ANOTHER REASON TO SEARCH FOR SOLAR g-MODES AND NEW LIMITS FROM SOLAR ELLIPTICITY MEASUREMENTS

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ABSTRACT. Observations of solar g-modes will teach us some gravitational physics. The present sensitivity of the Princeton Solar Distortion Telescope and other recent claims of solar g-modes suggest that these low frequency modes should be observable in shape measurements. From about 250 days and nearly 1000 hours of observations we find no evidence for significant spectral power that can be associated with g-modes and no sign of the elusive 160.01 min period solar oscillation.

1. NEW GRAVITATIONAL PHYSICS INTERESTS

We will undoubtedly learn a great deal about the solar interior from the long period g-modes if they are reliably observed, but there's another perspective from which their observation is interesting. I'm impressed that we (almost!) understand the sun to the extent that as we learn more about the oscillations we can treat the sun as a "detector." In particular, a good example of this are potential observations of low degree modes with periods near one hour. Such observations may yield interesting limits to a stochastic gravitational wave background (Boughn and Kuhn, 1984). It's also interesting that the low degree modes with periods of order an hour have a relatively large gravitational energy associated with their oscillation. In light of recent geophysical evidence (Holding and Tuck, 1984) that the effective gravitational constant may be larger on distance scales much larger than laboratory scales, a solar determination of G becomes interesting. Some hope that such a measurement can be disentangled from the mass uncertainty of the sun is gained by noting how the eigenfrequencies of the low order modes are affected by perturbations in the gravitational constant. Figure 1 shows the relative frequency shift, in terms of the fractional perturbation in G, expected for some low order \( \ell = 2 \) and \( \ell = 3 \) g- and p-modes. The figure suggests that a possible 1-percent effect seen in the geophysical data might be detected from the solar oscillation spectrum. At present such predictions are premature and more work must be done to constrain the density-gravitational constant product from other solar observational constraints.

2. LONG PERIOD SOLAR OSCILLATIONS

Several claims for detection of solar g-modes with periods between 2 and 10 hours have been made (e.g. Scherrer, 1984; Isaak, et al. 1984; Frohlich and Delache, 1984). The surface velocity amplitudes of these modes appear to be between 0.1 and 1 m/s but there is poor agreement on the excitation spectrum between different observers. On the other hand several groups have consistently claimed detection of a 160.01 min period solar oscillation (cf. Kotov, et al., 1984) also with a velocity amplitude near 1 m/s. All of these modes have been observed with low spatial resolution and are most likely of low spherical harmonic degree ($\ell \leq 2$).

It's notable that the surface amplitudes of these oscillations are comparable to 5-min p-mode amplitudes of about 15 cm/s (cf. Kuhn, 1984a). If the observations are correct then the energy per g-mode is roughly $10^8$ times larger than typical p-mode energies and of considerable curiosity.

3. SHAPE OBSERVATIONS

Except for the ACRIM data (cf. Frohlich and Delache, 1984) and
Hill's data (1985) most of the low frequency data are due to Doppler velocity observations. Since the period of the modes discussed in this paper are long compared to a radiative relaxation time at the photosphere (\(\approx 10^3\) s) the pressure and density perturbations due to the oscillation will be nearly isothermal. It is therefore reasonable to calculate the shape change by following photospheric brightness contours as they move due to mass motion of the oscillation displacement field below the photosphere.

The Distortion Telescope and oblateness data have been described in detail elsewhere (Kuhn, et al., 1985; Libbrecht and Kuhn, 1984, Dicke and Goldenberg, 1974). In brief, measurements of the deviation from a circular solar limb shape are obtained by measuring the flux from a solar image occulted by a slightly undersize circular occulting disk. The solar flux is measured in two broadband colors (.5\(\mu\) and .8\(\mu\)) and at 256 positions around the limb. Between 9 and 20 arcseconds of the limb extends beyond the occulting disk. The data are obtained with a specialized telescope now operating at the Mount Wilson Observatory.

A data set at one of 3 possible limb exposures and two simultaneous colors is obtained approximately once every 5 min. The flux measurements in 128 distinct position angles (only symmetric distortions about the image center may be observed) around the image are reduced to a displacement in arcseconds using a geometrical calibration. Corrections for optical imperfections of the telescope, mirror distortions, atmospheric transparency and refraction, and facular contamination are applied to the data before obtaining the two coefficients that describe the oblateness of the solar image. For the discussion below the oblateness is described by the coefficients D and V of a fit of \(D \sin 2\phi + V \cos 2\phi\) to the binned shape data. Here \(\phi\) measures the angle from the solar rotation axis projected onto the image plane. Thus each observation at a given limb exposure yields 4 coefficients -- a V and D term for each of the two color bands. Except for additional atmospheric corrections described by Dicke, et al. (1985) the analysis sketched above essentially follows the description in the references.

The two summers of observations yielded about 254 useful days of data. This amounted to about 11,000 5 min observations distributed between the three possible limb exposures and the two summers. After subtracting least-squares fit daily quadratic polynomials, to remove daily trends, the standard deviation of the residuals was approximately 24 milliarcseconds in D or V.

4. SEEING SOLAR OSCILLATIONS IN SHAPE DATA

The oblateness data will not be sensitive to \(l = 0\) or 1 oscillations. We may expect g-modes with \(l = 2, 3, \ldots\) to contribute to the observed power in the V or D coefficients. If the 160 min mode is, for example, an \(l = 2\) g-mode it will also contribute to the shape data.

For definiteness we will estimate the expected shape signal amplitude from the assumption that the 160 min mode is an \(n = 9, l = 2\) g-mode (the closest predicted frequency for low order modes). The velocity amplitude of an \(m = 0\) mode can be written
\[ \vec{v}_n(r, \theta) = V_n \cdot P_2(\cos \theta) \frac{U(r)}{r} \frac{\partial P_2(\cos \theta)}{\partial \theta} \theta \] \hspace{1cm} (1)

A model calculation (cf. Kuhn and Boughn, 1984) gives for the \( n = 9 \) mode \( V_n(R_{\text{sun}})/V_n(R_{\text{sun}}) = 1.14 \). The line-of-sight velocity \( V_{0n} \) can be related to the modal amplitude \( V_n \) from

\[ V_{0n} = S_n \cdot V_n \] \hspace{1cm} (2)

where \( V_{0n} \) is the intensity weighted line-of-sight velocity, \( \vec{V}_n \cdot \theta \), integrated over the solar disk. The factor \( S_n \) is easily evaluated for full disk observations and has been tabulated by Christensen-Dalsgaard and Cough (1982). For example \( S_9 = 1.3 \) for the radial-transverse velocity ratio given above for the \( n = 9, \ell = 2 \) mode. Other long period \( \ell = 2 \) modes have values of \( S_n \) within about 50\% of this value. Projecting the \( P_2(\cos \theta) \) term against \( \sin \theta \) or \( \cos \theta \) gives an approximate relation between the coefficient \( V \) and the observed full disk velocity amplitude, \( V_0 \), of a \( g \)-mode of frequency \( \omega \),

\[ V \approx 1.3 \frac{V_0}{\omega} \] \hspace{1cm} (3)

Thus, from the 160 min mode with an observed amplitude of about 1 m/s we expect an oblateness coefficient of \( V \approx 2 \times 10^3 \) m = 2.7 milliarcsec. If the oblateness residuals were uncorrelated such an oscillating shape signal would be observable with high signal-to-noise.

5. IS THE 160 MIN OSCILLATION VISIBLE?

In the following sections we address two questions: 1) Is there evidence of a 160.01 min solar oscillation in these data? and 2) Is there evidence of any other solar oscillating power in the oblateness residuals? These questions are complicated by the very uneven sampling in time of the data. For example in 1984 there were about 5500 observations between day number 107 and 274, spaced at about 5 min intervals. This represents only about 11\% coverage of the full interval with only an approximately even spaced 5 min sample interval domain. It's clear from the power spectrum of the window function for one of the data sets, plotted in Figure 2, that any mode structure will be highly aliased in the observed power estimates.
Figure 2. Power spectrum of the window function for a 1983 dataset.

Data from the two color channels are obtained simultaneously and are highly correlated. Thus to reduce the noise the colors are averaged before computing power spectra. By averaging the power spectrum from 3 possible limb exposures and both V and D coefficients we may hope to find a 160 min signal. This has been done in Figure 3 for 1983 and 1984 data separately, and combined. "Power spectra" are computed from the summed squared amplitude of least squares fits of sin ωt and cos ωt to the color averaged residuals.

Figure 3(a). Average of the least-squares spectra from the six 1983 datasets. 3(b). Same as 3(a) but for 1984 datasets. 3(c). All 12 spectra averaged from 1983 and 1984.
Figure 3c is an average of 12 "pseudo-spectra" and is our first approximation to the actual mean power spectrum. We note that at frequencies near 104.16 µHz (T = 160.01 min) all peaks are smaller than twice the mean power level. To the extent that this approximates the actual mean spectrum the probability distributions of signal in the presence of noise calculated by Groth (1975) are applicable here. We find that there is less than a 1% chance that the signal power at f = 104.16 µHz is larger than the mean noise power of about 0.4 milliarcsec². It's notable that there are prominent peaks in the averaged and yearly spectra at periods of 1/10, 1/9, 1/6, 1/5 and 1/3 days. Some 1 day⁻¹ frequency sidelobes of the stronger peaks are also prominent.

Since we know the frequency and phase of the 160.01 min signal we can also fit for it directly in the data. To simplify the analysis the data were again averaged by color and limb exposure and binned in half hour intervals to obtain a single time series for each V and D coefficient for each year. The phase and period described by Grec, et al. (1980) has been used to find the 160 min amplitude and fit error for 1983, 1984 and combined datasets, displayed in Table I. The results of a non-linear fit with frequency treated as an adjustable parameter are also displayed.

TABLE I. Least Squares Fits for 160.01 min Oscillation

<table>
<thead>
<tr>
<th></th>
<th>1983</th>
<th>1984</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>f = 104.16 µHz Amplitude [milliarcsec]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>0.4 ± 0.5</td>
<td>1.7 ± 0.8</td>
<td>0.3 ± 0.4</td>
</tr>
<tr>
<td>D</td>
<td>0.3 ± 0.5</td>
<td>0.6 ± 0.8</td>
<td>0.3 ± 0.4</td>
</tr>
<tr>
<td>Frequency [µHz]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>103.92 ± 0.02</td>
<td>104.23 ± 0.03</td>
<td>104.235 ± 0.003</td>
</tr>
<tr>
<td>D</td>
<td>104.18 ± 0.01</td>
<td>104.00 ± 0.05</td>
<td>104.171 ± 0.003</td>
</tr>
<tr>
<td>Amplitude [milliarcsec]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>1.3 ± 1.3</td>
<td>2.1 ± 2.0</td>
<td>1.7 ± 0.8</td>
</tr>
<tr>
<td>D</td>
<td>1.8 ± 1.3</td>
<td>1.3 ± 1.9</td>
<td>1.4 ± 0.8</td>
</tr>
</tbody>
</table>

Only the V coefficient of the 1984 data shows a marginally significant amplitude at about the 2σ level, but even this is probably due to the nearby strong peak at a period of 159.90 min. The results in Table I provide no evidence for the 160.01 min oscillation with an amplitude greater than, approximately, the fit uncertainty of 0.4 milliarcsec.

6. EVIDENCE OF OTHER g-MODES?

Several approaches have been considered to look for other discrete frequency power contributions to the data. Scherrer (1984) claimed detection of g-modes by searching for peaks
in the Stanford data using an iterative least-squares fitting technique (the "Clean" algorithm) and then noted that the peaks fit the asymptotic expression for the mode periods, \( T(n, \ell) \):

\[ T = T_0 \left( n + \frac{\ell}{2} - 1/4 \right) / \sqrt{\ell(\ell + 1)} \]  

where \( n \) and \( \ell \) are chosen separately for each mode to best fit the observed periods using a single value of \( T_0 \). With 14 power spectrum peaks with periods between 172 and 329 min Scherrer found a good fit with \( T_0 = 38.6 \) min and \( \ell \) and \( n \) values between 1 and 2, and 6 and 20. As a test of the uniqueness of such a fit we've simulated this procedure on 100 sets of random numbers evenly distributed in frequency between the corresponding limiting periods in Scherrer's data. Each set of 14 "peaks" is used to estimate a value of \( T_0 \) which minimizes the summed squared deviation of the period calculated from (4) and random peaks.

The integer \( n \) and \( \ell \) values are constrained to be less than or equal to 25 and 2 respectively and are adjusted to minimize the error for each trial \( T_0 \) value. A 1% resolution search between 30 and 50 minutes finds a value of \( T_0 \) which minimizes the summed error. The 100 trials yielded the \( T_0 \) and mean error distributions shown in Figure 4. Notice that such a fit is not very conclusive since an RMS error of about 3 minutes is often possible given the range of available parameters. From Scherrer's \( n \), \( \ell \) and \( T_0 \) values we calculate an RMS value of 3.0 minutes, from his data. Thus his \( T_0 \) provides no supporting evidence that the power spectrum peaks are of solar origin.

Figure 4(a). Distribution of the mean fit errors to the g-mode asymptotic period relation from 100 synthetic peaks lists. 4(b). Distribution of the minimizing parameter \( T_0 \) from the synthetic datasets.
A second approach for finding discrete power contributions is to reduce the aliased structure in the spectrum by deconvolving the window function from the observed power spectra. The data have been averaged over the 3 limb exposures and 2 colors and binned in half hour intervals for this purpose. Even so the "fill factor" on the evenly spaced half-hour domain is only 23% in 1983 and less in 1984. The technique we've applied to work around the gaps is described in Kuhn (1982, 1984b). By choosing a particular set of evenly spaced frequencies the window function effects can be minimized (the "swap" technique). Unfortunately the domain is too sparse for this method (and probably any other). Dummy data have been generated by adding unit Gaussian noise to a unit amplitude sinusoid of frequency 101 µHz. Figure 5 compares the least-squares and swap technique spectra for the 1983 domain and the above synthetic signal. It's notable that the swap algorithm helps some but that the sidelobes still dominate the observed "spectrum." We must rely on statistical techniques to deduce evidence of g-modes in the oblateness data.

Figure 5(a). Least squares power spectrum of a 101 µHz frequency signal plus Gaussian noise using the 1983 domain. 5(b). Same as 5(a) but using the "swap" technique to evaluate the power spectrum.

We take as a null hypothesis that the 1983 and 1984 power spectra
(Fig. 3) are the result of random uncorrelated normally distributed noise. If this is correct then the distribution of power from both years should be consistent with the power distribution generated from noise analyzed in the same way as the data. Thus we generate fake data-sets on the same domain as the actual data. The values are distributed normally from a distribution whose variance is scaled so that the mean fake power and actual power agree. The resulting cumulative power probability distributions are plotted in Figure 6. The agreement is quite good if the anamolous peak at $1/6$ day period in the 1984 data is ignored. The Kolmogorov-Smirnov test yields a useful quantitative description of the probability that both samples come from the same parent distribution. The K-S test is essentially a measure of the largest deviation between the two distributions (cf. Hollander and Wolfe, 1973). Applying the test to the 1983 and 1984 data separately yields statistic values of 0.34 and 0.86 respectively, indicating that we should accept the null hypothesis. In short we find no evidence of solar g-modes in oblateness power spectra of mean noise $0.4 \text{ milliarcsec}^2$.

![Cumulative Probability Graph](image)

Figure 6(a). Cumulative power distribution of the data in Fig. 3(a) (crosses) and the spectra of normally distributed white noise (circles). 6(b). Same as 6(a) except for 1984 data and time domain.
7. SUMMARY

There is no evidence of a 160.01 min solar periodicity in the oblateness data at a level corresponding to a g-mode velocity amplitude of about 0.2 m/s. Assuming the validity of previous 160.01 min signal claims this suggests that the oscillation is not an \( \ell = 2 \) mode but probably has \( \ell = 1 \). Further, we see no evidence of other g-modes to approximately the same level.

I'm grateful to K. G. Libbrecht who operated the telescope in 1983 and has been involved in the data analysis and to R. H. Dicke who has also contributed much to the analysis and numerous fruitful conversations.

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REFERENCES