POSTER PAPER:
THE ATMOSPHERE IN A THIN MAGNETIC FLUX TUBE

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ABSTRACT: We construct the equilibrium atmosphere inside a thin, vertical magnetic flux tube embedded in a grey external atmosphere in radiative and convective equilibrium. The temperature on the axis of the flux tube and the structure of the magnetic field are determined from the conditions of (i) pressure balance in horizontal layers, (ii) hydrostatic equilibrium in the vertical direction, and (iii) radiative equilibrium of the tube with the surrounding medium; thus the gas temperature is equal to the temperature associated with the intensity of the radiation field. The tube is assumed horizontally uniform and its temperature is characterized by its value on the tube axis; the depth dependence of the flux tube shape is taken into account in solving the transfer equation; the models described here make the two-stream approximation.

We discuss the difference between this kind of modelling and that of Spruit (1976), who determined the structure of the flux tube and the influence of the tube on the surrounding flux-free atmosphere from the diffusion equation for turbulent as well as radiative energy transport.

INTRODUCTION

Magnetic flux is not uniformly distributed over the solar surface but is concentrated in vertical structures of high field strength, ranging up to 2 kG. These structures are thought to be formed by the buoyancy instability of magnetic flux at the bottom of the convection zone, or by “flux-sweeping” near the surface layer of the convection zone. Large magnetic structures (with diameters exceeding about 1000 km) appear dark; smaller elements are brighter than their surroundings. These latter structures are idealized as thin vertical magnetic “flux tubes” with sharp boundaries between the high magnetic field and the field-free regions. The observed lifetime of these elements (with radius $R$) is much longer than the Alfvén time $\tau_A = R/V_A$.

There are two extreme points of view on the problem of the energy balance inside magnetic flux tubes; they are represented by the diffusion model and by the radiative transfer model.

DIFFUSION MODEL

The temperature structure of the atmosphere inside the flux tube is obtained from the energy transport equation in the diffusion approximation, $\nabla \cdot (K \nabla T) = 0$ (Spruit 1976, 1977), where $K$ includes the effects of both radiative transfer and turbulent diffusion. Outside the flux tube, the radiative part of the diffusion coefficient is given by $K_r = (16/3)(\sigma T^4/\kappa_n)$, where $\kappa_n$ is the Rosseland absorption coefficient and $T_r$ is the external temperature; the turbulent part is given by the standard expression (Spruit 1976) for the turbulent diffusion of heat. Inside the tube, the radiative coefficient is assumed equal to the outside coefficient, but shifted vertically by the Wilson depression, which is the geometrical distance between the layers where the optical depths inside and outside the tube are unity, i.e., $z_w = z(t_i = 1) - z(t_e = 1)$, where $z$ is the geometrical depth and $t_i$ and $t_e$ are the optical depths in the external and internal atmospheres, respectively. Thus, the diffusion coefficient inside the flux tube is given by $K_i(z + z_w) = K_r(z)$; an analogous expression holds for the turbulent diffusion coefficient. This procedure has been justified by the low contrast in faculae, which implies that $T_i(z_w) \approx T_i(0)$ (where $z = 0$ at $r_e = 1$).

The input parameters for this model construction are: the flux tube radius, the Wilson depression, and the heat flux at the bottom of the flux tube. The atmosphere far from the flux tube is an empirical solar model (HSRA, VAL, Spruit 1974).

The advantage of the method is that the flux tube structure is studied in two dimensions, with the influence of the tube on the surrounding atmosphere taken into account. Spruit's models cover the radius range from 80 km to 500 km. For narrower tubes the assumptions concerning the internal diffusion coefficients are likely to fail; for such tubes it may be appropriate to assume equality of
internal and external temperatures at a given physical depth, \textit{i.e.}, \( T_s(z) \approx T_e(z) \). The disadvantage of the method is that it solves the radiative transfer equation in the diffusion approximation. This is justified only when the gradient of the source function relative to optical depth is constant and the optical distance to boundaries is large compared to unity. These assumptions are not satisfied in a slender flux tube, where the source function gradient may jump by an order of magnitude or more at the tube wall.

**THE RADIATIVE TRANSFER MODEL**

The atmosphere in the flux tube satisfies (i) radiative equilibrium with the external atmosphere, which is assumed to remain unaffected by the presence of the magnetic flux tube, and (ii) (vertical) hydrostatic and horizontal pressure equilibrium for the internal gas. The gas external to the flux tube is in hydrostatic and in radiative or convective equilibrium according to the Schwarzschild criterion; its opacity is assumed to be grey and is due to \( \text{H}^- \) ions and H atoms in the third level. These constraints, together with the values of the tube radius (at \( r_s = 0 \)) and of the magnetic field strength (denoted in terms of the plasma-\( \beta \) at \( r_s = 0 \)), completely specify the structure of the atmosphere in the tube. The radiative transfer equation is solved in the two-stream approximation; since it is assumed that there is no convection inside the tube, the inside temperature is determined from the radiative equilibrium condition in the form \( J = B(T_i) \), where \( J \) is the mean intensity in the tube and \( B \) the Planck function.

Figure 1 shows the run of temperature \( T(\tau) \) in the flux tube and in the external atmosphere as a function of the corresponding optical depth for \( \beta = 0.1 \), for \( \beta = 0.25 \), and in the external atmosphere. Note that the flux tube temperature at the internal optical depth \( \tau_i = 1 \) is higher than the external temperature at the external depth \( \tau_e = 1 \); thus this tube appears brighter than the surrounding atmosphere, with the temperature contrast increasing with decreasing \( \beta \) (\textit{i.e.}, increasing magnetic field strength).

The structure of the atmosphere and that of the magnetic field in the flux tube (which satisfies magnetic flux conservation) are determined consistently by solving the constraint equations. In this modelling, the Wilson depression is a derived quantity. The analysis is (essentially) one-dimensional so that the influence of the tube on the surroundings cannot be examined. This restricts the validity of the theory to tubes with diameters less than about 100 km. The model completely neglects turbulent transport within the tube; whether this is important for such thin tubes may have to be decided by observations.

**COMPARISON OF THE MODELS**

A detailed comparison of the results obtained by these two ways of building models is not possible because they apply to different values of the parameters and because the physics considered is different; but it is possible to contrast the different assumptions concerning the input parameters characterizing the tube about (i) the vertical convective energy flux in the tube and (ii) the internal diffusion coefficients. Spruit (1976) has parametrized the convective flux in the tube by means of the parameter \( q \), with the range \( 0 < q < 1 \), the two limits corresponding to totally inhibited of completely unimpeded convection by the magnetic field. Ferrari \textit{et al.} (1985) on the other hand have assumed that there is no convective energy transport within the tube.

In order to judge Spruit's treatment of the internal diffusion coefficients we estimate the ratio of the radiative diffusion coefficients, \( \rho = K_s(z)/K_e(z + z_0) \). Note that Spruit (1976) assumes \( \rho \) to be identically equal to unity. A simple approximation for the internal temperature is \( T_i(z) \approx T_e(z) \) which agrees with numerical solutions to within a few percent, at least for thin flux tubes. For a given value of \( \beta \) the equation for horizontal pressure balance, \( n_i k_B T_i + B^2/(8\pi) = p_e \), between the tube interior and the external atmosphere then gives the zeroth-order value of the density, \( n_i \), and thus the ratio \( \rho \).

Figure 2 shows the ratio \( \rho \) as a function of \( \tau_e \); the continuous line represents the results obtained in the model of Ferrari \textit{et al.} 1985; the dashed line, the results with the approximation \( T_i(z) \approx T_e(z) \).
These solutions imply that Spruit's modelling assumption is not well satisfied; the variation of $\rho$ can amount to a factor of 10 or more (for $\beta = 0.1$, for example).

In Figure 3 shows the shift $(z_i - z_e)$ between the positions where the internal and external diffusion coefficients have the same value, i.e., $K_i(z_i) = K_e(z_e)$, for $\beta = 0.1$. The horizontal line gives the corresponding Wilson depression. Note the strong variation of the shift $(z_i - z_e)$ with depth in our model, a variation absent from Spruit's model by assumption; this behavior follows from the dependence of the radiative diffusion coefficient $K$ on density and temperature.

Figure 4 shows the Wilson depression as a function of the magnetic field strength $\beta$ in the model by Ferrari et al. (1985).

For thin flux tubes, where our model assumptions should be correct, we can use SOT observations of the temperature within tubes (via line strengths, for example) to estimate the plasma-$\beta$. Thus, SOT would permit the measurement of the magnetic field strength inside magnetic flux tubes.

REFERENCES