ON WIND-TYPE FLOWS IN ASTROPHYSICAL JETS. II. PROPAGATION OUTSIDE THE NUCLEUS AND THE CASE OF M87

A. Ferrari and E. Trussoni
Istituto di Fisica Generale, Università di Torino; and
Istituto di Cosmo-geofisica del Consiglio Nazionale delle Ricerche, Torino, Italy

R. Rosner
Harvard-Smithsonian Center for Astrophysics

AND

K. Tsinganos
Harvard-Smithsonian Center for Astrophysics; and Department of Physics, University of Crete, Heraklion, Greece

Received 1985 February 22; accepted 1985 July 18

ABSTRACT

We present solutions of the basic polytropic flow equations appropriate to the physical conditions of extragalactic jets well outside the acceleration region within their parent active galactic nuclei. The solutions are based on observationally determined galactic mass distributions and quasi-periodic brightness enhancements along the jets. We demonstrate that such a model is able to connect the observed morphology of jets to processes (including shocks or density condensations) which can give rise to the brightness enhancements, as well as to provide estimates for the jet flow speeds which are consistent with values implied by observations.

Subject headings: galaxies: individual — galaxies: jets — galaxies: nuclei

1. INTRODUCTION

Detailed morphology of extragalactic radio jets has become available from the high-resolution maps of the VLA and VLBI networks. Preliminary classifications have been attempted by Bridle and Perley (1984), and the following general features have been identified:

1. collimated outflows (jets) are often associated with AGN;
2. bright jets tend to be straight, retaining their directionality from the core to extended lobes for $10^4$ kpc;
3. opening angles are small, generally less than 10°;
4. quasi-periodic small amplitude spatial oscillations in the jet brightness distribution (knots) and the axis (wiggles) are found in most of those jets for which fine structures can be resolved; and
5. these fine structures are also seen in spectral and polarimetric measurements and, if available, at higher frequencies (i.e., optical and above).

All these characteristics are exemplified by the well-studied jet of M87 in Virgo, and some of them had been recognized since the original optical observations of Curtis (1918). We now have available very detailed maps, from which we can also measure the relative brightness and size of knots. Thus, this jet extends for about 1.5 kpc, has a projected opening angle on the plane of the sky of about 5°, and displays a sequence of six well-defined knots, three of which (D-E-F) are relatively weaker and closer to the galactic core, and three (A-B-C) are stronger, more extended and farther away (for more details see Nieto and Lelievre 1982, Biretta, Owens, and Hardee 1983, and Lelievre et al. 1984). The distance between knots varies between 120 and 200 pc. The change in regime between the first three and the last three knots is marked by a sharp feature in knot A, which has been interpreted as a strong shock (see Biretta, Owen, and Hardee 1983). It has also been argued that the constant opening angle from the core to knot A indicates that in this phase the jet has constant Mach number (the actual value of $M$ is very uncertain: $M \approx 25$ for free expansion, and $M \approx 3$ for the case of external confinement). In addition, some authors have suggested that the velocity itself and the sound speed may be constant as well, because the emissivity along the jet (apart from the emission enhancements at knots) is also quite uniform (Hardee 1984); although this emissivity refers to the nonthermal (relativistic) component of the flow plasma, equipartition arguments are used to predict that the thermal component is as uniform as is the relativistic component. If the equipartition assumption is accepted, then this suggests that the flow is isothermal in its initial propagation phase. In the following, the assumption that the flow is approximately isothermal will be used as a simplifying tool; however, our calculations have been carried out for the more general polytropic case.

There are many other jets that exhibit properties similar to those of M87. Two such representative examples are the much longer "pencil" jet ($l \approx 160$ kpc) in NGC 6251, and the far more distant ($d \approx 500$ Mpc) jet associated with 3C 273. The NGC 6251 jet has been recently studied in detail (Perley, Bridle, and Willis 1984): it is very straight, has an opening angle on the plane of the sky $\theta \approx 7°$, and shows regularly spaced knots and wiggles, but there is no definite evidence for a shock separating two regimes. The jet from the QSO 3C 263 also shows a knotty and wiggling structure, and has been recently studied in detail, in particular for its similarities with M87 (see Lelievre et al. 1984). Periodic knots have been detected and associated with shocklike structures for the jets associated with Cyg A (Perley, Dreher, and Cohen 1984), 3C 111 (Linsfield and Perley 1984) and the QSO 0800 + 608 (Shone and Browne 1985).

The models proposed so far to explain the formation of knots are of two types. Rees (1978) and Blandford and Königl (1980) have discussed how irregularities in the flow rate and inhomogeneities (clouds) in the plasma density, respectively, can give rise to internal shocks; at these shocks, particle acceleration and heating produce the observed brightness...
enhancement. It should be noticed that in this framework knots are transient structures, which are convected along the flow. However, this model does not provide any obvious explanation for the regularity of knot spacing.

Ferrari, Trussoni, and Zaninetti (1981, 1983) have instead proposed that the development of long-wavelength Kelvin-Helmholtz modes create localized, regularly spaced, compression regions which are convected along the jet direction, but at velocities below that of the jet material itself, and at which particle acceleration by interaction with short-wavelength MHD modes occurs. The periodicity of spatial oscillations is a natural outcome of the model, as this is linked to the wavelength of the modes with the largest growth rate: this wavelength can vary from a few up to several times the beam radius, depending on the physical parameters of the jets, namely density, intensity of the magnetic field, and presence of a smooth transition of velocity between the inner jet and the external medium (Ferrari, Trussoni, and Zaninetti 1983). In this paper we shall adopt a similar point of view, namely that the linearly unstable modes do grow to form finite amplitude oscillations; we shall then study the latter’s effect on the propagation of the jet well outside the nucleus. This study is carried out within the computational framework of a wind-type model which we recently proposed for the acceleration of supersonic jets inside the funnels of accretion disks orbiting massive black holes in the center of AGN (Ferrari 1984; Ferrari et al. 1985, hereafter Paper I). In particular, we have shown that a jet with a temperature $T \approx 10^8$ K can exit from a $m_0 \approx 10^4 M_\odot$ potential well with a typical Mach number $M \approx 5$. The acceleration is a synergistic result of the combination of radiation pressure within the funnel’s walls and the geometrical shape of the accretion funnel walls.

In this paper we address the problem of the propagation of jets through the interstellar medium of the associated galaxy in order to show how the model proposed may account for the the observed morphology when gravitational and dynamical interactions of the flow with the galactic matter distribution are taken into account. We shall take M87 and its jet as the prototype for the application of our model (although it may be that its features are exceptional) and shall use the parameters quoted in Table 1 for the numerical tests.

Before we proceed with the presentation of our model it is essential to point out and understand the sharp distinctions between our study and the numerical simulations of Woodward (1983) and Smarr, Norman, and Winkler (1984) Our treatment is "one-and-half"-dimensional in the sense that the governing flow equations are obtained by integrating across the jet cross section and therefore cannot in principle be used to study boundary effects and oblique shocks; it is also steady, so that time-dependent effects are by assumption eliminated. It has, however, several key advantages: the treatment is largely analytical, so that it is much simpler to understand the complex dependence of the flow on the ambient physical conditions; to the extent the geometrical model applies, the actual physical conditions can be modeled in a hierarchy of straightforward extensions and approximations (e.g., inclusion of gravity, departures from isothermality, etc.). The stationary solutions can be used as initial conditions for systematic studies of the time-dependent flow (see Habbal and Tsinganos 1983; Tsinganos, Habbal, and Rosner 1983). But perhaps more important is the fact that even our relatively (geometrically) simple steady model shows very complex behavior; and therefore in the spirit of economy of explanation, we prefer to ask how simple a model can be constructed which is physically plausible and in agreement with observations.

We shall not discuss the flow acceleration inside the accretion funnel, but refer to Paper I for this stage. The propagation will be studied assuming a polytropic equation of state for the gas inside the jet and starting at a distance $z_0 \approx 100$ pc from the core; this is the closest distance to the center of the galaxy for which we have both resolved imaging of the jet and information on the mass distribution of the galaxy. In fact, resolved imaging at radio frequencies has been obtained by VLBI techniques for distances down to less than 0.1 pc from the core; however analysis of the mass distribution on such scales will have to wait for the Space Telescope era. At a distance of about 100 pc the typical physical conditions of the flow may have changed from those connected to the inner regions by a transition zone lying between $\sim 0.1$ pc and $\sim 100$ pc, where the jet adjusts itself to the variation of its environment from the conditions of the deep dense core to those in the interstellar medium. This phase of the jet propagation involves huge flow expansion, and therefore requires (at least) a two-dimensional treatment. Assuming that the mass of the galaxy at the distance $z_0 = 100$ pc is $m(z_0) \approx 5 \times 10^9 M_\odot$, the range of temperature providing (in a polytropic wind-type flow) a jet Mach number $M \lesssim 3$ at $z_0 \approx 100$ pc is $T^{(0)} \approx 5 \times 10^{10} - 10^7$ K. Therefore, we shall assume that our picture for M87 is valid for distances $z > z_0 = 100$ pc, with $M \gtrsim 1$ at $z = z_0$.

In the following section we shall discuss the general equations that govern the motion of a nonrelativistic polytropic gas in a prescribed gravitational well within a jet of assigned (variable) cross section. The applications to the jet in M87 will be discussed in § III, and the main astrophysical implications as well as the assumptions of our model will be discussed in § IV, where we also present preliminary applications of these ideas to other similar jets.

### II. THE EQUATION FOR THE MACH NUMBER

As discussed in Paper I, the conservation laws for mass and momentum fluxes in polytropic flows (with index $\gamma$) along a channel of variable cross section can be combined into a single equation for the Mach number $M = V/V_s$, where $V_s$ is the sound speed. In particular, if we measure the distance $z$ along the channel’s axis in units of $z_0$, where $z = z_0$ is the position of the “base” of the flow, i.e., we use the dimensionless coordinate $Z = z/z_0$, measure the cross sectional area $A(Z)$ in units of its value $A_0$ at $z = z_0$, assume that the mass of the galaxy within $Z$ increases according to the law $m(Z) = m_{gal} Z^n$ (where $m_{gal}$ is the total mass within distance $z_0$), and assume no external input...
of momentum, we obtain the equation
\[ \frac{M^2 - 1}{2M^2} \frac{dM^2}{dZ} = \left(1 + \frac{\alpha - 1}{2} \right) \frac{M^2}{2M^2} \left\{ \frac{1}{A(Z)} \frac{dA(Z)}{dZ} - \frac{\alpha + 1}{2} \right\} \frac{HZ_{e}^{-2}}{h[1 + (\alpha - 1)M^2/2]} , \] (1a)
where
\[ H = 8.14 \times 10^9 m_{\text{rel}}, \quad h = \left[ \frac{M_0}{[A(Z)M]} \right]^{\alpha}, \]
\[ g(\alpha) = \frac{2(\alpha - 1)}{\alpha + 1}, \quad m_{\text{rel}} = \frac{m_{\text{el}}}{10^3 M_0}, \]
\[ V_s = 128(\Gamma T_6^{(0)})^{1/2} \text{ km s}^{-1}, \quad T_6 = \frac{T}{10^6 \text{ K}} ; \] (1b)
and all quantities with subscript and superscript "0" (i.e., \(M_0, T_6^{(0)}\)) are measured at the base of the flow \(z = z_0\). The dimensional quantities in equations (1) are derived under the assumption of an electron-proton plasma; if the beam is instead an electron-positron plasma (see Rees 1981), the quantities \(V_s, H\) must be multiplied by the factors \((m_p/m_e)^{1/2}\) and \(m_e/m_p\), respectively. For the monoatomic gas (local) adiabatic index, we adopt \(\Gamma = 5/3\) since we are assuming nonrelativistic temperatures. This form of the Mach number equation is similar to that derived by Kopp and Holzer (1976) and generalized by Bailyn, Rosner, and Tsinganos (1985), while in the isothermal case it reduces to that investigated by Habbal and Tsinganos (1983) in the context of solar wind studies.

Equation (1) can be easily integrated (see Kopp and Holzer 1976); its solutions depend on the number and nature of the critical points where the flow becomes supersonic. For that reason, we begin by investigating the position and the nature of these critical points for the case of a spherically expanding flow; in the next subsection we shall analyze the effects of perturbed boundaries of the jet on its propagation.

\( \alpha \) Spherical Expansion

For a jet with an opening angle \(\theta\) and a transverse radius \(r_0\) at \(z_0\) the dimensionless cross sectional area \(A(Z)\) is (see Fig. 1):
\[ A(Z) = \left[ (Z - 1) + \frac{r_0}{\theta z_0} \right]^2 \left( \frac{\theta z_0}{r_0} \right)^2 \right] , \quad \theta \ll 1 . \] (2a)

It follows that
\[ \frac{1}{A(Z)} \frac{dA(Z)}{dZ} = \frac{2}{Z + (r_0/\theta z_0) - 1} . \] (2b)

If \(\theta\) is assumed essentially constant in the propagation phase over large scales, as the observations discussed above suggest, it is easily seen that equation (2b) reduces to the standard expression for purely radially symmetric expansion by a trivial change of variable; in other words, the flow described by equation (2a) does expand spherically, but the flow cone apex is placed at a distance \((r_0/\theta) - z_0\) below the center of symmetry of the mass distribution (Fig. 1). Because the opening angle of jets is very narrow, we shall also simplify the problem by assuming a radially symmetric expansion law for the jet, with \(\theta = \) constant and \(r_0/\theta = z_0\).

Thus, for \(A(Z) = Z^2\), we obtain the location of the critical points by setting the right-hand side of equation (1) equal to zero:
\[ \frac{2}{Z_c} \left[ \frac{M_0}{Z_c^2} \right]^{\alpha(\theta)} - HZ_{e}^{-2} = 0 , \] (3a)
i.e.,
\[ Z_c = F^{n_1 - n} , \] (3b)
where
\[ F = \frac{H}{2M_0^{\alpha(\theta)}}, \quad 0 \leq n_1 = \frac{5 - 3\alpha}{\alpha + 1} \leq 1 . \] (3c)

It is evident that the location of the critical points depends exponentially on the two indices: \(n_1\), which determines the deviation of the flow from the adiabatic case \(\alpha = 5/3\) (for which \(n_1 = 0\)), and \(n\), which determines the deviation of the mass distribution from the point-mass case (for which \(n = 0\)). Note that \(0 \leq n_1 \leq 1\) and \(0 \leq n\). If \(n = 0\), equation (3a) yields the well-known X-type Parker critical point (Parker 1963):
\[ Z_{e}^{-\alpha} = \left[ \frac{H}{2M_0^{\alpha(\theta)}} \right]^{1/n_1} . \] (4)

In addition, if the flow is isothermal (\(\alpha = 1\)), the dependence of \(Z_c\) on \(M_0\) drops out because \(g(\alpha) = 0\) and \(n_1 = 1\), so that \(Z_{e}^{-\alpha} = H/2\) (Ferrari et al. 1984). Writing equation (3b) in the form
\[ Z_c = \left[ Z_c^{-\alpha} \right]^{n_1/(n_1 - n)} , \] (5)
one can easily see that, when \(Z_{e}^{-\alpha} > 1\), we obtain an X-type
critical point at \( Z = Z_c^p > Z_c^p \) for \( n < n_1 \), a single critical point at infinity \( (Z_c \to \infty) \) for \( n = n_1 \), and an O-type critical point at \( Z = Z_c^o < Z_c^p \) for \( n > n_1 \). Similarly, when \( Z_c^p < 1 \) (the case we shall in fact consider in detail in our application), we obtain an X-type critical point at \( Z = Z_c^x < Z_c^o \) for \( n < n_1 \), a single critical point at \( Z = 0 \) for \( n = n_1 \), and an O-type critical point at \( Z = Z_c^o < Z_c^p \) for \( n > n_1 \). This behavior follows from the physical fact that a smaller \( n \) implies a more centrally concentrated mass distribution and a smaller total mass, and hence a more constricted effective Laval nozzle. These results are obtained by linearizing our equations near the critical points (where \( M = 1 \)), i.e., \( M \approx 1 + \mu \) and \( Z \approx Z_c^o (1 + \eta) \) with \( \mu \ll 1 \) and \( \eta \ll 1 \), and using L'Hôpital's rule in equation (1). Thus we obtain the behavior of the critical solution from the equation

\[
\left[ \frac{d\mu}{d\eta} \right]^2 + v = 0, \quad v = \frac{2(n - n_1)}{\alpha + 1}.
\]

The nature of the critical point depends on the sign of \( v \) or, equivalently, the relative magnitudes of \( n \) and \( n_1 \) (see Paper I). The novel aspect of equation (6) is precisely in that \( n \neq 0 \) (as it would be for a point mass distribution), and therefore a richer set of solutions can become available. For example, if \( n > n_1 \), then there is no continuous solution from \( z = z_0 \) to \( z = \infty \) with a flow transition between the subsonic and supersonic regimes; instead, solutions cross the line \( M = 1 \) either "upward" (if \( M_0 < 1 \), i.e., they are subsonic at the base) or "downward" (if \( M_0 > 1 \) with \( dM/dZ \to \infty \), and then loop backward (in \( Z \)) around the O-type critical point (see Fig. 2). The physical explanation of this behavior is simply that for such a gravitational potential well, polytropic flows with \( \alpha = (5 - n_1) / (3 + n_1) \) and \( n < n_1 \) do not have sufficient internal energy, nor receive sufficient external energy, to escape the potential well with supersonic speed at infinity. This point is further discussed in § IIIb below. In the following we shall call these mathematical solutions "trapped."

In the limiting case \( n = n_1 \) (e.g., \( v = 0 \)), for which no critical point exists in the interval \( z_0 < z < \infty \), the solution topologies are determined by the sign of the right-hand side of equation (1). If it is negative, the solutions monotonically decrease (if \( M_0 > 1 \)) or increase (if \( M_0 < 1 \)) everywhere towards the line \( M = 1 \). If the right-hand side of equation (1) is positive, the solutions instead monotonically increase (if \( M_0 > 1 \)) or decrease (if \( M_0 < 1 \)) as \( z \to \infty \). Finally, if the right-hand side of equation (1) is zero, the flow is everywhere constant. To conclude, we see that even though the position of the critical points depends on \( M_0 \), the general behavior of the critical solutions is independent of \( M_0 \), and is instead fixed by the values of \( n \) and \( n_1 \). The mass distribution thus critically determines the behavior of the transonic solutions; in particular, we have shown that for \( n > n_1 \) no supersonic flow reaching infinity exists. On the other hand, for \( n < n_1 \) the position of the

![Fig. 2.—Wind type solutions (\( n < n_1 \); solid line) and trapped solutions (\( n > n_1 \); dotted line); \( n \) is the exponent of the mass distribution law](https://example.com/fig2.png)
critical points depends on the values of all the physical parameters, i.e., both the temperature and mass distributions. In Figure 3 we have plotted the position of the critical points against the gas temperature in the case of isothermal flows and for different values of the mass distribution exponent $n$ close to $n_1$; it appears that, for any given $n \approx n_1$, a lower temperature limit exists below which the critical point is rapidly shifted to infinity, a situation corresponding again to that of the "trapped" solutions.

b) Perturbed Flow Geometry

A jet propagating through an interstellar or intergalactic medium is very likely to undergo Kelvin-Helmholtz instability (K-H) whether confinement is by external gas pressure or magnetic field (cf. Ferrari 1984). For example, "reflected" modes, which are stable in planar geometry, are destabilized in cylindrical geometry (Ferrari, Trussoni, and Zaninetti 1981) and only for ultrarelativistic velocities and/or very strong magnetic fields do these modes tend to be stabilized. The unstable perturbations that can affect the beam walls are typically "wiggling" or "pinching" modes: in the former case, we expect oscillations of the jet's propagation direction about its axis, while in the latter the beam cross section periodically expands and contracts. We shall focus on the latter class of perturbations and assume that the observed periodic interval between knots is related to the wavelength $\lambda$ of the most unstable pinching mode; we then write the perturbed area in the form:

$$A(Z) = \left[Z + \epsilon_0 Z^m \sin \left(2\pi z_0 Z/\lambda\right)\right]^2,$$

where $\epsilon_0$ is the amplitude of the radial oscillation at $z_0$ in units of $r_0$, and $\lambda$ is taken to be of the order of the observed mean separation of the knots. In writing equation (7), we assume that the "corrugations" of the jet walls are not themselves in motion; in fact, this is not correct, as the K-H modes propagate at their own group velocity, which usually differs only slightly from their phase velocity (Payne and Cohn 1985). In the supersonic regime and for "reflected" modes, the phase velocity of the fastest growing modes lies between the sound speed and half of the speed of the jet matter itself. In the following, we assume that the perturbed jet walls (which move at that phase velocity) define the rest frame.

The term $Z^m$ accounts for the variation of the perturbation amplitude which can vary along the jet. This ansatz should be regarded as a purely phenomenological step, because, although the amplitude growth is assumed to be independent of $z$ for jets initially uniform along their axis direction, Hardee (1982) has shown that, in the case of (conically) expanding jets, the perturbation amplitude (as measured in units of the local radius) grows algebraically with $z$ in the linear instability regime. In particular, we note that for $m = 1$ the amplitude of the perturbations relative to the local radius of the jet is constant, so that we model Hardee's result for a jet with constant opening angle.

We shall show in the next section that perturbations of the jet surface affect the jet propagation even in the linear growth regime, i.e., even when we assume that the perturbation of the cross sectional area is much smaller than the total area ($2\epsilon_0 Z^m + 1 \ll Z^2, \epsilon_0 < 1$). In fact, as discussed in Habbal and Tsinganos (1983), the term $[1/A(Z)] [dA/dZ]$ in equation (1) is equivalent to momentum addition when positive, or momentum subtraction when negative, and any pattern different from standard spherical expansion may introduce new critical

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**Fig. 3.** Location of critical points for isothermal flows vs. temperature at the base of the flow for different values of the mass distribution power-law index $n$.
points to the Parker solution or, in our case, to the topologies calculated for a nonpointlike mass distribution.

An interesting consequence of the oscillating behavior of the beam's cross section given by equation (7) is the possibility of multiple transonic solutions, one of which is continuous from the jet's base to infinity, while the others involve standing shock transitions. These shock transitions occur between pairs of transonic solutions: an upstream solution, which is supersonic, coming either from the base or an inner X-type point, and a downstream solution, which is supersonic (via an X-type critical point) just downstream of the standing shock (see Fig. 5). The exact location of these shocks is found by considering the Rankine-Hugoniot conditions (see Paper I; Habbal and Tsinganos 1983; Bailyn, Rosner, and Tsinganos 1985):

$$M_2^2 = \frac{2 + (\alpha - 1)M_1^2}{2\alpha M_1^2 - (\alpha - 1)}.$$  (8)

Here $M_1$ and $M_2$ are the Mach numbers before and after the shock. Thus, if $M_1 (> 1)$ falls on an upstream transonic branch crossing an X-type critical point, while $M_2 (< 1)$ falls on a downstream transonic branch passing through another X-type critical point, a standing shock may form (see Fig. 5). Equation (8) is thus a necessary condition for shock formation; to establish a sufficient condition, it is necessary to consider the time-dependent evolution of the beam's cross sectional area, $A(Z, t)$, due to the growth of the Kelvin-Helmholtz instability, because whether a shock forms depends on the actual growth rate $1/\tau$ of the instability (Habbal, Tsinganos, and Rosner 1983; Tsinganos, Habbal, and Rosner 1983). For example, a large growth time scale $\tau$ (larger than the convective time scale of the flow $\tau_{\text{conv}}$) may allow a gradual steepening of the profile of the perturbations, and lead to a shock at the equilibrium positions described by equation (8). A smaller value of $\tau (\ll \tau_{\text{conv}})$ may prevent such steepening and shock formation (Tsinganos, Habbal, and Rosner 1983). In the next section we shall discuss how such shocks can in fact be associated with the morphology observed in M87.

III. RESULTS FOR THE JET OF M87

We are now in a position to apply our model to the jet associated with the giant elliptical galaxy M87. In particular our aim is to interpret the observed two propagation regimes separated by the brightest knot A (at a distance of about 700 pc from the nucleus): (1) the inner straight and conical section close to the galactic core, which expands at a constant opening angle, and has three well separated knots, and (2) the external section, whose three knots have roughly equal transverse dimensions and are connected by a diffuse emission trail. We note that the structure of knot A (and possibly of the innermost knot D) suggests the presence of shocks whose normal is almost parallel to the jet axis.

In Table 1 we summarize the standard values of the physical parameters used in the literature for this jet; from the data we can estimate an average spacing between knots $\lambda = 170$ pc. The values for $S_{\text{radio}}$ are from Biretta, Owen, and Hardee (1983). $L_{K}/L_{\text{tot}}(\%)$ is the knot-to-jet optical luminosity ratio from de Vaucouleurs and Nieto (1979); the remainder [i.e., $1 - (L_{K}/L_{\text{tot}})$] is in weaker structures.

In order to integrate the Mach number equation (1), we require the functional form for the gravitational potential. This we take from Sargent et al. (1978), who have studied the optical mass-luminosity relation for the core of M87. Extrapolation of their mass distribution is consistent with that derived by Fabricant, Lecar, and Gorenstein (1980) from X-ray observations of

![Fig. 4.—Mass distribution (in units of $10^9 M_\odot$) for the central region of M87 (after Sargent et al. 1978); crosses are observational points, and the curve is the power-law fitting used in this paper.](image-url)
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the extended halo. We thus use the following mass distribution as a function of the dimensional radial distance z (given in pc; see Fig. 4):

\[ m(z) = 1.2 \times 10^{8} \cdot z^{0.84} \, M_{\odot}, \quad 100 \leq z(\text{pc}) \leq 1500. \] (9)

Our basic assumptions are then as follows (their validity will be further discussed in the last section):

1. The "base" of the jet is assumed to lie at \( z_0 = 100 \) pc, with a local Mach number \( M_0 \approx 2.5 \); such initial boundary conditions are consistent with a wind solution for an isothermal flow with a temperature \( T(0) = 8.5 \times 10^{7} \) K whose critical point lies well inside the core, \( z \leq 1 \) pc (see Paper I).

2. The jet expands spherically (i.e., \( \theta = z_0/r_0 \)).

3. The jet's walls are unstable to fluid perturbations, which lead to variations of its cross sectional area with constant fractional maximum amplitude; thus we set \( m = 1 \) in equation (7).

The wavelength of the perturbation is chosen as \( \lambda = 170 \) pc, in accordance with the data summarized in Table 1.

\[ \text{a) Isothermal Flow} \]

We begin our discussion of the results by considering an isothermal flow (\( \alpha = 1 \)) which, as previously discussed, may apply especially to the propagation regime upstream of knot A. We have solved equations (1a) and (1b) for the Mach number \( M(z) \) over a wide range of values of \( \epsilon_0 \), the fractional change in jet radius \( \delta r_0/r_0 \) due to the perturbation at \( z = z_0 \), using standard numerical integration schemes. Some of the more interesting solution topologies are plotted in Figure 5, whose main features can be summarized as follows:

1. For \( \epsilon_0 \leq 0.015 \), i.e., \( \delta r_0/r_0 \leq 1.5\% \), we find only the continuous transonic solution, with slight oscillations in \( M(z) \) due to the perturbed cross section of the jet channel (this case is not illustrated as it follows trivially from Fig. 5a).

2. For \( 0.015 < \epsilon_0 \leq 0.225 \), multiple critical points appear (alternating X- and O-type critical points); as \( \epsilon_0 \) increases, they appear first at large \( Z \) where perturbations have larger amplitudes, then move closer to \( Z = 1 \) (see Fig. 5a). However, no shock transitions are possible between the first upstream transonic solution and the downstream transonic solutions crossing these new X-type points; instead, the solution pattern is very similar to that of case (1), but with an increased amplitude of oscillations.

3. For a very narrow range of \( \epsilon_0 \) (\( 0.225 < \epsilon_0 < 0.23 \)), the aforementioned increased amplitude of oscillations allows a shock transition ("strong" shock, i.e., with \( M_{\text{conn}}/M_0 > 1 \)) between the first upstream transonic solution and one of the downstream transonic solutions. For \( \epsilon_0 \geq 0.225 \), this shock connects the upstream transonic solution to the downstream solutions which pass through the most downstream critical point; the shock then shifts upstream as \( \epsilon_0 \) is increased slightly (see Fig. 5b). In particular, for \( \epsilon_0 \approx 0.230 \) the "strong" shock transition arises roughly at the distance of knot A. Further shock transitions ("weak" shocks, i.e., \( M_{\text{conn}}/M_0 \approx 1 \)) are also possible between various downstream critical solutions.

4. When \( \epsilon_0 \geq 0.23 \), the "strong" shock transition occurs between the initial upstream transonic solution and the downstream transonic solution which links with the most upstream (O-type) critical point (Fig. 5c).

5. The Mach number at \( z = 1200 \) pc is \( M \approx 3.7 \) for the continuous transonic solution, and \( 2 \leq M \leq 3 \) for the shocked transonic solutions; this yields velocities \( \bar{V} \) for \( z = 1200 \) pc \( \approx 1800 \) km s\(^{-1}\) and \( \approx 1000-1200 \) km s\(^{-1}\), respectively. These velocities are measured in the rest frame of the channel walls and must be transformed to the observer's frame before being compared with observations (the resulting changes are minor).

Our results indicate that the positions of knots can be associated either with jet compressions (when they are sufficiently strong), or with the shock transitions between critical solutions we have just focused on; this will be discussed further in the concluding section. For the moment, we restrict our attention to case (3) (Fig. 5b) and consider its correspondence with the morphological characteristics of the jet of M87. The "strong" shock at about 730 pc gives rise to a sharp transition in the flow pattern; this feature may be then associated with knot A, where the jet is thought to slow down drastically. The jet compressions upstream of knot A may be associated with the three inner blobs, while "weak shocks" between multiple solutions may generate the outermost knots. Thus case (3) provides a reasonable account of the major features of the jet morphology, which is based on a fairly straightforward description of the jet's hydodynamics.

Before ending this subsection on isothermal flows, we focus on two ingredients of our model which determine the above results quantitatively.

\[ \text{ii) Temperature and Initial Velocity} \]

The flow terminal velocity is proportional to the assumed (constant) temperature, but the asymptotic Mach number is insensitive to the temperature. For instance, if we adopt the parameters used in the case illustrated by Figure 5b, and set \( T = 10^{7} \) K and \( T = 4 \times 10^{7} \) K, respectively, the corresponding continuous supersonic solution has asymptotic values of \( M(z = 1200 \) pc) \( \approx 3.8 \) and \( M(z = 1200 \) pc) \( \approx 4.2 \), or velocities \( V(z = 1200 \) pc) \( \approx 1950 \) km s\(^{-1}\) and \( V(z = 1200 \) pc) \( \approx 4350 \) km s\(^{-1}\), respectively. Furthermore, the range for which we find shock transitions depends only slightly on the temperature: it is \( \epsilon_0 \approx 0.240-0.265 \) for \( T = 10^{7} \) K and \( \epsilon_0 \approx 0.270-0.315 \) for \( T = 4 \times 10^{7} \) K. The solution topologies for such \( \epsilon_0 \)'s are similar to those illustrated in Figure 5b; the discontinuous solutions reach slightly smaller asymptotic Mach numbers:

\[ M(z = 1200 \) pc) \( \approx 2.7-2.2 \) for \( T = 10^{7} \) K, and \( M(z = 1200 \) pc) \( \approx 3.2-2.3 \) for \( T = 4 \times 10^{7} \) K.

Finally, if we vary the flow velocity of the continuous supersonic solutions at the base z = \( z_0 \) (at fixed temperature), we find that the asymptotic Mach number of the continuous solution varies only slightly from \( M_0 \). Assuming the same physical parameters as in Figure 5b, we obtain for this continuous solution asymptotic values of \( M(z = 1200 \) pc) \( \approx 5.6 \) and \( V(z = 1200 \) pc) \( \approx 2700 \) km s\(^{-1}\) for \( M_0 = 5 \), and \( M(z = 1200 \) pc) \( \approx 10.3 \) and \( V(z = 1200 \) pc) \( \approx 5000 \) km s\(^{-1}\) for...
Fig. 5.—Solution topologies for isothermal flows. The heavy line represents the continuous wind solution which is supersonic at the base of the beam \((Z = 1)\), crosses indicate X-type critical points; circles, O-type critical points; and dashed lines, the position where the condition for shock transition is satisfied. The first three panels correspond to \(T_6(0) = 8.5, m_{gal}(z = 1 \text{ pc}) = 1.2 \times 10^8 \, M_\odot, n = 0.84\) and variable \(\epsilon_0\): (a) \(\epsilon_0 = 0.225\), no shocks; (b) \(\epsilon_0 = 0.230\), strong shock at 730 pc; (c) \(\epsilon_0 = 0.250\), strong shock at 220 pc. The remaining three panels correspond to \(T_6(0) = 8.5, m_{gal} = 1.2 \times 10^8 \, M_\odot, \epsilon_0 = 0.230\) and variable \(n\): (d) \(n = 0.60\), no shocks; (e) \(n = 0.95\), with shocks; (f) \(n = 1.1\), "trapped" solutions only.
Fig. 5—Continued

(c) $\epsilon_0 = 0.25$

(d) $n = 0.60$
Fig. 5—Continued

n = 0.95

n = 1.1
Thus, a larger base Mach number requires a larger value of $\epsilon_0$, so as to allow discontinuous solutions: shocks are found along the jet with $\epsilon_0 \approx 0.335 - 0.370$ for $M_0 = 5$, and $\epsilon_0 \approx 0.460 - 0.495$ for $M_0 = 10$. Note that, when $\epsilon_0$ is increased, the range of $\epsilon_0$ for which solution topologies similar to Figure 5b are found increases as well. In conclusion, the variations of $T$ and $M_0$ yield slight quantitative modifications of our results but do not affect the solution topologies qualitatively.

\subsection*{b) Polytropic Flows}

As $\alpha$ is increased, but with the same $M_0$ as in the isothermal case, then continuous transonic solution have larger asymptotic Mach number but smaller asymptotic velocity; the discontinuous transonic solutions have instead almost unchanged asymptotic Mach number, and shock transitions are found for smaller values of $\epsilon_0$ than in the isothermal case. As $\alpha \to 5/3$ the discontinuous solutions tend to be "trapped" (similar to the case of $n = n_i$ in isothermal conditions): upstream shock transitions which occur close to the base of the jet do not connect to downstream transonic solutions which escape to infinity: only for large $M_0$ and/or large $T_0$ can we again find solutions with shocks which reach infinity from the base $z = z_0$. This is consistent with the fact that a jet with $1 < \alpha \leq 5/3$ is not heated in isothermal conditions: upstream shock transitions which occur close to the base of the jet do not connect to downstream transonic solutions which escape to infinity: only for large $M_0$ and/or large $T_0$ can we again find solutions with shocks which reach infinity from the base $z = z_0$. This is consistent with the fact that a jet with $1 < \alpha \leq 5/3$ is not heated as effectively as an isothermal jet and therefore cannot escape the gravitational well supersonically unless its initial energy is already sufficiently high.

To be more specific, if we start with the same initial parameters of the case displayed in Figure 5b, then as $\alpha$ is increased, the asymptotic Mach number of the continuous transonic solution increases and its asymptotic velocity decreases; thus, for $\alpha = 4/3, 3/2$, and $5/3$ respectively, we find $M(z = 1200 \, \text{pc}) \approx 6.6, 9.3, 13.3$, and $V(z = 1200 \, \text{pc}) \approx 1350, 1250,$ and $1200 \, \text{km s}^{-1}$, respectively. The asymptotic Mach number of the discontinuous transonic solutions instead remains almost unchanged as $\alpha$ is increased, with velocities ranging between $\approx 500$ and $1000 \, \text{km s}^{-1}$.

The possible presence of shock transitions however does depend on the two parameters $\alpha$ and $\epsilon_0$. In the isothermal case, the shock conditions connecting the continuous solution with the downstream transonic branch arising from the most downstream O-type critical point is less than $n_1$. This mass distribution is however limited to any range of $Z$ for specific combinations of the parameters $\alpha$ and $\epsilon_0$. In summary, the effect of using polytropic indices $1 \leq \alpha \leq 5/3$ is to yield a wide range of values of $\epsilon_0$ for which discontinuous solutions can exist; this allows one to more easily vary the position of shocks along the jet to match observational constraints. Note again that the overall pattern of solutions that we have obtained is not critically dependent on the physical and morphological parameters that one assumes for M87, as long as the power law index $n$ for the mass distribution ($\epsilon_0 [9]$) is less than $n_1$. This mass distribution is however one of the main uncertainties of the present observations.

\section*{IV. SUMMARY AND CONCLUSIONS}

We have constructed a model in which the characteristic large-scale morphology of jets such as that in M87 can be described in terms of simple hydrodynamic effects within the flow. This model is based on the equations for hydrodynamic outflows that have wide application in the study of solar and stellar winds. In Paper I we described the initial acceleration phase of such a jet; here we have discussed the propagation phase at larger distances, for which morphological data are available for a number of objects. The combined effects of the galactic matter distribution and of the channel cross section perturbations due to fluid instabilities determine the effective shape of the nozzle within which the jet flows, and further determine the flow topologies. A major effect is the existence of multiple transonic solutions, which we propose can be used to interpret the morphology of extragalactic jets. In particular, we have discussed a specific application of our model for the M87 jet, for which the most complete observational data are available.

Our interpretation of the M87 jet morphology can be summarized as follows:

1. The brightness enhancements (knots) may be due to enhanced particle acceleration either in local compressions of the flow pattern or in shock transitions. In particular, a strong shock between the continuous transonic solution and one of the multiple solutions of our model occurs at the location of knot A, which is in fact the brightest knot (see Table 1) and displays a structure suggesting the presence of a discontinuity. In this scheme, the inner (upstream) knots would then be associated with flow compressions, and the outer (downstream) knots with weaker shock transitions between multiple critical solutions (Fig. 5b). If the inner knot D is also associated with a strong shock (i.e., the transition from the continuous transonic solution to the first multiple transonic solution), then our model does allow for a second transition at the location of knot A (Fig. 5c). These interpretations are based on the assumption that the hydrodynamic shocks and compressions do lead to enhanced local particle acceleration...
Fig. 6.—Solution topologies for polytropic flows. Symbols are the same as in Figure 5. Physical parameters are $T_e^{(0)} = 8.5, m_{gal} = 1.2 \times 10^8 M_\odot, \epsilon_0 = 0.230, n = 0.84$ and variable $\alpha$: (a) $\alpha = 4/3$, transonic solutions with shocks; (b) $\alpha = 3/2$, one transonic solution with shocks; (c) $\alpha = 5/3$, no transonic solutions.
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Fig. 6—Continued

(Ferrari 1984); an essential point is then that the efficiency of particle acceleration depends only slightly on the shock strength, which in our case is always relatively weak.

2. Once the appropriate solution topology is selected by fitting to the observed jet morphology (for M87), our model predicts velocities (depending on the chosen parameters) of the order of 1000–2000 km s\(^{-1}\) at the location of knot A in the perturbation rest frame, and therefore of the order of the values inferred from the observations after transformation to the external observer rest frame (Biretta, Owen, and Hardee 1983).

Detailed observations have also become available for other jets, which in many respects follow a pattern similar to M87, but can involve very different spatial scales. For instance, six knots have been studied in detail in 3C 111: these show spacings of \(\lambda \approx 10\) kpc \((H_0 = 100\) km s\(^{-1}\) Mpc\(^{-1}\)) for the inner three and 2\(\lambda\) for the external knots (Linfield and Perley 1984), while the strongest knot is located halfway along the jet. Unfortunately, very few data are available for the mass distribution of the central galaxy (see Sargent 1977); however, because the length of the jet \((\sim 100\) kpc) indicates that we are observing the propagation phase well outside the galaxy, we can apply our model to this object assuming that all the mass of the galaxy \(M_{\text{gal}} \approx 10^{12}\) \(M_\odot\) is concentrated in the center (i.e., \(n = 0\)). Assuming \(z_0 = 5\) kpc for the base of the jet, and a wavelength \(\approx 10\) kpc for the channel cross section perturbations, we find that a “strong” shock transition can form at the required distance (topologies similar to Fig. 6b) if the gas temperature is \(T^{(0)} \approx 10^7\) K and the fractional perturbation amplitude is \(\delta r_0/r_0 \approx 26\%\). The Mach number for the continuous transonic solution upstream of the shock is \(M \approx 4\) \((V \approx 4650\) km s\(^{-1}\)), and for the multiple transonic solutions downstream of the steady shock we found \(M \approx 2–2.6\) \((V \approx 2400–3000\) km s\(^{-1}\)). Again, the inner knots may be associated with flow oscillations and density compressions, while the external ones may correspond to shocks. The larger spacing between external knots may be due either to the fact that shocks are too weak to be detected, or are in fact totally absent.

More similar to M87 is the structure of the jet in Cyg A (Perley, Dreher, and Cohn 1984): five knots have been detected, separated at a regular spacing of \(\lambda \approx 11\) kpc; a change of regime is evident following the position of the fifth knot. In addition, the entire jet \((\sim 53\) kpc) falls within the giant elliptical galaxy as in the case of M87 (Fabbiano et al. 1979; Spinrad and Stauffer 1982). However, in this case the data are insufficient to adequately describe the galactic mass distribution in the form of equation (9): note that the parameter \(n\) is critical for our model, i.e., we need \(n < n_1\) for wind solutions to exist.

Finally, we also note that regularly spaced knots have been detected in the radio jets of NGC 6251 (Perley, Bridle, and Willis 1984) and of the QSO 0800+608 (Shone and Browne 1985), and in the optical jet of 3C 273 (Lelievre et al. 1984). The mass distribution of their associated galaxies is poorly known, however, and we defer any application of the model to the future.

We conclude with a discussion of the limitations of the various approximations which have been used. As pointed out...
above, the solution topologies depend critically on the galactic mass distribution and on the amplitude of the perturbation of the channel’s cross section. The equations used are based on the polytropic assumption; we prefer this approach to one based on solving the full energy equation because it makes explicit our ignorance of the actual jet heating processes (note that solutions with $\gamma$ close to unity are preferred by comparison with data) and consequently reduces the dimensionality of the parameter space of the solutions enormously.

The development of perturbations at the jet boundaries by Kelvin-Helmholtz instabilities requires that the jet be either thermally or magnetically confined, or that internal shears are present in the flow. While the question of confinement is still unresolved (for the case of M87, see Schreier, Gorenstein, and Feigelson 1982), it is noteworthy that optical luminosity variability represents a virtually simultaneous dimming of all knots, thus excluding any causal association with the central “engine.” Instead, the causal agent is likely to be a local instability which is in phase over large spatial scales; an example could be the previously described MHD shear instabilities (Ferrari, Trussoni, and Zaninetti 1981), for which the observed variability may be associated with a bulk variation of the boundary conditions for the jet. This supports the notion that the jet is in fact not freely expanding.

In closing, we note that our model is based on a time-independent treatment of the flow. In particular, we have derived only the necessary conditions for shock formation (i.e., where they could form). Time-dependent computations (Tsinganos, Habbal, and Rosner 1983) have shown that, at least in the case of the solar wind, shocks do form at the locations expected on the basis of the steady solutions. We intend to test our model by a time-dependent treatment of the flow using typical physical conditions of extragalactic jets in the future.

Support by the National Science Foundation grant AST 83-03522 and by the Italian Consiglio Nazionale delle Ricerche and Ministero della Pubblica Istruzione is acknowledged.

REFERENCES


Attilio Ferrari: Istituto di Fisica Generale, University of Torino, Corso Massimo d’Azeglio 46-I-10125 Torino, Italy

Robert Rosner: Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138

Edoardo Trussoni: Istituto di Cosmo-geofisica del C.N.R., Corso Fiume 4-I-10133 Torino, Italy

Kanaris Tsinganos: Department of Physics, University of Crete, Heraklion, Crete, Greece