energies.

With reasonable constraints, the hypothesis accounts satisfactorily for the lack of a marked decrease in the H content of the convective envelope; the He/H ratio after mixing is less than 20% above the normal value, provided that $Z(^{13}\text{C})_{\text{He}} \approx 0.05$, where $Z(^{13}\text{C})_{\text{He}}$ is the mass fraction of $^{13}\text{C}$ in the former He-burning shell. This same constraint translates to a limit on the mass of dredged-up material, $M_3$, namely, $M_3/M_{\text{CE}} < 0.06$ where $M_{\text{CE}}$ is the mass of the original convective envelope. Both $Z(^{13}\text{C})_{\text{He}}$ and $M_3$, which may be considered as the product of the mass dredged up following each thermal pulse and the number of pulses, are consistent with the available evolutionary models. Since the mass of dredged-up material is small, species such as $^{14}\text{N}$ which may have been consumed in the He shell suffer a negligible reduction in abundance when shell material is mixed with the envelope. Our estimates ignore the effects of mass loss, which may be quite severe (Knapp and Chang 1985).

One additional prediction is of interest. In the He shell the original CNO nuclei may be converted to $^{22}\text{Ne}$ via $^{14}\text{N}$ and $^{16}\text{O}$. If this occurs, dredge-up provides a fractional increase of the Ne/O ratio by about $8M_3/M_{\text{CE}}$, or $\leq 50\%$ for $M_3/M_{\text{CE}} < 0.06$. The increase observed in PNs is indeed close to 50%. Kaler (1983) gives Ne/O = 0.225 $\pm$ 0.01 for PNs which, with a cosmic abundance Ne/O $\approx 0.16$ (mean of values recommended by Grevesse 1984 and Anders and Ebihara 1982) represents an increase by 40%. This comparison suggests that $M_3/M_{\text{CE}}$ is close to the previously estimated upper limit. Note that the predicted increase refers to $^{22}\text{Ne}$, with $^{20}\text{Ne}$ providing (probably) the dominant isotope before dredge-up. Unfortunately, the isotopic ratios of Ne cannot be measured from a PN's emission lines.

Our working hypothesis fails to account for the abundances of the oxygen isotopes (Harris et al. 1986). The identical failure is found for MS and S stars (Harris, Lambert, and Smith 1985). Briefly, the abundances of $^{17}\text{O}$ and $^{18}\text{O}$ seem to be an order of magnitude less than predicted for AGB stars undergoing dredge-up following thermal pulses. Since the $^{16}\text{O}$ abundance in MS, S, and C stars is consistent with expectations, these AGB stars would appear to destroy or to dilute their $^{17}\text{O}$ and $^{18}\text{O}$. Their abundance is anticorrelated with the
neutron exposures as estimated from the $s$-process overabundances. Harris, Lambert, and Smith (1985) were unable to find a plausible explanation for the low $^{17}$O and $^{18}$O abundances. Their scenarios are redesigned by Harris et al. (1986) in the light of the now similar findings for these carbon stars, but no thoroughly acceptable explanation has been provided. We do not know whether the true explanation involves minor or major modifications to our working hypothesis. The simplest explanation (third dredge-up of $^{12}$C increasing $^{12}$C/$^{13}$C) appears to be definitely excluded by repeated measurements (Smith and Lambert 1985, 1986; this paper) of O abundances close to solar.

Although our results for the N abundances may be in error by a scale factor, they should reveal both the approximate dispersion in the abundance and significant correlations with other measures of the composition. One may hope to test, for example, theoretical suggestions that the base of the convective envelope during the intervals between thermal pulses may be hot enough for some $^{12}$C to be converted to $^{13}$C and $^{14}$N. Such a hot bottom convective envelope should result in a lower $^{12}$C/$^{13}$C ratio and a higher $[\text{N/H}]$. Our results (Fig. 41) show that $^{12}$C/$^{13}$C and [N/H] are not correlated and, hence, the convective envelopes have cool bottoms. Three of the $^{13}$C-rich stars (RY Dra, WZ Cas, Y CVn) are possible exceptions. The other three $^{13}$C-rich stars are among the least N-rich (also, least O- and C-rich) of the sample and, if a hot bottom convective envelope is to account for them, it must have a temperature sufficiently high to convert C, N, O to $^{14}$N and then to $^{18}$O and $^{22}$Ne. We speculate below on an alternative method of producing these stars.

An apparently surprising result of our study is that the C/O ratios are only slightly in excess of unity for most stars. The histogram of the C/O ratio in Figure 42 shows that 40% of our sample have C/O $< 1.05$ and 57% have C/O $< 1.10$. One is surprised that such a thin margin separates the carbon stars from their oxygen-rich progenitors. Our sample is not exceptional in this regard. After Gow's (1977) photometric estimates of the C/O ratio (see Appendix B) are recalibrated using our abundances for stars in common, the combined sample of 57 stars has a similar frequency distribution (see Fig. 42). However, the surprise is most probably unfounded. Scalo and Miller (1985) give frequency distributions for the $^{12}$C/O ratio in simulated populations of AGB stars for various initial mass functions, birthrate histories, initial metallicity, and a simple representation of the dredge-up and mass loss. These calculations show that the distribution is significantly flatter in a metal-poor system such as the Small Magellanic Cloud than in the Galactic disk. Our observed distribution agrees quite well with the predicted one for the disk and certainly does not fit the simulations for metal-poor populations. Carbon-rich envelopes may spawn copious amounts of graphite, which shroud the star, and, hence, our sample may be biased toward low C/O ratios.

d) The $^{13}$C-rich Stars

These stars with $^{12}$C/$^{13}$C $< 20$ fall outside the scope of the working hypothesis. As noted earlier, WZ Cas, with a significant overabundance of the $s$-process elements and severe enrichment of Li, may be set apart from the other four stars. It may be a more massive star with a hot bottom convective envelope with a potential to reduce the C/O ratio to below unity unless additional carbon is added. Both the time scale

![Figure 41](image1.png)

**Fig. 41.** - The $[\text{N/H}]$ and the $^{12}$C/$^{13}$C ratio are shown for the program stars. For the key to the symbols see Fig. 2.

![Figure 42](image2.png)

**Fig. 42.** - Distribution function for the $(^{12}$C + $^{13}$C)/$^{16}$O ratio. The top panel shows our sample: as in Fig. 38, a hatched box denotes a star with $^{12}$C/$^{13}$C $< 5$ and a stippled box a star with $5 < ^{12}$C/$^{13}$C $< 20$. The lower panel shows a sample of 57 stars taken from Gow (1977) (see Appendix B).
of H burning at the envelope's base and the interpulse period are such that marked changes in the spectrum of a star with C/O close to unity may be observable in 100 years or so.

With their lack of s-process enhancements, some (possibly all) of the remaining four 13C-rich stars would appear to have a different history from the stars to which our working hypothesis was applied. One possibility is that they are lower mass stars in which the 13C(a, α)16O neutron source was not ignited. Their low oxygen content as revealed by the mean [O/H] = −0.38 for the sample of five would seem to demand destruction of O by H burning at an earlier phase. If the low abundance is attributed to a low initial abundance, [Fe/H] = −0.8 is indicated (Clegg, Lambert, and Tomkin 1981), i.e., the stars are old disk or halo stars, an identification that is probably at odds with their kinematics and with the metal abundances of Table 5.

The observation that the 13C-rich stars in the Magellanic Clouds are among the most luminous of the carbon stars (Richer, Olander, and Westerlund 1979; Cohen et al. 1981) suggests that the CNO abundances cannot be raised by adopting higher surface gravities. There are less luminous 13C-rich stars in the Clouds (Richer 1981; Bessell, Wood, and Lloyd Evans 1983), but they are quite rare. Hydrogen burning is inadequate as the deus ex machina because the N abundance for three of the stars is also extremely low. This would seem to call for exposure in a He shell such that 14N is burned to 22Ne through 16O. The 13C-rich stars show a much higher 16O abundance than the more typical carbon stars (Harris et al. 1986), but not sufficiently high to be attributed to H burning. Since the temperature in the He shell must be low enough not to ignite the 22Ne as a neutron source, protection of 14N from 13N(a, α)16O is possible. The 13O would be a survivor from an anomalous H-burning layer (perhaps burned hydrodynamically as suggested by Harris et al.). Operation of H burning of O and He burning of N in tandem was recently suggested by Dominy, Wallerstein, and Suntzeff (1986) to explain their CNO abundances for SC stars which have an O deficiency similar to that of our 13C-rich stars. Two of the 13C-rich stars—RY Dra and Y Cyg—are quite N-rich, and destruction of O (and C) by the H-burning CNO cycles may be possible. However, termination of the burning may call for precise timing in order that the He/H ratio be preserved at approximately the normal value. One may prefer to look beyond the evolution of a single isolated star to a binary star.

At lower luminosity, the warm carbon or R-type stars are candidates as progenitors or descendants of the 13C-rich stars. Both groups lack the s-process enhancements that characterize other C-rich stars. Dominy (1984) finds [O/H] = −0.10, [C/H] = +0.4, [N/H] = +0.70, and 12C/ 16O = 9 from a trio of R stars. These results, and especially the carbon and oxygen abundances, seem to preclude the possibility that the 13C-rich cool carbon stars are evolved R-type stars. Mass transfer across a binary from a cool 13C-rich carbon star to a companion should create a carbon-enriched star of lower luminosity. The high carbon abundance of the R stars, [C/H] = +0.4, cannot be achieved by mixing slightly carbon-rich (C/O = 1.9) material with material having a solar-like (or smaller) ratio of C/O = 0.6. It would appear that the R-type and the 13C-rich cool carbon stars may be unrelated.

VI. CONCLUDING REMARKS

This analysis provides novel information on the chemical composition of 30 bright Galactic carbon stars. Attention was focused on the spectra obtainable through atmospheric windows between 1.5 and 2.4 μm. In this interval C2, CN, and CO contribute many lines, but the spectra permit the measurement of an adequate number of the weak lines so necessary in an abundance analysis. The H2 S(0) 1–0 line seen near 2.3 μm was employed as a consistency check, along with CN, NH, CH, and HCN lines measured from supplementary 3–4 μm spectra.

The hypothesis that the stars are hydrogen-poor leads to difficulties in interpreting the NH lines, and the low metal abundances. When the sensitivity of the H2 lines to the atmospheric structure, and, hence, to the line blanketing provided by polyatomic molecules, is appreciated, the observed S(0) 1–0 line is compatible with the claim that the He/H ratio is approximately normal.

With the exception of the 13C-rich stars, the remaining stars form a rather homogeneous sample with a slight deficiency of oxygen, very moderate overabundances of carbon relative to oxygen, and no significant enrichment of nitrogen. The 13C/ 16O ratios as provided by consistent measurements from CN and CO lines are in the range 20 < 13C/ 16O < 100. We suggest that the nitrogen abundances may be underestimated by rather large amounts, as a result of small (0.1 eV) errors in the adopted dissociation energies of the C2 and CN molecules. With the exception of the 16O/ 18O and 16O/ 17O ratios discussed by Harris et al. (1986), the results of our analysis are consistent with the hypothesis that the stars were created by the dredge-up of freshly synthesized 12C. The cool carbon stars' composition is consistent (with the possible errors in the N abundance taken into consideration) with that found for G, K, and M giants the carbon stars' progenitors, and the planetary nebulae, the carbon stars' descendants. The 13C-rich stars may have experienced the ravages of H burning and even He burning, due to a hot bottom convective envelope and mixing.

We hope that we have demonstrated, through infrared high-resolution spectroscopy and the application of model atmospheres, that carbon stars are forced to yield some of their secrets; the results of the analysis are expected to constrain theoretical scenarios for the production and evolution of these stars. Much remains to be done at infrared and other wavelengths; determinations of the abundances of lithium, and the heavy elements, are in progress. Many interesting problems require samples of fainter stars and application of spectrophotometry and photometry. As our calibration of Gow's (1977) molecular indices demonstrates (Appendix B), our analyses will serve to calibrate abundance studies by these other techniques.

Over the duration of this project, many individuals and organizations have provided support. D. L. Lambert thanks the John Simon Guggenheim Foundation for a Fellowship in 1980–1981. We thank Dr. James F. Dominy for his contributions to the molecular line lists, Ulf Ekberg for valuable contributions to the work on model atmospheres and for discussions of the HCN problem, Göran Hammarbäck for
discussions of line formation problems, Uffe Graae-Jørgensen for HCN line data, Dr. Mats Larsson for conversations on $j$-values and dissociation energies, and Tyrone Vieira for extensive work at the plotter. We thank Dr. George Wallerstein, the referee, for pointing out an inconsistency in the initial version of the paper. The project was supported by grants from the National Science Foundation (grant AST 83-16635 and its predecessors), the Robert A. Welch Foundation, and the Swedish Natural Science Research Council. Most of the necessary computer time was granted by the University of Uppsala.

APPENDIX A

BASIC ATOMIC AND MOLECULAR DATA

In this appendix, we comment on the sources of the basic data for molecules employed in the abundance analysis: the oscillator strengths, the line positions, and the dissociation energy. The data for the CO and NH vibration-rotation transitions were reviewed by Lambert et al. (1984) and are not discussed again here.

A rotational line, being a transition between the rotational levels $(A'\nu''\Sigma''J'p'')$ and $(A'\nu'S'J'p')$, has an absorption oscillator strength

$$f_{\text{line}} = \frac{(2 - \delta_{0,N''})}{(2 - \delta_{0,N+N''})} \frac{S_{J'J''}}{(2J''+1)} \frac{\sigma_{\nu''}}{\sigma_{\nu'}} R_{e}^{2},$$

where the band oscillator strength is

$$f_{\nu''} = \frac{8\pi^{2}mc\alpha_{\nu''}}{3\hbar} \frac{2 - \delta_{0,N+N''}}{2 - \delta_{0,N'}} \frac{q_{\nu''}}{q_{\nu'}} R_{e}^{2}.$$

The rotational line strength $S_{J'J''}$ is subject to a sum rule

$$\sum_{p'p''} \sum_{J'} S_{J'J''} = (2 - \delta_{0,N+N''})(2S+1)(2J''+1).$$

In the following text, $\Sigma S_{J'J''}$ is understood to refer to the above triple summation. Note that with the adopted definition of a rotational line, a $\Lambda$-doublet consists of two rotational lines, and the sum rule includes both lambda substates $p$. Larsson (1983) provides clear and consistent definitions of the rotational line strength and related quantities.

When a sum $\sum |R_{e}|^{2} = (2 - \delta_{0,N+N''})(2S+1)R_{e}^{2}$ is quoted,

$$f_{\nu''} = \frac{8\pi^{2}mc\alpha_{\nu''}}{3\hbar} \frac{q_{\nu''}}{q_{\nu'}} \frac{\Sigma R_{e}^{2}}{(2 - \delta_{0,N''})(2S+1)}.$$

Of course, the electronic transition moment $R_{e}$ is a function of internuclear separation; i.e., $R_{e}^{2} = R_{e}^{2}(r_{\nu'',\nu''})$. In the above expressions, $R_{e}$ is in atomic units.

I. THE C$_{2}$ MOLECULE

a) The Phillips System

i) Line Frequencies

Chauville, Maillard, and Mantz (1977, hereafter CMM) recorded and analyzed nine bands of the Phillips ($A\,^{1}\Pi_{u} - X\,^{1}\Sigma_{g}^{+}$) system, including the 0–2 band, which provides 80% of the lines on our list. Molecular constants derived by CMM were used to predict lines in the 0–2 and 1–3 bands; the 2–4 band was not considered because its lines are masked by strong CO lines. Positions for the lines of the isotopically substituted molecules $^{13}$C$_{2}$C and $^{13}$C$_{2}$ were predicted. A check is now provided by laboratory measurements of the 0–0 and 1–0 bands of $^{13}$C$_{2}$C and $^{13}$C$_{2}$ (Amiot and Verges 1983). Intercomparison of stellar spectra showing weak and strong $^{13}$C$_{2}$ lines and high and low $^{13}$C abundances also provides a useful check on the identifications of the Phillips system lines.

Our adopted line list included all lines which are unblended or only slightly blended in many stars. Many additional lines are clearly present.

ii) Oscillator Strengths

Experimental work on the oscillator strengths of Phillips system lines includes an analysis of solar absorption lines (Brault et al. 1982), analysis of emission lines from C$_{2}$ excited in a shock tube (Cooper and Nicholls 1975, 1976), measurements of radiative
lifetimes of vibrational levels in the $A^1\Pi_u$ state (Erman et al. 1982; Bauer et al. 1985, 1986), and a determination of the relative $f$-value of the Phillips and Swan systems using an absorption tube and high-resolution spectrometer (Davis et al. 1984). Other older experimental results are referenced by Erman et al. (1982).

The most accurate measurement of Phillips system oscillator strengths is surely the recent comparison of Phillips and Swan systems by Davis et al. (1984). A high-temperature furnace was used to provide $C_2$ absorption lines which were observed with a Fourier transform spectrometer. High spectral resolution and high S/N ratio are key features of the spectra. Ratios $f_{00}(\text{Swan})/f_{10}(\text{Phillips}) = 14.8 \pm 0.3$ and $f_{00}(\text{Swan})/f_{00}(\text{Phillips}) = 18.2 \pm 0.7$ were obtained. With $f_{00}(\text{Swan}) = 0.0239$ (Davis et al. adopt $f_{00} = 0.0251$), we obtain $f_{00}(\text{Phillips}) = (1.31 \pm 0.13) \times 10^{-3}$.

Brault et al. (1982) analyzed low-noise solar spectra and obtained an oscillator strength $f_{00} = (1.41 \pm 0.09 \times 10^{-3}$, where the uncertainty reflects only the scatter in the determinations from the individual lines and not the systematic errors associated with the adopted carbon abundance, the $C_2$ dissociation energy, and the model solar atmosphere. Fortunately, such systematic errors may be eliminated almost entirely by deriving relative oscillator strengths for the Phillips and Swan systems from solar lines and adopting the accurate experimental result $f_{00} = 0.0239$ for the Swan system (see Lambert 1978). By this method, Danks and Lambert (1983) obtained $f_{00} = (1.5 \pm 0.3) \times 10^{-3}$ for the Phillips system, which is in good agreement with Davis et al.'s recent result. A higher result, $f_{00} = (3.9 \pm 0.8) \times 10^{-3}$, from the shock-tube experiment is in marked disagreement with these results; Cooper and Nicholl's assumption of a constant transition moment is not a major source of error.

The two determinations of the radiative lifetimes are in fair agreement (Fig. 43). If we convert $\tau_v$ to $f_v/\tau_v$, using the predicted variation of $\Sigma[R^2_v]$ with internuclear separation (see below), we obtain $f_{00} = (1.5 \pm 0.1) \times 10^{-3}$ using the mean ratio between the measured and theoretical lifetimes (van Dishoeck 1983) for $v' = 1-4$ to obtain the scaling constant. A constant transition moment and $\tau_0 = 16.5$ $\mu$s as provided by smoothly extrapolating $\tau_v$ to $v' = 0$ gives $f_{00} = 1.7 \times 10^{-3}$. Fortunately, the lifetime measurements are quite consistent with Davis et al.'s accurate measurements of $f_{00}$ and $f_{10}$.

Ab initio calculations of the electronic transition moment of the Phillips system using the same theoretical approach (multireference double excitation with configuration interaction [MRD CI]), but differing atomic orbital basis sets, give results which are not consistent with the experimental consensus (see van Dishoeck 1983 and Chabalowski, Peyerimhoff, and Buenker 1983; Pouilly et al. 1983 obtain similar results from a less extensive calculation). In Figure 43 we compare predicted and observed radiative lifetimes. The theoretical value $f_{00} = 2.7 \times 10^{-3}$ (van Dishoeck 1983) falls between the value $f_{00} = 1.3 \times 10^{-3}$ (Davis et al. 1984) and the shock-tube result $f_{00} = 3.9 \times 10^{-3}$. The discrepancy between the ab initio predictions and the consensus of the recent measurements is disturbing. The good agreement between the two ab initio calculations is not to be taken as indicative of
their external precision because they use the same methods. It is to be hoped that other ab initio methods will be applied to the Phillips system.

Oscillator strengths for the infrared Δν = −2 bands based on \( f_{00} = 1.3 \times 10^{-3} \) and a constant \( \Sigma |R_j|^2 \) are \( f_{02} = 3.6 \times 10^{-4} \) and \( f_{13} = 6.0 \times 10^{-4} \). If the \( \Sigma |R_j|^2 \) computed by van Dishoeck is scaled to fit the above \( f_{00} \), we obtain \( f_{02} = 2.1 \times 10^{-4} \) and \( f_{13} = 3.0 \times 10^{-4} \). The two pairs differ because the ab initio \( \Sigma |R_j|^2 \) decreases with increasing internuclear separation. Since the true and the current ab initio \( \Sigma |R_j|^2 \) are unlikely to differ by a simple scaling factor, we shall adopt a compromise,

\[
f_{02} = 2.5 \times 10^{-4}, \quad f_{13} = 3.8 \times 10^{-4}.
\]

In the detailed discussion of their lifetime data, which became available at the conclusion of our analysis, Bauer et al. (1986) obtain \( f_{02} = 2.9 \times 10^{-3} \) for a trial \( \Sigma R_j^2 \) function providing a good fit to the lifetimes. An alternative function providing an almost equally good fit to the lifetimes gives \( f_{02} = 1.8 \times 10^{-3} \). Our adopted \( f_{02} \) value lies between these predictions.

The \( g_f \)-values for individual rotational lines are computed from

\[
g_f = f^j_{j''} \frac{S_{jj''}{a_{jj''}}}{2 \sigma_{jj''}},
\]

where the Hönl-London factors are \( S_p = 2J'' + 1 \), \( S_p = J'' - 1 \), and \( S_R = J'' + 2 \). We ignore the small rotational dependence of the Franck-Condon factors \( q_{jj''} \) (Dwivedi et al. 1978).

### b) The Ballik-Ramsay System

#### i) Line Frequencies

The 0–0 bandhead of the Ballik-Ramsay \( (b^3 \Sigma_g^{-} – a^3 \Pi_u) \) system at 5656 cm\(^{-1}\) is a pronounced feature in the spectra of many carbon stars, but because it is often very strong and is located at the edge of an atmospheric window, we rejected this region as a reliable indicator of the carbon abundance. As an alternative, we searched at \( \sigma \leq 5050 \) cm\(^{-1}\) for high rotational lines of the 0–0 and 1–1 bands of \( ^{13} \)C\(_2\). Our initial search was made by extrapolation from the published line lists (Ballik and Ramsay 1963; Amiot, Chauville, and Maillard 1979). Later, a list of predicted lines based upon published molecular constants became available (Amiot 1982). Although the high density of CN and other lines ensures that few C\(_2\) lines are unblended, sufficient lines were detected to guarantee that the extrapolation could be made to \( J'' \sim 90 \). Comparison of the spectra of C\(_2\)-strong and C\(_2\)-weak stars as well as the triplet structure of the bands were essential factors in establishing firm identifications.

#### ii) Oscillator Strengths

Experimental results for the oscillator strengths of the Ballik-Ramsay system are sparse. In their shock-tube study of several C\(_2\) band systems, Cooper and Nicholls (1975, 1976) obtained \( \Sigma |R_j|^2 = 0.65 \pm 0.15 \) a.u. for the \( \Delta \nu = 2 \) sequence at 11,970 Å corresponding to \( 0.6 \times 10^{-3} \leq f_{00} \leq 1.2 \times 10^{-3} \) or \( 0.33 \leq \Sigma |R_j|^2 \leq 0.66 \) a.u. at \( r = 2.54 \) a.u. An ab initio MRD CI calculation (Chabalowski, Peyerimhoff, and Buenker 1983) gives \( \Sigma |R_j|^2 \) at the upper limit of the above two experimental results (see Fig. 44). On the other hand, a SCF CI calculation (Cooper 1981) predicts \( \Sigma |R_j|^2 \) to run near the lower limit of these results. We adopt a
mean value of $\Sigma |R_j|^2 = 0.54$ a.u. (with $\pm 0.15$ a.u. as a likely uncertainty) at $r=2.54$ a.u., which translates to $f_{\infty} = 9.8 \times 10^{-4}$ (and $f_{11} = 2.7 \times 10^{-6}$), where we use the Franck-Condon factors computed by Krishna Swamy and O'Dell (1977).7

Since our selected lines come from high rotational levels, we investigated the rotational dependence of the Franck-Condon factors using wave functions computed for RKR potentials and the variation of $\Sigma |R_j|^2$ predicted by Cooper. We find at most a 10% reduction in the lines’ oscillator strength. Finally, the rotational line strengths at high $J$ approach closely the Hund’s case (b) expressions given by Schadee (1964). The sum rule is $\Sigma S_{2j''} = 6(2J+1)$, i.e., $S(Q_j) \sim S(Q_j) \sim 2J$.

c) Dissociation Energy of $C_2$

Among the spectroscopic methods providing the dissociation energy of $C_2$, the rotational predissociation of the $e \, 1 \Pi_g$ state appears to yield the most reliable result. Messerle and Krauss (1967a) defined the limiting curve for levels $v' = 0$ through $v' = 8$, and the extrapolated energy was less than 300 cm$^{-1}$ below the $v' = 8$ point. The dissociation products are believed to be $C(\Pi) + C(\Sigma)$ and, hence, the limiting energy for the $1 \Pi_g$ is readily converted to $D_0^0(C_2)$. The result is $D_0^0 = 6.11 \pm 0.04$ eV.

Extrapolations of the vibrational levels of $d \, 3 \Pi_g$ (the upper state of the Swan system) and $e \, 3 \Pi_g$ (the upper state of the Fox-Herzberg system) led Messerle and Krauss (1967b) to obtain $D_0^0 = 6.07$ and 6.09 eV, respectively. However, these results are probably uncertain because the extrapolations from the last observed level to the limit are large and the extrapolation formula is untested (Herzberg, Lagerqvist, and Malmberg 1969).

In their compendium of molecular constants Haber and Herzberg (1979) base their choice of $D_0^0 = 6.21$ eV on two experiments in thermochemistry; it is unclear why these authors eschew the spectroscopic determination based on the $e \, 1 \Pi_g$ state. Brewer, Hicks, and Krikorian (1962) measured the temperature dependence of the intensity of $C_2$ bands produced by a graphite furnace. Their final preferred result for the $C_2$ heat of formation ($\Delta H_0^0 = 195 \pm 5$ Kcal mole$^{-1}$), which was an average of their new results and published values by other authors, leads to $D_0^0 = 6.25 \pm 0.2$ eV. A later experiment by Kordis and Gingerich (1973) provided $D_0^0 = 6.17 \pm 0.11$ eV.

Our abundance analysis assumed $D_0^0 = 6.11$ eV, the spectroscopic value, but the implications of the adoption of $D_0^0 = 6.21$ eV are assessed.

II. THE CN MOLECULE

a) The Red System

i) Line Frequencies

The red ($A \, 2 \Pi - X \, 2 \Sigma^+$) system is perhaps the leading contributor of lines to a spectrum of a carbon star. Lines of a strength suitable for inclusion in an abundance analysis are found in the 2 $\mu$m $\Delta v = -2$ bands (and the $\Delta v = -3$ bands at 3–4 $\mu$m). Cerny et al. (1978) measured the positions of many lines up to a rotational quantum number $J'' \sim 60.5$ in the bands 0–2, 1–3, and 2–4. The 4–6 and higher series members provide detectable lines, but are of little use in an abundance analysis because they overlap the CO 2.3 $\mu$m first-overtone bands. A complete set of line positions for 0–2 through 4–6 and including the satellite lines was generated from the molecular constants (Kotlar, Field, and Steinfield 1980) with a program (Kotlar 1978) kindly provided by Field (1980). Line positions of $^{13}$C$^{14}$N lines were also predicted. Unpublished spectra of CN excited in an electrodeless discharge (Brault 1980) provided line frequencies for $^{12}$C$^{14}$N and $^{13}$C$^{14}$N lines to $J'' \sim 40.5$ in all bands of interest. Small corrections to the predicted positions were established from a comparison of predicted and laboratory line positions. For several bands, examination of the stellar spectra provided high-$J''$ lines and corrections to the predicted frequencies. Comparison of spectra for $^{13}$C-poor and $^{13}$C-rich stars was an essential step in the identification of $^{13}$C$^{14}$N lines.

ii) Oscillator Strengths

Thanks to recent experimental and theoretical work, reliable band oscillator strengths are now available for the red system. Laboratory determinations of the radiative lifetimes for vibrational levels in the $A \, 2 \Pi$ state are summarized in Figure 45. Measurements by Đurić, Erman, and Larsson (1978) and Katayama, Miller, and Bondybey (1979) are discussed by Sneden and Lambert (1982). Payne’s (1983, cited in Taherian and Slanger 1984) laser-induced fluorescence experiment gives $\tau_v = 7.2 \pm 1.0$ ms, in good agreement with the same group’s earlier result of $\tau_v = 7.3 \pm 1.5$ ms (Conley et al. 1980). These measurements, with Nishi, Shinohara, and Nahazaki’s (1982) time-of-flight experiment yielding $\tau_v = 7.1 \pm 0.6$ ms, show that the lifetime of the low vibrational levels is considerably longer than the $\tau_v = 4$ ms reported by Đurić, Erman, and Larsson (1978). Taherian and Slanger (1984) report radiative lifetimes for $\nu' = 0$ to $\nu' = 5$ obtained by observing excited CN radicals following laser-induced photodissociation of $C_2N_2$. These three recent experiments are quite consistent. Analysis of solar CN lines gives a set of $f_{\nu'\nu''}$ values from which $\tau_v$.

---

7 The $\Sigma |R_j|^2$ predictions for the Swan system are $\Sigma |R_j|^2 = 4.10$ a.u. (Cooper 1981) and 4.65 a.u. (Chabalowski, Peyerimhoff, and Buenker 1983) at the internuclear separation corresponding to the $r$-centroid for the 0–0 band. These results compare with $\Sigma |R_j|^2 = 3.54$ a.u. from the accurate measurements of the radiative lifetime (see Lambert 1978). Unfortunately, such a comparison is not a reliable guide to a scaling factor to be applied to the predictions for the Balilik-Ramsay system and, hence, we adopt the straight mean.
may be deduced. The absolute scale of these \( \tau_v \) estimates is rather uncertain for obvious reasons, and, hence, in Figure 45 we rescale published results (Sneden and Lambert 1982) to fit the laboratory results for high vibrational levels; the scaling factor is within the uncertainties contributed by the C and N abundances, \( D_0 \) (CN) and the model solar atmosphere. We note that the solar-based \( \tau_v \) values confirm that the lifetime lengthens for the low vibrational levels.

Results of two ab initio calculations are shown in Figure 45. The CASSCF predictions (Larsson, Siegbahn, and Ågren 1983) provide an excellent fit to the experimental and solar results and may indeed be of superior accuracy. The POL CI predictions (Cartwright and Hay 1982), which give systematically longer values of \( \tau_v \), are most probably less accurate than the CASSCF results (see Larsson, Siegbahn, and Ågren 1983).

In the light of the experimental and theoretical evidence represented in Figure 45, we adopt the CASSCF band oscillator strengths:

\[
\begin{align*}
\nu_1 &= 3.60 \times 10^{-4}, \\
\nu_2 &= 8.26 \times 10^{-4}, \\
\nu_3 &= 8.80 \times 10^{-4}.
\end{align*}
\]

The accuracy may be of the order of ±10%.

Hönl-London factors are normalized to \( \Sigma S_{J,J''} = 4(2J + 1) \), i.e., \( S(Q_1) - S(Q_2) \sim 2J \). The \( gf \)-value of a line is then

\[
\frac{S_{J,J''}}{2a_{J,J''}} \approx \frac{f_{\text{line}}}{f_{v,v''}} 
\]

where \( g = (2J'' + 1) \) for the absorption \( f_{\text{line}} \) value. A small correction, which was applied to account for the change in \( f_{v,v''} \) with rotational quantum number, was taken from Dwivedi et al (1978), who assumed that \( \Sigma |R_e|^2 \) is independent of internuclear separation. Use of the CASSCF \( \Sigma |R_e|^2 \) is unlikely to result in a significant revision.

### iii) Dissociation Energy of CN

The dissociation energy of CN continues to be a major source of uncertainty for an N abundance derived from stellar CN lines. Reviewers consistently note the large spread in the \( D_0 \) determinations (7.5 ≤ \( D_0 \) ≤ 7.9 eV), which for a constant C abundance translates for a carbon star to an N abundance uncertainty of a factor of about 30! For example, Stull and Prophet (1971) adopt a median value \( D_0 = 7.76 \pm 0.10 \) eV and remark that "considerable scatter still exists and several sources of error are probable."

Perhaps the most direct and reliable measurement is by Engleman and Rouse (1975), who measured the absorption lines of the violet system produced by dissociating cyanogen (C<sub>2</sub>N<sub>2</sub>) gas in a furnace. The measured equivalent widths with the accurately known oscillator strength demanded \( D_0 = 7.66 \pm 0.05 \) eV. Treffers (1975) performed a similar experiment on CN red system emission lines, to obtain \( f_{\text{line}} = (3.3 \pm 0.7) \times 10^{-3} \) for an assumed \( D_0 = 7.66 \) eV. Since his \( f_{\text{line}} \) is identical with the CASSCF value, \( D_0 \) must be close to 7.66 eV. However, Colket (1984) adds to the confusion by providing \( D_0 = 7.93 \pm 0.07 \) eV from a similar
experiment in which the CN molecules are produced in a shock tube. The solar red system lines with C and N abundances from other atomic and molecular indicators suggest $D_0^0 \sim 7.55 \pm 0.05 \text{ eV}$.

Larsson, Siegbahn, and Ågren (1983) investigated whether their CASSCF calculations could provide an accurate prediction of $D_0^0$. Their best calculation gave $D_0^0 = 7.18 \text{ eV}$, but when a scaling factor is derived from a comparison of analogous calculations and experiment for the $N_2^+$ ground state, the $D_0^0$ is increased to $D_0^0 = 7.52 \text{ eV}$. However, on recognizing that there is no strict justification for the scaling factor, Larsson et al. conclude that “one can probably safely state that $7.4 < D_0^0(\text{CN}) < 7.7 \text{ eV}.” In essence, the ab initio calculations cannot yet be used to resolve the conflict between the experimental results.

We adopt $D_0^0 = 7.60 \text{ eV}$ and explore the effects of uncertainty on the N abundance estimates.

### III. The H$_2$ Molecule

The three initial members of the S-branch of the 1–0 fundamental electric quadrupole vibration-rotation transition within the $X^1{^2}\Sigma^+_g$ state fall within the interval that, although rich in CN lines, is a potential source of weak atomic and molecular lines. The $S(0)$ line at 4497.84 cm$^{-1}$ is present and relatively unblended on our spectra. The $S(1)$ line at 4712.90 cm$^{-1}$, which is present and measurable through spectrum synthesis (Goorvitch, Goebel, and Augason 1980; Johnson et al. 1983) is not considered further here. The $S(2)$ line, the last S-branch member before the strong telluric bands commence at 5000 cm$^{-1}$, is too blended to warrant analysis.

Accurate ab initio calculations (Turner, Kirby-Docken, and Dalgarano 1977) provide a transition probability $A = 2.53 \times 10^{-7}$ s$^{-1}$ and an absorption oscillator strength $f = 9.37 \times 10^{-14}$ for the $S(0)$ transition. Laboratory measurements of line strengths for 1–0 H$_2$ lines confirm the ab initio calculations. Level population in H$_2$ in LTE reflect the nuclear spin statistical weights: $g_f = I(I+1)$ for even $J$-levels and $g_f = (I+1)(2I+1)$ for odd $J$-levels. These weights are customarily incorporated into the rotational partition function ($Q_{rot}$) by a symmetry factor $\sigma$, where $Q_{rot} = \sigma (kT/hcB)$ and $\sigma = \frac{1}{2}$ for a homonuclear diatomic molecule such as H$_2$, and $T/kB$ is the standard approximation for the rotational contribution to the partition function. Then the appropriate nuclear spin statistical weight to be combined with the absorption $f$-value is $g_f = I(I+1)/(2I+1)$ for $J$-levels and $g_f = \frac{1}{2}$ for the odd $J$-levels. For the $S(0)$ line with $J'' = 0$, one obtains $gf = 2.34 \times 10^{-14}$ with an accuracy better than 5%, which exceeds the accuracy of the stellar equivalent width measurements.

### IV. The CH Molecule

Lines from the fundamental 1–0 and 2–1 vibration-rotation bands of the $^2\Pi$ ground state of CH are present in 4 $\mu$m spectra of carbon stars. Frequencies of a precision adequate for identification of the stellar CH lines are obtainable from term values compiled by Botterud, Lofthus, and Veseth (1973). Accurate frequencies for many low 1–0 transitions were measured by Lubic and Amano (1984).

Oscillator strengths were derived from the electric dipole moment function (EDMF) computed in an ab initio calculation by Lie, Hinze, and Liu (1973; see their Table 2 and the “extended Cl” results). This EDMF and wave functions computed for the RKR potential for the $^2\Pi$ ground state provided the matrix elements $\langle \nu''J''|\mu(r)|\nu'J'\rangle$, where $\mu(r)$ is the EDMF. Such matrix elements include the “vibration-rotation interaction.”

The $gf$-value for a CH line is

$$gf = \frac{8\pi^2 m_\sigma}{3\hbar^2} S_{J'J''} |\langle \nu'J'|\mu(r)|\nu''J''\rangle|^2$$

$$= 4.70 \times 10^{-7} a_{J''} S_{J'J''} |\langle \nu'J'|\mu(r)|\nu''J''\rangle|^2,$$

where $a_{J''}$ is the line frequency (in cm$^{-1}$), the EDMF is given in Debyes, and $S_{J'J''}$ is the Hönl-London factor normalized as above, i.e., $S - J$ for each line comprising a $P$ (or $R$) branch quartet ($P_{le}, P_{lf}, R_{le}, R_{lf}$).

The extended CI EDMF is in rather good agreement with results provided by Meyer and Rosmus (1975) from the coupled electron pair approximation (CEPA). Matrix elements were not recomputed, but examination of the gradients $\frac{d\mu(r)}{dr}$ at the equilibrium separation ($r_e$) shows that the $gf$-values from the better CEPA calculation (CEPA-1; see Werner, Rosmus, and Reinsch 1983) are within 15% of the extended results. This comparison suggests that the accepted $gf$-values for the CH 1–0 lines have an accuracy of $\pm 20\%$.

### V. Atomic Lines

A search of the 2 $\mu$m region for useful atomic lines provided a list of about 20 candidates. This list was cut dramatically when the following requirements were imposed: the line should not be seriously contaminated in the carbon stars, the stellar line should not be stronger than $log(W_\nu/\alpha) = -4.8$, and the line should be measurable either in the solar spectrum (Hall 1972) or a high-resolution archival spectrum of the K5 III star α Tau, so that a determination of the $gf$-value is possible. The final list appears in Table 8.

In determining the $gf$-values, we required that the observed values of $W_\nu$ be reproduced in calculations with the Holweger-Müller (1974) solar model and a micromturbulence of 1 km s$^{-1}$, an appropriate value for the center of the solar disk (Blackwell and Shallis...
APPENDIX B

SPECTROSCOPY AND SPECTROPHOTOMETRY OF CARBON STARS

Spectrophotometry at low resolution is a tool for the rapid measurement of prominent molecular (and atomic) features in large samples of carbon stars. The measurements may be calibrated against either a sample analyzed at high dispersion or a set of low-resolution synthetic spectra. Gow (1977) published spectrophotometric indices for $^{12}$CN, $^{13}$CN, and $^{13}$CN obtained from scans covering the interval 5000–7000 \( \AA \) at a resolution of 20 \( \AA \). His sample of 61 cool carbon stars includes 29 of the 30 stars analyzed by us.

The measurements of the $^{12}$C bands are well correlated (Fig. 46): $^{12}$C$_{1}$ (Gow) \( \propto \) 0.5 $^{12}$W$_{a}$($^{12}$C$_{1}$), where Gow measured the depth of the Swan $\Delta v = -1$ bands at 5635 \( \AA \) and our lines were drawn from the Phillips system. In Figure 46 and the two subsequent figures, we identify the four $^{13}$C-rich stars (Y CVn, RY Dra, T Lyr, and WZ Cas) and the Mira variable R Lep. The $^{13}$C-rich star WZ Cas appears to deviate from the tight relation. Its position may reflect the facts that it has week C$_{1}$ lines and the relation between molecular column density and line/band intensity is nonlinear. The $^{13}$C-rich and C$_{2}$-strong stars RY Dra and T Lyr (also VX And) fit the mean relation, but Y CVn does not. As Gow’s index monitors $^{13}$C$_{2}$, but includes contributions from $^{12}$C$^{13}$C and $^{13}$C$^{13}$C, we expect $^{13}$C-rich stars to lie above the mean relation. For RY Dra and T Lyr, the $^{13}$C Swan lines may be so saturated that the isotopically substituted molecules contribute little to Gow’s C$_{2}$ index. The location of R Lep is plausibly attributed to variability of its spectrum.

A comparison (Fig. 47) of $^{12}$CN measurements shows an interesting result. Gow’s $^{12}$CN index for each of the four $^{13}$C-rich stars is significantly too small. The mean relation is well satisfied by the other stars (except R Lep): log $^{12}$CN(Gow) \( \propto \) log $^{12}$W$_{a}$($^{12}$CN). We suggest that one of the two “continuum” points at 5710 and 6780 \( \AA \) is depressed by $^{13}$CN lines. The molecular indices are the magnitude difference between the observed flux and the “continuum” flux of a blackbody curve fitted to the two “continuum” points. If either point is depressed by $^{13}$CN lines, the $^{13}$CN index will be reduced. (Also, the expected displacement in Figure 47 of Gow’s index monitors $^{13}$C$_{2}$, but includes contributions from $^{12}$C$^{13}$C and $^{13}$C$^{13}$C, we expect $^{13}$C-rich stars to lie above the mean relation. For RY Dra and T Lyr, the $^{13}$C Swan lines may be so saturated that the isotopically substituted molecules contribute little to Gow’s C$_{2}$ index. The location of R Lep is plausibly attributed to variability of its spectrum.)

Gow’s $^{13}$CN index provided by the flux at 5790 \( \AA \) is correlated with our mean $^{13}$W$_{a}$ (Fig. 48): log $^{13}$CN(Gow) is proportional to log $^{13}$W$_{a}$($^{13}$CN), with rather more scatter than is noted in Figure 47. As before, the $^{13}$C-rich stars, including VX And and T Lyr (also VX And) fit the mean relation, but Y CVn does not. As Gow’s index monitors $^{13}$C$_{2}$, but includes contributions from $^{12}$C$^{13}$C and $^{13}$C$^{13}$C, we expect $^{13}$C-rich stars to lie above the mean relation. For RY Dra and T Lyr, the $^{13}$C Swan lines may be so saturated that the isotopically substituted molecules contribute little to Gow’s C$_{2}$ index. The location of R Lep is plausibly attributed to variability of its spectrum.

In addition to these molecular indices, Gow derived spectral gradients and used a CO index (log CO) measured by Faÿ and Ridgway (1975) and a CN index (log CN) provided by Baumert (1972). Gow’s C/O ratios as calibrated against Kilston’s (1975) results do not correlate very well with ours: (C/O-1) = 0.18(C/O-1)$_{Gow}$ provides a rough conversion. However, Gow’s spectrophotometry is consistent and accurate enough to allow a calibration in terms of the abundances derived here. We find

\[
\begin{align*}
[\text{O}/\text{H}] & = 2.31 - 0.63(\log \text{CO}) - 0.46(\log ^{13}\text{CN}) + 0.64(\log ^{12}\text{CN}) - 1.09(\log \text{C}_2), \\
& \pm 0.63 \pm 0.24 \pm 0.24 \pm 0.31 \pm 0.32
\end{align*}
\]

\[
\begin{align*}
[\text{N}/\text{H}] & = -8.28 + 4.88(\log \text{CN}) - 1.05(\log ^{13}\text{CN}), \\
& \pm 1.28 \pm 0.73 \pm 0.23
\end{align*}
\]

\[
\begin{align*}
\log (\text{C}/\text{O}-1) & = -0.60 - 3.02(\log \text{CN}) + 1.58(\log ^{13}\text{CN}) + 1.62(\log \text{C}_2), \\
& \pm 1.87 \pm 0.97 \pm 0.53 \pm 0.57
\end{align*}
\]

\[
\begin{align*}
\log (^{13}\text{C}/^{12}\text{C}) & = -1.38 + 7.80(\log ^{13}\text{CN}) - 2.72(\log ^{13}\text{CN}) - 2.73(\log \text{C}_2), \\
& \pm 0.70 \pm 0.67 \pm 0.51 \pm 0.66
\end{align*}
\]

The resulting mean errors in the coefficients and intercepts are given in the line below each relation.

These relations reproduce our abundances with Gow’s photometry for our stars with mean errors of 0.11 in [O/H], 0.18 in [N/H], 0.21 in log (C/O-1), and 0.22 in log ($^{13}$C/$^{12}$C). Inclusion of more measures, such as Gow’s spectral gradients, do not improve the fits significantly. They clearly demonstrate that rather accurate CNO abundances may be derived from low-resolution data, provided that a set of stars with well-determined abundances are available for the calibration. The task of determining accurate $^{13}$C/$^{12}$C ratios from low-resolution scans is, however, difficult.
TABLE 8

<table>
<thead>
<tr>
<th>Element</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na I</td>
</tr>
<tr>
<td>Ca I</td>
</tr>
<tr>
<td>Fe I</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Line</th>
<th>(e) (cm(^{-1}))</th>
<th>(W_\alpha) (cm(^{-1}))</th>
<th>(\log g_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na I</td>
<td>4.35</td>
<td>4685(^b)</td>
<td>0.0099</td>
</tr>
<tr>
<td>Ca I</td>
<td>4.68</td>
<td>4413.1</td>
<td>0.014</td>
</tr>
<tr>
<td>Fe I</td>
<td>5.32</td>
<td>4466</td>
<td>0.0065</td>
</tr>
<tr>
<td></td>
<td>5.09</td>
<td>4491.1</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>5.06</td>
<td>4491.7</td>
<td>0.032</td>
</tr>
</tbody>
</table>

\(^a\)Assuming log \(\epsilon(Fe) = 7.50, \log \epsilon(Ca) = 6.34,\) and \(\log \epsilon(Na) = 6.32.\)

\(^b\)Uncertain identification.

Fig. 46.—\(^{12}\)C\(_2\) photometric index (Gow 1977) and the value of \(\langle W_\alpha \rangle\) for \(^{12}\)C\(_2\). The solid line is an approximate fit to the points. See text for comment on the identified \(^{12}\)C-rich stars and the Mira \(R\) Lep.

Fig. 47.—The \(^{13}\)C\(_2\) photometric index (Gow 1977) and the value of \(\langle W_\alpha \rangle\) for \(^{13}\)C\(_2\). See legend for Fig. 46 and the text for further comment.
Fig. 48.—The $^{13}$CN photometric index (Gow 1977) and the value of $\langle W_a \rangle$ for $^{13}$CN. See legend for Fig. 46, and the text for further comment.

REFERENCES
